A SIGNAL-THEORY BASED APPROACH FOR THE APPROXIMATION OF ELECTRON DEVICE CHARACTERISTICS

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ABSTRACT

A new approach to the approximation of nonlinear characteristics for look-up table based electron device models is proposed. It relys on the strong analogy between signal sampling and characteristic measurements over a discrete grid of bias points. On this basis, nonlinear function approximation can be carried out by using approaches similar to well-known techniques for reconstruction of continuous-time signals from a given sequence of samples. With respect to conventional interpolation techniques, the method proposed guarantees the accurate reproduction both of nonlinear characteristics and associated derivatives without non physical slopes or oscillating behaviour. Preliminary simulated and experimental results confirming the effectiveness of this approach are presented.

INTRODUCTION

One major problem associated with electron device (ED) modelling based on conventional nonlinear equivalent circuits is the technology dependence of such an approach. This happens since both the model definition (i.e., circuit topology) and the approximation of measured ED characteristics (i.e., nonlinear analytical expressions) are normally dealt with on the basis of specific hypotheses on the device structure and physics of operation, so that different models are needed for different types of devices.

In the last years, several general-purpose, "measurement-based" approaches to nonlinear device modelling have been proposed [1..10] with the aim of overcoming the above mentioned problems thus providing a technology-independent, predictive link between conventional measurements (i.e., DC characteristics plus bias- and frequency-dependent small-signal S-parameters) and large-signal performance prediction using nonlinear circuit simulators. Most of these models use look-up tables to store sets of characterisation measurements, or related data, as a function of device terminal voltages and operating frequencies. Obviously, since measurements are necessarily carried out only over a discrete grid of voltages (and frequencies), for circuit analysis purposes suitable interpolating functions are needed which correctly represent the ED characteristics not only over the discrete set of points corresponding to measurements, but also for any voltage value within the given operating region.

In this paper, a new approach to nonlinear characteristic approximation will be presented which is based on the strong analogy between signal sampling and nonlinear characteristic measurement over a discrete grid of bias points. According to this criterion, nonlinear function approximation can be carried out by using approaches which are similar (although somehow device-oriented) to the well-known methods for reconstruction of continuous-time signals from a given sequence of samples.

Some preliminary experimental and simulated results will also be presented and discussed.

APPROXIMATION OF NONLINEAR DEVICE CHARACTERISTICS

Any electron device model for large-signal circuit analysis requires the description (i.e., approximation) of the ED nonlinear dynamics in terms of suitable algebraic functions (e.g., the I/V and Q/V, or C/V characteristics for an equivalent circuit model or other nonlinear functions for mathematical models [2,3,9,10]). These characteristics can be identified, through suitable parameter extraction procedures, on the basis of conventional DC and small-signal, bias-dependent AC measurements carried out over a discrete grid of voltages (and frequencies).

In conventional technology-dependent approaches, different special-purpose function are used for each type of ED (e.g., the well-known exponential characteristics for diodes or BJTs, or the Curtice equations for FETs). These functions can give, for each type of device, a reasonable overall agreement with measurements after suitable parameter "fitting", which is normally carried out by using nonlinear optimisation algorithms.

In the following, with the aim of deriving technology-independent models, only general-purpose approaches for function approximation will be considered. For simplicity, only the problem of approximating the DC I/V characteristics will be dealt with, since the same considerations can obviously be done also for the description of charge/voltage, capacitance/voltage or other nonlinear ED characteristics. Thus, we consider the problem of approximating the DC characteristic F of a single port ED through a "general-purpose" approximating function $\widetilde{F}$:

$$i = F(v) \simeq \widetilde{F}(v, \underline{P}) \tag{1}$$

which can be "tailored" on the specific ED (i.e., the measured DC characteristic) by giving suitable values to the set $\underline{P}$ of its characterising parameters. To achieve adequate accuracy and flexibility on different types of devices, a large set of parameters $\underline{P}$ is required. In such conditions, different problems arise since model extraction necessarily involves also large numbers of measurements for ED characterisation. However, great flexibility also gives the possibility of accurately modelling nonlinear effects which are only "local" and non necessarily typical (e.g., "kink" effects in GaAs FETs).

