A NEW ANALYTICAL, CAD-ORIENTED MODEL FOR THE OHMIC AND RADIATION LOSSES OF ASYMMETRIC COPLANAR LINES IN HYBRID AND MONOLITHIC MIC'S

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ABSTRACT

The paper presents analytical approximations for the conductor, dielectric and radiation losses of asymmetric coplanar waveguides. The expression for the conductor losses is obtained through an extension of Owyang and Wu's conformal mapping approach [17]; the dielectric losses are estimated according to the so-called plane-wave approach [2]; finally, the analysis of radiation losses follows Rutledge's approach [19]. Comparisons with experimental results or numerical data derived from quasi-static or full-wave approaches are performed to validate the expressions presented.

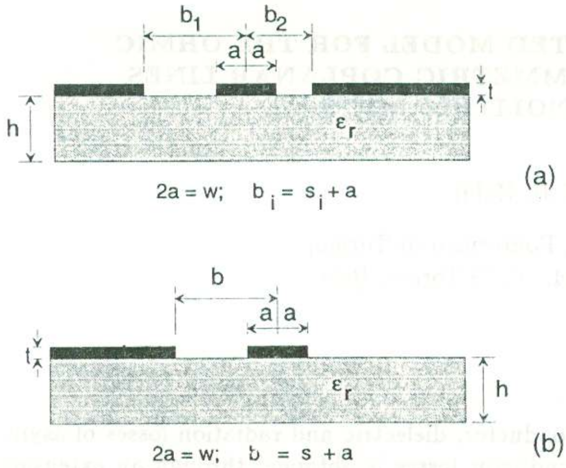
Keywords: Coplanar Lines, Ohmic Losses, Radiation Losses

1 INTRODUCTION

The quasi-TEM parameters of coplanar lines for hybrid or monolithic microwave integrated circuits (MMIC's) have been the object of extensive investigations; for a review of the available analytical approximations see *e.g.* [12, Ch.13] and references therein. Concerning *line losses*, the conductor and dielectric attenuation have been analyzed by means of quasi-static [10, 15] or full-wave [14] numerical methods. Analytical approximations have been proposed in the past for the conductor losses of *symmetric* coplanar lines in 1958 by Owyang and Wu [17] and later by Garg and Bahl [9]. Owyang and Wu's expression for the conductor attenuation is also reported, with some corrections, in [5] and in Hoffmann's reference book on MIC's [12]. No extension to asymmetric lines has been developed so far to the authors' knowledge. Radiation losses in coplanar resonators have been analyzed numerically in [5] and in [11, 10] starting from an approximation of the radiating electric or magnetic equivalent currents. In 1983 Rutledge *et al.* performed extensive analytical investigations on the radiation attenuation due to coupling with free-space radiation and TM or TE surface waves, and suggested analytical approximations to these parameters, again for the symmetric case. Rutlegde's approach was later extended in [4] and validated, at least for lines on thick substrates, by means of experimental comparisons.

In the present paper, CAD-oriented, analytical approximations are proposed for the attenuation of asymmetric coplanar lines. For the sake of brevity, only the Asymmetric Coplanar Waveguide (ACPW, see Fig.1(a) will be discussed in detail; a particular case of the ACPW is the asymmetric coplanar waveguide with a single lateral ground plane (ACPW_1, see Fig.1(b)). The expressions for the conductor attenuation are derived through a conformal mapping technique similar to Owyang and Wu's. Dielectric losses are approximated through the so-called plane-wave or Welch and Pratt's approach (see *e.g.* [12]), whereby the dielectric attenuation is a function of the effective permittivity of the line. For the radiation losses, the extension of the analytical approximations proposed by Rutledge to the asymmetric case is straightforward. An extended treatment of the conductor and dielectric losses of coplanar waveguides and striplines, where the underlying theory is fully developed, can be found in the forthcoming paper [8].

