A Nonlinear Dynamic Model for Performance Analysis of Large-Signal Amplifiers in Communication Systems

Domenico Mirri, Fabio Filicori, Gaetano Iuculano, and Gaetano Pasini

***Abstract*—A new nonlinear dynamic model of large-signal amplifiers based on a Volterra-like integral series expansion is described. The new Volterra-like series is specially oriented to the modeling of nonlinear communication circuits, since it is expressed in terms of dynamic deviations of the complex modulation envelope of the input signal. The proposed model represents a generalization, to nonlinear systems with memory, of the widely-used amplitude/amplitude (AM/AM) and amplitude/phase (AM/PM) conversion characteristics, which are based on the assumption of a practically memoryless behavior. A measurement procedure for the experimental characterization of the proposed model is also outlined.**

***Index Terms*—Communication systems, large-signal amplifiers, modeling approach, modified Volterra integral series, nonlinear dynamic systems.**

I. INTRODUCTION

CHARACTERIZATION and modeling of large-signal amplifiers is of basic importance in the design of communications systems. In fact, power amplifiers are among the most critical components in system design, owing to the need for optimal tradeoffs between different requirements, such as linearity, output power, bandwidth, power-added efficiency, etc. In particular, for correct performance analysis of communication systems, accurate prediction of nonlinear distortion and bandwidth limitations is strictly needed. At present, communication system design is based on the well-known amplitude/amplitude (AM/AM) and amplitude/phase (AM/PM) amplifier input/output characteristics, which enable the prediction of large-signal performance only in the presence of strictly narrow-band modulated signals. However, in many cases, modern communication systems involve modulated signals whose bandwidths are far from negligible with respect to the dynamic capabilities of the amplifiers, especially when these also include internal devices for distortion reduction through compensation of the nonlinear conversion characteristics. In such cases, a more complex nonlinear dynamic model, which takes into account the amplifier distortion, due to both the large amplitude and the large bandwidth of the input signal, is needed. To this aim, a special-purpose nonlinear modeling approach is proposed. It is based on a modified Volterra-like integral series expansion [1]–[5] which has also been expressed in a particular form specially oriented to nonlinear systems operating with modulated signals [6].

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The authors are with the University of Bologna, 40136 Bologna, Italy.
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II. NONLINEAR DYNAMIC MODEL

We assume that the output $u(t)$ of the large-signal amplifier can be expressed as a generic functional[1] $F|[\cdot]|$ of the input signal $s(t)$ which, according to the line-function symbolism [1], can be expressed in the following form:

$$u(t) = F\left|\left[\overset{+T_B}{\underset{0}{s(t-\tau)}} \right]\right|. \tag{1}$$

This equation simply means that the amplifier output $u(t)$ depends linearly or nonlinearly on the values of $s(t-\tau)$ over a sufficiently large "memory interval" $(0 \le \tau \le T_B)$ around the instant t at which the output is evaluated. In all cases of practical interest, the input signal $s(t-\tau)$, for any shift τ, can be described as a single carrier[2] with generic amplitude $|a(t-\tau)|$ and phase $\angle a(t-\tau)$ modulations with respect to τ in the following form:

$$\begin{aligned} s(t-\tau) &= 2\mathrm{Re}\left[a(t-\tau)\mathrm{e}^{\mathrm{j}2\pi f_0(t-\tau)}\right] \\ &= |a(t-\tau)| \sum_{\substack{i=-1\\ i\neq 0}}^{+1} \mathrm{e}^{\mathrm{j}i\left[2\pi f_0(t-\tau)+\angle a(t-\tau)\right]} \end{aligned} \tag{2}$$

where $a(t-\tau)$ is the equivalent complex modulation envelope

$$a(t-\tau) = |a(t-\tau)|\,\mathrm{e}^{\mathrm{j}\angle a(t-\tau)} \tag{3}$$

and f_0 the associated equivalent carrier frequency. On the basis of (2), (3), the amplifier response can be conveniently described by a Volterra-like integral series expansion [2], [3] in terms of the dynamic deviations of the signal $s(t-\tau)$ with respect to a convenient reference signal $\hat{s}(t,\tau)$

$$e(t,\tau) = s(t-\tau) - \hat{s}(t,\tau) \tag{4}$$

with $e(t,0)=0$. In this case, the reference signal is selected as an equivalent sinusoid with respect to τ

$$\begin{aligned} \hat{s}(t,\tau) &= 2\mathrm{Re}\left[a(t)\mathrm{e}^{\mathrm{j}2\pi f_0(t-\tau)}\right] \\ &= |a(t)| \sum_{\substack{i=-1\\ i\neq 0}}^{+1} \mathrm{e}^{\mathrm{j}i\left[2\pi f_0(t-\tau)+\angle a(t)\right]} \end{aligned} \tag{5}$$

[1]A functional is a real-valued function whose domain is a set of real functions. A simple example of a functional, on the set of integrable real functions defined on a prefixed domain D, is the integral on D.

[2]This is assuming that a single carrier with a generic modulation is not a strong limitation, since any multicarrier bandpass signal can be described as an equivalent single-carrier signal with appropriate amplitude and phase modulations.

whose amplitude and phase coincide with that of the input signal at the instant t at which the output $u(t)$ is evaluated; in fact, $\hat{s}(t,0)=s(t)$. Therefore, we can write

$$e(t,\tau)=2\mathrm{Re}\left\{[a(t-\tau)-a(t)]\,\mathrm{e}^{\mathrm{j}2\pi f_0(t-\tau)}\right\}. \quad (6)$$

By introducing the *dynamic deviation* $e(t,\tau)$ (4), (1) can be rewritten as follows:

$$u(t)=F\left|\left[\hat{s}(t,\tau)\overset{+T_B}{\underset{0}{+}}e(t,\tau)\right]\right|. \quad (7)$$

By expressing the functional of the two functions through a convenient series (see Appendix A), on the hypothesis that the bandwidth of the complex modulation envelope is so small as to make the product of the amplitude of the dynamic deviations $e(t,\tau_i)$ for each τ_i almost negligible in practice and by considering only the spectral components of the output signal component within the operating bandwidth $u_B(t)$ centred around f_0, it can be shown that the output signal can be expressed as follows [see (A28)]:

$$\begin{aligned}u_B(t)=&\,2\mathrm{Re}\left[\mathrm{e}^{\mathrm{j}2\pi f_0 t}a(t)H\left(f_0,|a(t)|\right)\right]\\&+2\mathrm{Re}\left[\mathrm{e}^{\mathrm{j}2\pi f_0 t}\int_0^{+T_B}h_1(\tau_1)\right.\\&\left.\times\left[a(t-\tau_1)-a(t)\right]\mathrm{e}^{-\mathrm{j}2\pi f_0\tau_1}\mathrm{d}\tau_1\right]\\&+2\mathrm{Re}\left[\mathrm{e}^{\mathrm{j}2\pi f_0 t}\int_0^{+T_B}g_1\left(\tau_1,f_0,|a(t)|\right)\right.\\&\left.\times\left[a(t-\tau_1)-a(t)\right]\mathrm{e}^{-\mathrm{j}2\pi f_0\tau_1}\mathrm{d}\tau_1\right]\\&+2\mathrm{Re}\left[\mathrm{e}^{\mathrm{j}2\pi f_0 t}a^2(t)\int_0^{+T_B}g_2^*\left(\tau_1,f_0,|a(t)|\right)\right.\\&\left.\times\left[a^*(t-\tau_1)-a^*(t)\right]\mathrm{e}^{\mathrm{j}2\pi f_0\tau_1}\mathrm{d}\tau_1\right]. \quad (8)\end{aligned}$$