Large sets of measurements do not necessarily represent a big problem when computer-controlled automatic instrumentation is used, while large numbers of parameters may lead to computer-intensive procedures for optimisation-based parameter extraction. However, since model identification is carried out just once (off-line) per each device, a relatively large computing cost can be tolerated, provided that reliable parameter extraction algorithms can be defined. Instead, computational cost for evaluation of (1) must be carefully considered, since iterative procedures involved in nonlinear circuit analysis and optimisation often require many repeated computations of the model equations. In this perspective, computing cost for function evaluation should be very limited, even with large numbers of parameters. Thus, computational efficiency and robustness of parameter extraction procedures, which should be above-all reliable, unambiguous and tolerant to measurement uncertainties, are the main requirements for CAD-oriented general-purpose approximating functions.

In general-purpose approximating functions, a linear dependence of the function $\widetilde{F}$ on the parameters P_k is normally assumed. More precisely, eqn. (1) is normally expressed in a form like:

$$i = F(v) \simeq \widetilde{F}(v, \underline{P}) = \sum_k P_k h_k(v) \tag{2}$$

that is a sum of contributions which are linearly dependent on the parameters P_k, but with a nonlinear dependence on v deriving from a suitable set of *"base functions"* h_k (which implicitly define a given type of approximating function). This is a convenient choice since, although a larger number of parameters may be possibly required to achieve the same accuracy as in the case of nonlinear parameter dependence, yet it has the great advantage of enabling for closed-form, reliable and unambiguous parameter extraction from measurements. In fact, parameter extraction can be carried out by solving for the unknown P_k's a set of linear equations where the known terms are represented by measured data. With linear parameter extraction procedures based on linear regression algorithms, robustness (i.e., unambiguous solution) can be easily guaranteed if a suitable, large enough set of measurements is used, which normally leads to a well-conditioned matrix in the mean-square solution of an overdetermined system of linear equations.

Most general-purpose approaches for function approximation in ED modelling are just special cases of (2). For instance, eqn. (2) takes the form of a polynomial when the base functions are defined as $h_k(v) = v^k$ and, consequently, the parameters P_k represent the polynomial coefficients. In such a case, good behaviour can be found for moderate nonlinearities while, for strong nonlinearities, the need for high order polynomial descriptions implies high computational cost and risks of spurious, non-physical oscillations.

A computationally efficient approach, which is often used in "measurement-based" ED modelling [1..10] as it can better represent strong nonlinearities, involves the use of look-up tables (i.e., tables storing measured data) in conjunction with suitable, low-order interpolating functions. In this case the parameters P_k coincide with measured "samples" $F(v^{[k]})$ of the DC characteristic (i.e., $P_k = F(v^{[k]})$), while the base functions h_k represent the "unit pulse response" of the interpolating function (i.e., the response of the interpolator to a virtual set of P_k's which are all zero except for one having a unit value). For instance, if Piece-Wise Linear (PWL) interpolation is considered, each base function h_k has the triangular shape shown in Fig.1. Besides its intrinsic simplicity and computational efficiency, this approach has the advantage of preserving the monotonic behaviour. Moreover, the extension to N-dimensional cases (multiport device characteristics) is quite straightforward [11].

One drawback of this approach, common to all "exact" interpolators, is that noise-like measurement errors are exactly reproduced in the model. Moreover, PWL interpolation of look-up table data is not differentiable and this may lead to convergence problems in nonlinear analysis or poor prediction of intermodulation distortion. The latter problems can be overcome by using cubic spline interpolators which, besides being computationally efficient, are twice differentiable. A major problem with splines is that reproduction of monotonic behaviour is not guaranteed (see the oscillating behaviour of the base function of a cubic spline interpolator in Fig.3). In particular, measurement errors and/or non suitable choices of the sample spacing with strong nonlinearity can lead to non physical oscillating behaviour [6].

