ELECTROMAGNETIC MODEL OF GAAS VCSEL'S WITH GAIN GIUDING

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ABSTRACT

An electromagnetic model of Vertical Cavity Surface Emitting Lasers (VCSEL) based on semiconductor compounds is developed; it relies on the rigorous solution of Maxwell equations, including the field confinement due to the gain guiding mechanism which strongly influences the noise properties. The solution of the problem is based on an integral equation of the Fredholm type whose eigenvalues are related to the threshold condition and eigenvectors give the field distributions.

INTRODUCTION

Vertical Cavity Surface Emitting Lasers (VCSEL) based on semiconductor compounds (mainly GaAs) have received much interest in recent years owing to many advantageous characteristics: high integrability, array features, low threshold, good optical beam, high modulation speed, etc. , Special issue (1).

A schematic representation of a typical device is shown in Fig.1: the structure includes two high reflectivity Bragg mirrors which define a λ cavity containing a multiple quantum well active region of thickness d. The structure is quasi-planar since the transverse dimensions ($2a$) are much larger than the cavity length.

The development of an electromagnetic model of the device presents some difficulties:

- the approximations commonly used in the theory of the classical open resonators can no longer be applied because the device length is much shorter than the transverse cross section,
- the mirror reflectivities play an important role,
- the optical field shape is determined by carrier injection that gives rise to gain guiding.

This implies that a rigorous solution of Maxwell equations must be carried out; in particular the most important consequence of the third point is that the Petermann factor must be introduced, Petermann (2), Henry (3), Duan et al. (4), Siegman (5), Grangier and Poizat (6); it strongly influences the noise performances.

THE MODEL

In order to simplify the analysis, a symmetrical device is considered so that only the region of positive z is accounted for and the symmetry plane (dashed line in Fig. 1) is replaced by a magnetic wall. Since the thickness of the active region is very small, its effect is equivalent to the boundary condition of a current sheet at z=0, that is in z=0:

$$\hat{z} \times \underline{H} = Y \underline{E}_t + \underline{J}_L \tag{1}$$

being $\underline{H}$ and $\underline{E}$ the magnetic and electric fields, the subscript t denotes the tangential component, $\underline{J}_L$ is a Langevin noise source, Henry (3), and:

$$Y = \frac{1}{2} j\omega \, \Delta\varepsilon \, d \tag{2}$$

where $\Delta\varepsilon$ is the contribution to the dielectric permittivity due to the quantum well active region, Debernardi et al. (8).

To obtain a rigorous solution the field is expanded in plane waves and the boundary condition (1) becomes an integral equation for the field distribution in z=0. By assuming that the transverse component of the propagation constant is much smaller that the longitudinal one, the plane waves are nearly TEM. In such a case linearly polarized solutions exist and the integral equation for the magnetic field component is:

$$H(\underline{\rho}) = -\frac{Y(\underline{\rho})}{4\pi^2} \int K(\underline{\rho}, \underline{\rho}') H(\underline{\rho}') d\underline{\rho}' + J_L(\underline{\rho}) \tag{3}$$

with the nucleus K:

$$K(\underline{\rho}, \underline{\rho}') = Z_0 \int Z(k_\rho) \exp(-j\underline{k}_\rho \cdot (\underline{\rho} - \underline{\rho}')) d\underline{k}_\rho \tag{4}$$

where Z_0 is the wave impedance in the resonator, Γ is the mirror reflection coefficient in z=L, $\underline{\rho}$ and $\underline{k}_\rho$ are bidimensional vectors in the z=0 plane, and:

$$Z(k_\rho) = \frac{1 + \Gamma \exp(-j2k_z L)}{1 - \Gamma \exp(-2jk_z L)}$$

is the normalized impedance at z=0.

For structures presenting circular symmetry it is convenient to introduce radial (r) and angular (φ) polar coordinates and solutions are found in the form $H(\underline{\rho}) = H(r, \varphi) = f(r) \exp(jm\varphi)$. Moreover we set $Y = Y_0 y(r)$ in order to put in evidence the carrier density dependence through Y_0 and the spatial profile $y(r)$.

Substituting H into eq (3), the function $f(r)$ must satisfy the equation:

$$f(r) = \Lambda \int K(r, r') f(r') dr' + \frac{1}{2\pi} \int J_L(r, \varphi) \exp(jm\varphi) d\varphi \tag{5}$$

where $\Lambda = -Y_0 Z_0$ and the nucleus becomes:

$$K(r, r') = \int Z(k) kr' J_m(kr') J_m(kr) dk \tag{6}$$

being J_m the Bessel function of the first kind of order m and k the radial component of the transverse wavevector.