According to this equation, the "in-band" output signal $u_B(t)$ can be computed as the sum of different terms. The first term represents the nonlinear memoryless contribution; the second represents the purely dynamic linear contribution and the last two the purely dynamic nonlinear contributions. The dynamic contributions are evaluated through a convolution integral expressed in terms of the dynamic deviations of the complex modulation envelope of the input signal (6). In particular, when the input signal is a nonmodulated signal carrier, i.e., $a(t)$ is a constant, according to (6), each dynamic deviation is identically zero so that the corresponding output in (8) is given only by the first term. It can be easily shown that the amplitude and the phase of $H(f_0,|a(t)|)$ simply correspond to the well-known and widely-used AM/AM and AM/PM amplifier characteristics. This means, in practice, that the AM/AM and AM/PM plots, which are the only data normally provided to characterize the large-signal amplifier response, simply represent a zero-order approximation with respect to the dynamic deviations of the complex modulation envelope $a(t)$, of the system behavior. Thus, in the presence of modulated signals, the commonly used AM/AM and AM/PM amplifier characterization is sufficiently accurate only when the bandwidth of the complex modulation envelope $a(t)$ is so small as to make the amplitude of the dynamic deviations $a(t-\tau_i)-a(t)$ for each τ_i almost negligible in practice. This constraint cannot be met in many practical cases, when dealing with large bandwidth modulated signals. In such conditions, for better accuracy, the generalized "black-box" modeling approach defined by (8) can be used, by taking into account more terms of the functional series expansion. In fact, even if the series has been truncated to the first order term $(n=1)$, considerable improvement in accuracy is achieved with respect to the "coarser" zero order approximation of the conventional AM/AM and AM/PM characteristics alone. In particular, the in-band output signal $u_B(t)$ can also be expressed, according to (8), in terms of the equivalent output complex demodulation envelope $b(t)$ in the following form:

$$u_B(t)=2\mathrm{Re}\left[b(t)\mathrm{e}^{\mathrm{j}2\pi f_0 t}\right] \quad (9)$$

where

$$\begin{aligned}b(t)=&\,a(t)H\left(f_0,|a(t)|\right)\\&+\int_0^{+T_B}h_1(\tau_1)\left[a(t-\tau_1)-a(t)\right]\mathrm{e}^{-\mathrm{j}2\pi f_0\tau_1}\mathrm{d}\tau_1\\&+\int_0^{+T_B}g_1\left(\tau_1,f_0,|a(t)|\right)\\&\times\left[a(t-\tau_1)-a(t)\right]\mathrm{e}^{-\mathrm{j}2\pi f_0\tau_1}\mathrm{d}\tau_1\\&+a^2(t)\int_0^{+T_B}g_2^*\left(\tau_1,f_0,|a(t)|\right)\\&\times\left[a^*(t-\tau_1)-a^*(t)\right]\mathrm{e}^{\mathrm{j}2\pi f_0\tau_1}\mathrm{d}\tau_1. \quad (10)\end{aligned}$$

By considering a discrete spectrum modulating signal, the corresponding complex modulation envelope $a(t)$ can be expressed in the following form:

$$a(t)=C+\sum_k C_k\mathrm{e}^{\mathrm{j}2\pi\nu_k t} \quad (11)$$

whose modulus is given by (12), as shown at the bottom of page. Therefore, the difference $a(t-\tau_1)-a(t)$ can be written as follows:

$$a(t-\tau_1)-a(t)=\sum_k C_k\mathrm{e}^{\mathrm{j}2\pi\nu_k t}(\mathrm{e}^{-\mathrm{j}2\pi\nu_k\tau_1}-1). \quad (13)$$

$$|a(t)|=\sqrt{a(t)a^*(t)}=\sqrt{|C|^2+\left|\sum_k C_k\mathrm{e}^{\mathrm{j}2\pi\nu_k t}\right|^2+2\mathrm{Re}\left[C^*\sum_k C_k\mathrm{e}^{\mathrm{j}2\pi\nu_k t}\right]} \quad (12)$$

By substituting into (10), we obtain

$$\begin{aligned} b(t) = {} & a(t)H(f_0,|a(t)|)+\sum_k C_k \mathrm{e}^{\mathrm{j}2\pi\nu_k t} \\ & \times \int_0^{+T_B} h_1(\tau_1)\left[\mathrm{e}^{-\mathrm{j}2\pi\nu_k\tau_1}-1\right]\mathrm{e}^{-\mathrm{j}2\pi f_0\tau_1}\mathrm{d}\tau_1 \\ & +\sum_k C_k \mathrm{e}^{\mathrm{j}2\pi\nu_k t} \\ & \times \int_0^{+T_B} g_1(\tau_1,f_0,|a(t)|)\left[\mathrm{e}^{-\mathrm{j}2\pi\nu_k\tau_1}-1\right]\mathrm{e}^{-\mathrm{j}2\pi f_0\tau_1}\mathrm{d}\tau_1 \\ & + a^2(t)\sum_k C_k^* \mathrm{e}^{-\mathrm{j}2\pi\nu_k t} \\ & \times \int_0^{+T_B} g_2^*(\tau_1,f_0,|a(t)|)\left[\mathrm{e}^{\mathrm{j}2\pi\nu_k\tau_1}-1\right]\mathrm{e}^{\mathrm{j}2\pi f_0\tau_1}\mathrm{d}\tau_1. \end{aligned} \tag{14}$$

Let us introduce the following incremental frequency functions, that is

$$\begin{aligned} & H_1(f_0+\nu_k,f_0) \\ & \quad = \int_0^{+T_B} h_1(\tau_1)(\mathrm{e}^{-\mathrm{j}2\pi\nu_k\tau_1}-1)\mathrm{e}^{-\mathrm{j}2\pi f_0\tau_1}\mathrm{d}\tau_1 \end{aligned} \tag{15}$$

$$\begin{aligned} & G_1\left(f_0+\nu_k,f_0,|a(t)|\right) \\ & \quad = \int_0^{+T_B} g_1\left(\tau_1,f_0,|a(t)|\right)\{\mathrm{e}^{-\mathrm{j}2\pi\nu_k\tau_1}-1\} \\ & \qquad \times \mathrm{e}^{-\mathrm{j}2\pi f_0\tau_1}\mathrm{d}\tau_1 \end{aligned} \tag{16}$$

$$\begin{aligned} & G_2\left(f_0+\nu_k,f_0,|a(t)|\right) \\ & \quad = \int_0^{+T_B} g_2\left(\tau_1,f_0,|a(t)|\right)(\mathrm{e}^{-\mathrm{j}2\pi\nu_k\tau_1}-1) \\ & \qquad \times \mathrm{e}^{-\mathrm{j}2\pi f_0\tau_1}\mathrm{d}\tau_1. \end{aligned} \tag{17}$$

Obviously, we verify that

$$\begin{aligned} & \lim_{\nu_k\to 0} H_1(f_0+\nu_k,f_0) \\ & \quad = \lim_{\nu_k\to 0} G_1\left(f_0+\nu_k,f_0,|a(t)|\right) \\ & \quad = \lim_{\nu_k\to 0} G_2\left(f_0+\nu_k,f_0,|a(t)|\right) \\ & \quad = 0. \end{aligned} \tag{18}$$

If we introduce these equations into (14), this last can be rewritten as follows:

$$\begin{aligned} b(t) = {} & a(t)H(f_0,|a(t)|) \\ & +\sum_k C_k \mathrm{e}^{\mathrm{j}2\pi\nu_k t}H_1(f_0+\nu_k) \\ & +\sum_k C_k \mathrm{e}^{\mathrm{j}2\pi\nu_k t}G_1(f_0+\nu_k,f_0,|a(t)|) \\ & + a^2(t)\sum_k C_k^* \mathrm{e}^{-\mathrm{j}2\pi\nu_k t}G_2^*(f_0+\nu_k,f_0,|a(t)|)\,. \end{aligned} \tag{19}$$

Some particular situations can be considered. First, when considering a low frequency modulating signal (i.e., the ν_ks are sufficiently small), according to (18), the large-signal model of (19) becomes

$$b(t) = a(t)H\left(f_0,|a(t)|\right). \tag{20}$$

This shows that the term $H(f_0,|a(t)|)$ alone completely describes the large-signal amplifier response when the bandwidth of the modulating signal is relatively small. It can be noted that the term $H(f_0,|a(t)|)a(t)$ corresponds to the well-known quasistatic amplifier characterization based on the AM/AM, AM/PM conversion plots. However, (20) also shows that only using the AM/AM and AM/PM conversion characteristics [i.e., only the term $H(f_0,|a(t)|)a(t)$] in order to predict the large signal amplifier response under wide-band modulating signals can lead to inaccurate results, since in such conditions, the contribution of the other three terms in (19) is no more negligible. In particular, the second term $\sum_k C_k \mathrm{e}^{\mathrm{j}2\pi\nu_k t}H_1(f_0+\nu_k)$ alone can take into account the amplifier dynamics with wide-band modulating signals, but only when the amplitude of the signal $s(t)$, or equivalently $|a(t)|$, is sufficiently small. In fact, for small $|a(t)|$, the contributions of the last two summation terms in (19) become negligible, according to (A16), (A23), and (A24), with respect to the first two terms. Thus, for small $s(t)$, or equivalently small $|a(t)|$, (19) becomes

$$b(t) = a(t)H_{\mathrm{SS}}(f_0) + \sum_k C_k \mathrm{e}^{\mathrm{j}2\pi\nu_k t}H_1(f_0+\nu_k) \tag{21}$$

where

$$H_{\mathrm{SS}}(f_0) = H(f_0,0) \tag{22}$$

is the small-signal amplifier transfer function at the carrier frequency f_0, while $H_1(f_0+\nu_k)$ clearly represents the difference between the values taken by the amplifier small-signal transfer function at the frequencies $f_0+\nu_k$ and f_0, respectively. The term $H_1(f_0+\nu_k)$ can be easily measured under small-signal operating conditions by using conventional signal generators and vector voltmeters. However, when wide-band large-amplitude modulated signals have to be dealt-with also the last two terms of (19) must be taken into account. To this aim, a special-purpose measurement procedure, involving wide-band signals and large-amplitude carrier, is needed for complete model identification.

