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Abstract

We propose a theoretical model to study individual lifestyle choices related to calorie intake and physical activity, depending on personal fitness level and body weight. The model builds on the rational eating literature and can generate a variety of behaviors that are consistent with the empirical evidence. In particular, we show that engaging in periods of a sedentary lifestyle can be a rational, utility-maximizing decision—a finding that is not present in the existing literature but is empirically widespread. Additionally, we show the possible existence of multiple equilibria and multiple indifferent lifestyles. The former justifies policy interventions to help individuals exit a self-reinforcing, but unhealthy equilibrium; the latter provides a theoretical basis for remediation plans that compensate for earlier unhealthy behaviors.

JEL codes: I10, D91
Keywords: Lifestyle, Obesity, Physical exercise, Remedial behavior, Sedentary behavior, Skiba point

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Summary

The global rise in obesity is a major public health concern that has been attributed to individual traits like impatience and time preferences, market forces that affect food affordability and availability, and social factors such as peer effects, social contagion, and evolving norms about body image.

Health economic theories suggest that lifestyle choices, including unhealthy behaviors, may result from rational intertemporal decision-making, where individuals weigh the present and future costs and benefits of their actions. Consequently, policy interventions addressing obesity have included taxation on calorie-dense foods and sugary drinks, incentives for physical activity, and educational campaigns promoting healthier lifestyles.

Despite extensive research on eating behavior, economic studies have paid comparatively less attention to physical activity. The World Health Organization (WHO) emphasizes the importance of regular exercise, recommending at least 150 minutes of activity per week for weight management and overall health. However, adherence to these guidelines remains low, with over 80% of adolescents and 27% of adults failing to meet the recommended levels. This has long-term consequences for individual health and places financial strain on healthcare systems.

This paper introduces a theoretical model exploring the interaction between rational eating and exercise decisions. It accounts for various lifestyle patterns and examines how body weight and physical activity evolve based on individual time and budget constraints. We show that three possible types of steady state can emerge: (1) an optimal weight with unrestricted eating and excessive exercise, (2) an overweight state with dietary restrictions and overexertion, and (3) an underweight state with excessive food consumption. The model also shows that, under certain conditions, adopting a sedentary lifestyle can be a rational choice. While inactivity may not be optimal over an entire lifetime, it can be an economically justified decision during specific periods. This finding aligns with empirical observations of individuals alternating between sedentary phases and active engagement in physical exercise. The cyclical nature of these behaviors resembles patterns seen in dieting and overeating.

The paper introduces the notion of "Indifferent Lifestyles," referring to different life trajectories that yield the same long-term utility despite following distinct paths. This provides a theoretical foundation for delayed health investments, contingent on the ability to commit to future corrective actions. Accordingly, individuals may postpone healthy behaviors and still achieve similar long-term health outcomes if they can precommitt to later compensate for previous unhealthy choices. To choose among indifferent lifestyles, we propose a novel selection criterion based on the fact that, although these trajectories offer the same utility over an infinite time horizon, they can be ranked over finite time horizons..

The model also provides insights into policy effectiveness. The impact of pricing interventions on food and exercise-related choices depends on an individual's initial health status. In cases where multiple equilibria exist, individuals may be unable to transition to a healthier state without external intervention. Additionally, small price changes can sometimes trigger substantial shifts in behavior rather than gradual adjustments, due to the presence of bifurcation points and regime switches. This

finding challenges the assumption that policy effects are always proportional and suggests that, in some cases, minor policy changes can lead to drastic behavioral responses.

Overall, this research contributes to the economic analysis of obesity by integrating physical activity into rational choice models and exploring the conditions under which individuals may adopt different health-related behaviors. The findings have important implications for policymakers, suggesting that interventions should account for the interdependence between eating and exercise decisions.

1 Introduction

The worldwide spread of the obesity epidemic is a public health concern that the economic literature has linked to several factors. These factors include individual characteristics, such as impatience and time preferences (Ikeda et al., 2010; Courtemanche et al., 2015; Stoklosa et al., 2018; Cobb-Clark et al., 2023), market dynamics and technological changes that influence the relative cost of calories and the availability of high-calorie products (Lakdawalla et al., 2005; Lakdawalla and Philipson, 2009; Dragone and Ziebarth, 2017), and factors that depend on social interactions, such as peer effects (Christakis and Fowler, 2007; Cohen-Cole and Fletcher, 2008), social contagion effects (Strulik, 2014) and social norms about physical appearance (Dragone and Savorelli, 2012; Dragone et al., 2016).[1]

Consistent with the view that incentives influence behavior and that even unhealthy lifestyles can result from rational intertemporal decision-making (Becker and Murphy, 1988; Dockner and Feichtinger, 1993; Levy, 2002; Dragone, 2009b), policy responses to the obesity epidemics have involved the introduction of taxes on calorie-dense food and beverages (Cawley, 2016; Cawley et al., 2020, 2021), incentives on physical exercise (Della Vigna and Malmendier, 2006; Charness and Gneezy, 2009; Kókai et al., 2022), as well as information and educational campaigns aimed at promoting healthy lifestyles (Mazzocchi et al., 2009).

While the main focus of the theoretical and empirical economic literature on obesity is on the intake of calories through eating or drinking, physical activity and exercise have been less investigated.[2] The WHO guidelines emphasize the benefits of regular exercise and an active lifestyle, suggesting incorporating physical activity into daily routines (World Health Organization, 2020).[3] Regular physical activity helps prevent weight gain and maintain a healthy body weight, with the effect being more evident when activity levels reach at least 150 minutes per week. Without dietary restriction, physical activity alone leads to an average weight loss of approximately 0.5 to 3.0 kg (Jakicic and Rogers, 2024). Yet, despite the recommendations from health organizations and practitioners, more than 80% of adolescents and 27% of adults do not meet the WHO's recommended levels of physical activity (World Health Organization, 2022). This not only impacts individuals throughout their lives but also increases financial pressure on health systems and society.

In this paper, we present a theoretical model to investigate the relationship between rational eating and physical exercise in a rational, forward-looking framework.[4] We show that the variety of individual lifestyles often observed about eating and physical activity can be rational outcomes, and we study how intertemporal patterns of body weight and exercise depend on individual characteristics, time constraints, and budget constraints.

Despite the general concern about increased physical inactivity in the population, we show that

[1] For excellent overviews, see Cawley (2004); Cawley and Ruhm (2012); Cawley (2015); Okunogbe et al. (2022).

[2] For notable exceptions, see Della Vigna and Malmendier (2006); Charness and Gneezy (2009); Yaniv et al. (2009); Strulik (2019).

[3] Since the 1980s, there has been a growing emphasis on the importance of being fit. The fitness industry has expanded, massively investing in marketing, opinion leaders, and role models to promote fitness as both a form of health investment and a productive factor for a healthy life (Pilgrim and Bohnet-Joschko, 2019; Scheerder et al., 2020). In 2022, the Wellness industry is estimated to be worth 5.6 trillion (Global Wellness Institute, 2023).