The conductor, dielectric and radiation attenuation of dielectric-supported coplanar lines depend on the *effective permittivity* ϵ_{eff} of the line, defined as $\epsilon_{\text{eff}} = (c_0/v_f)^2$, where v_f is the phase velocity on the line, c_0 the velocity of light *in vacuo*. An expression of this parameter as a function of the line geometry is therefore needed in the analysis of the ACPW losses. The characteristic impedance Z_c of coplanar lines supported by finite-thickness dielectric layers cannot be expressed in closed form; however, suitable approximations to Z_c have been proposed in the past through the so-called *partial capacitance* approach [6, 12], which yields an accurate approximation when the substrate thickness h is not smaller than the overall lateral ground plane spacing b_1+b_2. The partial capacitance approach can be implemented exactly in terms of complete elliptic integrals for all coplanar lines but for the ACPW with finite ground plane spacing. In this last case, an exact implementation requires the numerical evaluation of hyperelliptic



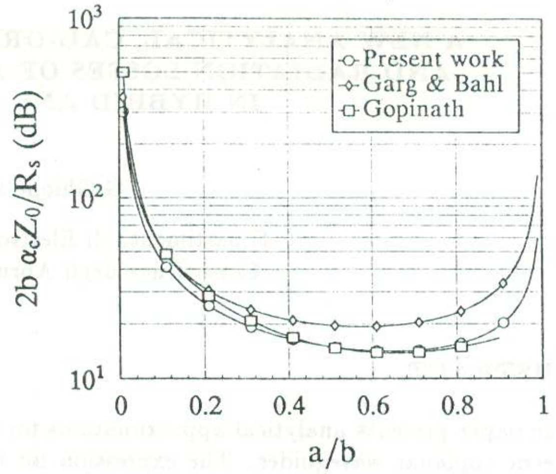


Figure 1: (a) Asymmetric Coplanar Waveguide (ACPW): $b_i = s_i + w/2$, $w = 2a$; (c) Asymmetric Coplanar Waveguide with one lateral ground plane ($ACPW_1$): $b = s + w/2$, $w = 2a$.

Figure 2: Normalized attenuation for CPW versus a/b: comparison between Eq.(11, Gopinath's approach [10] and the analytical approximation of [9] ($t = 5$ μm, $b = 600$ μm).

integrals [16]. Fouad Hanna and Thebault proposed in 1984 [3] an approximate implementation of the partial capacitance approach which, in most practical cases, is accurate enough, and will therefore be adopted in the paper. A more thorough discussion of the error introduced by this expression can be found in [8].

The paper is structured as follows. The line parameters, conductor and dielectric attenuation of the ACPW are discussed in Sec.2; Sec.2.1 is devoted to a review of the symmetric case, while Sec.2.2 deals with the line parameters of the coplanar waveguide with a single ground plane. Finally, Sec.3 presents a discussion of the radiation attenuation. Comparisons are presented with numerical results derived from other approaches, or experimental data. These suggest that, while the expressions for the conductor and dielectric losses are satisfactorily validated, radiation losses still pose problems, apart from limiting case in which the radiation mechanism is simple, like in lines with very thick substrates [4].

2 THE OHMIC LOSSES OF ASYMMETRIC COPLANAR WAVEGUIDES

The quasi-TEM characteristic impedance of the ACPW reads [8]:

$$Z_c^{\mathrm{ACPW}} = \frac{60\pi}{\sqrt{\epsilon_{\mathrm{eff}}^{\mathrm{ACPW}}}} \frac{K'(k)}{K(k)} \tag{1}$$

where $\epsilon_{\mathrm{eff}}^{\mathrm{ACPW}}$ is the effective permittivity of the line, K is the complete elliptic integral of the first kind, and:

$$k = \sqrt{\frac{2a(b_1+b_2)}{(b_1+a)(b_2+a)}} \tag{2}$$

while $K'(k) = K(k') = K(\sqrt{1-k^2})$. The effective permittivity $\epsilon_{\mathrm{eff}}^{\mathrm{ACPW}}$ can be approximated as [3]:

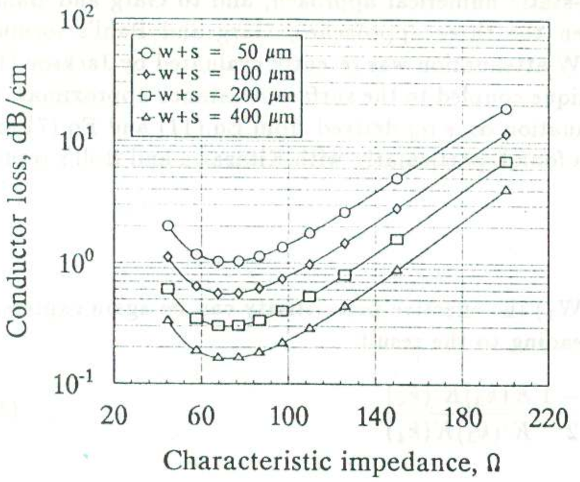
$$\epsilon_{\mathrm{eff}}^{\mathrm{ACPW}} \approx 1 + \frac{\epsilon_r - 1}{2} \frac{K'(k_1)K(k)}{K(k_1)K'(k)} \tag{3}$$

where:

$$k_1 = \sqrt{\frac{2\sinh(\pi a/2h)[\sinh(\pi b_2/2h) + \sinh(\pi b_1/2h)]}{[\sinh(\pi a/2h) + \sinh(\pi b_1/2h)][\sinh(\pi a/2h) + \sinh(\pi b_2/2h)]}}. \tag{4}$$

The attenuation due to conductor losses can be shown to take the following expression, in natural units (Np/m):

$$\alpha_c^{\mathrm{ACPW}} = \frac{R_s\sqrt{\epsilon_{\mathrm{eff}}^{\mathrm{ACPW}}}}{480\pi K(k)K'(k)} \left[\Phi(b_1 - a, k) + \Phi(b_2 - a, k) + \Phi(2a, k') - \Phi(b_1 + b_2, k')\right] \tag{5}$$



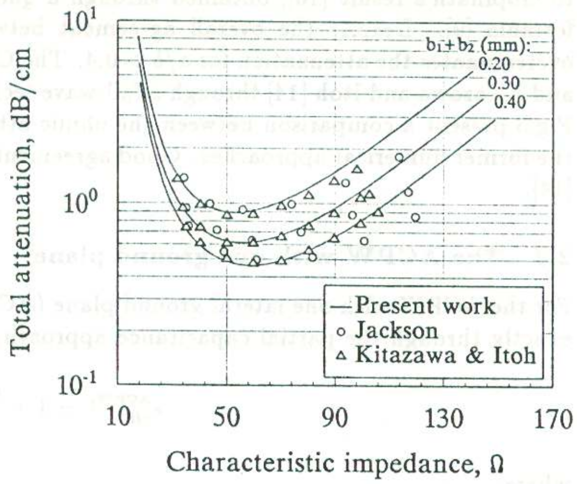


Figure 3: *Total loss of CPW on GaAs (ϵ_r=12.8, h = 100 μm, $\tan\delta$ = 0.0006) versus Z_c for several values of 2b, according to [13], [14] and Eq.(11). The operating frequency is 60 GHz and t = 3 μm, $R_s = 8.24 \times 10^{-3}\sqrt{f_{\rm GHz}}$ Ω (Cu, [12]).*

Figure 4: *Conductor loss of $ACPW_1$ on semi-infinite GaAs substrate (ϵ_r=12.8) as a function of the characteristic impedance, for several values of the parameter $w+s$ (spacing from strip edge to ground plane). The frequency is 20 GHz, and a 5 μm thick copper metallization was considered.*

where R_s is the surface resistance, and:

$$\Phi(\psi,\kappa) = \frac{1}{\psi}\left[\log\left(\frac{4\pi\psi\kappa}{t}\right) + \pi\right] \tag{6}$$

where t is the metallization thickness. The validity of Eq.(5) is restricted to the case in which t is suitably smaller than w and s_1, s_2 but larger than the skin penetration depth. Finally, the attenuation due to dielectric losses, α_d, can be expressed as (see *e.g.* [12]):

$$\alpha_d^{\rm ACPW} = \tan\delta\,\frac{\pi f}{c_0}\,\frac{1-1/\epsilon_{\rm eff}^{\rm ACPW}}{1-1/\epsilon_r}\sqrt{\epsilon_{\rm eff}^{\rm ACPW}} \tag{7}$$

where $\tan\delta$ is dielectric loss factor of the substrate and the attenuation is expressed in Np/m.

