# TECHNOLOGY INDEPENDENT NONLINEAR INTEGRAL MODEL OF MICROWAVE ELECTRON DEVICES

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## ABSTRACT

The paper describes a new technology-independent model for the large-signal performance prediction of electron devices. The Nonlinear Integral Model (NIM) is rigorously derived from the Volterra series through suitable modifications so that fast convergence can be achieved even under large-signal strongly nonlinear operation. In particular, the NIM, which can be directly used for Harmonic-Balance circuit analysis, enables the large-signal dynamic response of electron devices to be directly computed on the basis of data obtained either by conventional measurements or physics-based numerical simulations. This property makes the NIM particularly convenient for linking accurate device simulation based on carrier transport physics and Harmonic-Balance circuit analysis. Simulations and experimental results confirm the validity of the proposed modelling approach.

**Keywords:** Electron device modelling, nonlinear microwave circuits, Harmonic Balance

## 1. INTRODUCTION

The relevant advances in GaAs technology [1] nowadays makes it possible to develop MMICs where several components (e.g., transmission lines, capacitors, inductors, field-effect transistors, etc..) can be integrated in a single, small chip. In this context, numerical device simulation based on carrier transport physics [2, 3] is extremely important since device performance prediction is the basic step for reliable circuit design. Unfortunately, at present, numerical physics-based models of electron devices cannot be used in the framework of circuit analysis tools (which are typically based on Harmonic-Balance techniques [4]) since they require very large computing resources. For these reasons suitable intermediate "behavioural" models are needed for the computationally efficient linking of numerical physics-based device models with circuit analysis tools based on HB techniques. To this end, nonlinear equivalent circuits [1, 5, 6], based on an approximated description of the main physical phenomena in the device in terms of "lumped" circuit elements, can be derived from the results of numerical device simulation [6]. However, when large-signal operating conditions at microwave frequencies are considered, the identification of a nonlinear equivalent circuit is a relatively complex task. Owing to the presence of important nonlinear and parasitic phenomena, it can be very difficult to define the "lumped" circuit topology that best describes the electrical behaviour of an electron device. Moreover, the "lumped" representation itself may not be sufficiently accurate, especially at very high frequencies.

For the above reasons suitable approximation procedures, possibly based on numerical optimization, are used to determine the parameter values which best "fit" an equivalent circuit to the electrical device characteristics. The adoption of special-purpose procedures to extract equivalent circuit parameters from numerical device simulations has the disadvantage of introducing an "indirect", complex link between physical device parameters and circuit performance [3].

In this paper, a technology-independent mathematical approach [7]..[9] which does not require complex parameter extraction, is proposed for the circuit-oriented modelling of electron devices. The *"Nonlinear Integral Model"* (NIM), that is derived from the Volterra series by applying suitable considerations valid for most types of electron devices, allows the large-signal dynamic response of electron devices to be directly computed on the basis of bias- and frequency-dependent small-signal admittance parameters and DC characteristics; these can be either directly measured or computed by "off-line" physics-based device simulations.

## 2. THE NONLINEAR INTEGRAL MODEL

Circuit-oriented modelling of microwave electron devices is quite a complex task owing to the simultaneous presence of *nonlinear* and *dynamic* phenomena in the device. In particular, the terms *"nonlinear"* and *"dynamic"* imply that the current $i(t)$ at any time instant $t$ is nonlinearly dependent not only on the applied voltage $v(t)$ at the same instant, but also on past values $v(\tau)$ in the time interval $t - \tau_m \leq \tau < t$, where $\tau_m$ represents the practically finite duration of the "memory" associated to dynamic phenomena in the active device (e.g., charge-storage effects, carrier transit times, etc..).