VCSEL MODES

In order to study the device modes the noise source in (5) is omitted. The homogeneous eq.(5) is an eigenvalue problem; the complex eigenvalues allow to compute the mode gains and lasing frequencies. The eigenvalue related to the smaller gain (G) corresponds to the first lasing mode and through Y_0 it determines the required carrier density and operating frequency. The eigenvector can be used, by means of the plane wave expansion, to compute the optical field distribution inside and outside the resonator.

To put the integral equation into a standart form, the nucleus has been symmetrized and the variable k has been discretized (k_n, spacing Δk); then the integral equation has a solution of the form:

$$f(r) = y(r)\sum_n A_n J_m(k_n r) \tag{7}$$

and the eigenvectors A_n are determined by inverting the system:

$$A_n = \Lambda \sum_i R_{ni} A_i \tag{8}$$

with:

$$R_{ni} = k_n Z(k_n)\Delta k \int r' y(r') J_m(k_n r') J_m(k_i r') dr' \tag{9}$$

In the following results we refer to the device presented in Gulden et al. (7) which operates around $\lambda = 0.76\mu m$; the resonator is a $Al_{0.3}Ga_{0.7}As$ λ cavity, the active region contains 3 $Al_{0.12}Ga_{0.88}As$ QW and the mirrors are composed of 30 pairs of $\lambda/4$ $Al_{0.3}Ga_{0.7}As$ -AlAs layers.

In our model the mirrors are supposed to be infinitely extended in the transverse direction; the confinement is therefore due to the carrier injection into the active region. At increasing r the carrier density becomes smaller than the transparency value and $Y(r)$ represents an absorption.

In the numerical results we consider two simple cases in which the integral in (9) can be analytically solved: the gaussian and step profiles of $y(r)$.

In Fig. 2 we report square modulus and phase of the electric field in $z=0$ for the first two modes when $y(r)$ is a gaussian of width a=10 μm; the first lasing mode ($G = 3.42\ 10^{13}$ 1/s) corresponds to a circularly symmetric field distribution (m=0) while the other refers to m=1 ($G = 7.33\ 10^{13}$ 1/s). It is worth noting that the phase strongly varies with the radial coordinate; it's a typical effect of gain guiding that strongly influences the noise properties.

In Fig. 3 we compare the field distributions for the two considered profiles (both widths a=10 μm) for m=0; in the step distribution the sudden gain variation at the injected region boundary results in a magnetic field discontinuity and it provides a better electric field confinement.

Fig. 4a shows some electric field distributions corresponding to gaussian gain profiles for different values of a. Fig. 4b presents the modulus of the eigenvectors of eq. (8) versus radial wavevector. One can observe that bigger values of a result in better confined field distributions and that significant values of eigenvectors appear only for small k values.

VCSEL LINEWIDTH

Following the technique developed by Henry (3) and Duan et al (4), the evaluation of the laser linewidth requires the solution of the inhomogeneous eq (3); expressed directly in terms of the electric field and in the whole resonator it is given by:

$$E_\omega = \frac{J\ u}{W(\omega, N)} \tag{10}$$

where $J = \frac{1}{2\pi}\int J_L(r',\varphi')\exp(jm\varphi')f(r')\frac{r'}{y(r')}dr'd\varphi'$, $u(\rho,z)$ is the normalized spatial field distribution for the considered mode, which is obtained from the eigenvector through the plane wave expansion and:

$$W = -\frac{\Lambda_0 - \Lambda}{\Lambda_0} \tag{11}$$

is equivalent to the Wronskian defined in Duan et al (4). In eq. (11) Λ_0 is the eigenvalue corresponding to the $f(r)$ under analysis; for $\Lambda = \Lambda_0$, as already stated, in eq (10) $W=0$ gives rise to a pole which corresponds to the lasing condition.

In the presence of noise W has small fluctuations around the operating point related to frequency (ω) and carrier (N) changes; this fact is included through the corresponding derivatives W_ω and W_N.

The noisy electric field is expressed as:

$$E(\rho,z,t) = B\beta(t)u(\rho,z) + c.c. \tag{12}$$

where the normalization constant B is chosen so that $|\beta|^2$ represents the number of photons inside the cavity. Following a procedure similar to Duan et al. (4) it turns out that $\beta(t)$ satisfies the equation:

$$\frac{d\beta}{dt} + j\frac{W_N}{W_\omega}(N - N_0)\beta = F(t) = \frac{j}{2\pi W_\omega B}\int J\exp(j(\omega - \omega_0)t)d\omega \tag{13}$$

being ω_0 and N_0 the values in the operating condition.

The correlation coefficient of $F(t)$ is given by:

$$R = \frac{C^2}{32\pi^3|W_\omega B|^2}\int\left|\sum_n A_n J_m(k_n r)\right|^2 2D(r)dr \tag{14}$$

where C is the magnetic field normalization constant and $2D(r)$ is the correlation coefficient of J_L, Henry (2). The correlation coefficient R include self-consistently the effects of the so-called transverse and longitudinal Peterman factors, Grangier and Poizat (6).