[4] In the following, we will use eating behavior as a shorthand for calorie intake through food consumption and drinking.

choosing a sedentary lifestyle can be a rational decision. While such a choice may not be optimal over an entire lifetime, our model describes scenarios where it could be so during specific periods. This result is novel and matches the empirical observation of individuals alternating between periods of complete inactivity and periods of moderate or intense physical activity. This cyclical behavior also parallels behavioral patterns sometimes observed in dietary habits, particularly the oscillation between overeating and dieting (Atella and Kopinska, 2014; Lounassalo et al., 2021; Mathisen et al., 2023). It suggests that individuals make lifestyle decisions by jointly considering eating and physical activity. When this is the case, analyzing policies targeting calorie intake without accounting for potential interdependencies with physical activity may lead to incomplete or misleading conclusions.

We also show the possible existence of what we term *Indifferent Lifestyles*, which represent different "lifetime journeys" that start and end at the same points, provide the same intertemporal utility, but follow distinct paths.[5] The presence of indifferent lifestyle trajectories suggests that some individuals may have the flexibility to adopt healthy behaviors later in life while still achieving the same long-term outcomes as those who made healthier choices earlier. In this sense, the notion of indifferent lifestyles provides a rationale for delaying health investments if individuals can credibly commit to remediate previous unhealthy behavior later in life. To choose between indifferent lifestyles, we propose a novel selection criterion based on the idea that indifferent lifestyles—which are assessed over an infinite-time horizon— can be rankable over some finite-time horizon denoted as a reference. To the best of our knowledge, such a criterion is new in the literature.

The model we propose can be applied to make predictions about the impact of policy interventions targeting obesity. As an example, we consider the price of eating and of physical exercise, and we show that their effect on individual behavior may depend on the individual's current health condition. Specifically, there exist cases in which the current body weight and fitness condition can prevent reaching desirable outcomes, and cases in which small changes in prices can lead to drastic changes—as opposed to marginal adjustments—in individual behavior and long-run outcomes. The former case occurs when multiple equilibria arise and it justifies a policy intervention aimed at helping individuals reach a health outcome that is desirable, but cannot be reached. The latter result is due to the existence of bifurcation points and regime switches, and it suggests that the implicit assumption of a dose-response to a policy applies only in some cases, while in others there can be a drastic behavioral response to apparently minor policy interventions.

Our paper contributes to understanding the role of physical exercise and sedentary behavior within the framework of a rational choice model. We show that three possible types of steady states can emerge. In the first one, an individual maintains an optimal body weight, eats to satiation, and engages in excessive physical exercise. In the second one, the individual is overweight, follows a restrictive diet, and overexercises. In the third one, the individual is underweight and consumes food beyond satiation. By showing that these steady states can coexist, we extend the findings about the emergence of multiple equilibria and oscillatory behavior in eating models, adding to the models where consumption contributes to building a consumption capital, as in Becker and Murphy (1988); Dockner and Feichtinger (1993); Caputo and Dragone (2022), and adding to the results of Dragone (2009b),

[5] Formally, initial conditions with these properties denote Skiba curves. See Grass et al. (2008) for an introduction.

who considers eating choices and habit formation costs, Dragone (2009a) who considers effort exertion and fatigue accumulation, and Kuhn and Wrzaczek (2021), who study the stochastic escalation from being non-addicted to addiction.

Our model also highlights the role of corner solutions in individual behavior. While these solutions are less tractable than interior solutions, they are important for capturing empirically relevant behaviors, such as individuals choosing a sedentary lifestyle. From a health-based perspective, such behavior may not appear optimal, but it can be a rational outcome in a forward-looking, time-consistent framework that accounts for all consequences of individual choices on utility and health outcomes.

2 Theoretical framework

2.1 The model

Consider an optimal control model where the individual instantaneous utility function is $U(c, l, q, u, W, F)$ and depends on current calorie intake c, time at work l, leisure time q and exercising time u, as well as on body weight W and the individual fitness level F .

Body weight and fitness level are the key ingredients in our analysis and will be considered as state variables.[6] The law of motion for body weight is

$$\dot{W} = g(c, W, F) = c - \delta_w W F \tag{1}$$

with $\delta_w > 0$. As it is common in the literature, we assume that body weight increases with eating and decreases with body weight (see, e.g., Dockner and Feichtinger, 1993; Levy, 2002; Dragone, 2009b; Strulik, 2014). Differently from the previous contributions, we add an interplay between body weight and fitness level. Specifically, based on the results from some clinical studies (see, e.g., Shook et al., 2014; Shook et al., 2014; Arnold et al., 2021), we assume the daily calorie consumption rate depends on the individual fitness level. The intuition is that a higher fitness level goes along with a higher proportion of muscles in relation to body weight. Hence, for a given body weight, a more fit body consumes more energy.[7]

We assume that the fitness level increases with exercising time. Based on the evidence about the decreasing marginal returns of training, we assume that exercising contributes more to the fitness level when the individual has a poor fitness level.[8] For concreteness, we consider the following:

$$\dot{F} = f(u, F) = \frac{u}{F} - \delta_F F, \tag{2}$$

where the depreciation rate $\delta_F > 0$ describes the fact that the fitness condition decreases if one does

[6] Variables names are summarized in Table 1.

[7] One could also assume that physical exercise directly reduces body weight by "burning" calories, as in, e.g., Strulik (2023). Here we explore an alternative perspective by focusing on changes in the basal metabolic rate that endogenously depend on the individual's fitness level and body weight.

[8] Consider, for instance, a runner who can run 10 km in 60 minutes. Then it is relatively easy to increase training efforts to improve the personal best by one minute. If, however, the runner is able to run 10 km in 30 minutes (which is only a few minutes above the men's world record), it is hardly possible to improve by one minute at all.

Functions	$U(\cdot)$	instantaneous utility function
	$g(t)$	dynamics of body weight
	$f(t)$	dynamics of fitness level
Control variables	$c(t)$	eating
	$u(t)$	exercising time
	$l(t)$	time at work
	$q(t)$	leisure time
State variables	$W(t)$	body weight
	$F(t)$	fitness level
Adjoint variables	$\lambda(t)$	adjoint variable of body weight
	$\mu(t)$	adjoint variable of fitness level

Table 1: List of functions and variables

not exercise.

At each point in time, the individual faces the following time and budget constraints, respectively,

$$T = l + q + u \tag{3}$$

$$wl = p_u u + p_c c \tag{4}$$

where T is the time endowment to be allocated between labor, leisure and physical exercise, w denotes the wage rate of labor, p_u is the cost of exercising (such as gym memberships or equipment), and p_c is the price of eating. The time and budget constraints (3) and (4) can be combined to obtain the full income constraint (Becker, 1965)

$$wT = p_c c + (p_u + w)u + wq. \tag{5}$$

The left-hand side represents the time endowment valued at the wage rate for labor. The right-hand side allocates this full income among eating, exercise, and leisure. By assumption, eating takes no time, hence its cost is solely determined by its price. The cost of exercising includes both its market price and the opportunity cost of time evaluated at the market wage of labor. The cost of leisure is only given by its opportunity cost w.