In order to overcome these problems, which are typical of polynomial-like exact interpolation, a different approach, based on signal sampling theory, has been developed. The basic aim is eliminating spurious oscillations and reducing noise-like measurement errors through band-limited approximating functions which are intrinsically "smooth" (no "jumps" or anomalous "ringings" both in functions and derivatives) and can be evaluated with good computational efficiency. This new approach derives from the strict analogy between the approximation of nonlinear characteristics measured over a discrete voltage "grid" and the reconstruction of time-domain waveforms from sampled data. In particular, according to this analogy where voltage-domain corresponds to a virtual time-domain, I/V measurements represent samples of the DC characteristic in the voltage space (analogously to the time-domain samples of a signal), while function reconstruction (i.e., interpolation of samples) can be carried out in the same way as in the case of sampled signals, that is by low-pass filtering the equivalent "sampled" function. More precisely, by considering for simplicity a uniform sampling (i.e., $v^{[k]} = k\Delta v$), the approximating function $\widetilde{F}(v)$ can be obtained through the linear convolution of the low-pass filter (LPF) pulse response h and the voltage domain samples of F:

$$\widetilde{F}(v) \doteq F_S(v) * h(v) = \Delta v \sum_k P_k h(v - k\Delta v) \tag{3}$$

$$\text{where} \quad F_S(v) \doteq \Delta v \sum_k P_k \delta(v - k\Delta v) \quad \text{and} \quad P_k = F(k\Delta v)$$

δ being the Dirac function. This shows that the base function h for nonlinear function approximation can be considered as the pulse response of an equivalent LPF. Thus, according to signal sampling theory, if the actual nonlinear characteristic $F(v)$ is (practically) band-limited with bandwidth B (in our analogy bandwidth actually corresponds to nonlinearity, so that practically limited bandwidth involves "smooth" nonlinearity), the DC characteristic can be "exactly reconstructed", that is

$$F(v) \equiv \widetilde{F}(v) = \Delta v \sum_k P_k h(v - k\Delta v) \quad \text{with} \quad P_k = F(k\Delta v) \tag{4}$$

provided that an adequate sampling step Δv is chosen (i.e., $B \leq \frac{1}{2\Delta v}$ according to the Nyquist criterion) and h coincides with the pulse response of an ideal LPF with cut-off frequency equal to the function bandwidth B.

The above approach, with the same exactness property, can be easily generalised to the N-dimensional case. For instance, in a two-dimensional case the DC current measurements $P_{k,r} = F(k\Delta v_1, r\Delta v_2)$ of a two-port device represent 2D samples[1], over a two-dimensional voltage grid. In such conditions, the 2D approximating function can be expressed in the form:

$$\widetilde{F}(v_1, v_2) = \Delta v_1 \Delta v_2 \sum_{k,r} P_{k,r} h_{2D}(v_1 - k\Delta v_1, v_2 - r\Delta v_2) \quad \text{with} \quad P_{k,r} = F(k\Delta v_1, r\Delta v_2) \tag{5}$$

[1] A perfect analogy can be found, for instance, with the 2D space-domain sampling in digital image processing.

where h_{2D} is the pulse response of a 2D-LPF used for function reconstruction after sampling, as in the one-dimensional case. The bandwidths B_1 and B_2 can still be defined, which represent limits on function nonlinearities (i.e., "smoothness" constraints) with respect to variables v_1 and v_2. Analogously to the 1D case, ideally "exact" reconstruction of the DC characteristic is still guaranteed when h_{2D} represents the pulse response of an ideal 2D-LPF with cut-off frequencies equal to B_1 and B_2, provided that sampling steps are adequately small (i.e., $B_1 \leq \frac{1}{2\Delta v_1}$ and $B_2 \leq \frac{1}{2\Delta v_2}$).
It should be noted that the pulse response of an ideal 2D-LPF (or, in practice, a good approximation of it), can be conveniently defined as the product of 1D pulse responses of two ideal (or well approximated) 1D-LPF's with given 1D bandwidths B_1 and B_2. Thus, the given nonlinear characteristic can be exactly reconstructed according to the expression:

$$F(v_1, v_2) \equiv \widetilde{F}(v_1, v_2) = \Delta v_1 \Delta v_2 \sum_{k,r} P_{k,r} h_1(v_1 - k\Delta v_1) h_1(v_2 - r\Delta v_2) \qquad (6)$$

$$\text{with} \quad P_{k,r} = F(k\Delta v_1, r\Delta v_2)$$

This shows an interesting property of the finite-bandwidth sampling approach to nonlinear function approximation: a suitable choice of the one-dimensional base function h (i.e., choice of suitable LPF pulse response or, equivalently, transfer function) leads to a general purpose N-dimensional interpolator.
Clearly, an ideal LPF cannot be used, but, for computing efficiency requirements in circuit analysis, a Finite Impulse Response (FIR) filter, with small normalised duration of pulse response, must be used. In fact, the number of terms to be considered in the summations in (6) becomes smaller for shorter FIR. Obviously, the FIR approximation introduces some errors (small with a reasonably good FIR LP-filter). In other words we do not obtain an exact interpolator but a function $\widetilde{F}$ approximating the DC characteristic. However, the presence of an LPF in data processing also involves some important advantages, like "smooth" function approximation and measurement "noise" reduction through "averaging" of measured samples.

CHOICE OF A SUITABLE FIR FILTER APPROXIMATION

According to the considerations above, a Finite Impulse Response filter must be used for function reconstruction after sampling. In particular, the choice of a suitable base function for nonlinear characteristic approximation could be based on well-known FIR filter synthesis approaches. However, in the FIR choice also some additional constraints, like the absence of spurious nonlinearities in function approximation (e.g., no spurious oscillations), must be taken into account. These important constraints, deriving from specific problems in nonlinear modelling, can be more precisely expressed in terms of linear exactness (i.e., linear $\widetilde{F}(v)$ with linear sequence of P_k's) and monotonical correctness (i.e., monotonic $\widetilde{F}(v)$ with monotonic sequence of P_k's). Moreover, also fast asymptotic exactness should be guaranteed (e.g., approximation error decreasing at least quadratically for $\Delta v \to 0$).
A possible way (often used in digital image processing) of defining a FIR approximation of an ideal LPF is based on a Gaussian transfer function, since its shape can adequately approximate the rectangular transfer function of an ideal LPF, while its correspondent pulse response, which has the nice property of having a still Gaussian, fast decaying shape, can be truncated with practically negligible errors. This leads to a FIR LP-filter with relatively small width, which can give reasonable results with suitable "oversampling" (i.e., $1/2\Delta v$ sufficiently larger than the Nyquist limit), but does not fully satisfy the above requirements on linear exactness. These constraints can be satisfied, instead, by simply considering an LPF transfer function defined as $H = sinc^n(f_v)$ where f_v is the Fourier transform variable analogous to the conventional frequency in the time/voltage analogy. The $sinc^n$ function corresponds to the cascade of n elementary filters, each having the same rectangular pulse response with unit area and width equal to Δv. In Fig.2, the $sinc^n$ transfer function is represented for different values of n.
It can be shown that the "linear exactness" and "monotonical correctness" properties of the $sinc^n$-based approximation are guaranteed for $n \geq 2$; moreover, also asymptotical exactness is guaranteed with quadratically decreasing error: $|\widetilde{F}(v) - F(v)| = O(\Delta v^2)$. In particular, for $n = 2$ the FIR response has a triangular shape, which corresponds to a linear interpolator; this is obviously linearly exact but