Since eq (13) has the same form as in Duan et al (4), the linewidth of the laser emission can be computed as:

$$\Delta f = \frac{R}{4\pi|\beta|^2}(1+\alpha_e^2) \qquad (15)$$

with the effective $\alpha_e = -\frac{\Re(W_N / W_\omega)}{\Im(W_N / W_\omega)}$.

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REFERENCES

[1] Several papers in: IEEE J. Selected topics in quantum electronics, Special issue on semiconductor laser, 1, 1995

[2] K. Petermann: `Calculated spontaneous emission factor for double-heterostructure injection lasers with gain-induced waveguiding', IEEE J.QE-15, pp. 566-570, 1979

[3] C.H. Henry: `Theory of spontaneous emission noise in open resonators and its application to lasers and optical amplifiers', Journal of Lightwave Tech. 4, pp. 288-297, 1986

[4] G. Duan, P. Gallion, G. Debarge: `Analysis of the phase-amplitude coupling factor and spectral linewidth of distributed feedback and compose-cavity semiconductor lasers', IEEE J.QE-26, pp. 32-44, 1990

[5] A.E. Siegman: `Excess spontaneous emission in non-hermitian optical systems. II Laser oscillators', Phys. Rev. A, 39, pp. 1264-1268, 1989

[6] P. Grangier, J.P. Poizat: `A simple quantum picture for the Petermann excess noise factor', European Phys. Journal D, 1, pp. 1-8, 1998

[7] K.H. Gulden, M. Moser, S. Lusher and H.P. Schweizer: `High performance near-IR (765nm) AlAs/AlGaAs vertical cavity surface emitting lasers', Electronics Letters, 31, pp. 2176-2178, 1995

[8] P. Debernardi, A. Pisoni, G. P. Bava: `Quantum Well nonlinear optical response including valence band mixing and coulomb effects', IEEE Journal of Quantum electronics, 30, pp.93-107, 1994

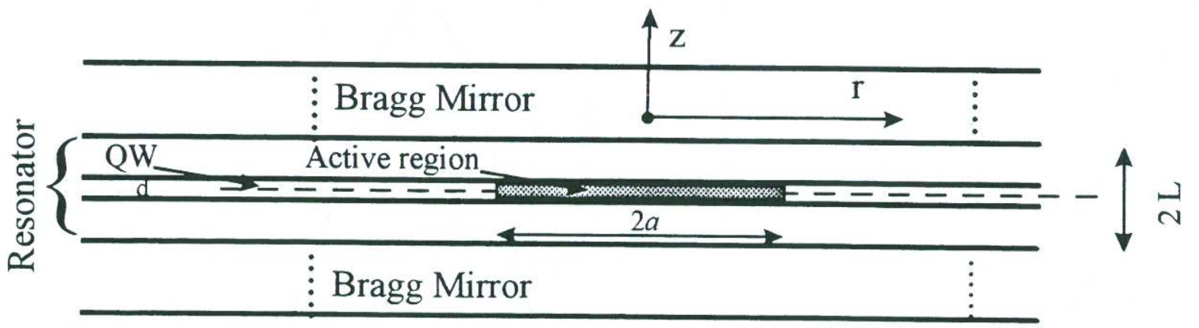

Figure 1: *Schematic representation of a VCSEL with the definition of the reference system.*