The individuals' objective is to maximize intertemporal utility, which is given by the sum of instantaneous utility $U(\cdot)$, discounted by the subjective discount rate r_1 and the survival probability $S(t)$. The latter is assumed to follow an exponential distribution with mortality (hazard) rate r_2 and, consequently, life expectancy $\frac{1}{r_2}$. Hence, the individual discounts the instantaneous utility $U(\cdot)$ by the combined rate $r := r_1 + r_2$, and solves the following problem:

$$\max_{c(t),u(t),q(t),l(t)} \int_0^\infty e^{-rt} U(c(t), u(t), q(t), l(t), W(t), F(t))\mathrm{d}t \tag{6}$$

$$\text{s.t. } T = l(t) + q(t) + u(t) \tag{7}$$

$$wl(t) = p_u u(t) + p_c c(t) \tag{8}$$

$$\dot{F}(t) = f(u(t), F(t)) \tag{9}$$

$$\dot{W}(t) = g(c(t), W(t), F(t)) \tag{10}$$

$$W(0) = W_0 > 0, \qquad F(0) = F_0 \geq 0 \tag{11}$$

Given the fixed mortality rate contained in the discount rate, the problem is formulated over an infinite time horizon. This implies that it is optimal for individuals to make time-consistent plans as if they could live indefinitely. This approach is commonly adopted in the economic literature (e.g., Yaari, 1965; Becker and Murphy, 1988; Mas-Colell et al., 1995; Dragone and Raggi, 2021) as it facilitates the analysis of optimal trajectories converging towards steady-state equilibria. However, note that such equilibria will be reached with probability zero because agents will in fact die in finite time with probability one.

2.2 Types of steady state

The model is solved by applying the Maximum Principle (see Grass et al., 2008). We first consider general formulations of the utility function and the laws of motions of body weight and fitness level, and we study the features of the first-order conditions for optimality. Then we consider specific formulations for the laws of motions, and we show the types of steady states that the model can produce.

As a preliminary step, it is useful to isolate the values of q and l from the time and budget constraints (7) and (8). By substituting these values into the instantaneous utility function $U(c, l, q, u, W, F)$, a more compact objective function $\mathcal{U}(c, u, W, F)$ can be obtained. This modified objective function

$$\mathcal{U}(c, u, W, F) := U\left(c, u, T - u - \frac{p_u u + p_c c}{w}, \frac{p_u u + p_c c}{w}, W, F\right) \tag{12}$$

depends only on two control variables (eating c and physical exercise u) and two state variables (body weight W and fitness level F).

For later reference, we label as a static optimum the combination of eating, physical exercise, and body weight such that the associated partial derivatives $\mathcal{U}_i$ are zero, for $i \in \{c, u, W\}$.[9] Since this point denotes a satiation (or bliss) point, we denote the case in which $\mathcal{U}_c > 0$ as eating below satiation (or adhering to a diet), and $\mathcal{U}_c < 0$ as overeating. Similarly, $\mathcal{U}_u > 0$ denotes underexercising, and

[9] To simplify the notation, the time arguments will be omitted and the partial derivatives of the utility function and the laws of motion will be denoted with subscripts. Note that the static optimum also depends on variables that influence the time and budget constraints, such as prices and the wage rate.

$\mathcal{U}_u < 0$ denotes overexercising relative to the static individual optimum (which could be zero if one does not enjoy physical activity). Finally, $\mathcal{U}_W > 0$ denotes being underweight, $\mathcal{U}_W < 0$ indicates being overweight, and $\mathcal{U}_W = 0$ denotes being at the optimal body weight. For the fitness level, instead, we assume that "more is better," i.e. $\mathcal{U}_F = U_F > 0$ for all F.

The Hamiltonian function corresponding to the individual problem with the modified utility function is:

$$\mathcal{H} = \mathcal{U}(c, u, W, F) + \lambda g\,(c, W, F) + \mu f\,(u, F) \tag{13}$$

where λ and μ denote the adjoint variables of body weight W and fitness level F, respectively. These adjoint variables are the shadow prices of body weight and fitness level, and they describe the marginal impact of one additional unit of the corresponding state variable on lifetime utility. At any time t, the first-order conditions for eating and physical exercise are, for an internal solution,

$$\mathcal{U}_c = -\lambda g_c; \qquad \mathcal{U}_u = -\mu f_u \tag{14}$$

hence,

$$\frac{\mathcal{U}_c}{\mathcal{U}_u} = \frac{\lambda}{\mu}\frac{g_c}{f_u}. \tag{15}$$

The above expression implies that, if an internal solution exists, it is such that the marginal rate of substitution between consumption and physical exercise is proportional to the ratio of the shadow prices of body weight and fitness level. The proportionality factor g_c/f_u is the dynamic analog of the marginal rate of transformation between consumption and physical exercise.[10]

Using (1) and (2), the following holds in a steady state $(c^{ss}, u^{ss}, W^{ss}, F^{ss})$:

$$\mathcal{U}_W = -\sigma_1 \mathcal{U}_c \tag{16}$$

$$\mathcal{U}_F = -\sigma_2 \mathcal{U}_c - \sigma_3 \mathcal{U}_u \tag{17}$$

where $\sigma_1 = \delta_W F^{ss} + r > 0$; $\sigma_2 = \delta_W W^{ss} > 0$ and $\sigma_3 = (2\delta_F + r)\, F^{ss} \geq 0$. See Appendix A.1 for details.

Since $\sigma_1 > 0$, condition (16) implies that the marginal utility of body weight and eating must have opposite sign (see Caputo and Dragone, 2022). Since $\mathcal{U}_F > 0$, the right-hand side of condition (17) must be positive, which requires that either $\mathcal{U}_c$ or $\mathcal{U}_u$ are negative (or both). Hence, the following holds:

Proposition 1 (Types of steady state) *In a steady state, one of the following three cases can occur:*

1. *The individual eats up to satiation, overexercises and has an optimal body weight,*

2. *The individual is on a diet, overexercising and overweight,*

3. *The individual is overeating and underweight*

[10] For an analog result in the context of the life-cycle literature, see, e.g., Shepard and Zeckhauser (1984); Murphy and Topel (2006); Kuhn et al. (2015).

The first case describes a steady state where the agent reaches the bliss point in terms of eating, physical exercise, and body weight. The second scenario corresponds to an equilibrium in which an overweight individual must exercise more than would be desirable and must restrict caloric intake to avoid further weight gain. This case is the theoretical counterpart of the empirical evidence concerning the obesity epidemics. The third case describes a scenario in which an underweight agent must overeat to prevent further weight loss. This underweight condition may result from inefficient nutrient absorption, insufficient caloric intake, a high metabolic rate, or other physiological factors which prevent gaining weight despite overeating.

3 Lifestyles and health outcomes

Contoyannis and Jones (2004) define a lifestyle as *"a set of behaviors which are considered to influence health and generally involve a considerable amount of free choice"*. In our context, individual lifestyles are represented by intertemporal optimal paths of eating and physical exercise that solve the intertemporal problem (6) to (11).

