System-Level Simulation of a Noisy Phase-locked Loop

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***Abstract*— This paper presents a compact model of a noisy phase-locked loop (PLL) for inclusion in a time-domain system simulation. The phase noise of the reference is modeled as a Wiener process, and the phase noise contribution of the voltage-controlled oscillator (VCO) is described as an Ornstein-Uhlenbeck process. The model is applied to phase error modeling for a 60 GHz OFDM system including correction of the common phase error. A close agreement is observed between the time-domain simulation and a frequency-domain model.**

I. Introduction

Integrated phase-locked loops have found a wide range of applications including clock generation in microprocessors, clock and data recovery circuits in fiber-optic receivers and generation of the sampling clock in analog-to-digital converters (ADC). The phase noise performance of integrated PLLs is especially critical in RF synthesizers for 60 GHz WLAN [1] and automotive radar at 77 GHz, which has motivated this work.

Many publications on PLL noise modeling have appeared during the last few years [2]-[12] mainly focussed on behavioral circuit simulation. For system simulations a more abstract PLL model is required to minimize the simulation time and the required knowledge on circuit level. In [13]-[15] phase noise has been discussed in the context of OFDM. These noise models are simple and allow a time-efficient modeling with a system simulator. However, they are oversimplified, since they disregard the combination of high-pass filtering and low-pass filtering of noise in a PLL.

This paper describes, how the PLL output phase can be generated for behavioral system modeling with realistic model parameters. We take into account VCO phase noise as well as phase noise of the reference due to white noise sources. In addition, we consider an RF synthesizer for a 60 GHz OFDM system including correction of the common phase error. The numerical approach is verified by comparison of the simulated jitter with analytical results from the literature.

II. Phase Noise in PLLs

In a PLL the VCO output phase is divided by N and compared with a reference phase in a phase-frequency detector (PFD). A charge pump current proportional to the phase error is produced, low-pass filtered and applied to the control input of the VCO (Fig. 1). We will first describe the PLL output noise assuming a noisy reference

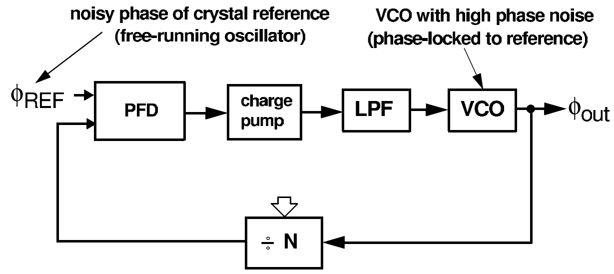

Fig. 1. Schematic view of a charge-pump PLL. The noisy VCO output is divided by N and phase-locked to a relatively clean reference to define the output frequency and to clean the VCO phase noise spectrum.

and an ideal VCO. Subsequently, we will consider a noisy VCO driven by an ideal reference.

The output voltage of an oscillator can be written as

$$V_{\rm out}(t) = V_0 \cos[\omega_0 t + \phi_{\rm out}(t)]\,, \tag{1}$$

where V_0 is the amplitude, $f_0 = \omega_0/(2\pi)$ is the oscillation frequency, and $\phi_{\rm out}(t)$ is the excess phase. In a PLL the phase noise of the reference is low-pass filtered, while the VCO noise is high-pass filtered [16] resulting in the total phase error

$$\phi_{\rm out}(t) = {\rm N}\int_0^\infty dt' h_{\rm LPF}(t')\phi^{\rm REF}(t-t') + \int_0^\infty dt' h_{\rm HPF}(t')\phi^{\rm VCO}(t-t')\,, \tag{2}$$

where $\mathrm{N}\,h_{\rm LPF}$ and $h_{\rm HPF}$ are the linear phase responses at the PLL output referred to the phase noise source. Note that $\mathrm{N}\,h_{\rm LPF}(t')$ implies a multiplication by the division ratio N, which enhances the reference noise.

