README file

Dataset Title: "**QUANTHEM. Data for the asymmetry dynamics of random quantum circuits. Version 1**."

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Dataset Content

The dataset contains data generated in the framework of Horizon Europe ERC QUANTHEM project. The data were the result of numerical computations. The data represent the simulated dynamics of entanglement asymmetry under random quantum circuits with different geometric connectivity.

The data are presented in the pre-print:

F. Ares, S. Murciano, P. Calabrese, and L. Piroli, *Entanglement Asymmetry Dynamics in Random Quantum Circuits*, arXiv:2501.12459 (2025)

Dataset Documentation

The dataset consists of a .zip archive, named **QUANTHEM_AsymmetryDynamics.zip**, containing 29 tabular quantitative data files saved in .txt format and a README file saved in .pdf format (**README_QUANTHEM.pdf**).

The data correspond to the numerical evaluation of analytic formulas describing the dynamics of entanglement asymmetry under random quantum circuits.

We consider two types of random quantum circuit architectures: local "brickwork'' circuits and 2-local quantum circuits with arbitrary connectivity. In each case, the two-qubit gates are drawn randomly from the Haar measure. The numerical data do not correspond to classical simulation of the quantum circuit, but they are obtained from evaluating analytic formulas describing the entanglement asymmetry dynamics, where the average over the random gates is taken analytically.

The 3 data files named:

QUANTHEM_local_full_L_X_exact.txt

(with X=50,74,100)

contain the values of the averaged Rényi-2 entanglement asymmetry for a local random brickwork circuit, as a function of the discrete time step (or "circuit depth"). Periodic boundary conditions are assumed. The data correspond to the entanglement asymmetry of the full system [L=X is the total number of qubits]. The values are numerically exact: they were computed by evaluating an analytic formula where the average over the random circuits is taken analytically.

The 3 data files named:

QUANTHEM_local_full_L_X_analytic.txt

(with X=50,74,100)

contain the predictions for the Rényi-2 entanglement asymmetry, in a local random brickwork circuit, as a function of the discrete time step (or "circuit depth"). The data correspond to the entanglement asymmetry of the full system [L=X is the total number of qubits]. The values are obtained evaluating an analytic formula which is exact only in the limit of infinte L, and only approximately accurate for finite L.

The 3 data files named:

QUANTHEM_local_L_X_l_Y_exact.txt

(with X=80,120,200, Y=60,96,150)

contain the values of the averaged Rényi-2 entanglement asymmetry for a local random brickwork circuit, as a function of the discrete time step (or "circuit depth"). Periodic boundary conditions are assumed. The data correspond to the entanglement asymmetry of a subsystem of l=Y qubits [L=X is the total number of qubits]. The values are numerically exact: they were computed by evaluating an analytic formula where the average over the random circuits is taken analytically.

The 3 data files named:

QUANTHEM_local_L_X_l_Y_analytic.txt

(with X=80,120,200, Y=60,96,150)

contain the predictions for the Rényi-2 entanglement asymmetry, in a local random brickwork circuit, as a function of the discrete time step t (or "circuit depth"). The data correspond to the entanglement asymmetry of a subsystem of l=Y qubits [L=X is the total number of qubits]. The values are obtained evaluating an analytic formula which is exact only in the limit of infinte L.

The data file named:

QUANTHEM_local_subsystem_exact.txt

contains the values of the averaged Rényi-2 entanglement asymmetry for a local random brickwork circuit, as a function of the discrete time steps (or "circuit depth"). Periodic boundary conditions are assumed. The data correspond to the entanglement asymmetry of a subsystem of l=(2/5)*L qubits [L is the total number of qubits]. Different columns correspond to increasing values of L= 10,20,40. The values are numerically exact: they were computed by evaluating an analytic formula where the average over the random circuits is taken analytically.

The data file named:

QUANTHEM_local_subsystem_vs_subsize_exact.txt

contains the values of the averaged Rényi-2 entanglement asymmetry for a local random brickwork circuit, as a function of the subsystem size. Periodic boundary conditions are assumed. The data correspond to the entanglement asymmetry in a system of L=20 qubits, and different subsystem syzes. Different columns correspond to increasing values of the circuit depth t = 3,4,5,6. The values are numerically exact: they were computed by evaluating an analytic formula where the average over the random circuits is taken analytically.