The most general, rigorous modelling approach for nonlinear dynamic systems is the well-known Volterra series [7, 10]. By considering for simplicity a single-port device[1] its large-signal current/voltage response can be expressed in the time domain by means of the Volterra series in the form:

$$i(t) = \sum_{n=1}^{\infty} \int \cdot\cdot \int_{t-\tau_m}^{t} h_n(t-\tau_1, .., t-\tau_n) v(\tau_1)..v(\tau_n) d\tau_1..d\tau_n \quad (1)$$

where the integration interval is obviously limited to the duration $\tau_m$ of the memory effects in the device and the multi-dimensional impulse responses $h_n(\tau_1, \ldots, \tau_n)$, which are known as Volterra kernels, describe the dynamic nonlinear characteristics of the device. Sum (1), in order to be practically useful, should be extended to a finite, possibly small, number of terms. Unfortunately, when the amplitudes of the signals become large with respect to the nonlinearities of the device, the number of terms which makes (1) sufficiently accurate, increases and the Volterra approach becomes quite impractical [7]. For these reasons this modelling approach can be efficiently used only when weakly nonlinear operation is considered. Moreover, at microwave frequencies, the lack of instrumentation for the measurement [7, 11] of the kernels, limits the practical usefulness of the Volterra series.

A considerable improvement of the convergence properties of (1) can be obtained by rewriting (1) as a series expansion in terms of the *"dynamic deviations"* $e(\tau_i, t) = v(\tau_i) - v(t)$ between the voltages $v(\tau_i)$ at the time instants $\tau_i < t$ and the voltage $v(t)$. In particular, it can be shown [9] that (1) can be written in the following form:

$$i(t) = F_{DC}\{v(t)\} + \sum_{k=1}^{\infty} \int \cdot\cdot \int_{t-\tau_m}^{t} g_k\{v(t), t-\tau_1, .., t-\tau_k\} e(\tau_1, t)..e(\tau_k, t) d\tau_1..d\tau_k \quad (2)$$

where the functions $F_{DC}$ and $g_k\{v(t), t-\tau_1, \ldots, t-\tau_k\}$ can be expressed in terms of the Volterra kernels by:

$$F_{DC}\{v(t)\} = \sum_{n=1}^{\infty} v^n(t) \int \cdot\cdot \int_{t-\tau_m}^{t} h_n(t-\tau_1, .., t-\tau_n) d\tau_1..d\tau_n \quad (3)$$

and:

$$g_k\{v(t), t-\tau_1, .., t-\tau_k\} = \sum_{n=k}^{\infty} v^{(n-k)}(t) \binom{n}{k} \int \cdot\cdot \int_{t-\tau_m}^{t} h_n(t-\tau_1, .., t-\tau_n) d\tau_{k+1}..d\tau_n \quad (4)$$

where $\binom{n}{k} = n!/(n-k)!k!$

[1]The same considerations can be made for multi-port devices [9].

In (2), the nonlinear function $F_{DC}$ clearly coincides with the static characteristic of the device since, in DC steady-state operation, the terms $e(\tau_i, t) = v(\tau_i) - v(t)$ in the integrals are identically zero. So, the strictly dynamic phenomena are described by the multidimensional convolution integrals in terms of the voltage-controlled multidimensional impulse-responses $g_k$, which are *nonlinearly* dependent on the *instantaneous* value assumed by the voltage $v$.

Equation (2) provides fast convergence when the dynamic deviations $e(\tau_i, t)$ are small enough since, in such conditions, the multi-dimensional integrals quickly decrease; this happens even in the presence of large-voltage amplitudes, provided that the practically finite duration $\tau_m$ of memory effects is sufficiently small. This condition, as will be shown in the following, is satisfied for the most common types of electron devices in typical analog applications. In particular, even under large-signal and strongly nonlinear operating conditions, the dynamic deviations $e(\tau_i, t)$ are almost always small enough to make negligible the integrals of dimension greater than one in (2); this leads to the following equation:

$$i(t) = F_{DC}\{v(t)\} + \int_{t-\tau_m}^{t} g_1\{v(t), t-\tau\}[v(\tau) - v(t)]d\tau \tag{5}$$

which defines[2] the Nonlinear Integral Model [9] and consists of a single-fold convolution (nonlinearly controlled by the instantaneous voltage $v(t)$) with respect to the voltage dynamic deviations $e(\tau, t) = v(\tau) - v(t)$.

