

Performance Function for Time-Jittered Equispaced Sampling Wattmeters

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Abstract—This paper evaluates the effect of time-jitter in the equally spaced sampling wattmeters on the hypothesis of equal effects in the two channels and a jitter uncorrelated with the input signals. It is shown that time-jitter, which is a random fluctuation with respect to the nominal sampling time, introduces a frequency limitation which is evaluated together with that due to the sampling strategy and filtering algorithm. The theoretical results are compared with the simulated ones.

I. INTRODUCTION

RANDOM fluctuation with respect to the nominal sampling instant is commonly called timing-jitter. Its effect on the amplitude and phase estimates of sampled signals [1], [2] and on the recovery of a signal from noise by averaging [3] has been studied by different authors. This paper investigates the effect of time-jitter in digital wattmeters based on equally spaced sampling techniques on the hypothesis of equal random fluctuations in the voltage and current channels and a jitter uncorrelated with the input signals. Section II summarizes the properties of the equally spaced synchronous, quasynchronous and asynchronous sampling strategies in terms of accuracy and bandwidth [4], [5]. Section III quantifies the effect of time-jitter and expresses the estimate of the output as a function of the sampling instants. Section IV gives the expressions of the asymptotic parameters which characterize the estimate of the output of the instrument [6], [7] and compares the theoretical values with the experimental results.

II. EQUALLY SPACED SAMPLING STRATEGIES

Let us assume that the instantaneous power $p(t)$ is periodic with period T_1 ; therefore it can be represented by the Fourier series:

$$p(t) = \sum_{q=-\infty}^{+\infty} P_q e^{j2\pi q f_1 t} = P_0 + \sum_{\substack{q=-\infty \\ q \neq 0}}^{+\infty} P_q e^{j2\pi q f_1 t} \quad (1)$$

where: $f_1 = 1/T_1$, $P_{-q} = P_q^*$, and P_0 is the mean value of $p(t)$, i.e., the value of the measurand. The distribution of power in the signal $p(t)$ as a function of frequency can be deduced from the power density spectrum $S_p(f)$:

$$S_p(f) = \sum_{q=-\infty}^{+\infty} |P_q|^2 \delta(f - q f_1) \quad (2)$$

where $\delta(f)$ is the Dirac function.

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For the digital implementation of a wattmeter, the instantaneous power $p(t) = v(t)i(t)$ is evaluated, at any discrete time instant t_i , by multiplying the sampled values of the instantaneous voltage and current. On the hypothesis of an adequate resolution, the digital multiplication does not introduce any relevant error, and the sampling procedure can be assumed directly effected on the instantaneous power when the main aim is the theoretical study of the errors due to the sampling strategy and the filtering algorithm.

The sampling techniques here considered are of an equally spaced type, i.e., any sampling instant t_i differs from the preceding one by a constant lag T_c :

$$t_i = t_{i-1} + T_c = \tau_0 + iT_c \quad (3)$$

where i is an integer, T_c is the sampling interval, and τ_0 is the shift between the initial sampling instant and the time origin of P_0 . In the synchronous case τ_0 is related to the instantaneous power $p(t)$ through the synchronizing circuit, while in the asynchronous one it is independent.

The discrete output of the instrument is an estimate of P_0 and can be deduced by forming a weighted average of the last N successive values of the instantaneous power (finite impulse response filter, FIR):

$$\hat{P}_k = \sum_{i=0}^{N-1} a_i p(t_{k-i}) \quad (4)$$

where each output is marked with an integer k . The estimate \hat{P}_k continuously changes in an asynchronous sampling strategy, while it remains unchanged in the synchronous one ($\hat{P}_k = P$) because the N sampling instants of any successive average are always the same.

By substituting (1) and (3) into (4) we can write:

$$\begin{aligned} \hat{P}_k &= \sum_{q=-\infty}^{+\infty} P_q \sum_{i=0}^{N-1} a_i e^{j2\pi q f_1 t_{k-i}} \\ &= P_0 \sum_{i=0}^{N-1} a_i + \sum_{\substack{q=-\infty \\ q \neq 0}}^{+\infty} P_q e^{j2\pi q f_1 (\tau_0 + kT_c)} H(q f_1 T_c) \end{aligned} \quad (5)$$

where $H(fT_c)$ is the frequency response of the finite impulse response (FIR) filter used; this response is periodic of period f_c . In order to obtain a scale factor equal to one in (5), the sum of the N coefficients a_i must be equal to one. Obviously the coefficients a_i must be selected to obtain a frequency response which adequately attenuates the contribution to \hat{P}_k of each

spectral component of $p(t)$. By assuming $a_i = 1/N$ (rectangular window) we obtain the well-known Dirichlet-Kernel formula:

$$H(fT_c) = e^{-j\pi fT_c(N-1)} \frac{\text{sinc}(fNT_c)}{\text{sinc}(fT_c)} \quad (6)$$