not differentiable. In order to have "smoother", differentiable approximating functions, larger values of n can be adopted (with $n \geq 2$, we have $(n-2)$ times differentiable interpolation). However, large values of n also involve larger computing effort in function evaluation, since the corresponding pulse response width is $n\Delta v$, while the quality of the $sinc^n$-based approximation of the LPF transfer function response is not comparable with the fast exponential decay of a single Gaussian filter. Thus, it seems preferable to combine the best qualities of both approaches, by adopting a "hybrid" filter consisting of the cascade of Gaussian and $sinc^n$ (with low n, $n \geq 2$) transfer functions. In particular, the simplest case of $n = 2$ leads to the FIR filtering characteristic shown in Fig.3, giving quite a good approximation of an ideal LPF and a relatively "short" pulse response, which can be easily expressed analytically as the convolution of triangular and Gaussian functions. This leads to a simple, yet quite efficient, "smooth" and non-oscillating interpolator, which can also be interpreted as the Gaussian filtering of a PWL interpolator and can be used also for non uniform measurement grids. In practice PWL interpolation gives linear exactness, no spurious oscillations and quadratically decreasing errors, while the highly effective "smoothing" of Gaussian filtering (besides preserving these features) provides both differentiability and measurement "noise" reduction. With exact (i.e., non-truncated) Gaussian filtering, the approximating function would be infinitely differentiable, while, in theory, only first-order differentiability is achieved after Gaussian-pulse-response truncation. However, reasonable truncation criteria easily lead to practically negligible discontinuities in higher-order derivatives.

PRELIMINARY EXPERIMENTAL RESULTS

Preliminary experimental validation was carried out by adopting the above described approach for the 2D Band-Limited (BL) approximation of the DC characteristics of a GaAs MESFET. The agreement between measurements and BL approximated DC characteristics (see Figs.4 to 6) is very good and the high regularity and accuracy of approximation is also confirmed by the plots in Figs.7, 8 where the first order derivatives of the parameters G_m and G_d w.r.t. the gate voltage are plotted both for a conventional cubic spline interpolator and the new BL approximator.
Finally, the accuracy improvement in HB circuit simulation using the Finite Memory nonlinear Model (FMM) proposed in [10] in conjunction with the new BL approximator of ED characteristics is evident in Fig.9, where three harmonic components of the output power for a large-signal GaAs MESFET amplifier under mild nonlinear operations at 5GHz are plotted. The agreement between measurements and performance predicted using the FMM with the new BL approximator is definitively better (in particular for the third-order harmonic) with respect to results obtained on the basis of PWL interpolation of ED characteristics.

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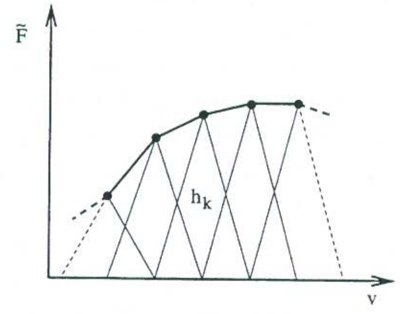


Figure 1: Approximating function $\widetilde{F}$ based on PWL interpolation. The base functions h_k have a triangular shape.

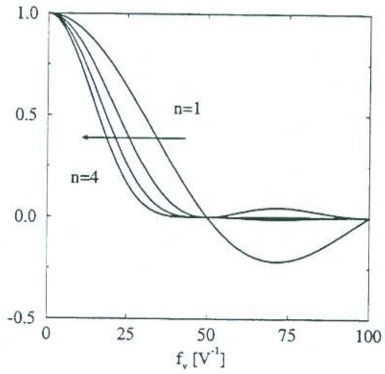


Figure 2: $sinc^n$ LPF transfer function for different values of n.

