

# Modeling, Analysis and Classification of a PA based on Identified Volterra Kernels

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**Abstract**—This article presents the modeling of a microwave Power Amplifier (PA) in the almost linear and compression operation modes. An in-band quasi-white noise real-valued signal is used as input for the identification process to excite every possible source of nonlinearity. A segment of the input-output measurement data is processed to generate an initial Parallel Cascade Wiener Model (PCWM). The model is cross-validated with the entire measurement signal. The first order Volterra kernel is extracted in order to obtain an estimation of the amplifier’s memory. A new model is generated and its Volterra kernels up to the second order are estimated to apply the Structural Classification Methods (SCM). The result of this process is a suitable block-structure for the final amplifier model. The optimized model is intended to be numerically robust having a high identification percentage based on a variance figure of merit. This resulting model can be used for simulation of linearization systems or even in further identification processes.

## I. INTRODUCTION

With so many emerging transistor technologies and fabrication methods it is very difficult to develop a general behavioral model which can be used with every component. The necessary physical parameters are not available in commercial cases since this constitutes the industry’s know-how. However, they are the basis to formulate precise mathematical expressions or parametric models that contain algebraic or differential equations. These facts lead to the use of inductive and data driven amplifier models in many cases.

In this scope, Neural Networks (NN) based models are a novel and an efficient approach [1] and represent the connectionist modeling methodology [2]. The main disadvantage is a lack of parsimony and interpretability compared to parametric models. It has been shown that a good choice of the NN structure should follow directly from the application and the prior knowledge of the physical system to be modeled [3]. This implies some restrictions to the design of NN when no prior information about and the number of neurons and their organization inside a structure is available [4].

To overcome these NN disadvantages the modular approach was employed. It combines parametric and nonparametric methodologies providing flexible arrangements of block-structured models (Wiener model (Linear-Nonlinear), Hammerstein model (Nonlinear-Linear) and Wiener-Hammerstein model (Linear-Nonlinear-Linear)) that are connected in parallel [5]. The overall output is a reliable model that can predict the amplifier’s behavior and its Volterra kernels.

Thus the process of achieving the amplifier model consist of two tasks: First, a suitable model structure has to be selected and then the model parameters can be estimated.

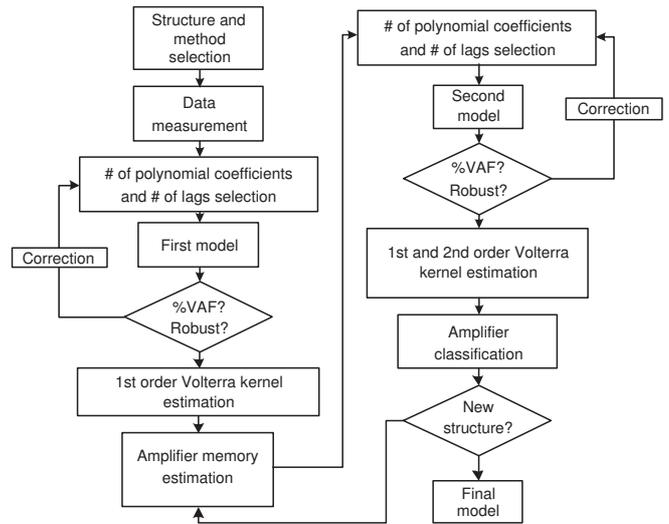


Fig. 1. Algorithm used in the identification process

Based on an initial model the structure is found by applying the structure classification method [6]. Finally, the coefficients of the resulting model can be extracted.

The objective of this paper is to present a model identification algorithm as shown in Fig. 1. At the beginning the estimated first order Volterra kernel from an initial model is evaluated, providing an insight into the amplifier’s memory effects. By the use of this parameter a second model is calculated which is afterwards used to classify the system’s structure. Then the gathered information on memory and structure is used for the extraction of the final model.

## II. BACKGROUND

This section presents the block structure used in the initial model, the discrete Volterra series and the figure of merit applied to validate the extracted results.

### A. Parallel Cascade Wiener Method

At the start of the identification process (Fig. 1) a model must be generated to estimate the amplifier memory. As no prior information was available, a flexible structure and a system identification method should be selected for this task.

