actual predictor characteristics is unnecessary. The problem considered here is prediction with two possible prediction delays. The desired signals are: (1) $d^{(1)}(n) = \sin(2\pi/64(n+3))$, and (2) $d^{(2)}(n) = \sin(2\pi/64(n+6))$. The two versions of the desired outputs are associated with the $U^{(1)}(n) \equiv [000\ 011\ 111]$ and $U^{(2)}(n) \equiv [111\ 100\ 000]$ input patterns, respectively.

During training, the input signal, the desired outputs, and the corresponding input patterns were repetitively applied. The weighting coefficients of the neural network were adjusted by using the on-line version of the backpropagation algorithm described by (3), (5), (7), and (8). Fig. 3(b) shows the decrease of the average prediction error during the repetitive trials. It is important to note that during training, the neural network controlled resonator-bank structure *learned* the frequency characteristics of the two predictors and *associated* them with the corresponding input patterns.

V. Summary

A new adaptive processing structure was presented in this paper. The neural network controlled resonator-bank structure offers an attractive alternative for implementing signal processing and control systems whose dynamic characteristics have to be adjusted if changes are detected in the environment. An important feature of the proposed system is that the required behavior can be achieved by on-line training when a desired dynamic response is associated with patterns in observed signals. The first results of the experimental analysis of the structure are encouraging; the system is able to approximate a wide range of nonlinear dynamic behavior.

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Frequency Stability in Resonator-Stabilized Oscillators

FABIO FILICORI AND GIORGIO VANNINI

Abstract —**A simplified stability analysis of resonator-stabilized oscillators is carried out by using the describing function approach. On this basis a criterion for the evaluation and optimization of the frequency stabilization introduced in an oscillator by a resonating element with a large quality factor is proposed. In particular, a frequency-stabilization index, which can be conveniently used in the design of highly stable oscillators, is defined. The validity of this performance index has been verified in the design of microwave oscillators using dielectric resonators as frequency-stabilizing elements.**

I. Introduction

Good frequency stability in oscillators is usually achieved by inserting into the circuit highly stable resonating elements (e.g., dielectric resonators, resonating cavities, quartz crystals, mechanical-type resonators, etc.). The presence of a resonator characterized by a large quality factor, however, is a necessary but not always sufficient condition for the good frequency stability of an oscillator. In fact, if the circuit is not suitably designed, the actual frequency of oscillation, which does not necessarily coincide with the highly stable intrinsic resonance frequency of the resonating element, could also be considerably dependent on other circuit elements (e.g., active device, power supply, load, etc.) that are subject to "drift" phenomena related to temperature, aging, or other possible sources of perturbation. This may happen, in particular, when the resonator is subject to strong "loading" effects deriving from other circuit elements. Thus, in the design of resonator-stabilized oscillators, special attention should be given to the problem of minimizing the frequency sensitivity to the parameters of all the circuit elements other than the resonator.

Accomplishing this task through a detailed analysis of the frequency sensitivity to various possible sources of perturbation can be quite difficult; to this aim, in fact, the dependence of the parameters of all the circuit elements on temperature or other external factors should be adequately modeled. This is probably the main reason why optimization of frequency stability is not directly dealt with in many oscillator design approaches [1]–[4], especially when very high operating frequencies make accurate component modeling a very complex task.

In this paper, through a simplified stability analysis based on the describing function approach, a suitable criterion for the global evaluation of the frequency stability with respect to circuit perturbation is proposed; in particular, a frequency-stabilization index is defined that allows for a quantitative evaluation of the stabilizing effect introduced in an oscillator by a resonating element and provides an effective criterion for the optimization of frequency-stability performance.

The validity of the proposed criterion is discussed by analyzing the frequency-stability performance of microwave oscillators stabilized by means of dielectric resonators.

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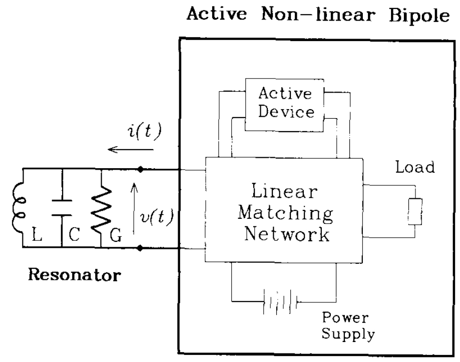

Fig. 1. Schematic circuit diagram of a resonator-stabilized oscillator.