2.1 The symmetric case

In the symmetric coplanar waveguide (CPW) one has $b_1 = b_2 = b$. By applying transformation rules for elliptic inegrals one obtains for the characteristic impedance the customary Wen's expression [20]:

$$Z_c^{\rm CPW} = \frac{30\pi}{\sqrt{\epsilon_{\rm eff}^{\rm CPW}}}\frac{K'(k_s)}{K(k_s)}, \tag{8}$$

where $k_s = a/b$. The effective permittivity reads, according to the partial capacitance approximation [6, 12]:

$$\epsilon_{\rm eff}^{\rm CPW} = 1 + \frac{\epsilon_r - 1}{2}\frac{K'(k_s)}{K(k_s)}\frac{K(k_2)}{K'(k_2)} \tag{9}$$

where:

$$k_2 = \frac{\sinh(\pi a/2h)}{\sinh(\pi b/2h)}. \tag{10}$$

In the symmetric case the general expression for the conductor attenuation can be shown to reduce to Owyang and Wu's formula [17, 12, 6]:

$$\alpha_c^{\rm CPW} = \frac{R_s\sqrt{\epsilon_{\rm eff}^{\rm CPW}}}{480\pi K(k_s)K'(k_s)(1-k_s^2)}\left\{\frac{1}{a}\left[\pi + \log\left(\frac{8\pi a(1-k_s)}{t(1+k_s)}\right)\right] + \frac{1}{b}\left[\pi + \log\left(\frac{8\pi b(1-k_s)}{t(1+k_s)}\right)\right]\right\}. \tag{11}$$

For the dielectric attenuation the general expression holds with the appropriate effective permittivity.

In Fig.2 the normalized attenuation derived from Owyang and Wu's approach (Eq.(11)) is compared to Gopinath's result [10], obtained through a quasi-static numerical approach, and to Garg and Bahl's formula [9]. Despite the overall agreement between the three approaches, Garg and Bahl's formula overestimates the attenuation for $a/b > 0.4$. The CPW attenuation was recently evaluated by Jackson [13] and Kitazawa and Itoh [14] through a full-wave technique coupled to the surface resistance approximation. Fig.3 present a comparison between the ohmic attenuation $\alpha_c + \alpha_d$ derived from Eq.(11) and Eq.(7) and the former numerical approaches. Good agreement is found, particularly with Kitazawa and Itoh's results [14].

2.2 The ACPW with one ground plane

For the ACPW with one lateral ground plane (ACPW_1) the effective permittivity can be again expressed exactly through the partial capacitance approach, leading to the result:

$$\epsilon_{\mathrm{eff}}^{\mathrm{ACPW}_1} = 1 + \frac{\epsilon_r - 1}{2} \frac{K(k_3)K'(k_4)}{K'(k_3)K(k_4)} \tag{12}$$

where:

$$k_3 = \sqrt{\frac{2a}{b+a}} \tag{13}$$

$$k_4 = \sqrt{\frac{\exp(2\pi a/h) - 1}{\exp[\pi(b+a)/h] - 1}}. \tag{14}$$

This expression is equivalent to the one in [12], Sec.13.4. For the characteristic impedance and line attenuation the general expressions for the ACPW apply in the limit $b_1 = b$ and $b_2 \to \infty$.

The behaviour of the conductor losses of $\epsilon_{\mathrm{eff}}^{\mathrm{ACPW}_1}$ is shown in Fig.4. Since the ACPW_1 has a higher impedance than the ACPW, the impedance value leading to the minimum conductor loss is near 60—70 Ω. Conversely, the 50 Ω line is close to a minimum-loss geometry for the symmetric CPW on a GaAs substrate.