It can be noted that (10) corresponds to the truncation of a modified Volterra expansion expressed in terms of the dynamic deviations $a(t-\tau)-a(t)$ of the input signal modulation envelope $a(t)$. In comparison with the classical Volterra series approach, the modified one has the advantage of allowing for series truncation to single-fold convolution integrals even when strongly nonlinear operation is involved, provided that the system to be described has a relative short memory time with respect to the modulating signal bandwidth. More details on the different truncation properties of the classical and modified Volterra series approaches can be found in [2]–[5].

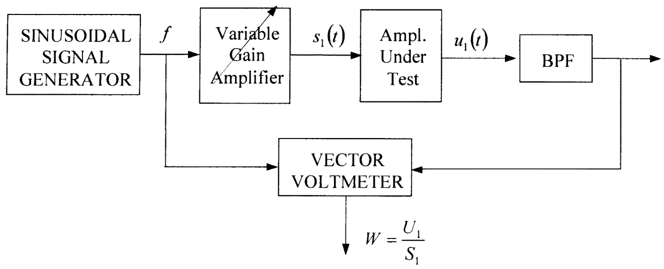

Fig. 1. Measurement setup for the experimental characterization of the terms $H(f_0, |a(t)|)$ and $H_1(F_0 + \nu_k)$ in the large-signal model of (19) of the amplifier under test (AUT).

III. Measurement Procedures for Model Identification

In order to completely identify the large-signal dynamic model of (19), the four complex functions $H(f_0, |a(t)|)$, $H_1(f_0+\nu_k)$, $G_1(f_0+\nu_k, f_0, |a(t)|)$, and $G_2(f_0+\nu_k, f_0, |a(t)|)$ must be experimentally characterized, through suitable measurement procedures for any given frequency f_0 and a sufficiently wide set of values for ν and $|a(t)|$. As it has been outlined in the previous section, the terms $H(f_0, |a(t)|)$ and $H_1(f_0 + \nu_k)$ can be experimentally identified by using a conventional measurement setup like the one of Fig. 1. This consists of a generator providing a sinusoidal signal at a given frequency f, whose amplitude $|a(t)|$ can be controlled by a variable amplifier/attenuator. A vector voltmeter provides the measured ratio $W = U_1/S$ between the phasor U for the corresponding first harmonic of the output signal $u(t)$ and the phasor S associated to the sinusoidal input signal $s(t)$; higher-order harmonics are eliminated by the bandpass filter (BPF).[3] In particular, according to (20), on the hypothesis of a zero bandwidth modulating signal, the term $H(f_0, |a(t)|)$ can be characterized for any value of f_0 and $a(t)$, by simply applying an input signal $s_1(t)$ with amplitude $|S_1|$ and frequency $f = f_0$, and measuring with the vector voltmeter the ratio $W = U_1/S_1 = H(f_0, |a(t)|)$ (Fig. 1). Instead, the term $H_1(f_0 + \nu_k)$ can be characterized, for any value of $f_0 + \nu$, by applying a small-amplitude sinusoidal signal

$$s_1(t) = \mathrm{Re}[S_1 \mathrm{e}^{\mathrm{j}2\pi f t}] \tag{23}$$

with frequency $f = f_0 + \nu$; this signal can also be represented in the form defined by (2) and (11) by letting $C = 0$ and $C_k = 0$ for any k except $C_1 = S_1$. In such conditions, according to (21), the term $H_1(f_0 + \nu)$ can be simply computed as

$$H_1(f_0 + \nu) = W(f_0 + \nu) - H(f_0, 0) \tag{24}$$

once the ratio $W = U_1/S_1$ has been measured by the vector voltmeter.

As far as the terms $G_1(f_0 + \nu_k, f_0, |a(t)|)$ and $G_2(f_0 + \nu_k, f_0, |a(t)|)$ in the model of (19) are concerned, their experimental identification cannot be carried out by using the single-frequency measurement setup in Fig. 1. In fact, for a full characterization (i.e., for both positive and negative frequency shifts ν), the more flexible setup shown in Fig. 2, which enables for any type of sinusoidal modulation for a given carrier at frequency f_0, is needed. This consists of a complex envelope modulator which, by applying suitable signals at its two inputs $a_R(t)$ and $a_1(t)$, can implement any complex modulation of the carrier $v_c(t) = \mathrm{Re}[V_0 \mathrm{e}^{\mathrm{j}2\pi f_0 t}]$ provided by a sinusoidal oscillator (without loss of generality in the following we will assume $V_0 = 1$ V). The two baseband modulating signals $a_R(t)$ and $a_I(t)$ are obtained from a second generator, which provides a sinusoidal modulating signal $v(t) = \mathrm{Re}[V \mathrm{e}^{\mathrm{j}2\pi\nu t}]$ at any frequency ν. In the following, we will assume that the modulating signal amplitude $|V|$ is small enough, so that the system response is linear with respect to the modulating signal $v(t)$. It can be noted, owing to the superposition of the effects in small signal operation, that there is no loss of generality in considering a purely sinusoidal modulating signal $v(t)$. It should also be noted that linearization, with respect to the modulating signal, does not undermine the possibility of testing the amplifier dynamic nonlinearity, since this will be detected by suitably varying the amplitude of the carrier, as it will be discussed in the following. In fact, the carrier amplitude $|C| = \sqrt{C_R^2 + C_I^2}$, with $C = C_R + \mathrm{j}C_I$, can be controlled through the "biasing" signals C_R and C_I in the circuit consisting of two sum blocks and two linear blocks with transfer functions D_1 and D_2 which provide the two signals at the input of the complex envelope modulator

$$\begin{aligned} a_R(t) &= C_R + \mathrm{Re}[D_1 V \mathrm{e}^{\mathrm{j}2\pi\nu t}] \\ &= C_R + \mathrm{Re}\left[D_1^* V^* \mathrm{e}^{-\mathrm{j}2\pi\nu t}\right] \\ &= C_R + \frac{1}{2}\left(D_1 V \mathrm{e}^{\mathrm{j}2\pi\nu t} + D_1^* V^* \mathrm{e}^{-\mathrm{j}2\pi\nu t}\right) \end{aligned} \tag{25}$$

$$\begin{aligned} a_I(t) &= C_I + \mathrm{Re}[D_2 V \mathrm{e}^{\mathrm{j}2\pi\nu t}] \\ &= C_I + \mathrm{Re}\left[D_2^* V^* \mathrm{e}^{-\mathrm{j}2\pi\nu t}\right] \\ &= C_I + \frac{1}{2}\left(D_2 V \mathrm{e}^{\mathrm{j}2\pi\nu t} + D_2^* V^* \mathrm{e}^{-\mathrm{j}2\pi\nu t}\right). \end{aligned} \tag{26}$$

Therefore, we can write

$$\begin{aligned} a(t) &= a_R(t) + \mathrm{j}a_I(t) \\ &= C + \frac{D_1 + \mathrm{j}D_2}{2} V \mathrm{e}^{\mathrm{j}2\pi\nu t} + \frac{D_1^* + \mathrm{j}D_2^*}{2} V^* \mathrm{e}^{-\mathrm{j}2\pi\nu t}. \end{aligned} \tag{27}$$

According to (2), the signal $s(t)$ applied at the amplifier input can be expressed as follows:

$$s(t) = 2\mathrm{Re}\left[a(t)\mathrm{e}^{\mathrm{j}2\pi f_0 t}\right] = 2\mathrm{Re}\left[a_R(t)\mathrm{e}^{\mathrm{j}2\pi f_0 t} + \mathrm{j}a_I(t)\mathrm{e}^{\mathrm{j}2\pi f_0 t}\right]. \tag{28}$$

The output signal $u(t)$ given by (7) is bandpass filtered in order to eliminate all the spectral components which are out of the signal band of interest (i.e., all the side bands around the harmonics of f_0) so that the complex envelope demodulator (see Fig. 1) can provide the two baseband signals

$$\begin{aligned} b_R(t) &= B_R + \mathrm{Re}[W_1 V \mathrm{e}^{\mathrm{j}2\pi\nu t}] \\ &= B_R + \frac{1}{2}\left(W_1 V \mathrm{e}^{\mathrm{j}2\pi\nu t} + W_1^* V^* \mathrm{e}^{-\mathrm{j}2\pi\nu t}\right) \end{aligned} \tag{29}$$

$$\begin{aligned} b_I(t) &= B_I + \mathrm{Re}[W_2 V \mathrm{e}^{\mathrm{j}2\pi\nu t}] \\ &= B_I + \frac{1}{2}\left(W_2 V \mathrm{e}^{\mathrm{j}2\pi\nu t} + W_2^* V^* \mathrm{e}^{-\mathrm{j}2\pi\nu t}\right) \end{aligned} \tag{30}$$

[3] In the following, we will assume that the BPF is almost ideal all over the band of the signal $s(t)$ which have to be used as the input of the large-signal model of (19). In practice, non idealities of the BPF can be compensated by a suitable calibration procedure.