The data file named:

QUANTHEM_nonlocal_L_60.txt

contains the values of the averaged Rényi-2 entanglement asymmetry for a non-local random circuit, as a function of the time. The data correspond to the entanglement asymmetry in a system of L=60 qubits, and different subsystem sizes. Different columns correspond to increasing values of the subsystem size, l = 15,45. The values are numerically exact: they were computed by evaluating an analytic formula where the average over the random circuits is taken analytically.

The data file named:

QUANTHEM_nonlocal_subsystem_vs_subsize.txt

contains the values of the averaged Rényi-2 entanglement asymmetry for a non-local quantum circuit, as a function of the subsystem size. The data correspond to the entanglement asymmetry in a system of L=60 qubits, and different subsystem sizes. Different columns correspond to increasing values of the time t = 1,2,4,8,10. The values are numerically exact: they were computed by evaluating an analytic formula where the average over the random circuits is taken analytically.

The data file named:

QUANTHEM_nonlocal_subsystem_vs_time.txt

contains the values of the averaged Rényi-2 entanglement asymmetry for a non-local random circuit, as a function of time. The data correspond to the entanglement asymmetry in systems of different total number of qubits (L) and different subsystem sizes (l). The first column corresponds to the time, while the other columns correspond to:

(L,l) = (80,10), (80,30), (80,40), (120,15), (120,45), (120,60)

The values are numerically exact: they were computed by evaluating an analytic formula where the average over the random circuits is taken analytically.

The 2 data files named:

QUANTHEM_equilibration_time_ratio_X.txt

(with X= 0.75, 0.80)

contain the values of the equilibration time as a function of the total system size, for a non-local random circuit. The equilibration time is computed as the time needed for the asymmetry to reach its infinite-time limit, up to an error ϵ=0.1. The subsystem size is chosen to be X*L with X= 0.75, 0.80.

The 2 data files named:

QUANTHEM_equilibration_time_log_ratio_X.txt

(with X= 0.20, 0.25)

contain the values of the equilibration time as a function of the total system size, for a non-local random circuit. The equilibration time is computed as the time needed for the asymmetry to reach its infinite-time limit, up to an error ϵ=0.01. The subsystem size is chosen to be X*L with X= 0.20, 0.25.

The 2 data files named:

QUANTHEM_equilibration_time_vs_l_log_L_X.txt

(with X= 80, 120)

contain the values of the equilibration time as a function of the subsystem size for a non-local random circuit. The equilibration time is computed as the time needed for the asymmetry to reach its infinite-time limit, up to an error ϵ=0.01.

The 2 data files named:

QUANTHEM_local_subsystem_exact_qudit_d_X.txt

(with X= 3,4)

contain the values of the averaged Rényi-2 entanglement asymmetry for a local random brickwork circuit, as a function of the discrete time steps (or "circuit depth"). Periodic boundary conditions are assumed. The data correspond to the entanglement asymmetry of a subsystem of l qudits, with local Hilber-space dimension d=2,3. Different columns correspond to increasing values of the subsystem size, l = 2,4,6. The total number of qudits L=20. The values are numerically exact: they were computed by evaluating an analytic formula where the average over the random circuits is taken analytically.

The 4 data files named:

QUANTHEM_local_charged_moment_alpha_X_l_Y.txt

(with X= 0, pi_2; Y=2,6)

contain the values of averaged charged moments as a function of t for values of $\alpha = 0, \pi/2$ (alpha=0,pi_2), where α is the angle variable of the charged moment. The data correspond to a local brickwork circuit with periodic boundary conditions. In each file, the first column is the number of time steps, while the other columns provide results for d=2,3,4 respectively.

Methodology

All the data were obtained by numerical computations. In all cases, we evaluated analytic formulas where the average over the Haar random gates are taken exactly, so the data are not the result of a sampling procedure (therefore, they do not come with sampling errors). The formulas are derived using a mapping to a statistical mechanics model, as explained in the pre-print

F. Ares, S. Murciano, P. Calabrese, and L. Piroli, *Entanglement Asymmetry Dynamics in Random Quantum Circuits*, arXiv:2501.12459 (2025)

Notes

The data are further explained and presented in the pre-print:

F. Ares, S. Murciano, P. Calabrese, and L. Piroli, *Entanglement Asymmetry Dynamics in Random Quantum Circuits*, arXiv:2501.12459 (2025)