NOISE MODELLING OF DEVICES UNDER NONLINEAR OPERATION

François DANNEVILLE, Alain CAPPY

Institut d' Electronique et de Microelectronique du Nord
UMR CNRS 9929
Cite Scientifique
Avenue Poincare- BP 69
59652 Villeneuve d'Ascq Cédex - France

Abstract

This paper presents a general method for the calculation of the noise correlation matrix of devices under nonlinear operation. It is based on a perturbation analysis of the large-signal noiseless steady state and it constitutes a generalization of the impedance field method. This method is applied for the calculation of the noise correlation matrix of a HEMT Gate Mixer.

I. Theory

Let us consider a non linear device with several electrodes E_k (Fig.1).

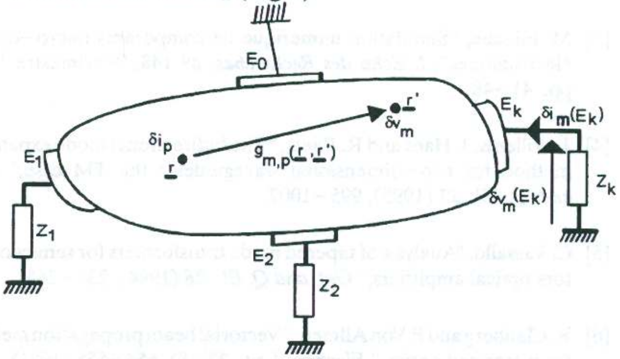

Fig.1 : Non Linear device with several electrodes.

Each electrode is connected to a linear circuit. The device is driven by a large signal of fundamental angular frequency ω_o. We suppose that the large signal steady state is known. Due to the large signal, the physical quantities such as the current density $j(\underline{r},t)$, the electrostatic potential $v(\underline{r},t)$ and the carrier density $n(\underline{r},t)$ are periodic functions of fundamental frequency ω_o at any point of the device. For small perturbations of the steady state, the device can be replaced by a linear multifrequency network described by its conversion matrix.

Let us now consider a noise source $\delta i_p(\underline{r})$ at angular frequency $\omega+p\omega_o$ located in a volume $d\Omega$ at a point $\underline{r}$ in the device. At any point $\underline{r}'$ of the device, $\delta i_p(\underline{r})$ produces a noise voltage $\delta v_m(\underline{r}')$ at frequency $\omega+m\,\omega_o$ given by:

$$\delta v_m(\underline{r}') = \sum_p g_{m,p}(\underline{r}',\underline{r}) i_p(\underline{r}) d\Omega \tag{1}$$

where $g_{m,p}(\underline{r},\underline{r}')$ is just a form of Green's function. Formula (1) can be applied at any point and especially at the device electrodes E_k. $g_{m,p}(\underline{r},\underline{r}')$ can be deduced from a perturbation of the large signal steady state as long as a physical description of the device is known. Assuming spatially uncorrelated noise sources, the cross spectrum $\langle\delta V_m(\underline{r}')\delta V_n^*(\underline{r}')\rangle$ can be deduced from (1) and integration over the entire device:

$$\left\langle \delta V_m(\underline{r}')\delta V^*{}_n(\underline{r}')\right\rangle = \sum_p \sum_s \int_\Omega g_{m,p}(\underline{r},\underline{r}')\left\langle \delta i_p(\underline{r})\delta i_s^*(\underline{r})\right\rangle g_{n,s}^*(\underline{r},\underline{r}')d\Omega \tag{2}$$

This expression is valid for any $\underline{r}'$ and so it can be applied for any electrode. Formula (2) can be considered as a generalized form of Shockley's impedance field method to devices under non linear operation [1]. At this step, a discussion on the microscopic noise source $\langle\delta i_p(\underline{r})\delta i^*_s(\underline{r})\rangle$ is necessary. The case of white noise sources has been only considered.