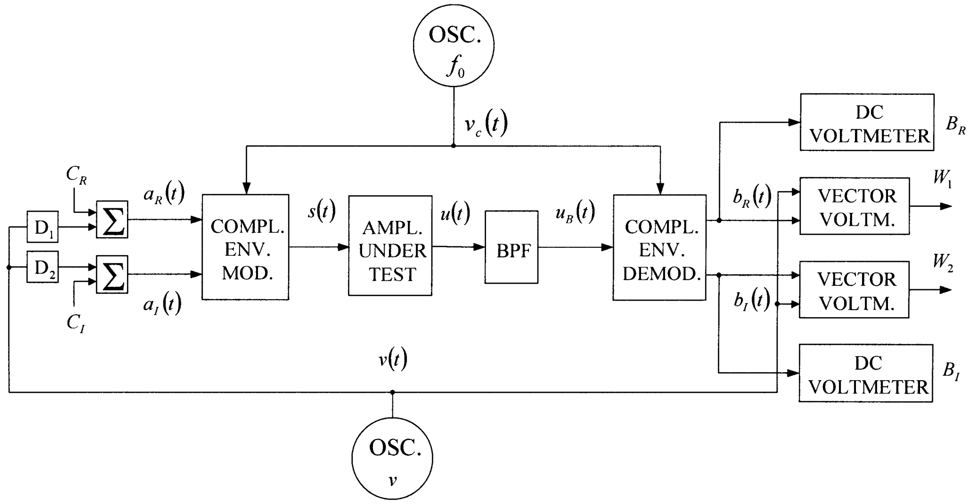

Fig. 2. Measurement setup for the experimental characterization of the terms $G_1(f_0+\nu_k, f_0|a(t)|)$ and $G_2(f_0+\nu_k, f_0, |a(t)|)$ in the large-signal model of (19) of the AUT.

which represent, respectively, the real and imaginary parts of the complex envelope $b(t) = b_r(t) + \mathrm{j}b(t)$ of the in-band output signal $u_B(t)$. Thus, the dc components B_R and B_I can be measured by a dc voltmeter, while the two in-band transfer functions W_1 (or W_2) represent the ratios of the phasor associated to the sinusoidal components of $b_R(t)$ [or $b_I(t)$] and $\nu(t)$. Consequently, from (29) and (30), we deduce

$$\begin{aligned} b(t) &= b_R(t) + \mathrm{j}b_I(t) \\ &= B + \frac{W_1 + \mathrm{j}W_2}{2} V \mathrm{e}^{\mathrm{j}2\pi\nu t} + \frac{W_1^* + \mathrm{j}W_2^*}{2} V^* \mathrm{e}^{-\mathrm{j}2\pi\nu t}. \end{aligned} \tag{31}$$

In order to characterize the terms $G_1(f_0+\nu_k, f_0, |a(t)|)$ and $G_2(f_0+\nu_k, f_0, |a(t)|)$ in (19), for positive values of the modulating frequency ν, the test signal

$$a(t) = C + C_1 \mathrm{e}^{\mathrm{j}2\pi\nu t} \tag{32}$$

with sufficiently small amplitude of the complex, term C_1 will be used, so that the inputs of the modulator can be expressed as

$$a_R(t) = C_R + \mathrm{Re}[C_1 \mathrm{e}^{\mathrm{j}2\pi\nu t}] \tag{33}$$

$$\begin{aligned} a_I(t) &= C_I + \mathrm{Im}[C_1 \mathrm{e}^{\mathrm{j}2\pi\nu t}] \\ &= C_I + \mathrm{Re}[-\mathrm{j}C_1 \mathrm{e}^{\mathrm{j}2\pi\nu t}]. \end{aligned} \tag{34}$$

Consequently, by also taking into account (25) and (26), the transfer functions D_1 and D_2 in Fig. 2 must satisfy the conditions

$$D_1 V = C_1 \tag{35}$$

$$D_2 V = -\mathrm{j}C_1 \tag{36}$$

or equivalently

$$D_2 = -\mathrm{j}D_1 \tag{37}$$

so that the complex modulation envelope of the input signal $s(t)$ becomes

$$a(t) = C + D_1 V \mathrm{e}^{\mathrm{j}2\pi\nu t} \tag{38}$$

where C, D_1, and $V = V_R + \mathrm{j}V_I$ are complex quantities. As the amplitude of the modulating signal V is chosen small enough in order to consider as significant only the linear contributions around $V_R = V_I = 0$, the model of (19) can be linearized with respect to $\nu(t)$, so that the corresponding complex modulation envelope $b(t)$ of the output signal $u_B(t)$ can be expressed (see Appendix B) in the following form:

$$\begin{aligned} b(t) \cong\, & CH(f_0, |C|) + D_1 V \mathrm{e}^{\mathrm{j}2\pi\nu t} \\ & \times \Bigg[\frac{|C|}{2} H'(f_0, |C|) + H(f_0, |C|) \\ & \quad + H_1(f_0+\nu, f_0) + G_1(f_0+\nu, f_0, |C|)\Bigg] \\ & + D_1^* V^* \mathrm{e}^{-\mathrm{j}2\pi\nu t} C^2 \\ & \times \left[\frac{1}{2|C|} H'(f_0, |C|) + G_2^*(f_0+\nu, f_0, |C|)\right]. \end{aligned} \tag{39}$$

By comparing this equation with (31), we can write

$$B = CH(f_0, |C|) \tag{40}$$

$$\begin{aligned} \frac{W_1{}' + \mathrm{j}W_2{}'}{2} = D_1 \Bigg[& \frac{|C|}{2} H'(f_0, |C|) + H(f_0, |C|) \\ & + H_1(f_0+\nu, f_0) \\ & + G_1(f_0+\nu, f_0, |C|)\Bigg] \end{aligned} \tag{41}$$

$$\begin{aligned} \frac{W_1^{*\prime} + \mathrm{j}W_2^{*\prime}}{2} = D_1^* C^2 \Bigg[& \frac{1}{2|C|} H'(f_0, |C|) \\ & + G_2^*(f_0+\nu, f_0, |C|)\Bigg]. \end{aligned} \tag{42}$$

Where W_1' and W_2' have been measured, for a set of values of $|C|$ and ν for a given carrier frequency f_0, by using the setup in Fig. 2 under the first test condition (i.e., for $D_1 = -\mathrm{j}D_2$). Equations (41) and (42) provide a simple way of extracting the unknown value of the complex functions H', H_1, G_1, and G_2^*, which completely characterize, together with H, the large signal dynamic model defined by the (14). In fact, since according to (B4), (B5), and (B6), it results

$$\lim_{\nu\to 0} H_1 = \lim_{\nu\to 0} G_1 = \lim_{\nu\to 0} G_2^* = 0 \tag{43}$$

so that (42) becomes

$$\frac{2|C|}{C^2}H'(f_0,|C|)=\frac{1}{D_1^*}\left[\lim_{\nu\to 0}\frac{W_1^{*\prime}+jW_2^{*\prime}}{2}\right]. \tag{44}$$

This shows that the unknown term H' is easily derived from measurements carried out under the first test condition with a small modulating signal having a sufficiently small frequency.