III. The Wiener Process and the Noise of the Reference

The Wiener process was applied to the phase of narrowband oscillators by Stratonovich to calculate the output spectrum which represents a Lorentz function [17]. More recently, this model was used to calculate the jitter of free-running oscillators [18]. The Wiener process can be described by the stochastic differential equation [19]

$$\frac{\rm d}{{\rm d}t}\phi(t) = F(t)\,, \tag{3}$$

where $F(t)$ is white Gaussian noise with the auto-correlation function (ACF)

$$< F(t+\tau)F(t) >= 2D_\phi \delta(\tau)\,, \tag{4}$$

and D_ϕ is the phase diffusivity. Period jitter and absolute jitter are given by [18]

$$\sigma_T = \sqrt{\frac{4\pi D_\phi}{\omega_0^3}} = \sqrt{\frac{D_\phi}{2\pi^2 f_0^3}}\,, \tag{5}$$

and

$$\sigma_{\mathrm{abs}} = \sqrt{n}\sigma_T\,. \tag{6}$$

In order to solve the differential equation (3) numerically, all quantities are to be calculated on a discrete time grid

$$t_{\mathrm{k}} = \mathrm{k}\,\Delta t \;\; (\mathrm{k} = 0, 1, 2, ...)\,. \tag{7}$$

In this case, the standard deviation of the noise force is, according to (4), given by

$$\sigma_F = \sqrt{\frac{2\,D_\phi}{\Delta t}}\,. \tag{8}$$

For white noise sources we can calculate the phase diffusivity from the single-sideband phase noise according to [18]

$$D_\phi = \frac{S_{\mathrm{SSB}}}{2}\omega^2 = S_{\mathrm{SSB}}\,2\pi^2 f^2\,, \tag{9}$$

where the phase noise must be taken at a frequency offset f from the carrier f_0 in the region where $S_{\mathrm{SSB}} \propto 1/f^2$ corresponding to a -20 dB/decade slope of $\pounds = 10\log(S_{\mathrm{SSB}})$.

IV. The Ornstein-Uhlenbeck Process and the VCO Device Noise

The Ornstein-Uhlenbeck process is described by the Langevin equation [19]

$$\frac{\mathrm{d}}{\mathrm{d}t}\phi(t) + \omega_L \phi(t) = F(t)\,, \tag{10}$$

where the auto-correlation function (ACF) of the Gaussian noise force is given by

$$< F(t+\tau)F(t) >= 2D_\phi \delta(\tau)\,. \tag{11}$$

From the steady-state ACF

$$< \phi(t+\tau)\phi(t) >= \frac{D_\phi}{\omega_L}\exp\left(-\omega_L|\tau|\right) \tag{12}$$

we obtain the *two-sided* PSD of ϕ, which is the Fourier transform of the ACF according to the Wiener-Khintchine theorem and reads

$$S_\phi(\omega) = \frac{2D_\phi}{\omega^2 + \omega_L^2}\,. \tag{13}$$

Figure 2 shows four realizations of the Ornstein-Uhlenbeck process. In contrast to the Wiener process, this process has a stationary solution.

The phase noise spectrum of a free-running VCO is high-pass filtered in a PLL, which gives for a second-order charge-pump-PLL model [16]

$$S_\phi^{\mathrm{PLL}}(\omega) = \frac{2D_\phi}{\omega^2}\left|\frac{\omega^2}{\omega^2 - 2j\zeta\omega_n\omega - \omega_n^2}\right|^2\,, \tag{14}$$

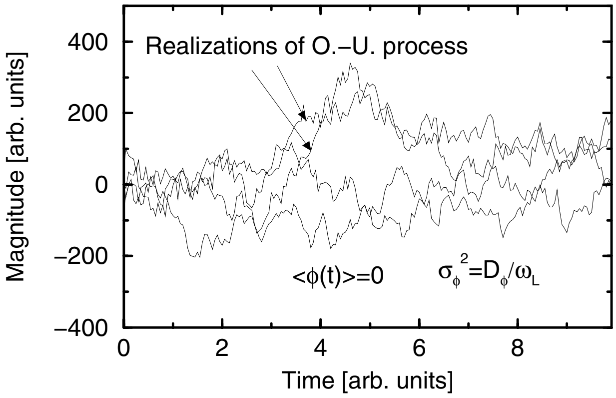

Fig. 2. Four realizations of the Ornstein-Uhlenbeck process generated by (10).

where ζ is the damping factor and $\omega_n = 2\pi f_n$ the natural angular frequency. Integrating the phase noise and multiplying the result by $T/(2\pi) = 1/\omega_0$ results in the absolute jitter given by [12]

$$\sigma_{\mathrm{abs}}^{\mathrm{PLL}} = \frac{f}{f_0}\sqrt{\frac{S_\phi^{\mathrm{VCO}}}{8\pi\zeta f_n}} \approx \frac{f}{f_0}\sqrt{\frac{S_{\mathrm{SSB}}^{\mathrm{VCO}}}{8\pi\zeta f_n}}\,. \tag{15}$$