To get insight into the lifestyles compatible with our model, it is convenient to choose a specific utility function. Following the literature on rational addiction (Becker and Murphy, 1988; Dockner and Feichtinger, 1993) and on obesity and addictive consumption (Naik and Moore, 1996; Hauck et al., 2020), we choose a linear-quadratic specification for the instantaneous individual utility function. Specifically, we consider

$$U = a_c c + a_u u + a_W W + a_F F + a_q q + \frac{a_{cc}}{2}c^2 + \frac{a_{uu}}{2}u^2 + \frac{a_{WW}}{2}W^2 + \frac{a_{FF}}{2}F^2 + a_{cW}cW + a_{uW}uW \quad (18)$$

with $a_c, a_W, a_F, a_q > 0$. To capture the possibility that physical exercise may be either disliked or pleasurable, parameter a_u can take any sign (Dragone, 2009a).

We assume (18) is concave and that $a_{cc}, a_{uu}, a_{WW}, a_{FF} \leq 0$. The last two terms of (18) imply that body weight interacts with both eating and exercising time. Specifically, past consumption increases the marginal utility of current consumption, $a_{cW} > 0$, a property Dragone and Ziebarth (2017) referred to as taste formation and that, in the context of Becker and Murphy (1988)'s theory of rational addiction, is known as reinforcement. Term $a_{uW}uW$, with $a_{uW} < 0$, describes the realistic possibility that the marginal utility of exercising is a decreasing function of body weight. The linearity and separability of leisure time q is meant to simplify the analysis, so that price shocks produce changes in consumption and physical exercise only through substitution effects, without income effects being involved (Becker and Murphy, 1988; Dragone and Vanin, 2022, 2025).

After substitution of equations (7) and (8) into (18) we obtain

$$\mathcal{U} = \alpha_c c + \alpha_u u + a_W W + a_F F + a_q T + \frac{a_{cc}}{2}c^2 + \frac{a_{uu}}{2}u^2 + \frac{a_{WW}}{2}W^2 + \frac{a_{FF}}{2}F^2 + a_{cw}cW + a_{uW}uW \quad (19)$$

where

$$\alpha_c := a_c - \frac{p_c}{w}a_q, \qquad \alpha_u := a_u - \left(\frac{p_u}{w} + 1\right)a_q. \quad (20)$$

Hence, if an internal solution exists, it satisfies:

$$c^* = -\frac{1}{a_{cc}}\left(\alpha_c + a_{cW}W + \lambda\right) \tag{21}$$

$$u^* = -\frac{1}{a_{uu}}\left(\alpha_u + a_{uW}W + \frac{\mu}{F}\right). \tag{22}$$

Optimal consumption c^* increases with the marginal incentives of eating α_c, with body weight W, and with its adjoint variable λ. Optimal physical exercise u^* depends on the marginal incentives of exercising α_u, body weight, the fitness level and its adjoint variable μ. Everything else equal, an increase in body weight W increases eating but reduces physical exercise (recall that $a_{cc}, a_{uu} < 0$). The former result is due to the reinforcing nature of eating so that past and present consumption are positively correlated (see, in the context of rational addiction models, Becker and Murphy, 1988). The latter occurs because, when body weight is higher, exercising becomes more effortful (because $a_{uW} < 0$). Since the fitness level increases the basal metabolic rate, it is optimal to exercise less when the individual becomes fitter, everything else equal and provided increasing the fitness level increases intertemporal utility ($\mu > 0$).

Based on (21) and (22), the following holds

Remark 1 (Sedentary behavior) *Optimal physical inactivity is more likely when the marginal utility of exercising a_u and the wage rate w are lower, and when the marginal utility of leisure time a_q, the price of exercising p_u, and body weight W are higher.*

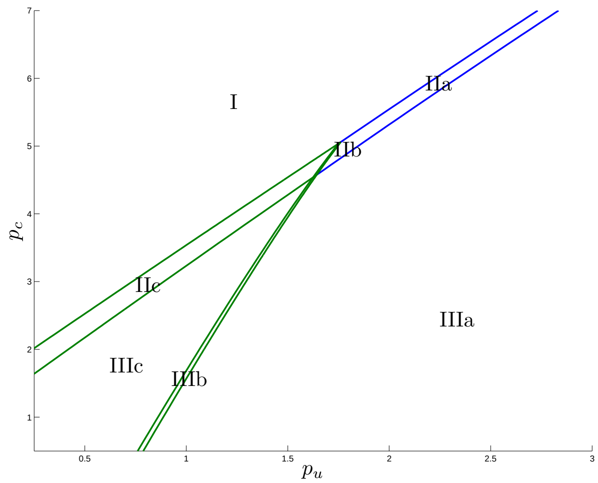
This proposition highlights a previously overlooked result: choosing not to exercise can be optimal, at least during certain periods of life. This outcome may be the consequence not only of a high monetary cost and a low taste for exercise, but also of the interaction between the physical cost of exercising and body weight, as well as the opportunity cost of time spent exercising instead of engaging in other leisure activities or work.

3.1 The past matters: history-dependence and multiple equilibria

In this section, we further study the solution of our model through numerical simulations, showing that unique steady states, limit cycles, or multiple long-run equilibria can emerge. When there are multiple equilibria, the solution becomes history-dependent, meaning that the steady state an individual can reach depends on their initial body weight and fitness level. As a result, individuals may find themselves trapped in an undesirable equilibrium, even if they recognize that a different equilibrium would be better. When this occurs, a policy intervention—a "Big Push"—aimed at shifting individuals out of the undesirable equilibrium may be warranted.

As noted in the Introduction, the prices of eating and physical exercise are key factors influencing obesity trends and are frequently targeted by policy interventions through, e.g. taxes on high-calorie foods and beverages or subsidies for gym memberships. Accordingly, we consider the prices space (p_u, p_c) to classify the number and type of steady states that can arise in an optimal solution. In the bifurcation diagram in Figure 1, underweight steady states (denoted as W^-) occur in Regions I and

Bifurcation diagram



Region	Long-run solution	Type of solution
I	W^-	globally optimal
IIa	W^- and W^+	locally optimal
IIb	W^-, W^+ and limit cycle	locally optimal
IIc	W^- and limit cycle	locally optimal
IIIa	W^+	globally optimal
IIIb	W^+ and limit cycle	locally optimal
IIIc	limit cycle	globally optimal

Figure 1: Bifurcation diagram for different values of p_u and p_c. Underweight steady states emerge in Regions I and II; overweight steady states emerge in Regions IIa, IIb and III. The symbol W^+ denotes an overweight steady state; W^- an underweight steady state. Blue curves denote heteroclinic connections between equilibria (no limit cycle is involved); Green curves denote heteroclinic connections between a limit cycle and an equilibrium. Parameters as described in footnote 11.

II, while overweight steady states (denoted as W^+) appear in Regions IIa, IIb, and III.[11]

Consider the case in which the price of eating p_c is very large (say, $p_c = 6$) and the cost of exercising is small, which amounts to considering Region I. With such a parametric configuration, there exists a unique underweight equilibrium. The equilibrium is globally optimal, which means that it will be reached for any initial condition of body weight and fitness condition. As the price of exercising increases, however, an overweight equilibrium also emerges (Region IIa) and, for even higher cost of exercising (Region IIIa), it remains the only reachable equilibrium. This result is consistent with the notion that the increase in the cost of exercising can determine obesity.