In order to complete the spectrum analysis, we must consider also when the frequency is negative $(-\nu)$; to this end, we use as test signal the quantity

$$a(t)=C+C_1\mathrm{e}^{-\mathrm{j}2\pi\nu t}. \tag{45}$$

By considering the second expressions of (33) and (34), we deduce

$$D_1^*V^*=C_1 \tag{46}$$
$$D_2^*V^*=-\mathrm{j}C_1. \tag{47}$$

Consequently, $D_2^*=-\mathrm{j}D_1^*$, or equivalently

$$D_2=\mathrm{j}D_1. \tag{48}$$

By substituting into (45), we obtain

$$a(t)=C+D_1^*V^*\mathrm{e}^{-\mathrm{j}2\pi\nu t}. \tag{49}$$

Proceeding in a similar manner as in the previous case with positive frequency ν, we can deduce the new expression of $b(t)$ obtained from (39) by substituting ν, D_1, and V, respectively, with $-\nu$, D_1^*, and V^*. Consequently, we obtain

$$\begin{aligned}b(t)\cong{}&CH(f_0,|C|)+D_1^*V^*e^{-j2\pi\nu t}\\&\times\left[\frac{|C|}{2}H'(f_0,|C|)+H(f_0,|C|)\right.\\&\left.\quad+H_1(f_0-\nu,f_0)+G_1(f_0-\nu,f_0,|C|)\right]\\&+C^2D_1Ve^{j2\pi\nu t}\\&\times\left[\frac{1}{2|C|}H'(f_0,|C|)+G_2^*(f_0+\nu,f_0,|C|)\right]\end{aligned} \tag{50}$$

and comparing again with (31), we obtain

$$B=CH(f_0,|C|) \tag{51}$$
$$\frac{W_1+\mathrm{j}W_2}{2}=D_1C^2\left[\frac{1}{2|C|}H'(f_0,|C|)+G_2^*(f_0-\nu,f_0,|C|)\right] \tag{52}$$
$$\begin{aligned}\frac{W_1^*+\mathrm{j}W_2^*}{2}={}&D_1^*\left[\frac{|C|}{2}H'(f_0,|C|)(1-\mathrm{j})+H(f_0,|C|)\right.\\&+H_1(f_0-\nu,f_0)\\&\left.+G_1(f_0-\nu,f_0,|C|)\right].\end{aligned} \tag{53}$$

In the particular case of $V=0$, i.e., $a(t)=C$ into (27), from (39) or (50), we obtain

$$b(t)=CH(f_0,|C|). \tag{54}$$

In this condition, the real and imaginary part of $H(f_0,|C|)$ can be measured with two dc voltmeters. By applying (54) for different values of $|C|$, we can estimate $H'(f_0,|C|)$. In the general case of V sufficiently low, for each value of $|C|$, the two complex quantities W_1 and W_2 can be measured.

A preliminary validation of the above outlined characterization procedure has been presented in [7]. The results show that performance predictions can be achieved which are more accurate than those provided by the conventional AM/AM AM/PM characterization approach.

IV. Conclusion

A nonlinear dynamic model of a power amplifier for communication systems, which takes into account the amplifier distortion due both to the large amplitude and the large bandwidth of the input signal, has been proposed. The model derives from the truncation of a new modified Volterra series expression formulated in terms of dynamic deviations of the complex modulation envelope which describes the input signal. This model can be considered as a generalization of the conventional AM/AM and AM/PM conversion characteristics which are commonly used to describe the nonlinear amplifier behavior for quasistatic memoryless operating conditions under the hypothesis of a relatively narrow-band input modulating signal. The proposed approach has the advantage of making the narrow-band modulation constraint much less restrictive. Preliminary validation results [7] have shown that significant accuracy improvement can be achieved in comparison with the conventional AM/AM and AM/PM approach. The proposed approach has been presented with special emphasis on communication circuits in the presence of modulated signals. However, it can be potentially applied also to other types of nonlinear dynamic systems operating with bandpass signals. In fact, any bandpass signal can be mathematically described as an equivalent modulation of a virtual carrier frequency allocated within the signal bandwidth. Further work will be devoted to the extension of the proposed modeling approach to the characterization of other types of nonlinear dynamic systems.

Appendix A

It can be shown [1] that the functional of the sum of two functions $\varphi(x)$ and $\psi(\mathrm{x})$ of the same real variable x can be expressed through the following series:

$$\begin{aligned}F\left|\left[\varphi(x)\overset{b}{\underset{a}{+}}\psi(x)\right]\right|={}&F\left|\left[\overset{b}{\underset{a}{\varphi(x)}}\right]\right|+\sum_{n=1}^{+\infty}\frac{1}{n!}\int_a^b\cdots\int\\&\times F^{(n)}\left|\left[\varphi\overset{b}{\underset{a}{(x)}};\xi_1,\ldots\xi_n\right]\right|\left[\prod_{i=1}^{n}[\psi(\xi_i)]\right]\mathrm{d}\xi_1\ldots\mathrm{d}\xi_n\end{aligned} \tag{A1}$$

where $F^{(n)}|[\varphi\overset{b}{\underset{a}{(x)}};\xi_1,\ldots.\xi_n]|$, with $\xi_1\ldots\ldots\xi_n$ convenient parameters, represents the nth derivative of the original functional $F|[\overset{b}{\underset{a}{\varphi(x)}}]|$. This formula gives an extension of the Taylor series to functions which depend on all the values of another function. On this basis (7) can be rewritten as follows:

$$u(t) = F\left|\left[\overset{+T_B}{\underset{0}{\hat{s}(t,\tau)}}\right]\right| + \sum_{n=1}^{+\infty}\frac{1}{n!}\int_0^{+T_B}\cdots\int \times F^{(n)}\left|\left[\overset{+T_B}{\underset{0}{\hat{s}(t,\tau)}};\tau_1\ldots\tau_n\right]\right|\left[\prod_{i=1}^{n}e(t,\tau_i)\right]\mathrm{d}\tau_1\ldots\mathrm{d}\tau_n. \quad \text{(A2)}$$

For the traditional Volterra series [1], [4], the functional $F|[\hat{s}(t,\tau)]$ can be expressed as follows:

$$\begin{aligned} F\left|\left[\overset{+T_B}{\underset{0}{\hat{s}(t,\tau)}}\right]\right| &= F\left|\left[\overset{+T_B}{\underset{0}{0}}\right]\right| + \sum_{m=1}^{+\infty}\frac{1}{m!}\int_0^{+T_B}\cdots\cdots\int F^{(m)} \\ &\quad\times\left|\left[\overset{+T_B}{\underset{0}{0}};\tau_1,\ldots\tau_m\right]\right|\left[\prod_{p=1}^{m}\hat{s}(t,\tau_p)\right]\mathrm{d}\tau_1\ldots\mathrm{d}\tau_m \\ &= y_0 + \sum_{m=1}^{+\infty}\frac{y_m(t)}{m!} \end{aligned} \quad \text{(A3)}$$

where

$$y_0 = F\left|\left[\overset{+T_B}{\underset{0}{0}}\right]\right| \quad \text{(A4)}$$

$$y_m(t) = \int_0^{+T_B}\cdots\cdots\cdots\int h_m(\tau_1,\ldots\tau_m) \times\prod_{p=1}^{m}\hat{s}(t,\tau_p)\mathrm{d}\tau_1\ldots\mathrm{d}\tau_m \quad \text{(A5)}$$

being

$$h_m(\tau_1\ldots\tau_m) = F^{(m)}\left|\left[\overset{+T_B}{\underset{0}{0}};\tau_1\ldots\tau_m\right]\right| \quad \text{(A6)}$$

the mth kernel of the Volterra series.

Each functional $F^{(n)}|[\hat{s}(t-\tau);\tau_1,\ldots\tau_n]|$, which is the nth derivative of the functional $F|[\hat{s}(t,\tau)]|$, can be expressed through the traditional Volterra series kernels, as follows [1]:

$$\begin{aligned} &F^{(n)}\left|\left[\overset{+T_B}{\underset{0}{\hat{s}(t,\tau)}};\tau_1,\ldots\tau_n\right]\right| \\ &= F^{(n)}\left|\left[\overset{+T_B}{\underset{0}{0}};\tau_1,\ldots\tau_n\right]\right| + \sum_{m=1}^{\infty}\frac{1}{m!}\int_0^{+T_B}\cdots\int F^{(n+m)} \\ &\quad\times\left|\left[\overset{+T_B}{\underset{0}{0}};\tau_1,\ldots\tau_{n+m}\right]\right|\left[\prod_{p=1}^{m}\hat{s}(t,\tau_{n+p})\right]\mathrm{d}\tau_1\ldots\mathrm{d}\tau_{n+m} \\ &= h_n(\tau_1,\tau_2\ldots\tau_n) + \sum_{m=1}^{\infty}\frac{1}{m!}\int_0^{+T_B}\cdots\int h_{n+m}(\tau_1\ldots\tau_{n+m}) \\ &\quad\times\left[\prod_{p=1}^{m}\hat{s}(t,\tau_{n+p})\right]\mathrm{d}\tau_1\ldots\mathrm{d}\tau_{n+m}. \end{aligned} \quad \text{(A7)}$$