The PCWM (Fig. 2) combines following favorable properties: It is computationally efficient even for high-order models with large memory-bandwidth products, it allows the direct extraction of the Volterra kernels and it offers the convenience

to use different methods for the identification of the linear and nonlinear blocks [7]. The main disadvantage: The model is still sensitive to noise in the data when too many paths are used [2]. Consequently, a proper selection of the paths and the order of the nonlinearity should be made to assure low noise and good convergence, respectively.

The PCWM output can be represented as:

$$y(t) = \sum_{p=1}^P \sum_{q=0}^Q c_p^{(q)} \left( \sum_{\tau_1=0}^{T-1} \cdots \sum_{\tau_q=0}^{T-1} \mathbf{h}_p(\tau_1) \cdots \mathbf{h}_p(\tau_q) \cdot u(t - \tau_1) \cdots u(t - \tau_q) \right) \quad (1)$$

where  $P$  is the total number of paths,  $Q$  denotes the maximum order of the polynomial used,  $c_p^{(q)}$  are the polynomial coefficients,  $T$  is the memory length,  $t$  and  $\tau_q$  are discrete indexes of the sampling interval and  $\mathbf{h}_p(\tau)$  is the impulse response of the index  $p$  path. The polynomials used are the Tchebychev Polynomials. Its basis function are bounded between  $[-1,1]$  for inputs between  $[-1,1]$ . The model input signals were all normalized to  $[-1,1]$ .

### B. Volterra Kernels

Volterra kernels are the complete and reliable descriptors of the system's function. Their estimation from input-output data is the objective of the system identification task in the nonparametric context [2].

The finite, discrete Volterra series model is given by [9]:

$$y(t) = \sum_{q=0}^Q \sum_{\tau_1=0}^{T-1} \cdots \sum_{\tau_q=0}^{T-1} \mathbf{h}^{(q)}(\tau_1, \dots, \tau_q) \cdot u(t - \tau_1) \cdots u(t - \tau_q) \quad (2)$$

where  $\mathbf{h}^{(q)}$  is the kernel of order  $q$ ,  $t$  and  $\tau$  are discrete indexes of the sampling interval and  $T$  is the memory length.

The minimal number of samples (lags) along each dimension of the kernel required to represent it in discrete time is given by [2]:

$$M \geq 2 \cdot B_s \cdot \mu \quad (3)$$

where  $B_s$  (Hz) denotes the system bandwidth and  $\mu$  is the effective kernel memory or the correlation time over which the kernel has significant values.

### C. Model Evaluation

The statistically based figure of merit used in this article to specify how well a model describes an unknown system is the *percent variance accounted for* [8]:

$$\%VAF = 100 \left( 1 - \frac{\text{var}(y - \hat{y})}{\text{var}(y)} \right) \quad (4)$$

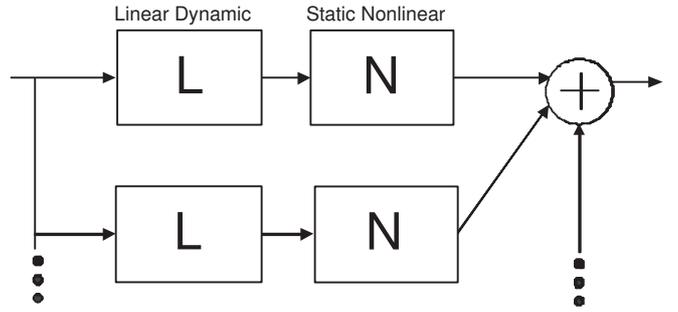


Fig. 2. Parallel Cascade Wiener Model

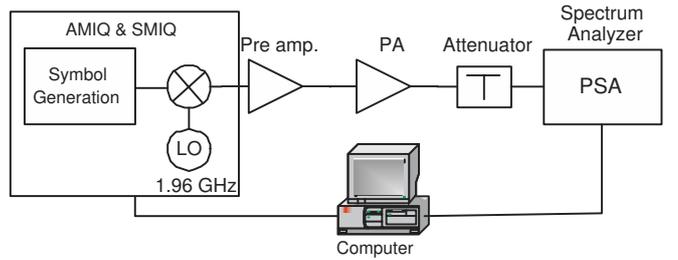


Fig. 3. Measurement System Used

## III. MEASUREMENT SETUP

The measurement system is presented in Fig. 3. The input signal was loaded in the I/Q Modulation Generator R&S AMIQ and up converted by a Vector Signal Generator R&S SMIQ. The preamplifier was a Minicircuits ZHL-42W. The class AB main amplifier has the following nominal characteristics: Frequency range of 1.93–1.96 GHz, maximum output power of +48 dBm and 36 dB gain. The PA output signal measurement was performed at 1.96 GHz using an Agilent Performance Spectrum Analyzer (PSA) and processed by the Agilent 89600 Signal Analysis Software in a workstation. Different output power levels were measured, to check the behavior of the estimated models in such situations.