which is a periodic function assuming a value equal to one for fT_c integer and, in the first period, values equal to zero for $fT_c = i/N$, with $i = 0, 1, \dots, N-1$. Therefore, by using a synchronous sampling strategy in which f_1T_c coincides with one of the zeroes of $H(fT_c)$:

$$f_1T_c = \frac{r}{N}. \quad (7)$$

Selecting r as a positive integer having no common submultiple with N , only the spectral components of $p(t)$ multiple integer of N contribute to the output [4], i.e., when $q = zN$ (z integer) in (5):

$$\hat{P} = P_0 + \sum_{\substack{z=-\infty \\ z \neq 0}}^{+\infty} P_{zN} e^{j2\pi z r T_0 f_c}. \quad (8)$$

The output becomes independent from k and the bandwidth, relative to $p(t)$, results therefore in $(N-1)f_1T_c$ and N in (7) can be selected independently provided that the signals do not change in the observation (or summation) interval NT_c , i.e., the interval in which the N samples are taken, the interval which is equal to an integer number of periods T_1 ($NT_c = rT_1$).

By imposing for every $q \neq zN$ the condition

$$|H(qf_1T_c)| \leq \frac{1}{N} \quad (9)$$

the contribution to \hat{P}_k of the corresponding spectral components is not null as in the synchronous sampling strategy, but it can be assumed negligible if N is adequately high. Equation (8) still being approximately valid, this asynchronous sampling strategy has been called quasisynchronous. It can be shown that (9) is satisfied when:

$$|qNf_1T_c - \langle qNf_1T_c \rangle| \leq |qf_1T_c - \langle qf_1T_c \rangle| \quad (10)$$

where the symbol $\langle x \rangle$ means the nearest integer to x . On the hypothesis of $f_1T_c < 0.5$, the condition relative to the fundamental component ($q = 1$) becomes [5]:

$$|NT_c - r_1T_1| \leq T_c \quad \text{for } f_1T_c < 0.5 \quad (11)$$

where r_1 is the nearest integer to NT_c/T_1 . This condition must be compared with that relative to the synchronous sampling strategy (7). In the quasisynchronous case f_1 and T_c are uncorrelated; therefore (7) cannot be satisfied. We can nevertheless conveniently adjust N so that the observation interval NT_c becomes as close as possible to an integer number of periods T_1 . Once N is fixed, the bandwidth is evaluated determining the minimum value of q which does not satisfy (10).

In the asynchronous case the bandwidth is instead limited by the periodicity of the function $H(fT_c)$, i.e., f_c , because the product fT_c can assume any value. Therefore the FIR filter must be selected in such a way to minimize $|H(fT_c)|$ within this interval.

III. THE EFFECT OF TIMING JITTER IN THE EQUALLY SPACED SAMPLING STRATEGIES

Time-jitter introduces random fluctuations with respect to the nominal sampling instants. On the hypothesis of common random fluctuations in the voltage and current channels, the effect of these fluctuations can be analyzed by adding to the second member of (3) the quantity X_iT_c , where X_iT_c is the i th deviation from the nominal sampling time (the random variables are written in boldface). The output of the instrument can therefore be evaluated by assuming in (4):

$$t_{k-i} = \tau_0 + (k-i + X_{k-i})T_c. \quad (12)$$

The nuisance or incidental parameter X_{k-i} is the i th of a set of independent random variables having a continuous distribution with characteristic function $\Phi(fT_c)$, null mean value and variance σ^2 .

A generic discrete output of the instrument is identified by the labeling mark k , an integer which can be considered randomly picked up from a sequence of $2h+1$ possible successive values ($-h \leq k \leq +h$), each one of the outputs having an equal chance of being selected [6], [7]. In the synchronous case the shift τ_0 depends on the synchronizing circuit; in the asynchronous one, instead, it is strictly related to the turn-on instant of the instrument. In both cases τ_0 is *a priori* unknown, and its actual value can be considered a realization of a continuous set of values that are equally likely to fall anywhere within some generic time interval ($-T/2, +T/2, T$ being unknown). Therefore we introduce τ_0 as a continuous random variable uniformly distributed in a time interval T and k as a discrete random variable uniformly distributed in the interval $2h+1$. Obviously, the variability of the output of the instrument when the sampling strategy is synchronous without time-jitter is due solely to the hypothesized variability of τ_0 [7].

When each random variable X_i is expressly generated by the instrument with a continuous uniform distribution in an interval $\pm T_c/2$, (12) describes a random sampling strategy whose properties were previously studied by the authors [8].