By introducing (A3), (A5), and (A7) into (A2), we can write

$$u(t) = y_0 + u^{(1)}(t) + u^{(2)}(t) + u^{(3)}(t) \quad \text{(A8)}$$

where

$$\begin{aligned} u^{(1)}(t) &= \sum_{m=1}^{\infty}\frac{1}{m!}\int\cdots\overset{+T_B}{\underset{0}{}}\cdots\int h_m(\tau_1\ldots\tau_n) \\ &\quad\times\left[\prod_{p=1}^{m}\hat{s}(t,\tau_p)\right]\mathrm{d}\tau_1\ldots\mathrm{d}\tau_m \end{aligned} \quad \text{(A9)}$$

$$\begin{aligned} u^{(2)}(t) &= \sum_{n=1}^{\infty}\frac{1}{n!}\int\cdots\overset{+T_B}{\underset{0}{}}\cdots\int h_n(\tau_1\ldots\tau_n) \\ &\quad\times\left[\prod_{i=1}^{n}e(t,\tau_i)\right]\mathrm{d}\tau_1\ldots\mathrm{d}\tau_n \\ &\cong \int_0^{+T_B} h_1(\tau_1)e(t,\tau_1)\mathrm{d}\tau_1 \end{aligned} \quad \text{(A10)}$$

$$\begin{aligned} u^{(3)}(t) &= \sum_{n=1}^{\infty}\frac{1}{n!}\sum_{m=1}^{\infty}\frac{1}{m!}\int\cdots\overset{+T_B}{\underset{0}{}}\cdots\int h_{n+m}(\tau_1\ldots\tau_{1+m}) \\ &\quad\times\left[\prod_{p=1}^{m}\hat{s}(t,\tau_{n+p})\right]\left[\prod_{i=1}^{n}e(t,\tau_i)\right]\mathrm{d}\tau_1\ldots\mathrm{d}\tau_{n+m} \\ &\cong \sum_{m=1}^{\infty}\frac{1}{m!}\int\cdots\overset{+T_B}{\underset{0}{}}\cdots\int h_{1+m}(\tau_1\ldots\tau_{1+m}) \\ &\quad\times\left[\prod_{p=1}^{m}\hat{s}(t,\tau_{1+p})\right]e(t,\tau_1)\mathrm{d}\tau_1\ldots\mathrm{d}\tau_{1+m}. \end{aligned} \quad \text{(A11)}$$

The components $u^{(2)}(t)$ and $u^{(3)}(t)$ have been approximated, by considering uniquely $n=1$, on the hypothesis that the bandwidth of the complex modulation envelope is so small as to make the product of the amplitude of the dynamic deviations $e(t,\tau_i)$ for each τ_i almost negligible in practice. Now, we consider only the spectral components within the operating bandwidth ($\pm f_0$), so that we have

$$u_B(t) = u_B^{(1)}(t) + u_B^{(2)}(t) + u_B^{(3)}(t). \quad \text{(A12)}$$

In order to obtain the spectral contribution $u_B^{(1)}(t)$ within the operating bandwidth, we must develop into (A9) the product [see (5)]

$$\prod_{p=1}^{m}\hat{s}(t,\tau_p) = |a(t)|^m\sum_{\substack{i_1=-1\\ i_1\neq 0}}^{+1}\cdots\cdot\sum_{\substack{i_m=-1\\ i_m\neq 0}}^{+1}\mathrm{e}^{-\mathrm{j}2\pi f_0(i_1\tau_1+\ldots.i_m\tau_m)}. \quad \text{(A13)}$$

Its contribution within the operating bandwidth, by introducing $a(t) = |a(t)|e^{j\angle a(t)}$ [see (3)], is

$$\begin{aligned}
&\left\{\prod_{p=1}^{m} \hat{s}(t,\tau_p)\right\}_B \\
&= a(t)e^{j2\pi f_0 t}\,|a(t)|^{m-1} \sum_{\substack{i_1=-1\\ i_1\neq 0\\ i_1+i_2+\ldots i_m=+1}}^{+1} \cdots \sum_{\substack{i_m=-1\\ i_m\neq 0}}^{+1} \\
&\quad\times e^{-j2\pi f_0(i_1\tau_1+\ldots i_m\tau_m)} + a^*(t)e^{-j2\pi f_0 t}\,|a(t)|^{m-1} \\
&\quad\times \sum_{\substack{i_1=-1\\ i_1\neq 0\\ i_1+i_2+\ldots i_m=-1}}^{+1} \cdots \sum_{\substack{i_m=-1\\ i_m\neq 0}}^{+1} e^{j(i_1+\ldots+i_m)\left(2\pi f_0 t+\angle a(t)\right)} \\
&\quad\times e^{-j2\pi f_0(i_1\tau_1+\ldots i_m\tau_m)} \\
&= 2\,|a(t)|^{m-1}\,\mathrm{Re}\left[a(t)e^{j2\pi f_0 t} \sum_{\substack{i_1=-1\\ i_1\neq 0\\ i_1+\ldots i_m=1}}^{+1} \cdots \sum_{\substack{i_m=-1\\ i_m\neq 0}}^{+1} \times e^{-j2\pi f_0(i_1\tau_1+\ldots+i_m\tau_m)}\right].
\end{aligned} \tag{A14}$$

The last passage is due to the fact that each term of the first double sum (relative to $i_1 + \ldots + i_m = 1$) it corresponds, also since the result must be real, its global conjugate in the second double sum (relative to $i_1 + \ldots + i_m = -1$). It is important to emphasise that the contributions within the operating bandwidth are non null only for odd values of m. Finally, we can write

$$\begin{aligned}
u_B^{(1)}(t) &= 2\sum_{\substack{m=1\\ m\ \text{odd}}}^{+\infty} \frac{|a(t)|^{m-1}}{m!} \int_0^{+T_B}\!\!\cdots\!\int h_m(\tau_1,\ldots,\tau_m) \\
&\quad\times \mathrm{Re}\left[a(t)e^{j2\pi f_0 t} \sum_{\substack{i_1=-1\\ i_1+\ldots+i_m=1}}^{+1} \cdots \sum_{i_m=-1}^{+1} \times e^{-j2\pi f_0(i_1\tau_1+\ldots+i_m\tau_m)}\mathrm{d}\tau_1\ldots\mathrm{d}\tau_m\right] \\
&= 2\mathrm{Re}\left[a(t)e^{j2\pi f_0 t} H\left(f_0, |a(t)|\right)\right]
\end{aligned} \tag{A15}$$

where we have assumed

$$\begin{aligned}
&H\left(f_0, |a(t)|\right) \\
&= \sum_{m=1}^{\infty} \frac{|a(t)|^{m-1}}{m!} \sum_{\substack{i_1=-1\\ i_1\neq 0\\ i_1+\ldots+i_m=1}}^{+1} \cdots \sum_{\substack{i_m=-1\\ i_m\neq 0}}^{+1} \times H_m(i_1 f_0,\ldots,i_m f_0)
\end{aligned} \tag{A16}$$

$$\begin{aligned}
&H_m(i_1 f_0,\ldots,i_m f_0) \\
&= \int \cdots \overset{+T_B}{\underset{0}{}} \cdots \int h_m(\tau_1,\ldots\tau_m) \times e^{-j2\pi f_0(i_1\tau_1+\ldots i_m\tau_m)}\mathrm{d}\tau_1\ldots\mathrm{d}\tau_m
\end{aligned} \tag{A17}$$

which are complex quantities. The second component $u^{(2)}(t)$ (A10) is already in bandwidth; in fact, by remembering (6), we obtain

$$\begin{aligned}
u^{(2)}(t) &\cong 2\mathrm{Re}\left\{e^{j2\pi f_0 t}\int_0^{+T_B} h_1(\tau_1)[a(t-\tau_1)-a(t)]e^{-j2\pi f_0\tau_1}\mathrm{d}\tau_1\right\} \\
&= u_B^{(2)}(t).
\end{aligned} \tag{A18}$$