For the microscopic white noise sources (i.e. thermal noise, diffusion noise), a noise power can be generated at any frequency $\omega+p\omega_o$. The microscopic noise source is given by [2]:

$$\left\langle \delta i(\underline{r})\delta i^*(\underline{r}')\right\rangle = 4\,q^2 \Delta f\, D(\underline{r}) n(\underline{r})\, \delta(\underline{r}-\underline{r}') \tag{3}$$

where $D(\underline{r})$ is the diffusivity and $n(\underline{r})$ the carrier density. Under non linear operation the product $D(\underline{r}).n(\underline{r})$ is a periodic function of fundamental frequency ω_o. Assuming that the random function $\delta i(t)$ is simply modulated by the large signal, the cross spectrum $\langle\delta i_k(\underline{r})\delta i_l(\underline{r})\rangle$ is given by [3] :

$$\left\langle \delta i_k(\underline{r})\delta i_l^*(\underline{r}')\right\rangle = 4\,q^2\, \Delta f\, H_{k-l}\, \delta(\underline{r}-\underline{r}') \tag{4}$$

where H_{k-l} is the (k-l)th harmonic of the function $D(\underline{r},t).n(\underline{r},t)$. These coefficients are given by the large signal analysis.

II. Application to a HEMT Gate Mixer

The theoretical analysis described in the preceeding section was applied to FETs, in order to study a HEMT Gate Mixer (using a pseudomorphic HEMT, Lg=0.25 μm, W=120 μm). A large signal model described in [4] provides the expressions of the non-linearities for G_m, G_d and C_{gs}. Ref. [4] describes also the behavior of the mixer.

To avoid a microscopic two dimensional modeling, the device is considered as a noisy nonlinear active line [5] (Fig. 2).

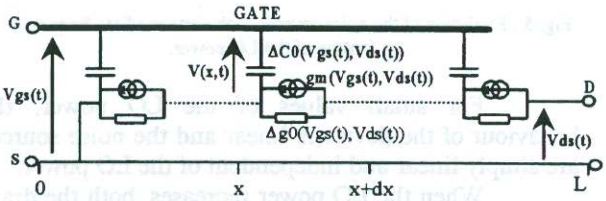

Fig.2 : FET nonlinear model using the uniform active line concept

(i) Conversion matrix calculation

We assume that the different elements of the line, g_m (the transconductance), Δg_0 (the local channel conductance) and Δc_0 (the local gate-to-channel capacitance) are functions of the time dependent gate-to-source $V_{gs}(t)$ and drain-to-source voltage $V_{ds}(t)$. The expressions of Δg_0 and Δc_0 are related to the expressions of $G_m(V_{gs},V_{ds})$, $G_d(V_{gs},V_{ds})$, and $C_{gs}(V_{gs},V_{ds})$ [5]. A large signal signal steady state $V_{gs}(t)$ and $V_{ds}(t)$ is imposed to the line. The nonlinear differential equation for V(x,t), the gate-to-channel voltage, is given by:

$$g_0(t)L\frac{d^2V(x,t)}{dx^2} - g_m(t)\frac{dV(x,t)}{dx} - \frac{1}{L}\frac{d(c_0(t).V(x,t))}{dt} = 0 \qquad (5)$$

In this equation, L is the length of the line, $g_0.L$ is the line channel conductance, c_0/L the line gate-to-channel capacitance.

Since the elements g_m, g_0, c_0 are periodic functions of the fundamental angular frequency ω_0 (the Local Oscillator frequency), they can be written as:

$$g_m(t) = \sum_k g_{m_k} e^{jk\omega_0 t} \qquad (6)$$

$$g_0(t) = \sum_k g_{0_k} e^{jk\omega_0 t} \qquad (7)$$

$$c_0(t) = \sum_k c_{0_k} e^{jk\omega_0 t} \qquad (8)$$

If we now consider a noisy line, the noise signals are supposed to be small with respect to the steady state; consequently, the noise analysis is nothing but a perturbation analysis of the large signal noiseless steady state. If equation (5) is linearized, we obtain a differential equation for the voltage fluctuations:

$$g_0(t)L\frac{d^2(\delta V(x,t))}{dx^2} - g_m(t)\frac{d(\delta V(x,t))}{dx} - \frac{1}{L}\frac{d(c_0(t).\delta V(x,t))}{dt} = 0 \qquad (9)$$

Due to the mixing, the voltage fluctation $\delta V(x,t)$ along the line can be written:

$$\delta V(x,t) = \sum_n \delta V_n(x)\, e^{j(n\omega_0+\omega)t} \qquad (10)$$

with ω corresponding to the intermediate frequency of the mixer.