For an overdamped PLL one can neglect ω_n^2 in (14) which yields

$$S_\phi^{\mathrm{PLL}}(\omega) = \frac{2D_\phi}{\omega^2}\frac{\omega^2}{\omega^2 + \omega_L^2} \tag{16}$$

with $\omega_L = 2\zeta\omega_n$. We conclude that the VCO noise is filtered by a first-order high-pass of bandwidth $\omega_L = 2\pi f_L$ (in rad/s). The absolute jitter is [9]

$$\sigma_{\mathrm{abs}}^{\mathrm{PLL}} = \frac{f}{f_0}\sqrt{\frac{S_\phi^{\mathrm{VCO}}}{4\pi f_L}} \approx \frac{f}{f_0}\sqrt{\frac{S_{\mathrm{SSB}}^{\mathrm{VCO}}}{4\pi f_L}}\,. \tag{17}$$

Comparison of (15) and (17) shows that the second-order loop has the same absolute jitter as the first-order loop if we have $f_L = 2\zeta f_n$. This allows to model a second-order loop (also if not overdamped) as a first-order loop, if the steady-state absolute jitter is the crucial parameter in the system. The spectrum (16) is identical to (13) which shows that the Langevin equation (10) adequately describes VCO noise filtering in a first-order PLL.

Figure 3 illustrates that for a PLL, in contrast to a free-running oscillator, the ensemble average of the timing error converges to a steady-state value, where the time constant is the inverse PLL loop bandwidth $\tau_L = 1/\omega_L$ as derived by McNeill [2].

V. Algorithm for Numerical Generation of PLL Output Voltage

The recommended procedure for the generation of the PLL output voltage is as follows.

- Determine the phase noise $\pounds$ [dBc/Hz] of the reference from measurement or data sheet.
- Calculate $S_{\mathrm{SSB}} = 10^{\pounds/10}$ and D_ϕ^{REF} from (9).
- Generate white Gaussian noise with the standard deviation (8) on a time grid (7).

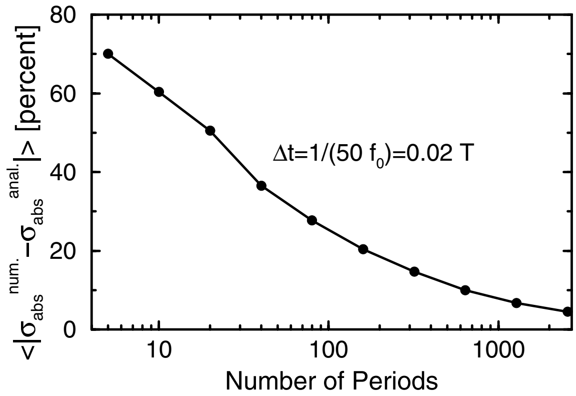

Fig. 3. Deviation of numerical absolute rms jitter from analytical result (17). The numerical value was obtained by averaging a large number of Ornstein-Uhlenbeck process realizations generated by (10).

- Integrate the differential equation (3), low-pass filter the result to obtain the first contribution $\phi_1(t) = \mathrm{N}\,\omega_L \int_0^\infty \mathrm{d}t' \exp(-\omega_L t')\phi(t-t')$ to the PLL phase error.
- Determine the phase noise $\pounds$ of the VCO by measurement, simulation or from data sheet.
- Calculate $S_{\mathrm{SSB}} = 10^{\pounds/10}$ and D_ϕ^{VCO} using (9).
- Generate white Gaussian noise with the standard deviation (8) on a time grid (7).
- Integrate the differential equation (10) to obtain the second contribution to the PLL phase $\phi_2(t)$.
- Substitute $\phi_{\mathrm{out}}(t) = \phi_1(t) + \phi_2(t)$ into (1).

This algorithm represents a recipe for an easy but realistic description of the noisy PLL output voltage from the oscillator phase noise and the loop bandwidth.

VI. Phase Error Simulation for OFDM

In an OFDM system the carriers are separated by the sub-carrier spacing $(\Delta f)_{\mathrm{car}} = 1/T_u$, where T_u is the useful part of the symbol length, that is, the length of the Fourier transformation interval [13]-[15]. The so-called common phase error (CPE) in an OFDM system can be eliminated by subtracting the mean phase for every symbol. This operation can be modeled in the frequency domain by a high-pass filter [20]. Combining this high-pass filter with filtering due to PLL action, we obtain for the weighted PLL phase noise spectrum

$$S_{\mathrm{PLL}} = \left(S_{\mathrm{VCO}}|1 - H_{\mathrm{LPF}}|^2 + N^2 S_{\mathrm{REF}}|H_{\mathrm{LPF}}|^2\right) \times \left[1 - \mathrm{sinc}^2(fT_u)\right] \,, \tag{18}$$

where the "sinc" function is defined by $\mathrm{sinc}(x) = \sin(\pi x)/(\pi x)$. The high-pass filter $[1 - \mathrm{sinc}^2(fT_u)]$ significantly improves the integrated phase jitter, if the sub-carrier spacing $1/T_u$ is larger than the loop bandwidth f_l [21].