IV. PERFORMANCE ANALYSIS

The output quantity $\hat{P}_k = f(\tau_0, \mathbf{X}_k)$ is a function of the random variables k , τ_0 and the vector \mathbf{X}_k of the N random variables X_{k-i} used to obtain the output labeled with k . An appropriate characterization of the output uncertainty can be obtained by evaluating the statistical parameters of \hat{P}_k , i.e., the mean value $M\{\hat{P}_k\}$ and the mean square error $M\{(\hat{P}_k - P_0)^2\}$. In order to incorporate all the *a priori* chances into the instrument's performance, in the asynchronous case the number of the output states, i.e., $2h+1$ must be sufficiently large and theoretically tend to infinity; in the synchronous one it is the excursion of the initial shift τ_0 , i.e., T , which must instead tend to infinity [7]. Therefore, in both cases, we consider the asymptotic statistical parameter, i.e., the asymptotic mean $\bar{P} = \underline{M}\{\hat{P}_k\}$ and the asymptotic mean square error:

$$E^2 = \underline{M}\{(\hat{P}_k - P_0)^2\} = \underline{M}\{\hat{P}_k^2\} - 2P_0\underline{M}\{\hat{P}_k\} + P_0^2. \quad (13)$$

On the hypothesis of time-jitter uncorrelated with the input signals, it can be shown [7], [8] that $\bar{P} = P_0 \sum_{i=0}^{N-1} a_i$ and:

$$E^2 = 2P_0^2 \left[1 - \sum_{i=0}^{N-1} a_i \right] + 2 \sum_{q=1}^{+\infty} |P_q|^2 W^2(qf_1 T_c). \quad (14)$$

The first term of the second member represents the asymptotic bias, and the second one the asymptotic variance. On the hypothesis that $\sum_{i=0}^{N-1} a_i = 1$, the output of the instrument is asymptotically unbiased, and the asymptotic mean square error coincides with the asymptotic variance. In this hypothesis, by referring to (2), (14) can be rewritten as follows:

$$E^2 = \int_{-\infty}^{+\infty} S_p(f) W^2(fT_c) df - P_0^2 \quad (15)$$

with $W^2(0)$ being equal to 1. This equation shows that the asymptotic mean square error can be obtained by multiplying the power density spectrum of $p(t)$, excluding the dc component, with the function $W^2(fT_c)$ which weights the contribution of each spectral component of the power density spectrum; for this reason we call $W^2(fT_c)$ the weighting function. Therefore the behavior of this last as a function of frequency completely describes the performance of the instrument in terms of accuracy and bandwidth.

By assuming the FIR filter is fitted with a rectangular window with $a_i = 1/N$ (6), the weighting function can be written as follows [7], [8]:

$$W^2(fT_c) = \frac{1}{N} (1 - |\Phi(fT_c)|^2) + |\Phi(fT_c)|^2 \frac{\text{sinc}^2(NfT_c)}{\text{sinc}^2(fT_c)}. \quad (16)$$

For time-jitter-free sampling, $|\Phi|$ is equal to one and, consequently, $W^2 = \text{sinc}^2(NfT_c)/\text{sinc}^2(fT_c)$. With time-jitter present, $|\Phi|$ becomes less than one and a function of frequency; therefore, the first term of the sum in (16) results not null and variable with frequency while, in the second term, the effect of the digital filter is attenuated. When jitter is uniformly distributed in the range $\pm bT_c$, we have $\Phi(fT_c) = \text{sinc}(2bfT_c)$. Fig. 1 shows the weighting functions of the equally spaced sampling strategies without jitter (dashed line) and with a jitter uniformly distributed with $\pm b = \pm 0.1$ (continuous line); the value $N = 10$ was selected uniquely to make clearer the different shapes of the two functions. This figure shows that time-jitter leads to values of the relative minima increasing with frequency and values of the relative maxima decreasing with frequency.

In an equally spaced synchronous sampling strategy the second term of the sum in (16) is null for $q \neq zN$, while the first one, in the presence of time-jitter, is not null; in this case the contribution of the spectral components for which the condition $q \neq zN$ is verified must also be investigated to limit the asymptotic mean square error. Therefore, by imposing a maximum value of the weighting function, an upper limit of the frequency of the wattmeter can be deduced from (16) or Fig. 1, once the jitter distribution is fixed.