3 RADIATION LOSSES

Radiation in uniform lines occurs because coupling occurs with surface waves or free-space radiation. In the ACPW the dielectric substrate, topped by the lateral ground planes, supports TM and TE surface waves. The fundamental mode is the TM_0 while the TE_0 mode is under cut-off in typical MMIC's (substrate thickness and operating frequency). A discussion of parasitic coupling with surface waves can be found *e.g.* in [12, Chap. 15] and in [19]; sinchronous coupling is not needed for energy loss, since this can take place also obliquely with respect to the line axis. Several techniques are available to analyze the radiation losses from finite-extent structures (*e.g.*, resonators), according to approaches which are well known in the field of planar antenna theory. In the exact, self-consistent approach the current distribution is the numerical solution of an integral equation and radiation is evaluated through a Green's function technique. Suitable assumptions can be made on the strip current distribution or slot electric field of resonators, thereby enabling to derive the radiated power by separately evaluating the free space and surface wave contribution through the spectral-domain Green's function approach [5]. A simplified version of this approach, based on the assumption of transversally constant current or field, can be found in [11, 10]. Finally, Rutledge *et al.* [19] directly evaluated the coupling to free-space radiation for a line supported by a thick dielectric substrate by first computing the power radiated by a magnetic or electric line current and then integrating the radiating source on the line cross-section. The same technique is applied to evaluating the power coupled to TM or TE surface waves. Rutledge's approach was given some improvement in [4] where coplanar lines on thick substrates were considered in the THz range, and comparison were presented between measurements and computed free-space radiation attenuation. A similar approach is applied to surface wave losses of coplanar striplines in [18]. Despite the good agreement shown in [4], the use of Rutledge's approach in MMIC's is not straightforward, since MMIC lines are not thick enough to lead to prevailing free-space losses, nor thin enough to restrict the losses to purely surface waves. All in all, it can be surmised that, in moderately thin lines, most radiated power is carried by the fundamental TM modes, while in moderately thick lines the free space radiation

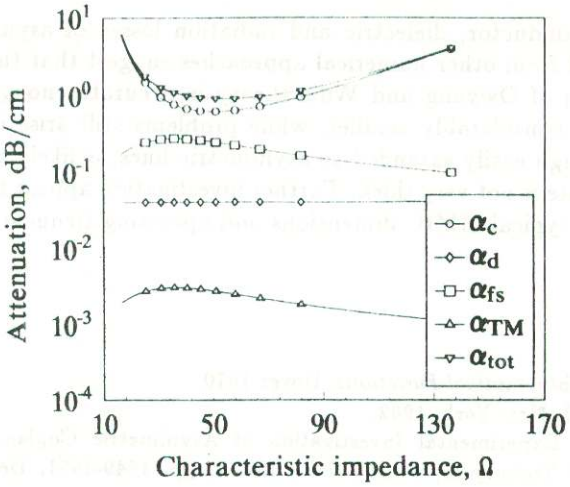


Figure 5: *Behaviour of conductor, dielectric, free-space and TM surface-wave radiation attenuation for symmetric CPW on GaAs ($h = 300\ \mu m$, $b = 100\ \mu m$, $t = 3\ \mu m$, gold metallization) versus the line impedance. The frequency is $f = 30$ GHz.*

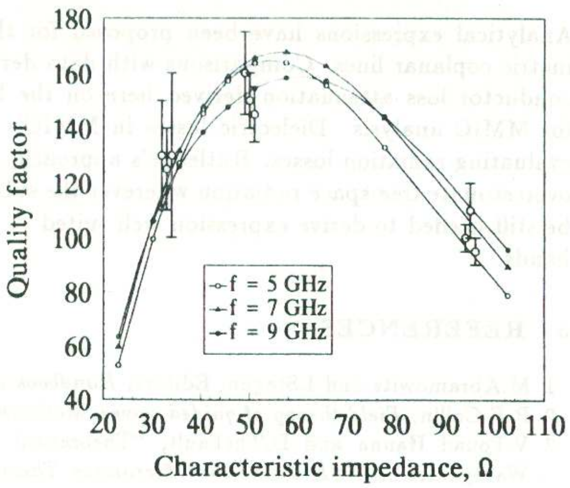


Figure 6: *Quality factor of $\lambda/2$ CPW resonator on GaAs substrate versus the line impedance ($h = 635\ \mu m$, $2b = 1200\ \mu m$, $t = 5\ \mu m$, gold metallization) versus the line impedance. The measured data are from [10], Fig.5.*

attenuation formula proposed by Rutledge overestimates losses. Experimental validation is fairly difficult in complex environments like realistic or even packaged MMICs because radiation losses are influenced by the presence of ground planes, metallic housing etc.