## IV. AMPLIFIER IDENTIFICATION

### A. Initial Models

The first model was developed using a band limited noise input signal of 1.25 MHz bandwidth at a very low average input power level (26-dB back off). The total data time was about 1 ms. For the first approach a model with 2500 taps was used, which limits the maximum amplifier memory to 25  $\mu$ s.

A similar model was developed for the same amplifier, but operating with four times the bandwidth of the first signal.

The models were evaluated by comparing their outputs with the experimental ones. The %VAF and time domain plots were used to evaluate the model (compare Fig. 1). For the cross-validation an input signal different from the one used for identification was applied, to avoid noise modeling as well as system dynamics when the model is over parameterized [10].

The first order Volterra kernels obtained from these models are shown in Fig. 4. The faster changes presented by the kernel of the system with higher bandwidth reveals the higher difficulty to find a model.

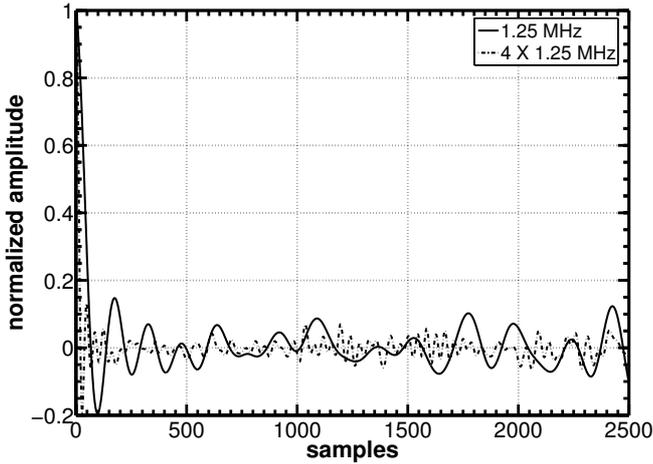


Fig. 4. First-order kernels estimated from the amplifier using different bandwidths.

### B. Memory Estimation

The system memory length could be roughly estimated observing Fig. 4 to maximum of 350 samples or  $3.5 \mu\text{s}$ . After this estimated point the first order kernels show no exponential decaying but some oscillations around zero that could be caused by modeling noise. This value limits the number of lags used in the next models. It was determined squaring the kernels and calculating where they concentrate approximately 90% of the total energy.

The product  $2 \cdot B_s \cdot \mu$  considered for the first case (1.25 MHz) was 35 lags and for the second case ( $4 \times 1.25$  MHz) 100 lags.

These values are important information obtained from data, since no prior knowledge about the amplifier's memory was known before.

### C. Second models

1) 1.25 MHz Bandwidth: A PCWM with five paths was obtained from cross correlated data with a maximum polynomial order of 4. This model was developed for low power (LP) – in this case +22 dBm total output power (26 dB back-off) – and tested with higher power levels also to check its validity. Near the maximum amplifier's output power, referred in this article as high power case (HP), the model did not work well as observed by the %VAF results and the PDF measurements output. The %VAF was lower and the measured data output PDF has changed from a Gaussian-like to an OQPSK-like distribution [11], similar to the measurement results presented in Fig. 7. Then it was necessary to design another model with higher order polynomials to meet the new data characteristics. At last the complete characterization of the amplifier was achieved with 2 distinct models, one for the LP case and another one for the HP case.

2)  $4 \times 1.25$  MHz Bandwidth: The maximum order of the polynomials was 5 for LP and 7 for the HP case. These values were obtained observing the %VAF and the robustness of the model. The order was increased due to the necessity to represent higher order distortion components present on the output signal.