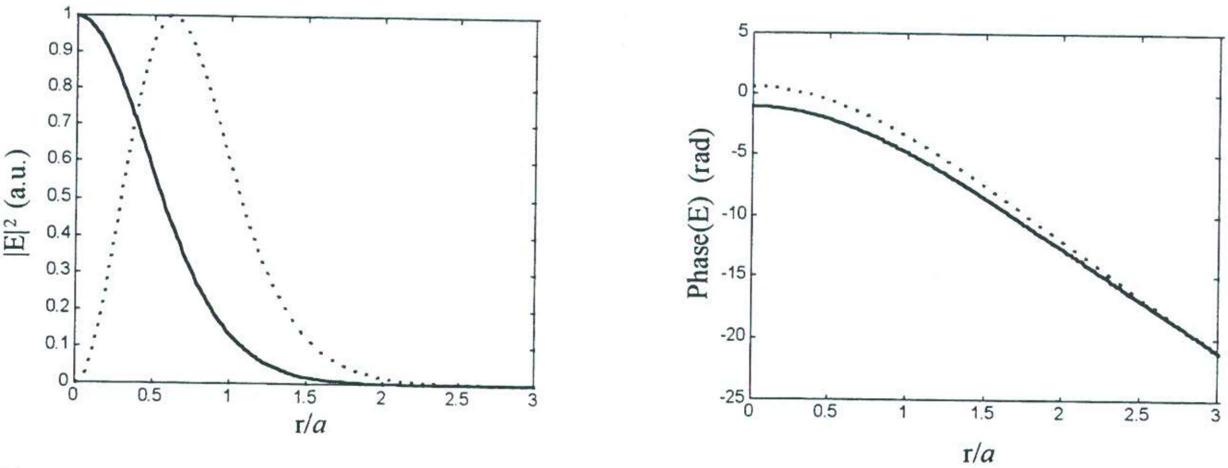

Figure 2: *Squared modulus of the electric field (left) and phase (right) for the first two modes; gaussian y(x) (a= 10 μm). Abscissa normalized to a.*

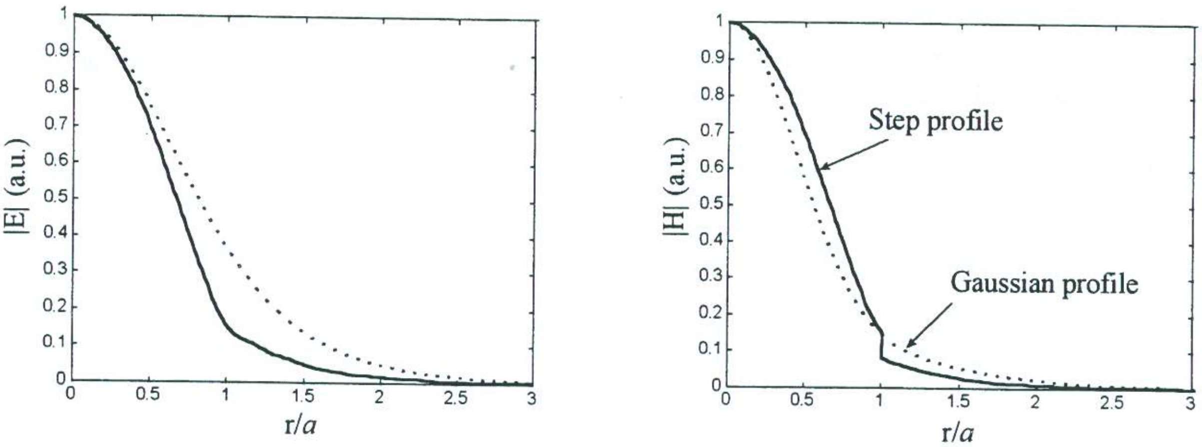

Figure 3: *Modulus of the electric (left) and magnetic field (right) for step and gaussian profiles of y(r) with a=* 10 *μm. Abscissa normalized as in Fig. 2*

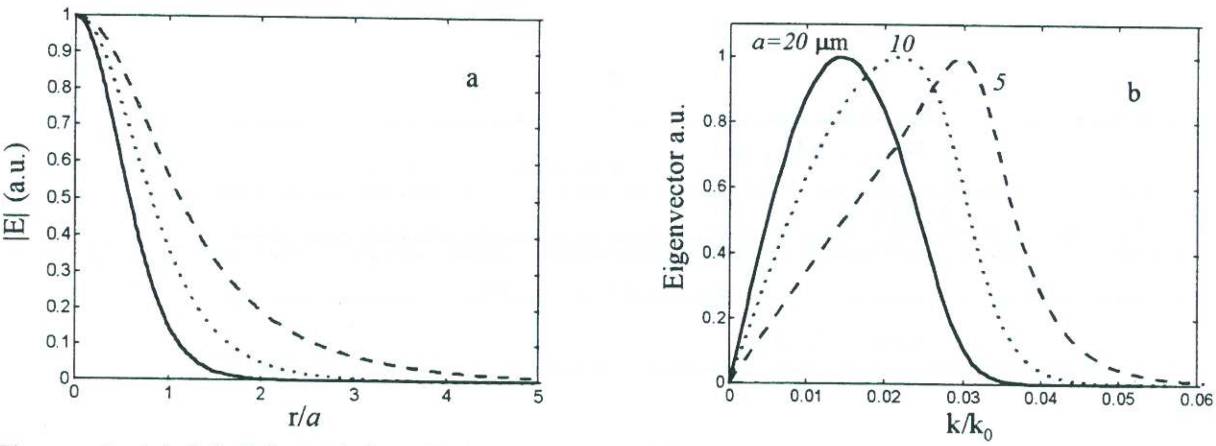

Figure 4: **(a)** *Modulus of the electric field for gaussian y(r) with 3 different widths. Abscissa normalized on the different widths.* **(b)** *Modulus of the corresponding eigenvectors vs k / k_0 (k_0 is the propagation constant inside the cavity).*