Introducing (6-8) and (10) into (9), a differential equation for each harmonic of the voltage fluctuations $\delta V(x,t)$ can be readily obtained:

$$\sum_k g_{0_{n-k}} L^2 \frac{d^2(\delta V_k(x))}{dx^2} - g_{m_{n-k}} L \frac{d(\delta V_k(x))}{dx} - j(n\omega_0+\omega)c_{0_{n-k}}(\delta V_k(x)) = 0 \qquad (11)$$

Equation (11) can be solved numerically for the calculation of the frequency convertion relation between $\delta V_{n,i}$ (the n^{th} Fourier component of the voltage fluctuations at position i) and the k^{th} fourier component of the voltages fluctuations applied at the input and/or output of the line $\delta V_{k,0}$ and $\delta V_{k,N}$:

$$\delta V_{n,i} = \sum_k A_{n,k}\, \delta V_{k,0} + \sum_k B_{n,k}\, \delta V_{k,N} \qquad (12)$$

From (12), the frequency conversion matrix for the voltage fluctuations along the line can be easily defined. This matrix allows to calculate the frequency conversion matrix in any representation of the device (admittance conversion matrix C_Y and/or impedance conversion matrix C_Z, for an abcissa comprised between 0 and x), which is necessary to determine the noise correlation matrix of the device.

(ii) Noise Correlation Matrix calculation

The theory developped in Section I was applied to calculate the noise correlation matrix. The problem encountered is to calculate the influence of a microscopic nonlinear noise source located between x and x+dx at the gate and drain electrodes for given gate-to-source and drain-to-source terminations. A voltage-current representation with an open circuit between gate and source and a short circuit between drain and source has been chosen (Fig. 3).

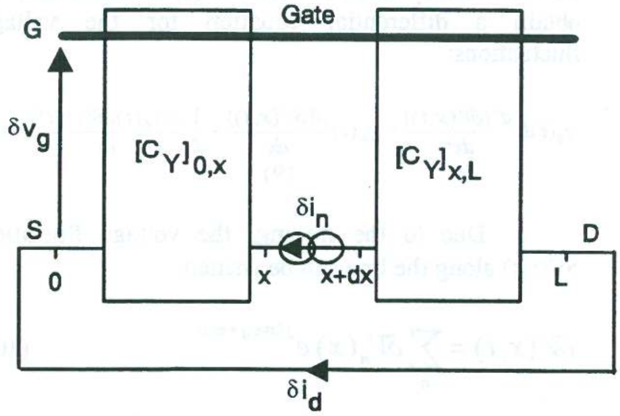

Fig.3 : Circuit used for the noise calculation.

After calculation, the FET mixer can be replaced by a linear multifrequency network described by its noise correlation matrix. By simply adding the noisy linear circuit for each frequency of interest (Fig. 4), the calculation of the noise figure for the mixer can be easily performed.

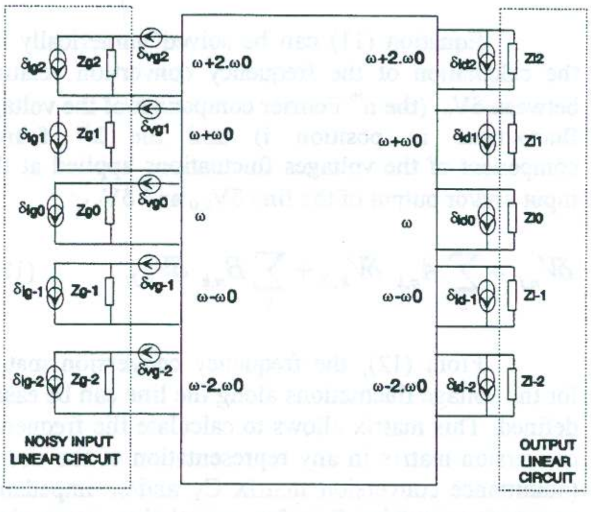

Fig. 4 : Linear multifrequency network associated with its correlation matrix.