Consider now the case of a low price of eating (say, $p_c = 1$) and a low cost of exercising (Region IIIc). In such a case, a limit cycle emerges, that is, an outcome in which a stable oscillatory pattern of eating, exercising, fitness level and body weight emerges.[12] As in the previous case, when the cost of exercising increases an overweight steady state emerges (Region IIIb), until the limit cycle disappears and only the overweight steady state remains (Region IIIa).

The more complicated case of an intermediate price of eating will be considered in Section 4 in the context of policy interventions. For now, it suffices to observe that a unique steady state exists when one of the two prices is high and the other one is low (Regions I and IIIa). In the remaining regions, the difference between p_c and p_u is relatively small, which favors the coexistence of multiple steady states

[11] Parameters: $a_u = 3$, $a_c = 7$, $a_F = 1.5$, $a_W = 0.5$, $a_q = 1$, $a_{cc} = -5$, $a_{uu} = -0.5$, $a_{ww} = -0.07$, $u_{FF} = 0$, $a_{cw} = 1$, $w = 1$, $\delta_w = 0.1$, $\delta_F = 0.4$, $r_1 = 0.02$ and $r_2 = 0.01$. These values are provided for illustrative purposes only and have not been calibrated or validated using real-world data. Recall from (20) that the linear terms α_u and α_c negatively depend on prices, hence a higher price corresponds to a lower value of the corresponding α.

[12] For a similar result in other health-related models, see Dockner and Feichtinger (1993)'s rational eating model and Cawley and Dragone (2024)'s harm reduction model.

and, possibly, limit cycles. The equilibrium ultimately attained depends on the initial conditions. In Region IIa, for instance, convergence to either the underweight or overweight equilibrium is possible. This outcome is reasonable, given that Region IIa lies between Regions I (underweight steady state) and IIIa (overweight steady state). If an individual starts with a large weight and low fitness condition (i.e., $W(0)$ is large and $F(0)$ is small) they will eventually settle into the overweight steady state. This entrapment results mainly from two factors. First, the positive reinforcement parameter α_{cW} implies that a heavier individual derives greater utility from consumption. Second, as indicated by expression (1), a less fit individual burns fat too slowly to counteract weight gain. On the contrary, individuals starting with low body weight and high fitness condition will ultimately reach the underweight steady state.

3.2 Indifferent lifestyles and remedial behavior

In this section we emphasize a so far overlooked result in the literature on obesity: there exist initial conditions such that the same steady state can be reached through multiple paths that are indifferent in terms of the associated value function.

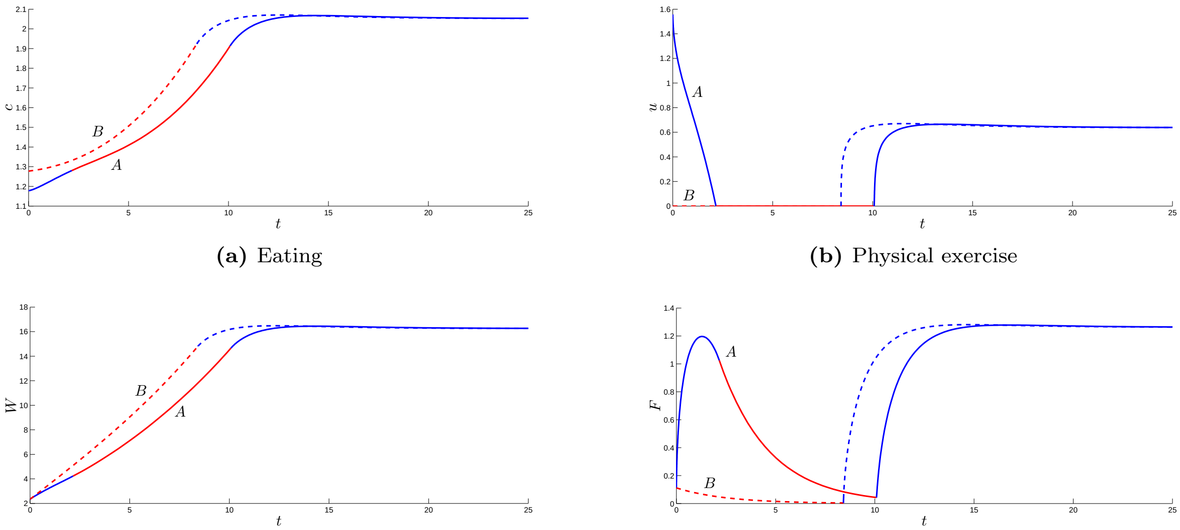
Consider, for instance, the time paths in Figure 2. They share the same initial and terminal conditions (panels c and d), they are solutions of the intertemporal problem (6)-(11), and it can be shown that they entail equal values of the maximized intertemporal utility function when computed at $t = 0$. Hence, they are indifferent with respect to the associated value functions. In a sense, they are the dynamic analog of indifference curves in static consumer theory, wherein different combinations of consumption goods yield the same utility levels. Accordingly, we label these trajectories as *Indifferent Lifestyles*.[13]

Figure 2 depicts an example of two indifferent lifestyles. They are indifferent when considering the associated value functions, but they represent different intertemporal behaviors. In the early periods, trajectory B (dashed line) describes higher levels of eating (panel a) and a prolonged period of no physical activity (depicted in red, panel b) compared to trajectory A (solid line). This leads to a higher body weight (panel c) and lower fitness condition (panel d). To compensate for the initial lack of physical activity, prevent excessive weight gain, and obtain a sufficient fitness level, the individual in trajectory B engages in physical activity at an earlier age compared to trajectory A. Physical activity, improves the fitness condition and slows down the rate of accumulation of body weight. Ultimately, both trajectories converge to the same body weight and fitness conditions, albeit through different paths over the lifecycle.

This example illustrates two main results. First, periods of sedentary behavior can be optimal during certain phases of an individual's lifetime. Second, the choice to delay physical exercise can be fully remediated later in life, although it may require more effort and an earlier start to catch up to

[13] The set of initial conditions with these features is known as a Skiba curve (Skiba, 1978). For an introduction to the topic, see Grass et al. (2008). See Skiba (1978) and Dechert and Nishimura (1983) for seminal works studying Skiba curves in the context of economic models framed as optimal control problems, particularly the one-sector optimal growth problem with a convex-concave production function. In the following, we consider a parametric configuration that results in a unique overweight steady state (Region IIIa in Figure 1). The phase diagram showing the trajectories of the state variables associated to the two indifferent lifestyles A and B is presented in Figure 4 in the Appendix.