In order to understand why the voltage-controlled expression (5) can correctly describe even the strongly nonlinear response of an electron device, a few considerations, besides the experimental and simulated results given in Section 3, are needed. To this end, it should be taken into account that any spectral component $V(\omega)$ of the voltage $v(t)$ gives a contribution to the dynamic deviation $v(\tau) - v(t)$ which can be expressed in the form $Re\{V(\omega)[e^{j\omega\tau} - e^{j\omega t}]\}$; clearly, this contribution may have a small amplitude, even for a large amplitude of $V(\omega)$, provided that the *"short-term" memory* condition:

$$max(t-\tau) = \tau_m << \frac{1}{\omega} \tag{6}$$

holds for any angular frequency $\omega$ where large-amplitude voltage spectral components (i.e., large enough to stimulate nonlinear behaviour) are present. Thus, it can be said that the NIM (5) is valid provided that $\tau_m << \frac{1}{B_{LS}}$, where $B_{LS}$ represents the *large-signal bandwidth* of the applied voltage (that is, the maximum angular frequency where spectral components $V(\omega)$ are present with an amplitude large enough to directly stimulate nonlinear behaviour in the electron device). It should be noted that simply considering the maximum angular frequency $\omega_{max}$ of $v(t)$ (i.e., the maximum frequency where non negligible spectral components can be found) would have been more restrictive than strictly necessary. In fact, in order to guarantee the validity of (5), inequality (6) must be satisfied only for large-amplitude spectral components; no constraint needs to be imposed on those spectral components whose amplitude is not large enough to autonomously generate nonlinear behaviour[3], although it is not negligible for accurate circuit analysis (owing to important reactive phenomena).

Under typical large-signal operating conditions, the amplitude of the higher-order signal spectra components decreases quite quickly with increasing frequency, so that $B_{LS}$ does not usually exceed the second- or third-order harmonic component of the fundamental frequency; consequently, for typical applications of electron devices, $B_{LS}$ will not generally be much higher (i.e., it has the same order of magnitude) than the maximum operating frequency given by the manufacturer.

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[2] Equation (5) can be directly used in the case of multi-port devices [9] by interpreting the functions $F_{DC}$, $g_1$ and $i(t)$, $v(t)$ respectively as matrices and vectors of suitable dimensions.

[3] It must be emphasized that (5) is intrinsically "exact" in small-signal operation, since, under such conditions, it reduces to the linear convolution integral.

The "short-term" memory condition $\tau_m << \frac{1}{B_{LS}}$ is verified for most types of electron devices when described in a *voltage-controlled form*. This property, which can be intuitively explained by considering that nonlinear dynamic effects in electron devices are mainly associated with voltage-controlled charge-storage phenomena or very short transit times, is also confirmed by experimental evidence. For instance, the duration of the current transient responses to voltage pulses applied either to the source or gate terminals of GaAs MESFETs, is generally much smaller than the inverse of the typical operating frequency given by the manufacturer (see for instance Fig. 1 where the drain current response to an ideal voltage pulse applied to the gate is shown for typical GaAs MESFETs).

Similar results can be found for other electron devices (e.g., bipolar transistors, junction diodes, etc...) provided a *voltage-controlled description* is used and the *intrinsic* device (i.e., the "active" part of the device) is not subject to dominant parasitic effects. In the case of packaged discrete devices, the package parasitics may slow down the impulse response of the device and make the assumption of short-term memory not completely acceptable for very high operating frequencies; in such conditions good accuracy can still be achieved with the NIM provided that conventional techniques for "de-embedding" from the linear parasitics are used [1].

The Nonlinear Integral Model can also take into account some other phenomena which do not belong to the class of "short-memory" effects, such as the low-frequency dispersion of electrical characteristics in GaAs electron devices. In particular, it must be observed that, under small-signal operation, the NIM is **intrinsically** exact whatever frequency of operation is considered, thus also for low-frequency dispersion phenomena. As far as the direct influence of these phenomena on the large-signal performance of an electron device is concerned, theoretical considerations and experimental results confirming the validity of the NIM are given in [12].