$\omega/(2\pi)$: Intermediate Frequency (IF = 4 GHz)

$\omega_0/(2\pi)$: Local Oscillator Frequency (LOF = 24.5 GHz)

$(\omega+\omega_0)/(2\pi)$: Radio Frequency (RF = 28.5 GHz)

(iii) Results

The variations of the noise sources at the intermediate frequency is shown in Fig.5 as a function of the LO power.

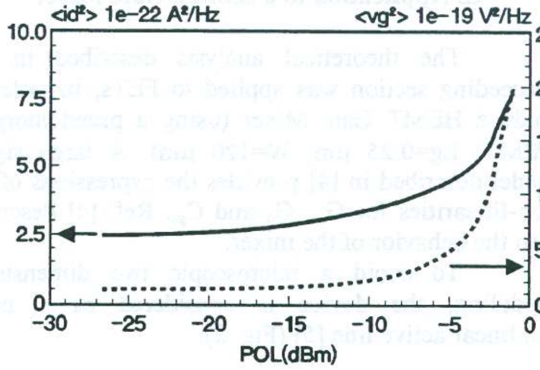

Fig. 5 : Evolution of the noise sources at the intermediate frequency as a funtion of the LO power.

For small values of the LO power, the behaviour of the device is linear and the noise sources are simply linear and independent of the LO power.

When the LO power increases, both the drain noise current and the gate noise voltage increase; the frequency conversion mechanism occurs in this case.

The evolution of both the noise figure and the conversion gain as a function of the LO power is shown in Fig. 6.

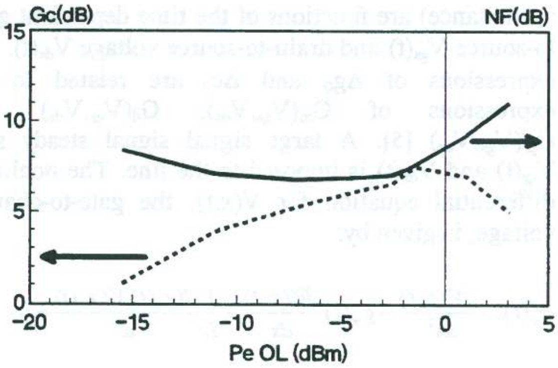

Fig. 6 : Evolution of the noise figure and the conversion gain as a function of the LO power.

The device is biased near pinch-off. For small levels of the LO power, the noise temperature is close to the room temperature and the Noise Figure decreases to reach a minimum. For higher values of the LO power, both the noise temperature and the frequency conversion mechanism occurs and the noise figure increases. Note that the minimum of the noise figure is not found for the maximum of the conversion gain; this result is in accordance with experimental results [6].

Finally, the variation of the Noise Figure is shown in Fig.7 as a function of the average value of the drain current (the DC drain current).

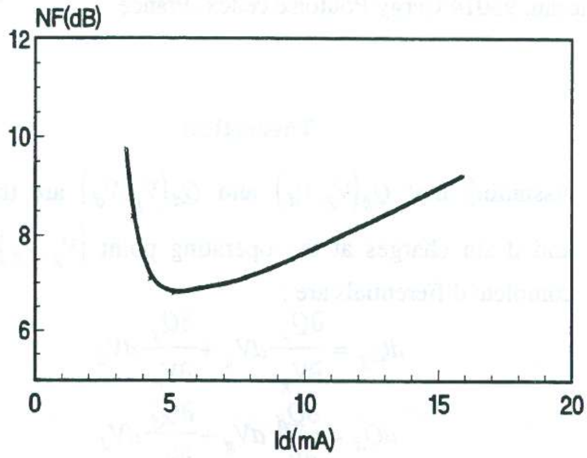

Fig. 7 : Evolution of the Noise Figure as a function of the average value of the drain current.

It is clear that the variation of the Noise Figure is similar as those observed in the linear case.

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Conclusion

A generalization of the impedance field method is proposed for calculating the noise correlation matrix of devices under non linear operation. This method is general and can be applied to any device. It has been used in the case of a HEMT Gate Mixer and the results are in good agreement with experimental data.

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