Optimal time paths of two indifferent lifestyles



(a) Eating

(b) Physical exercise

(c) Body weight

(d) Fitness level

Figure 2: Time trajectories of eating, physical exercise, body weight and fitness level associated to the indifferent lifestyles A (solid line) and B (dashed line) described in Figure 4. The trajectories have the same initial and terminal conditions of body weight and fitness level (panels c and d). Red portions of the trajectories denote periods of no physical exercise, as indicated in panel (b). Parameters as described in footnote 11, with $p_c = 4.5$, $p_u = 3$.

the health condition of an individual who begins exercising at a younger age.[14]

Since indifferent lifestyles originate from the fact that multiple, equivalent optimal trajectories exist, one may wonder how an individual would choose between them. We propose a criterion based on the idea that the time of death is uncertain, but it will ultimately be at some (random) finite-time T. Over the time-horizon from zero to T, the utility experienced may differ from the value function computed over an infinite time horizon (which is the relevant one when the time of death is uncertain at all t). A possible time-breaking criterion can be introduced based on the ranking of the finite-horizon utility experienced across alternative lifestyles over T:

Definition 1 (Finite-horizon utility) *The finite-horizon utility associated to lifestyle L from time zero to T is*

$$P(L,T) := \int_0^T e^{-r_1 t}\mathcal{U}(\mathcal{L}(t))\,dt, \qquad T \in [0,\infty)\,, \tag{23}$$

where $\mathcal{L}(t) = (c(t), u(t), W(t), F(t))$ denotes, for each t, the vector of state and optimal control variables associated to lifestyle L.

[14] For empirical investigation on remedial behavior and remedial policies in the context of skills formation, see Cunha and Heckman (2007); Cunha et al. (2010). For the role of early health conditions in shaping future outcomes and behavior, see Currie and Almond (2011); Campbell et al. (2014); Conti et al. (2016); Baranov et al. (2020); Hendren and Sprung-Keyser (2020); Carneiro et al. (2021).

The finite-horizon utility in (23) represents the intertemporal utility actually experienced by the individual from time zero to T, discounted at the subjective rate of time preference r_1. It does not account for the mortality rate r_2 because the terminal time is treated as fixed, as if the individual were certain to die exactly at T.

While the assumption of a deterministic time of death may seem strong, it is not uncommon in individual lifecycle models in health economics and in labor supply models (Grossman, 1972; Heckman and MaCurdy, 1980; Dalgaard and Strulik, 2014). Here, it is useful to introduce the following tie-breaking criterion to select among indifferent lifestyles:

Definition 2 (Selection among indifferent lifestyles) *Given two indifferent lifestyles A and B, we say that A is preferred to B over the period t_1 and t_2 if $P(A,T) \geq P(B,\tau)$ for $T \in [t_1, t_2]$.*

The above definition is consistent with the notion that, if the time of death turns out to be T, one would experience no regret if lifestyle A has been chosen over lifestyle B. Accordingly, although ex-ante—when T is uncertain—the two lifestyles are indifferent, the two alternative lifestyles are no longer indifferent from a perspective in which T is known.[15]

Figure 5 in the Appendix illustrates the proposed selection criterion by drawing the finite-horizon utility for the indifferent lifestyles A and B described in Figure 2. The figure shows that an individual is better off with lifestyle A during the first 10 years and better off with B thereafter. Consequently, an individual would prefer lifestyle A if the relevant time horizon T falls within the first 10 years ($T \in [0, 10]$), whereas lifestyle B would be chosen for a longer time horizon ($T \in [10, \infty)$).

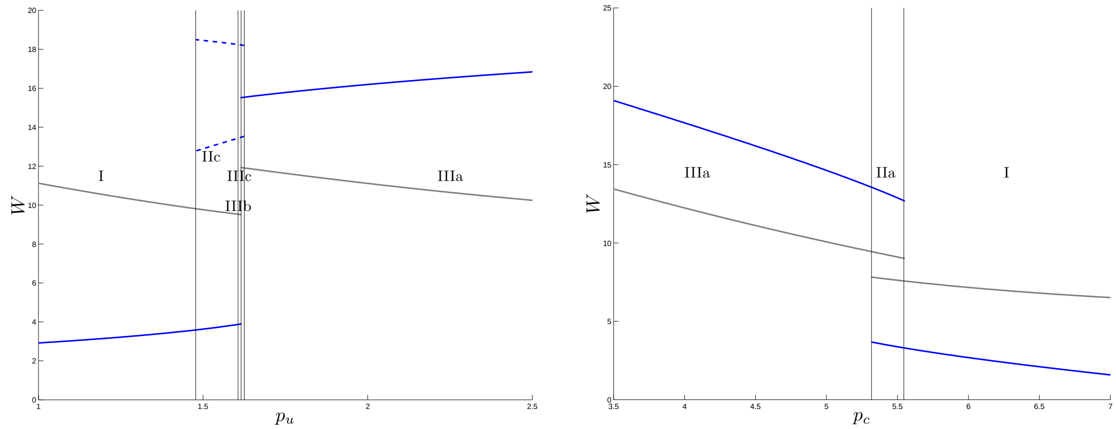
4 Policy interventions and regime changes

Governments often discuss and implement market-based instruments to address the growing obesity epidemic. Policy measures such as subsidies for physical activity and taxation on energy-dense foods are commonly discussed. In this section, we show how these policies affect body weight.

The effect on the steady-state equilibrium of changes in the cost of physical activity p_u is depicted in Figure 3a, while changes in the cost of calorie intake are considered in Figure 3b. The blue curves represent the set of long-run outcomes (steady states with saddle-point stability or limit cycles) in correspondence of each price, everything else being equal. As a reference, the set of optimal body weights in the static problem is indicated by the gray (non-vertical) curves. Hence, long-run outcomes above this curve correspond to an overweight outcome, while those below it indicate being underweight.

Figure 3a shows that body weight monotonically increases with the cost of exercising. Specifically, if the cost is sufficiently low, the individual will become underweight for any initial condition. If, instead, the cost of physical exercise is high enough, the individual will converge to the overweight steady state, irrespective of the initial conditions. These results are intuitive. It is interesting to note, however, that there exists an intermediate range of values of p_u where both an overweight and an underweight steady state coexist. In Figure 3a, this range corresponds graphically to the

[15] Our results hold both when the time T is exogenously given and when it is endogenously set, as in, e.g., Grossman (1972); Dalgaard and Strulik (2014).



(a) Changing the cost p_u of exercising

(b) Changing the cost p_c of eating

Figure 3: Long run optimal body weight as a function of the cost of exercising (left panel) and eating (right panel). Parameters as described in footnote 11. Solid blue lines denote steady-state values, dashed lines the minimum and maximum body weight described by a limit cycle. Grey solid (non vertical) lines depict the optimum of the static problem. See Figure 1 for a description of the different types of steady states.

area between the vertical lines, which denote bifurcation values of p_u. Since in this range there exist multiple equilibria, which one will be reached depends on the initial level of body weight and fitness. For example, in Region II, for low initial body weight the individual is predicted to reach an underweight condition, while for high initial body weight the long-run outcome is predicted to be oscillating over time, without ultimately converging to any specific steady-state value. As the price increases only the overweight equilibrium survives. When this is the case, the past does not matter, as for a high enough price of physical exercise the overweight equilibrium is reached for any initial fitness and body weight condition.