Integrating the spectrum, we obtain the rms phase error given by [12]

$$\sigma_\phi\,[\mathrm{rad}] = \sqrt{2\int_0^{B/2} \mathrm{d}f\, S_{\mathrm{PLL}}(f)}\,. \tag{19}$$

The upper integration limit is half the bandwidth of the whole OFDM band. This corresponds to the "middle-carrier" weighting function as representative for the whole OFDM signal [20]. A PLL is usually modeled as a second-order system. For the charge-pump PLL under consideration the LPF transfer function is given by [16]

$$H_{\mathrm{LPF}}(s) = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\,, \tag{20}$$

where ζ is the damping factor, and ω_n is the natural angular frequency. As explained above, the transfer function can be approximated by a first-order filter according to

$$H_{\mathrm{LPF}}(s) \approx \frac{\omega_l}{s + \omega_l}\,, \tag{21}$$

where the loop bandwidth (in rad/s) is given by $\omega_l = 2\zeta\omega_n$. Equation (19) in conjunction with (18) and the filter function (20) or (21), respectively, allows the effective phase error σ_ϕ to be calculated from circuit parameters and the carrier spacing.

We have calculated the rms phase for a 60 GHz RF synthesizer with a phase noise of -90 dBc/Hz at 1 MHz frequency offset and a divider ratio of N=1024 as presented in [1]. The calculation was done both in the time domain and in the frequency domain for three different phase noise levels of the crystal reference. We used an FFT period of T_u =640 ns corresponding to a sub-carrier spacing of 1.5625 MHz. Due to this large bandwidth the neglection of $1/f$-noise in the model is justified, since it is eliminated by the high-pass filter in (18). Figure 4 shows the rms phase error for the first-order model both from time-domain simulation (symbols) and from the frequency-domain model given by (19) shown as solid lines. The close agreement demonstrates the high accuracy of the simulations and the frequency-domain model for predicting the phase error after common-phase error correction.

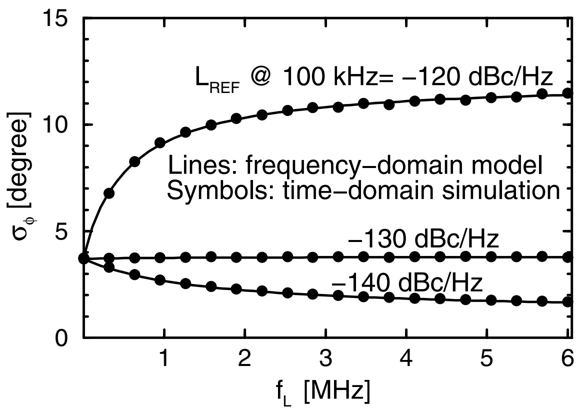

Fig. 4. RMS phase error after correction of common phase error from frequency domain model (19) and from time-domain simulation as a function of the loop bandwidth.

VII. Conclusion

We have presented and verified an algorithm to generate the PLL output voltage for behavioral system simulations in the time domain. We have included the two most important noise sources in an integrated PLL, namely, white noise in the reference oscillator and white noise in the voltage-controlled oscillator. While the first is low-pass filtered in a PLL, the second is high-pass filtered. The numerical parameters are derived from oscillator phase noise and the loop bandwidth ω_L. The noise behavior of a second-order PLL with the natural angular frequency ω_n and damping constant ζ can be approximated by a first-order PLL, if we identify ω_L with $2\omega_n\zeta$. We applied the model to a 60 GHz RF synthesizer for an OFDM system, where the oscillator noise parameters are based on measured data. The simulated rms phase error agrees with the integrated phase noise spectrum including a weighting function for the removal of the common phase error. Our model facilitates realistic bit-error-rate calculations in communication systems. It can be used to estimate the VCO phase noise required to meet given system specifications. This will help to choose an appropriate technology (InP, GaAs-HEMT, GaAs-HBT, SiGe-HBT, BiCMOS, CMOS) for a given system, where a trade-off between phase noise performance and cost must be considered.

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