3) LP signals with HP models: It was observed that when the HP models were tested with LP signals they weren't satisfactory when compared with the already obtained LP models. The estimated higher order polynomial coefficients for the HP

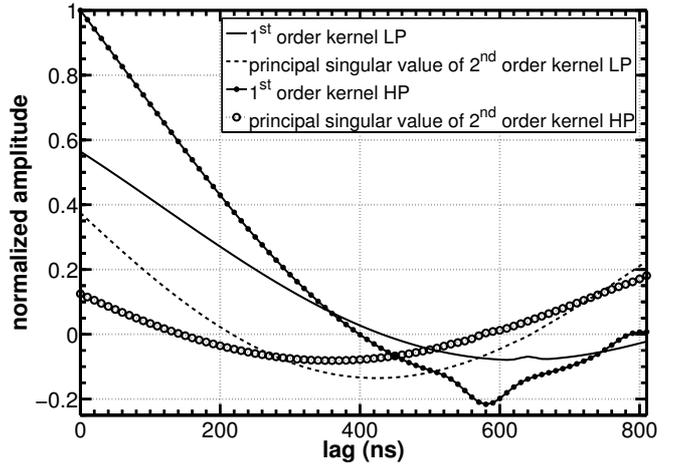


Fig. 5. Test for a Wiener Structure applied to the LP and HP levels case for the 1.25 MHz input bandwidth.

model have generated undesired higher order components that haven't contributed to the %VAF (overparameterized model).

### D. Amplifier Classification

To verify if the system could be treated as a PCWM the Singular Value Decomposition (SVD) of the second order Volterra kernel obtained from the identified models was calculated. Its principal singular value (PSV) was compared to the first order Volterra kernel and should be identical, what is a necessary but not a sufficient condition. This procedure is described at the SCM [6]. Both cases (LP and HP) for the 1.25 MHz and  $4 \times 1.25$  MHz input bandwidth were analyzed:

1) 1.25 MHz Input Bandwidth: The results for the 1.25 MHz input bandwidth are shown in Fig. 5. The first order kernel and the second order Volterra kernel's PSV at the LP case are not identical but at least similar up to 450 ns, what justifies the use of the PCWM.

In the HP case the low similarity between the first order kernel and the second order kernel's PSV is obvious. The first order kernel was moved up and the second order Volterra kernel's PSV was moved down, lowering the possibility of using the same model as in the LP case. This helps to understand why the model used in the LP case was not valid for HP case.

2)  $4 \times 1.25$  MHz Input Bandwidth: The results for the  $4 \times 1.25$  MHz input bandwidth are shown in Fig. 6. For the first model (LP) the curves show some similarities although the second order Volterra kernel's PSV was shifted to the left.

In the last case, the first order kernel presented a very long memory due to the HP output. At 500 ns and after 860 ns the second order Volterra kernel's PSV showed an unexpected behavior with a considerable expansion against the first order kernel. All these factors have contributed to diminish the %VAF. They have also changed the classification of the amplifier as in the HP case with 1.25 MHz input bandwidth to an undefined structure.

### E. Final Models

1) 1.25 MHz Input Bandwidth: The final models used 82 lags and presented a VAF of 99.1% for the LP case and 95.3% for the HP case. The system has shown no similarities in the HP case with any structure that it was tested for (LN, NL and

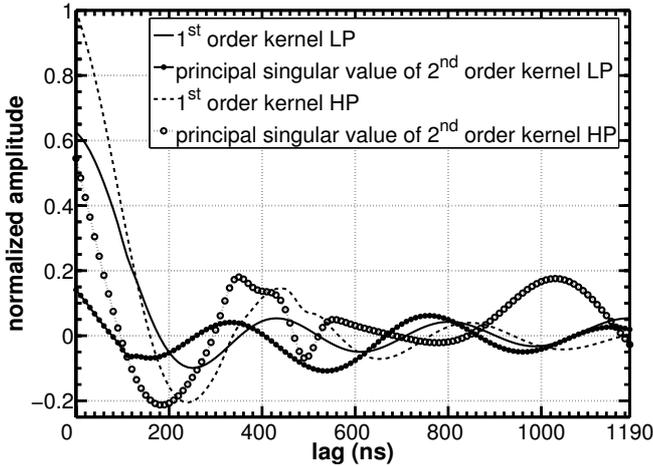


Fig. 6. Test for a Wiener Structure applied to the LP and HP case with  $4 \times 1.25$  MHz input bandwidth.