The above considerations support the notion that one of the causes of the obesity epidemics is the increased cost of physical exercise (Lakdawalla et al., 2005). They also provide a rationale for government interventions that reduce the cost of exercising through, e.g., direct subsidies to physical activity (Yaniv et al., 2009) to reduce body weight in the population. We contribute to this literature by showing that the effects of such policies can either be progressive or *drastic*. The effects are expected to be drastic when the new cost of exercise crosses the region defined by the bifurcation values. In this case, small changes in the price of exercise can lead to large jumps in individual behavior and outcomes. This happens because, at the bifurcation points, new equilibria and the associated optimal paths emerge. An individual "jumping" on these new paths will abruptly change individual behavior and transition to a new lifestyle trajectory, depending on their current body weight and fitness condition.

The effect of the price of eating on body weight features the opposite dynamics (Figure 3b). For high values of p_c, the individual is underweight. However, as the price of eating decreases, the individual gets more and more body weight, until getting overweight. This is consistent with the observation that in modern society the reduced cost of acquiring calories has played a major role in determining the obesity epidemics (Lakdawalla and Philipson, 2007; Cawley, 2015). In analogy to the previous case, there exists a range of prices in which two steady states coexist: an underweight equilibrium, and an overweight one. They are optimally local, in that the specific body weight that different individuals reach depends on their initial body weight conditions (region IIa in Figure 1). In the range of prices between the two vertical lines of Figure 3b, one would expect a bimodal distribution of body weights. However, as the price of eating becomes even lower, only the overweight steady-state equilibrium survives.

5 Conclusions

In this paper, we propose a model to study lifestyle choices related to eating and physical exercise. We show that the interaction between health conditions and individual choices can generate diverse trajectories of eating behavior, body weight, physical activity, and fitness levels over the lifecycle. In the simplest cases, these trajectories follow monotonic patterns in which a gradual increase in body weight is accompanied by a gradual decline in physical activity over time. In other cases, the trajectories exhibit oscillatory behavior. This result matches the empirical observation of people experiencing periods of their lives in which they are physically active and fit, followed by periods in which they are inactive, as well as the evidence about cyclical patterns of dieting and body weight.

We show that sedentary behavior can be optimal during certain periods of an individual's life and that compensating for an initial lack of physical activity with increased exercise later in life can be effective and optimal. In the long-run, three possible types of equilibria can be reached. In the first one, an individual maintains an optimal body weight, eats to satiation, and engages in excessive exercise. In the second one, the individual is overweight, follows a restrictive diet, and overexercises. In the third one, the individual is underweight and consumes food beyond satiation.

We show that these types of equilibria—such as being underweight and fit or overweight and unfit—can coexist for the same individual. The specific equilibrium reached depends on the individual's initial body weight and fitness conditions. Due to history-dependence, individuals may thus be trapped in a suboptimal equilibrium of poor health, even when they recognize that a different, healthier equilibrium would be more desirable. In such cases, policy interventions may be necessary to assist individuals in leaving an unhealthy, yet self-reinforcing equilibrium.

The concept of indifferent lifestyles—defined as equivalent trajectories that begin from the same initial conditions and reach the same terminal body weight and fitness outcomes—offers a theoretical justification for remediation plans that fully compensate for earlier unhealthy behaviors. This aligns with evidence from clinical applications, where remedial strategies, such as progressive intensity training and structured exercise routines tailored to an individual's fitness level, have shown potential in mitigating the long-term effects of prolonged sedentary behavior (Jakicic and Rogers, 2024). Although the conclusion about the effectiveness of remedial behaviors is encouraging, it is important to emphasize that it relies on the implicit assumption that individuals can precommit to compensating for a previously unhealthy lifestyle. This assumption overlooks the evidence regarding the empirical difficulties individuals face in adhering to well-intentioned lifestyle plans, due to the role of timeinconsistent preferences (Strotz, 1955; Laibson, 1997), temptation, and costly self control (Gul and Pesendorfer, 2001, 2004; Loewenstein and O'Donoghue, 2004). Controlled experiments have shown that gym attendance can effectively increase when using modest financial incentives (Della Vigna and Malmendier, 2006; Charness and Gneezy, 2009; Carrera et al., 2020) or behavioral nudges such as weekly reminders (Calzolari and Nardotto, 2017). How these results persist over time and contribute to the formation of healthy habits remains an open question for further research.

Finally, the model we propose can be used to understand how market prices, subsidies for physical exercise, or taxes on energy-dense foods influence the equilibria individuals reach, along with the associated lifestyle trajectories. In particular, we show that when bifurcation points exist, small changes in the economic environment can lead to abrupt shifts in an individual's behavioral response and health outcomes. This contrasts with the notion that the effects of the policies are continuous, meaning small changes to exogenous variables (e.g., in the price of physical exercise) lead to marginal adjustments in behavior. Instead, when the policy variable crosses a bifurcation point, a small change can trigger a large, discrete shift in behavior and in the health outcome that will be reached in the long run.

During the preparation of this work Davide Dragone used ChatGPT for proof editing. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

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A Appendix

A.1 Necessary optimality conditions

Consider the current-value Hamiltonian function

$$\mathcal{H} = \mathcal{U}(c, u, W, F) + \lambda g\,(c, F, W) + \mu f\,(u, F) \tag{24}$$

The corresponding first-order conditions are

$$\frac{\mathcal{H}}{\partial c} = \mathcal{U}_c + \lambda g_c = \mathcal{U}_c + \lambda \tag{25}$$

$$\frac{\mathcal{H}}{\partial u} = \mathcal{U}_u + \mu f_u = \mathcal{U}_u + \frac{\mu}{F} \tag{26}$$

The adjoint variables change over time as follows

$$\dot{\lambda} = \lambda\,(r - g_W) - \mathcal{U}_W = \lambda\,(r + \delta_W F) - \mathcal{U}_W \tag{27}$$

$$\begin{aligned} \dot{\mu} &= \mu\,(r - f_F) - \lambda g_F - \mathcal{U}_F \\ &= \mu\left(r + \delta_F + \frac{u}{F^2}\right) + \lambda\delta_W W - \mathcal{U}_F \end{aligned} \tag{28}$$

given the transversality conditions:

$$\lim_{t\to\infty} e^{-rt}\lambda(t) = \lim_{t\to\infty} e^{-rt}\mu(t) = 0. \tag{29}$$

Differentiating (25) and (26), replacing the values of λ and μ that solve the focs, and using (27) and (28) yields

$$\dot{c} = \frac{1}{\mathcal{U}_{cc}}\left[\sigma_1\mathcal{U}_c + \mathcal{U}_W - \mathcal{U}_{cW}\dot{W}\right] \tag{30}$$

$$\dot{u} = \frac{1}{\mathcal{U}_{uu}}\left[\frac{\sigma_2}{F}\mathcal{U}_c + \frac{\sigma_3}{F}\mathcal{U}_u + \frac{\mathcal{U}_F}{F} - \mathcal{U}_{uW}\dot{W}\right] \tag{31}$$

where $\sigma_1 = \delta_W F + r > 0$; $\sigma_2 = \delta_W W > 0$ and $\sigma_3 = (2\delta_F + r)\,F \geq 0$.