## 2.1 Harmonic-Balance circuit analysis

When microwave applications are considered, circuit analysis is more conveniently carried out in the frequency domain by means of Harmonic-Balance techniques [4]. It can easily be shown [9] that the Nonlinear Integral expression (5) can be written, by applying well-known properties of the Fourier transform, in the Harmonic-Balance-oriented form:

$$i(t) = F_{DC}\{v(t)\} + \sum_{k=-M}^{+M} \widetilde{Y}\{v(t),\omega_k\} V_k e^{j\omega_k t} \quad \text{with: } \widetilde{Y}\{v(t),\omega\} = \int_0^{\tau_m} g\{v(t),\tau\}[e^{-j\omega\tau}-1]d\tau \quad (7)$$

where the $V_k$'s represent the harmonic components of the voltage $v(t)$ at the angular frequencies $\omega_k$ (with $\omega_0 = 0$).

The term $\widetilde{Y}$ in (7) represents a voltage-dependent *"dynamic" admittance* which describes the *purely dynamic* phenomena in the device, since $\widetilde{Y}\{v(t),0\} = 0$. The dynamic admittance $\widetilde{Y}$ is nonlinearly dependent only on the instantaneous voltage $v(t)$; this is justified by the hypothesis of short-term memory and is consistent with similar assumptions used in the derivation of some nonlinear equivalent circuits [1].

The non-linearly voltage-controlled *dynamic admittance* $\widetilde{Y}\{v(t),\omega\}$ can be easily identified since it is related to the conventional bias-dependent small-signal admittance $Y$ by a very simple relation. In fact, by considering a small sinusoidal voltage superimposed on a given DC bias voltage $V_B$, linearization of (7) with respect to the small-signals leads to the following expression:

$$\widetilde{Y}\{v,\omega_k\} = Y[V_B,\omega_k] - g_{DC}[V_B] \quad \text{with } v = V_B \quad (8)$$

where $g_{DC} = (dF_{DC}/dv)_{V_B}$ is the DC differential conductance of the device.

Equation (8) shows that the identification of the mathematical model proposed here is really straightforward as it can be carried out **directly**, without requiring numerical fitting procedures, on the basis of static characteristics and small-signal admittance parameters measured for several DC bias conditions and operating frequencies over the range of interest. Since this electrical characterization can easily be obtained when a suitable tool for the physics-based simulation of semiconductor devices is available [2, 3], the NIM can be used as a computationally efficient link between accurate physics-based device simulation and HB circuit analysis algorithms.

The model proposed is particularly suitable when the designer is concerned with computing the sensitivity $\mathcal{S}_P^B = \partial B/\partial P$ of a circuit performance $B$ with respect to a process-dependent parameter $P$. To this end, it must be emphasized that the NIM allows the closed-form expression (7) to be used for computation of the large-signal response of an electron device, **directly** in terms of DC characteristics and small-signal admittance parameters. Thus the computation of the sensitivity $\mathcal{S}_P^B$ requires only knowledge of the sensitivities of the DC characteristics and $Y$-parameters with respect to the process-dependent parameter $P$. These sensitivities can be efficiently obtained when suitable tools for physics-based simulation of active devices are available [13].

When an equivalent circuit model is adopted to predict the large-signal response of an electron device, the computation of the sensitivity $\mathcal{S}_P^B$ is not a simple task since it requires calculating the sensitivities $s_P^C = \partial C/\partial P$ of model parameters $C$ with respect to the process-dependent parameter $P$. Such a computation can be quite cumbersome, since the determination of the parameters $C$ of the equivalent circuit is carried out by means of numerical fitting procedures, and a direct link between such parameters and process-dependent variables is not available.

## 3. MODEL VALIDATION

In order to verify the validity of the approach proposed, a comparison of the results obtained in circuit analysis by means of the NIM and those obtained by means of physics-based simulations has been carried out.

In particular, a medium power X-band GaAs MESFET with $0.4\mu m$ gate length, grown by Molecular Beam Epitaxy, has been considered. Physics-based numerical simulations [2, 6] have been carried out in order to obtain a complete DC characterization and the small-signal admittance parameters of the MESFET for several bias points (about 500 in the $V_{GS0}$, $V_{DS0}$ space) and for 10 harmonic frequencies (with fundamental at 10 GHz). On the basis of this characterization and by means of (7), obviously considered as a matrix representation, the HB analysis of a class-AB amplifier at the frequency of 10 GHz loaded with a 50 $\Omega$ impedance has been carried out. The nonlinearly voltage-controlled dynamic admittance $\tilde{Y}\{v_{GS}(t), v_{DS}(t), \omega\}$ which describes the purely dynamic phenomena in the device, was computed according to (8), for each pair of instantaneous voltages $v_{GS}(t)$, $v_{DS}(t)$, occurring in the HB analysis, by using piece-wise linear interpolation techniques in the two-dimensional $V_{GS0}$, $V_{DS0}$ space.