A synchronous wattmeter was simulated by assuming $f_1 = 100$ Hz and $T_c = 20 \mu\text{s}$. From (7) we deduce $N = 500$

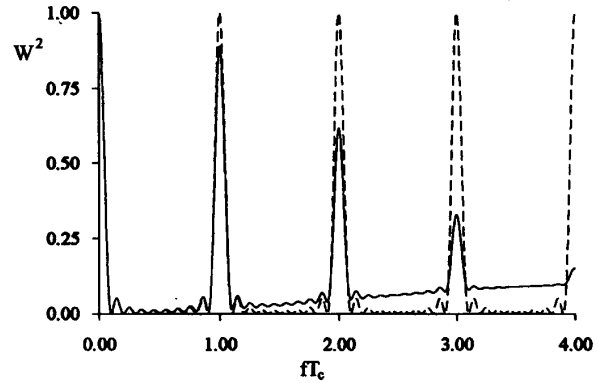


Fig. 1. Weighting function of a synchronous sampling strategy without jitter (dashed line) and jitter uniformly distributed in the interval $\pm bT_c = \pm 0.1T_c$ (continuous line) as a function of fT_c ($N = 10$).

and $r = 1$; therefore only the harmonic components of $p(t)$ at frequencies equal to or multiple integer of $Nf_1 = 50$ kHz contribute to the mean square error. With a time-jitter present, the weighting function must also be investigated at the other harmonic components. On the hypothesis of a uniform distribution with $\pm b = \pm 0.01$, the upper limit of the frequency (f_{max}) for two prefixed values of the root of the weighting function (10^{-3} and 10^{-4}) was theoretically deduced from (16). By assuming an instantaneous power with the unique harmonic component at frequency f_{max} of amplitude equal to the fundamental one, the asymptotic values of P_0 and W were estimated by considering fifty different outputs and comparing with the theoretical values (Table I).

Due to time-jitter, the sampling instants are randomly generated; therefore the accuracy can be increased with additional measurements. Table I also gives the results when the summation intervals are multiple integers of the previous one. For $N \geq 4 \cdot 500$ and $W = 10^{-3}$ the upper limit of frequency is imposed by the second term in (16), instead of the first one as in the previous cases. Obviously, this limit can be overcome by selecting a different value of T_c so that r can be an integer greater than one in (7). The experimental results were in very good agreement with the theoretical ones.

V. CONCLUSIONS

This paper evaluated the effect of time-jitter in the equally spaced sampling wattmeters on the hypothesis of equal random fluctuations in the voltage and current channels and a jitter uncorrelated with the input signals. After a brief overview of the properties of the synchronous, quasisynchronous and asynchronous sampling strategies in terms of uncertainty and bandwidth, the effect of time-jitter, which introduces random fluctuations with respect to the nominal sampling instants, was investigated. The asymptotic variance of the output was deduced, and it was shown that the instrument's accuracy can be described by a function which weights each component of the power density spectrum of the instantaneous power. This function, depending on the variance of the random variable associated with the time-jitter and on the number of the sampling instants in the summation interval, allows

TABLE I
COMPARISON BETWEEN THEORETICAL VALUES AND THEIR
ASYMPTOTIC ESTIMATES IN A TIME-JITTERED SYNCHRONOUS
SAMPLING WATTMETER ($T_c = 20 \mu\text{s}$ WHEN $f_1 = 100 \text{ Hz}$)

N	W	f_{max} (kHz)	$\Delta W = W_{\text{fmax}} - \hat{W}_{\text{fmax}}$	$\Delta P_0 = P_0 - \hat{P}_0$	\hat{P}_0
500	$1 \cdot 10^{-3}$	30.8	$-1.8 \cdot 10^{-5}$	$3 \cdot 10^{-5}$	1.00003
	$1 \cdot 10^{-4}$	3.0	$2.5 \cdot 10^{-7}$	$-4 \cdot 10^{-6}$	0.999996
2·500	$1 \cdot 10^{-3}$	43.5	$-7.0 \cdot 10^{-7}$	$3 \cdot 10^{-5}$	1.00003
	$1 \cdot 10^{-4}$	4.3	$-1.7 \cdot 10^{-6}$	$9 \cdot 10^{-7}$	1.000009
4·500	$1 \cdot 10^{-3}$	49.9	$-1.0 \cdot 10^{-5}$	$2 \cdot 10^{-5}$	1.000002
	$1 \cdot 10^{-4}$	6.1	$-4.7 \cdot 10^{-6}$	$-9 \cdot 10^{-7}$	0.9999991
10·500	$1 \cdot 10^{-3}$	49.9	$3.0 \cdot 10^{-6}$	$1 \cdot 10^{-7}$	1.000001
	$1 \cdot 10^{-4}$	9.7	$2.0 \cdot 10^{-6}$	$-2 \cdot 10^{-6}$	0.999998
20·500	$1 \cdot 10^{-3}$	49.9	$-3.6 \cdot 10^{-6}$	$8 \cdot 10^{-6}$	1.000008
	$1 \cdot 10^{-4}$	13.7	$-5.3 \cdot 10^{-6}$	$-1 \cdot 10^{-6}$	0.999999

the bandwidth due to time-jitter to be deduced, both in the synchronous and quasisynchronous sampling strategies.

The experimental results, obtained with a simulated wattmeter, confirmed the validity of the proposed analysis.

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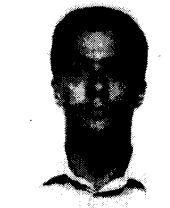
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