In steady state, $\dot{c} = \dot{u} = \dot{W} = \dot{F}$, which holds when

$$\mathcal{U}_W = -\sigma_1\mathcal{U}_c \tag{32}$$

$$\mathcal{U}_F = -\sigma_2\mathcal{U}_c - \sigma_3\mathcal{U}_u \tag{33}$$

Using the specific functional forms proposed in the main text, the Hamiltonian becomes

$$\begin{aligned} \mathcal{H} &= \alpha_c c + \alpha_u u + a_W W + a_F F + a_q T + \frac{a_{cc}}{2}c^2 + \frac{a_{uu}}{2}u^2 + \frac{a_{WW}}{2}W^2 + a_{cw}cW + a_{uW}uW \\ &\quad + \lambda\,(c - \delta_w FW) + \mu\left(\frac{u}{F} - \delta_F F\right). \end{aligned} \tag{34}$$

By taking the first derivative we obtain the following first-order conditions for the control variables:

$$\frac{\mathcal{H}}{\partial c} = \alpha_c + a_{cc}c + a_{cw}W + \lambda \tag{35}$$
$$\frac{\mathcal{H}}{\partial u} = \alpha_u + a_{uu}u + a_{uW}W + \mu\frac{1}{F}, \tag{36}$$

which, for interior solutions, reduces to

$$c = -\frac{1}{a_{cc}}\left(\alpha_c + a_{cw}W + \lambda\right) \tag{37}$$
$$u = -\frac{1}{a_{uu}}\left(\alpha_u + a_{uW}W + \mu\frac{1}{F}\right). \tag{38}$$

The dynamics of the costate variables are

$$\dot{\lambda} = (r + \delta_w F)\,\lambda - a_W - a_{WW}W - a_{cw}c - a_{uW}u \tag{39}$$
$$\dot{\mu} = \left(r + \frac{u}{F^2} + \delta_F\right)\mu - a_F + \lambda\delta_w W \tag{40}$$

A.2 Proof of Remark 1

Using the transversality conditions (28) and solving backward the adjoint equations (40) gives the following expressions for the adjoint variables:

$$\lambda(t) = \int_t^\infty e^{-rs}\left(a_W + a_{WW}W + a_{uW}u + a_{cw}c\right)\mathrm{ds} \tag{41}$$
$$\mu(t) = \int_t^\infty e^{-\left(r+\frac{u}{F^2}\right)s}\left(a_F - \lambda\delta_w W\right)\mathrm{ds}, \tag{42}$$

From (38) it follows that $u(t) = 0$ is optimal if

$$-\frac{1}{a_{uu}}\left(\alpha_u + a_{uW}W + \mu\frac{1}{F}\right) \leq 0. \tag{43}$$

Let us assume for simplicity that equality only holds at one t (and not on an interval). Then the above expression becomes

$$-\frac{1}{a_{uu}}\left(\alpha_u + a_{uW}W + \mu\frac{1}{F}\right) < 0. \tag{44}$$

As $a_{uu} < 0$ and $a_{uW} < 0$ the following expression is a necessary condition:

$$\alpha_u + a_{uW}W < -\mu\frac{1}{F}, \tag{45}$$

which means that the static incentives for exercising (left hand side) are lower than the dynamic consequences on the fitness condition, as measured by its adjoint variable μ (right hand side).

To prove the assertion of Remark 1, consider three cases:

(a) $\mu < 0$. This requires $\lambda > 0$ (see 42), which in turn needs a positive a_{cw} and/or a_W, i.e., high body weight is appreciated as it causes a high direct utility and a high marginal utility with respect to consumption.

(b) $\mu > 0$ but W high enough.

- $\lambda > 0$: means a positive a_{cw}, as in case (a).
- $\lambda < 0$. Consider the contrary, i.e., $a_{cw} < 0$ (so no addition). Then $\lambda < 0$ for sure, which implies a low c by the first order condition. This further implies a low W, which contradicts (45). Thus, again $a_{cw} > 0$ and/or $a_W > 0$.

(c) $\alpha_u < 0$: if the exercising causes a (strong enough) disutility the lhs can become smaller than the rhs.

A.3 Additional figures

Phase diagram of optimal trajectories of body weight and fitness condition

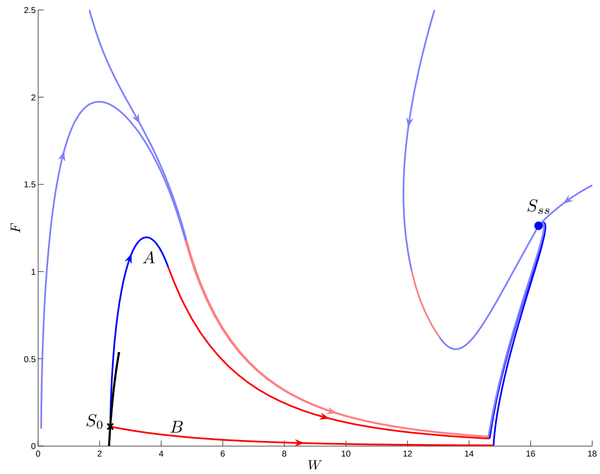

Figure 4: Phase diagram in the body-weight/fitness-condition space for $p_c = 4.5$, $p_u = 3$ and the parameters described in footnote 11. All optimal trajectories converge to the unique overweight steady state S. Trajectories A and B correspond to indifferent lifestyles with equal initial and terminal conditions and equal value function. Red portions of the trajectories denote periods of no physical exercise ($u = 0$); blue portions denote periods where some physical activity ($u > 0$) is optimal. The Skiba curve is in black.

Finite-horizon utility associated to the indifferent lifestyles A and B

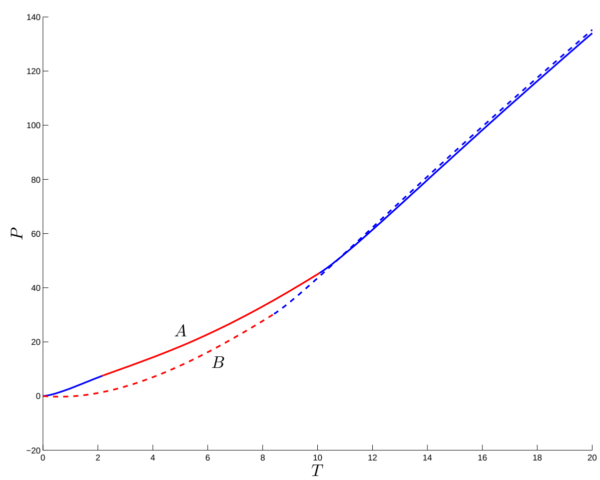

Figure 5: Time paths of finite-horizon utility, as defined in eq. (23), corresponding to the indifferent lifestyles A and B depicted in Figure 4. Lifestyle A dominates B when the agent is "younger", and the reverse holds at older age. Since the two paths describe indifferent lifestyles (and the survival rate does not depend on body weight or on the fitness condition), they coincide when $T \to \infty$. Path A: solid line; path B: dashed line. Red portions of the trajectories denote periods of no physical exercise. Parameters as described in footnote 11, with $p_c = 4.5$, $p_u = 3$.

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