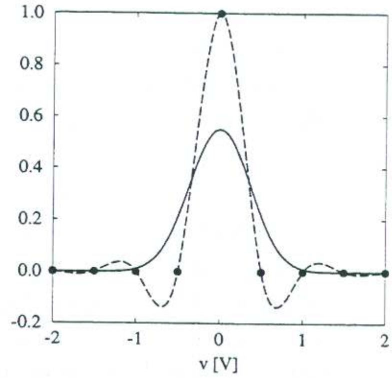
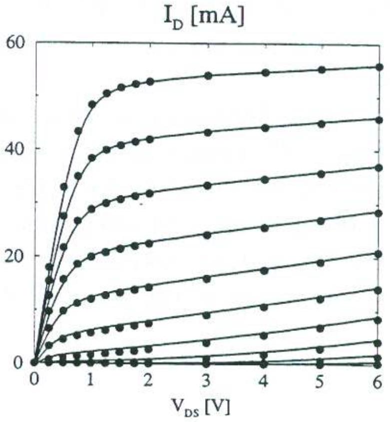
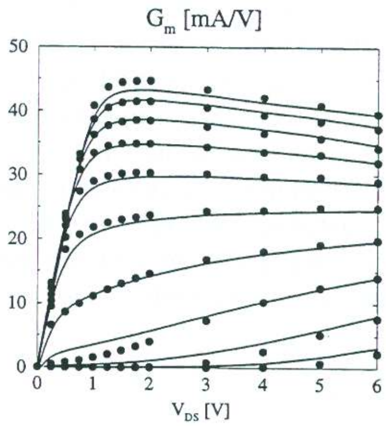
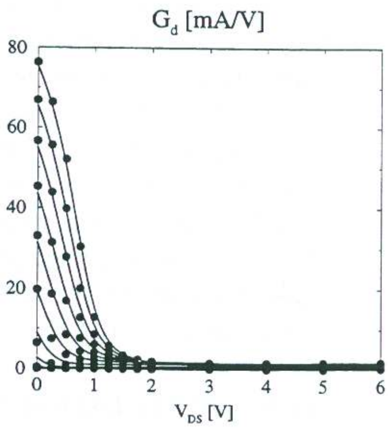


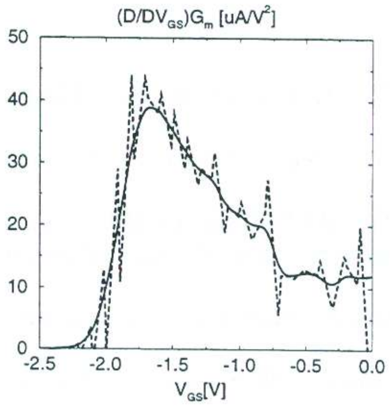
Figure 3: Pulse response of a Gaussian/triangular filter (—) compared with the oscillating base function of a cubic spline interpolator (- - -).

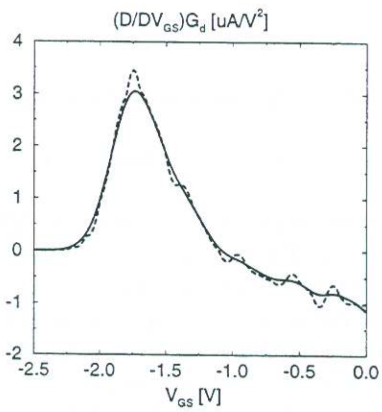






Figs. 4,5,6: 2D band-limited approximation (—) of measured (•) DC characteristics for a GaAs MESFET.





Figs. 7,8: G_m and G_d derivatives w.r.t. V_{GS}: 2D band-limited approximation (—) and conventional splines (- - -).

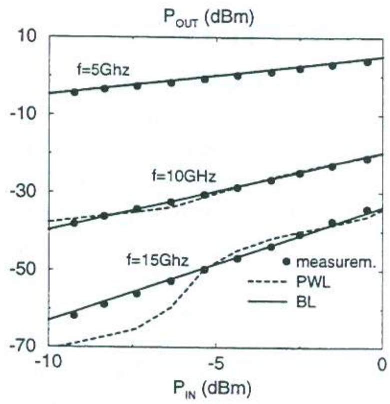


Fig.9: Performance prediction for a nonlinear amplifier using the model [10] in conjunction with the BL approximator and conventional PWL.