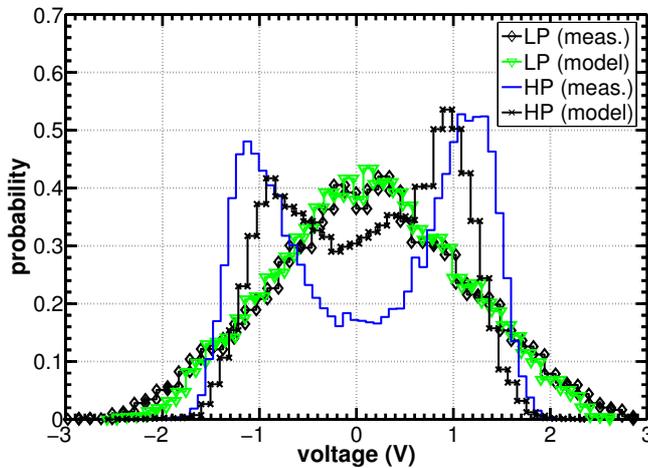


Fig. 7. PDFs of the amplifier output signal in the low and high power case, with  $4 \times 1.25$  MHz input bandwidth, where nonlinearities make difficult to find a better model. All amplitudes were normalized with respect to the standard deviation.

LNL), leaving the choice open. The Wiener cascade structure was selected. Nevertheless, the %VAF was not so dramatically reduced (see table I).

2)  $4 \times 1.25$  MHz Input Bandwidth: In this case, 120 lags were used and the VAF was 97.6% for the LP case and 73.9% for the HP case. As already commented in the previous section the HP case %VAF was not so high. This indicates that further studies are required in order to obtain more accurate models. The relevant results are summarized in Table I.

The results of the estimated output PDFs for the LP and HP case are presented in Fig. 7.

The baseband normalized spectral results from these models are presented in Fig. 8. It can be seen that at HP the model couldn't represent every intermodulation product that degrades the signal.

## V. CONCLUSION

This article presented the estimation of PA models at LP and HP case and their equivalent Volterra kernels. The models were analyzed in the time, frequency domain and with PDFs showing acceptable results when compared with measured data. The influence onto the kernels due to the memory

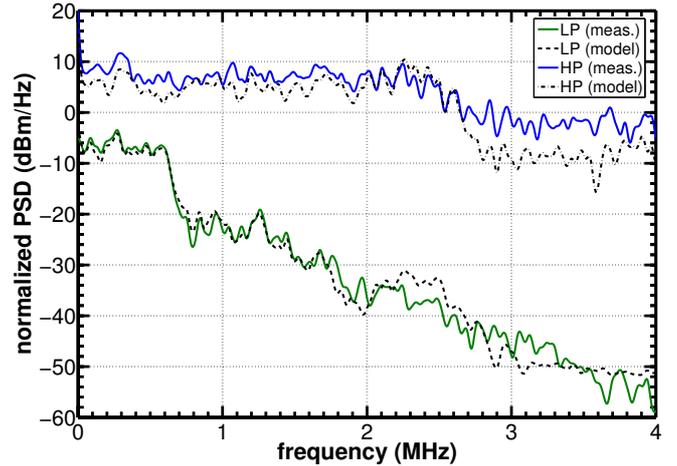


Fig. 8. Output baseband measured PSD and model estimated output PSD for the 1.25 MHz input signal bandwidth and  $4 \times 1.25$  MHz at HP. The curves are normalized with respect to the standard deviation and shifted for a better visualization.

| Input Signal BW     | lags | Polynomial order | %VAF         |
|---------------------|------|------------------|--------------|
| 1.25 MHz            | 82   | 4 (LP)<br>5 (HP) | 99.1<br>95.3 |
| $4 \times 1.25$ MHz | 120  | 5 (LP)<br>7 (HP) | 97.6<br>73.9 |

TABLE I

SUMMARY OF THE DIFFERENT MODELS AND RESULTS

effects was observed, showing that at higher power levels the memory effects can accumulate changing the classification of the estimated equivalent amplifier model.

## ACKNOWLEDGEMENT

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