Figure 2 shows the time-varying waveform of the drain current $i_D(t)$ obtained by the HB circuit analysis for the above-mentioned amplifier. In the same figure, the results obtained by carrying out the same analysis by means of the same physics-based full two-dimensional simulator, at the cost of a very long computation time, is also drawn. The good agreement between circuit analysis carried out on the basis of the NIM and the large-signal prediction obtained for the same circuit by means of an accurate physics-based numerical simulator, confirms that the Nonlinear Integral Model can be used as an accurate, computationally efficient tool for linking numerical device simulators with HB analysis algorithms.

An experimental validation has also been carried out for the Nonlinear Integral Model. In particular, validation tests have been carried out at the frequency of 2 GHz on a large-signal amplifier using a GaAs MESFET. Figure 3 shows plots of the output power associated with different order harmonics vs the input power. Also in this case, the agreement between the results provided by the model proposed and large-signal measurements confirms the accuracy of the NIM.

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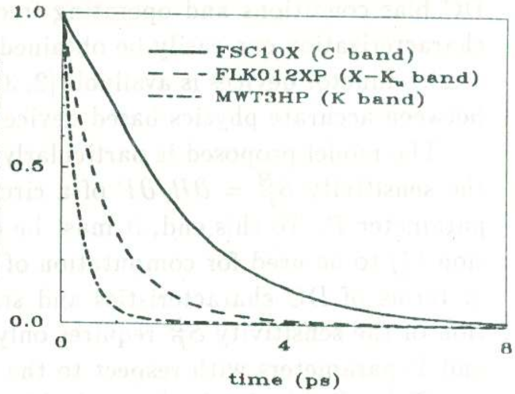


Figure 1: Drain current responses to a gate voltage pulse for GaAs MESFETs.

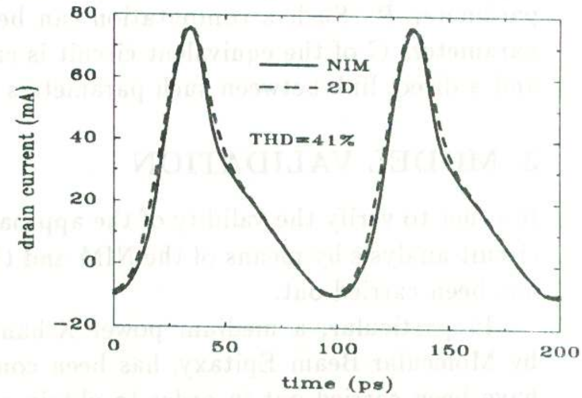


Figure 2: Drain current in a GaAs MESFET amplifier with sinusoidal input at 10 GHz ($P_{IN}$ =13 dBm, $V_{GS0} = -3V$, $V_{DS0} = 4V$). The Total Harmonic Distortion is 41%.

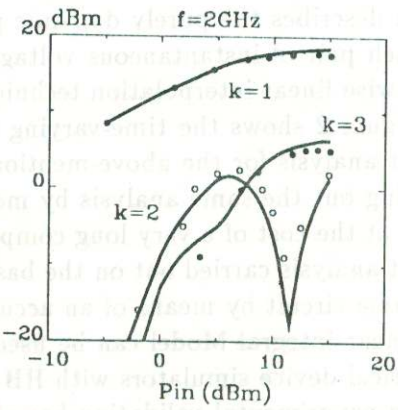


Figure 3: Plots of output powers (—: NIM, ∘•: measurements) associated with different order harmonics ($k = 1, 2, 3$) vs input power $P_{IN}$ for a GaAs MESFET amplifier ($V_{GS0} = -0.4V$, $V_{DS0} = 4V$) with sinusoidal input at 2 GHz.