Rutledge's theory and its extensions can be readily extended to the case of asymmetric coplanar waveguides, at least when asymmetry is not large. In the limit of the ACPW_1 the radiation mechanism is mixed (one radiating strip and one radiating slot) thereby requiring the development of special-purpose expressions. Wherever the asymmetry is moderate, Rutledge's approach holds for asymmetric lines provided that [19, Eq.(34)] is evaluated in the asymmetric case and the corrections proposed in [4] are applied. Since the TE_0 radiation is likely to be of little interest in MMIC's, only the free-space and TM surface wave attenuation will be discussed. The free-space radiation attenuation becomes:

$$\alpha_{\rm ACPW}^{\rm fsr} = \frac{\pi^3(1-\epsilon_{\rm eff}^{\rm ACPW}/\epsilon_{\rm r})^2}{\sqrt{\epsilon_{\rm eff}^{\rm ACPW}/\epsilon_{\rm r}}}\,\frac{2a^2\epsilon_{\rm r}^{3/2}f^3}{c_0^3K(k)K'(k)}\left[K(\sqrt{k_rk_l})-\Pi(k_r,\sqrt{k_rk_l})-\Pi(k_l,\sqrt{k_rk_l})\right]^2, \qquad (15)$$

where $k_r = (b_1 - a)/(b_1 + a)$, $k_l = (b_2 - a)/(b_2 + a)$ and $\Pi(n, k)$ is the complete elliptic integral of the third kind:

$$\Pi(n,k) = \int_0^{\pi/2}\frac{d\theta}{(1-n\sin^2\theta)\sqrt{(1-k^2\sin^2\theta)}}. \qquad (16)$$

For symmetric lines the result reported in [4, Eq.(9a)] is obtained by making use of [1, Eq.(17.7.22)]. Similarly, for the attenuation due to coupling to TM_0 surface waves in the ACPW one has:

$$\alpha_{\rm ACPW}^{\rm TM} = \frac{4\pi^2a^2\sqrt{2\epsilon_{\rm r}}}{\sqrt{1+\epsilon_{\rm eff}^{\rm ACPW}/\epsilon_{\rm r}}}\,\frac{(1-\epsilon_{\rm eff}^{\rm ACPW}/\epsilon_{\rm r})^{3/2}\sin\theta_d f^2}{h_e^3c_0^2K(k)K'(k)}\left[K(\sqrt{k_rk_l})-\Pi(k_r,\sqrt{k_rk_l})-\Pi(k_l,\sqrt{k_rk_l})\right]^2, \qquad (17)$$

where, having defined as $\epsilon_{\rm eff}^{\rm TM}$ the effective permittivity of the TM_0 surface wave, one has $\sin\theta_d = \sqrt{\epsilon_{\rm eff}^{\rm TM}/\epsilon_r}$ while the effective waveguide height h_e is defined in [19, Eq.(47-49)].

An example of radiation loss evaluation carried out according Rutledge's formulae is shown in Fig.5 for a MMIC CPW at a moderately high frequency. Radiation losses are far smaller than conductor losses, and free-space radiation is likely to be overestimated since, as already stated, Rutledge's theory for this parameter implies a semi-infinite substrate. This conclusion is also supported by the experimental comparison shown in Fig.6, where radiation losses had to be reduced to 30% in order to obtain a good agreement with measurements.

4 CONCLUSIONS

Analytical expressions have been proposed for the conductor, dielectric and radiation losses of asymmetric coplanar lines. Comparisons with data derived from other numerical approaches suggest that the conductor loss attenuation derived here on the basis of Owyang and Wu's theory is accurate enough for MMIC analysis. Dielectric losses in MMIC's are considerably smaller, while problems still arise in evaluating radiation losses. Rutledge's approach, though easily extended to asymmetric lines, is likely to overestimate free-space radiation wherever the substrate is not very thick. Further investigation appear to be still needed to derive expression well suited to the typical MMIC dimensions and operating frequency bands.

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