By now taking into account $u^{(3)}(t)$ (A11), we observe that, by recalling (6), we have

$$\begin{aligned}
&\prod_{p=1}^{m} \hat{s}(t,\tau_{1+p})e(t,\tau_1) \\
&= 2\mathrm{Re}\left[\prod_{p=1}^{m} \hat{s}(t,\tau_{1+p})\left(a(t-\tau_1)-a(t)\right)e^{j2\pi f_0(t-\tau_1)}\right]
\end{aligned} \tag{A19}$$

due to the fact that $\hat{s}(t,\tau_{1+p})$ is real. On the other hand, in analogy with (A13), by introducing (5), we obtain

$$\begin{aligned}
&\prod_{p=1}^{m} \hat{s}(t,\tau_{1+p})e(t,\tau_{1+p}) \\
&= 2\,|a(t)|^m \\
&\quad\cdot \mathrm{Re}\left[e^{j2\pi f_0 t}\left(a(t-\tau_1)-a(t)\right)e^{j2\pi f_0\tau_1}\right. \\
&\quad\times\left(\sum_{\substack{i_2=-1\\ i_2\neq 0}}^{+1} \cdots \sum_{\substack{i_{m+1}=-1\\ i_{m+1}\neq 0}}^{+1} e^{j(i_2+\ldots+i_{m+1})\left(2\pi f_0 t+\angle a(t)\right)}\right. \\
&\quad\left.\left.\times e^{-j2\pi f_0(i_2\tau_2+\ldots i_{m+1}\tau_{m+1})}\right)\right].
\end{aligned} \tag{A20}$$

By considering only the contributions with respect to t within the operating bandwidth ($\pm f_0$), we have

$$\begin{aligned}
&\prod_{p=1}^{m} \hat{s}(t,\tau_p)e(t,\tau_{1+p}) \\
&= 2\,|a(t)|^m\,\mathrm{Re}\left[e^{j2\pi f_0 t}\left(a(t-\tau_1)-a(t)\right)e^{-j2\pi f_0\tau_1}\right. \\
&\quad\times\left(\sum_{\substack{i_2=-1\\ i_2\neq 0\\ i_2+\ldots+i_{m+1}=0}}^{+1} \cdots \sum_{\substack{i_{m+1}=-1\\ i_{m+1}\neq 0}}^{+1} \left.\times e^{-j2\pi f_0(i_2\tau_2+\ldots i_{m+1}\tau_{m+1})}\right)\right] \\
&\quad+ 2\,|a(t)|^m\,\mathrm{Re}\left[e^{-j2\pi f_0 t}\left(a(t-\tau_1)-a(t)\right)e^{-j2\pi f_0\tau_1}\right. \\
&\quad\times\left(\sum_{\substack{i_2=-1\\ i_2\neq 0\\ i_2+\ldots+i_{m+1}=-2}}^{+1} \cdots \sum_{\substack{i_{m+1}=-1\\ i_{m+1}\neq 0}}^{+1} \times e^{-j2\pi f_0(i_2\tau_2+\ldots i_{m+1}\tau_{m+1})}\right) \\
&\quad\left.\times e^{-j2\angle a(t)}\right].
\end{aligned} \tag{A21}$$

Obviously, the non-null contributions derive uniquely from even values of m, in order to verify the equalities $i_2 + \ldots + i_{m+1} =$

0 (referring to $-f_0$) and $i_2+\ldots.+i_{m+1}=-2$ (referring to $-f_0$). Now, by recalling (3), for which we can write

$$\mathrm{e}^{-\mathrm{j}2\angle a(t)}=\frac{a^*(t)}{|a(t)|} \tag{A22}$$

we obtain

$$\begin{aligned}\prod_{p=1}^{m}&\hat{s}(t,\tau_p)e(t,\tau_{1+p})\\ &=2\,|a(t)|^m\,\mathrm{Re}\left[\mathrm{e}^{\mathrm{j}2\pi f_0 t}\left(a(t-\tau_1)-a(t)\right)\mathrm{e}^{-\mathrm{j}2\pi f_0\tau_1}\right.\\ &\quad\times\left(\sum_{\substack{i_2=-1\\ i_2\neq 0\\ i_2+\ldots+i_{m+1}=0}}^{+1}\ldots.\sum_{\substack{i_{m+1}=-1\\ i_{m+1}\neq 0}}^{+1}\right.\\ &\quad\left.\left.\times\mathrm{e}^{-\mathrm{j}2\pi f_0(i_2\tau_2+\ldots.i_{m+1}\tau_{m+1})}\right)\right]\\ &\quad+2\,|a(t)|^{m-2}\left(a^*(t)\right)^2\\ &\quad\times\mathrm{Re}\left[\mathrm{e}^{-\mathrm{j}2\pi f_0 t}\left(a(t-\tau_1)-a(t)\right)\mathrm{e}^{-\mathrm{j}2\pi f_0\tau_1}\right.\\ &\quad\times\left(\sum_{\substack{i_2=-1\\ i_2\neq 0\\ i_2+\ldots+i_{m+1}=-2}}^{+1}\ldots.\sum_{\substack{i_{m+1}=-1\\ i_{m+1}\neq 0}}^{+1}\right.\\ &\quad\left.\left.\times\mathrm{e}^{-\mathrm{j}2\pi f_0(i_2\tau_2+\ldots.i_{m+1}\tau_{m+1})}\right)\right].\end{aligned} \tag{A23}$$

From (A11), by taking into account (A21), and so considering only the contribution of $u^{(3)}(t)$ within the operating bandwidth, we can write

$$\begin{aligned}u_B^{(3)}(t)=&\,2\mathrm{Re}\left[\mathrm{e}^{\mathrm{j}2\pi f_0 t}\int_0^{+T_B}\left(a(t-\tau_1)-a(t)\right)\mathrm{g}_1\right.\\ &\left.\times\left(\tau_1,f_0,|a(t)|\right)\mathrm{e}^{-\mathrm{j}2\pi f_0\tau_1}\mathrm{d}\tau_1\right]\\ &+2\mathrm{Re}\left[\mathrm{e}^{-\mathrm{j}2\pi f_0 t}\left(a^*(t)\right)^2\int_0^{+T_B}\left(a(t-\tau_1)-a(t)\right)\right.\\ &\left.\times\mathrm{g}_2\left(\tau_1,f_0,|a(t)|\right)\mathrm{e}^{-\mathrm{j}2\pi f_0\tau_1}\mathrm{d}\tau_1\right]\end{aligned} \tag{A24}$$

where

$$\begin{aligned}\mathrm{g}_1(\tau_1,f_0,|a(t)|)=&\sum_{m=1}^{\infty}\frac{|a(t)|^{2m}}{2m!}\sum_{\substack{i_2=-1\\ i_2\neq 0\\ i_2+\ldots i_{2m+1}=0}}^{+1}\ldots\sum_{\substack{i_{2m+1}=-1\\ i_{2m+1}\neq 0}}^{+1}\\ &\times H_{2m+1}^{(3)}(\tau_1,i_2f_0,\ldots,i_{2m+1}f_0)\end{aligned} \tag{A25}$$

$$\begin{aligned}\mathrm{g}_2(\tau_1,f_0,|a(t)|)=&\sum_{m=1}^{\infty}\frac{|a(t)|^{2m-2}}{2m!}\sum_{\substack{i_2=-1\\ i_2\neq 0\\ i_2+\ldots i_{2m+1}=-2}}^{+1}\ldots\sum_{\substack{i_{2m+1}=-1\\ i_{2m+1}\neq 0}}^{+1}\\ &\times H_{2m+1}^{(3)}(\tau_1,i_2f_0,\ldots,i_{2m+1}f_0)\end{aligned} \tag{A26}$$

and

$$\begin{aligned}H_{2m+1}^{(3)}&(\tau_1,i_2f_0,\ldots,i_{2m+1}f_0)\\ &=\int\ldots\overset{+T_B}{\underset{0}{}}\ldots\int h_{2m+1}(\tau_1,\ldots\tau_{2m+1})\\ &\quad\times\mathrm{e}^{-\mathrm{j}2\pi f_0(i_2\tau_2+\ldots+i_{2m+1}\tau_{2m+1})}\\ &\quad\times d\tau_2\ldots d\tau_{2m+1}\end{aligned} \tag{A27}$$

which are complex quantities. Being

$$\begin{aligned}H_{2m+1}^{(3)}&(\tau_1,-i_2f_0,\ldots,-i_{2m+1}f_0)\\ &=H_{2m+1}^{(3)^*}(\tau_1,i_2f_0,\ldots,i_{2m+1}f_0)\end{aligned}$$

we have

$$\mathrm{g}_2\left(\tau_1,-f_0,|a(t)|\right)=\mathrm{g}_2^*\left(\tau_1,f_0,|a(t).|\right)$$

By taking into account (A15), (A18), (A24), and, in the second term of this last by substituting the real part of the complex quantity with the real part of its conjugate, the "in-band" output signal (A12) can, therefore, be expressed as follows:

$$\begin{aligned}u_B(t)=&\,2\mathrm{Re}\left[\mathrm{e}^{\mathrm{j}2\pi f_0 t}a(t)H\left(f_0,|a(t)|\right)\right]\\ &+2\mathrm{Re}\left[\mathrm{e}^{\mathrm{j}2\pi f_0 t}\int_0^{+T_B}h_1(\tau_1)\left[a(t-\tau_1)-a(t)\right]\right.\\ &\left.\times\mathrm{e}^{-\mathrm{j}2\pi f_0\tau_1}d\tau_1\right]\\ &+2\mathrm{Re}\left[\mathrm{e}^{\mathrm{j}2\pi f_0 t}\int_0^{+T_B}\mathrm{g}_1\left(\tau_1,f_0,|a(t)|\right)\right.\\ &\left.\times\left(a(t-\tau_1)-a(t)\right)\mathrm{e}^{-\mathrm{j}2\pi f_0\tau_1}d\tau_1\right]\\ &+2\mathrm{Re}\left[\mathrm{e}^{\mathrm{j}2\pi f_0 t}a^2(t)\int_0^{+T_B}\mathrm{g}_2^*\left(\tau_1,f_0,|a(t)|\right)\right.\\ &\times\left(a^*(t-\tau_1)-a^*(t)\right)\\ &\left.\times\mathrm{e}^{\mathrm{j}2\pi f_0\tau_1}d\tau_1\right].\end{aligned} \tag{A28}$$

Appendix B

In this sense, by referring to (27) and (38), we develop the modulus of $a(t)$ around $V=0$, that is

$$\begin{aligned}|a(t)|&=\sqrt{a(t)a^*(t)}\\ &=\sqrt{|C|^2+|D_1|^2|V|^2+2\mathrm{Re}[C^*D_1V\mathrm{e}^{\mathrm{j}2\pi\nu t}]}\\ &\cong|C|+\frac{1}{|C|}\mathrm{Re}[C^*D_1V\mathrm{e}^{\mathrm{j}2\pi\nu t}].\end{aligned} \tag{B1}$$

Besides, by considering negligible the non linear terms in V, we can write

$$\begin{aligned}a^2(t)&=(C+D_1V\mathrm{e}^{\mathrm{j}2\pi\nu t})(C+D_1V\mathrm{e}^{\mathrm{j}2\pi\nu t})\\ &\cong C^2+2CD_1V\mathrm{e}^{\mathrm{j}2\pi\nu t}.\end{aligned} \tag{B2}$$

By introducing these approximations and (38) into (19), we obtain

$$\begin{aligned}b(t)\cong&\left(C+D_1V\mathrm{e}^{\mathrm{j}2\pi\nu t}\right)H\\ &\times\left(f_0,|C|+\frac{1}{|C|}\mathrm{Re}[D_1C^*V\mathrm{e}^{\mathrm{j}2\pi\nu t}]\right)\\ &+D_1V\mathrm{e}^{\mathrm{j}2\pi\nu t}H_1(f_0+\nu,f_0)+D_1V\mathrm{e}^{\mathrm{j}2\pi\nu t}G_1\\ &\times\left(f_0+\nu,f_0,|C|+\frac{1}{|C|}\mathrm{Re}[D_1C^*V\mathrm{e}^{\mathrm{j}2\pi\nu t}]\right)\\ &+C^2D_1^*V^*\mathrm{e}^{-\mathrm{j}2\pi\nu t}G_2^*\\ &\times\left(f_0+\nu,f_0,|C|+\frac{1}{|C|}\mathrm{Re}[D_1C^*V\mathrm{e}^{\mathrm{j}2\pi\nu t}]\right)\end{aligned} \tag{B3}$$

where we have assumed

$$H_1(f_0+\nu, f_0) = \int_0^{+T_B} h_1(\tau_1)(\mathrm{e}^{-\mathrm{j}2\pi\nu\tau_1}-1)\mathrm{e}^{-\mathrm{j}2\pi f_0\tau_1}d\tau_1 \quad \text{(B4)}$$

$$G_1\left(f_0+\nu, f_0, |C|+\frac{1}{|C|}\mathrm{Re}[C^*D_1V\mathrm{e}^{\mathrm{j}2\pi\nu t}]\right) = \int_0^{+T_B} \mathrm{g}_1\left(\tau_1, f_0, |C|+\frac{1}{|C|}\mathrm{Re}[C^*D_1Ve^{\mathrm{j}2\pi\nu t}]\right) \times \{\mathrm{e}^{-\mathrm{j}2\pi\nu\tau_1}-1\}\mathrm{e}^{-\mathrm{j}2\pi f_0\tau_1}d\tau_1 \quad \text{(B5)}$$

$$G_2\left(f_0+\nu, f_0, |C|+\frac{1}{|C|}\mathrm{Re}[C^*D_1V\mathrm{e}^{\mathrm{j}2\pi\nu t}]\right) = \int_0^{+T_B} \mathrm{g}_2\left(\tau_1, f_0, |C|+\frac{1}{|C|}\mathrm{Re}[C^*D_1V\mathrm{e}^{\mathrm{j}2\pi\nu t}]\right) \times (\mathrm{e}^{-\mathrm{j}2\pi\nu\tau_1}-1)\mathrm{e}^{-\mathrm{j}2\pi f_0\tau_1}d\tau_1 \quad \text{(B6)}$$

with $G_1(f_0, f_0, |C| + (1/|C|)\mathrm{Re}[C^*D_1V\mathrm{e}^{\mathrm{j}2\pi\nu t}]) = 0$, $G_2(f_0, f_0, |C| + (1/|C|)\mathrm{Re}[C^*D_1V\mathrm{e}^{\mathrm{j}2\pi\nu t}]) = 0$, and $H_1(f_0, f_0) = 0$. Let us now again develop (B3) around $V = 0$; by recalling that we consider negligible the nonlinear terms in V, we deduce

$$b(t) \cong CH(f_0,|C|) + \frac{C}{|C|}H'(f_0,|C|)\,\mathrm{Re}[D_1C^*\mathrm{e}^{\mathrm{j}2\pi\nu t}V] + D_1V\mathrm{e}^{\mathrm{j}2\pi\nu t}[H(f_0,|C|)+H_1(f_0+\nu,f_0) + G_1(f_0+\nu,f_0,|C|)] + C^2D_1^*V^*\mathrm{e}^{-\mathrm{j}2\pi\nu t} \times G_2^*(f_0+\nu,f_0,|C|) \quad \text{(B7)}$$

where

$$H'(f_0,|C|) = \left.\frac{\mathrm{d}H(f_0,x)}{\mathrm{d}x}\right|_{x=|C|}.$$

The second term of this equation can be rewritten as follows:

$$\frac{C}{|C|}H'(f_0,|C|)\,\mathrm{Re}[D_1C^*\mathrm{e}^{\mathrm{j}2\pi\nu t}V] = \frac{C}{2|C|}H'(f_0,|C|)\left(C^*D_1V\mathrm{e}^{\mathrm{j}2\pi\nu t}+CD_1^*V^*\mathrm{e}^{-\mathrm{j}2\pi\nu t}\right). \quad \text{(B8)}$$

By introducing this expression into (B7), after simple manipulations, we obtain

$$b(t) \cong CH(f_0,|C|) + D_1V\mathrm{e}^{\mathrm{j}2\pi\nu t} \times \left[\frac{|C|}{2}H'(f_0,|C|)+H(f_0,|C|) + H_1(f_0+\nu,f_0)+G_1(f_0+\nu,f_0,|C|)\right] + D_1^*V^*\mathrm{e}^{-\mathrm{j}2\pi\nu t}C^2 \times \left[\frac{1}{2|C|}H'(f_0,|C|)+G_2^*(f_0+\nu,f_0,|C|)\right]. \quad \text{(B9)}$$

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Domenico Mirri was born in Italy in 1936. He received the electronic engineering degree from the University of Bologna, Bologna, Italy.

He is currently a Full Professor of electronic measurement at the University of Bologna. His current research interest is in the areas of digital measurement instruments, devices metrological characterization, and biomedical measurements.

Fabio Filicori was born in Italy in 1949. He received the degree in electronic engineering from the University of Bologna, Bologna, Italy, in 1964.

He is currently a Full Professor of applied electronics at the University of Bologna. His current research interests are in the areas of nonlinear circuit analysis and design, electronic devices modeling, digital measurement instruments, and power electronics.

Gaetano Iuculano was born in Italy in 1938. He received the degree in electronic engineering from the University of Bologna, Bologna, Italy.

He is currently a Full Professor of electrical measurements at the University of Florence, Florence, Italy. His current research interests are in calibration applications, reliability analysis and life testing for electronic devices and systems, statistical analysis, and digital measurement instruments.

Gaetano Pasini was born in Italy in 1964. He received the degree in electronic engineering from the University of Bologna, Bologna, Italy.

He is currently a Researcher in electrical measurement. His research activity is mainly focused on digital signal processing in electronic instruments, power measurements, and characterization of nonlinear systems with memory.