II. Stability Analysis

The electrical response of different types of resonators can be approximated, in most cases, by an equivalent lumped LC resonating circuit; consequently, a resonator-stabilized oscillator can be described by the schematic circuit diagram shown in Fig. 1 where, without loss of generality, a parallel-type resonator has been considered.[1]

In order to evaluate the stability properties of an oscillator, a circuit analysis can be carried out at the connection port between the equivalent resonating circuit and the active nonlinear bipole (ANB) (see Fig. 1), which includes, besides the active device, load, and power supply, all the remaining circuit components. Oscillator analysis is intrinsically nonlinear but can be quite easily carried out at this port because, owing to the high quality factor of the resonator, the voltage $v(t)$ can be assumed to be almost sinusoidal, so that $v(t) \simeq a\cos 2\pi ft$. Thus the amplitude a_o and the frequency f_o of the oscillating voltage can be computed with sufficient accuracy by solving the phasor equation:

$$Ga + jGQ_u\left(\frac{f}{f_u} - \frac{f_u}{f}\right)a = F(a) \tag{1}$$

where $f_u = 1/2\pi\sqrt{LC}$ and $Q_u = 2\pi f_u C/G$ are, respectively, the unloaded resonance frequency and quality factor of the resonator; $I = F(a) = F_R(a) + jF_I(a)$ is a nonlinear complex function that defines the phasor I of the first-order harmonic[2] of the current $i(t)$ in the ANB as a function of the amplitude a (zero phase can be assumed) of the almost sinusoidal voltage $v(t)$. The plots in Fig. 2 show typical curves of the real and imaginary parts of the nonlinear function F for a correctly designed oscillator.

Equation (1) is a describing-function-like representation that allows for a straightforward stability analysis by applying Loeb's criterion [5], [6]. According to this criterion, in fact, the condition that guarantees instability of the solution $a = 0$ of (1) (i.e., the "self-starting capability" of the oscillator) can be approximated, in the hypothesis of a relatively large quality factor Q_u, by the inequality

$$K_i = \frac{g_i}{G} > 1, \qquad \text{with } g_i = \left(\frac{\partial F_R}{\partial a}\right)_{a=0}. \tag{2}$$

K_i can be interpreted as a *small-signal instability factor* that characterizes the self-starting capability of an oscillator. Stability conditions for the large-signal solution (a_o, f_o), instead, can be approximated in the same hypothesis by the inequality

$$K_a = \frac{g_o}{G} < 1, \qquad \text{with } g_o = \left(\frac{\partial F_R}{\partial a}\right)_{a=a_o} \tag{3}$$

where K_a represents a *large-signal amplitude stability factor* that characterizes the stability of the steady-state oscillation. Inequalities (2) and (3) are a good approximation of the stability condition defined by Loeb's criterion [5], [6] when, as is quite common in practice, the quality factor of the resonating element is large enough to make the frequency-dependence of the small- and large-signal admittances of the ANB negligible with respect to that of the resonator.

The stability factors K_i and K_a, which are defined in terms of the differential conductances g_i and g_o associated to the nonlinear function F (see Fig. 2), characterize the basic stability properties of an oscillator. In particular, the constraints $K_i > 1$ and $K_a < 1$, in addition to a given minimum acceptable value for the output power, are the basic requirements in any oscillator design procedure; such design constraints, however, are not sufficient to guarantee good performance in a resonator-stabilized oscillator, where minimization of the frequency sensitivity to circuit perturbation should also be considered as one of the main design goals. To this aim, a suitable criterion for the quantitative evaluation of the frequency-stabilizing effect introduced by the resonator is needed.

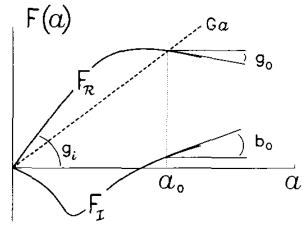

Fig. 2. Plots of the real and imaginary parts of the current-voltage characteristic F of an ANB versus the voltage amplitude a.

III. The Frequency-Stabilization Index

The magnitude of the frequency-stabilizing effect introduced in an oscillator by a resonating element can be evaluated by computing the frequency sensitivity to perturbations (e.g., a temperature variation) in the ANB. Any perturbation, in fact, gives modified values $p_{o1} + \Delta p_1, p_{o2} + \Delta p_2, \cdots, p_{oN} + \Delta p_N$ for a number N of parameters $p_1, p_2, \cdots, p_N$ that characterize the components of ANB;[3] the parameter variations $\Delta p_1, \Delta p_2, \cdots, \Delta p_N$ lead to a modified characteristic $\tilde{F}$ of the ANB and, consequently, to a different steady-state oscillation with amplitude $\tilde{a}_o = a_o + \Delta a$ and frequency $\tilde{f}_o = f_o + \Delta f$. In particular, for small perturbations, the amplitude and frequency variations Δa

[1] All the following considerations are still valid in the case of series-type resonators, provided that the roles of currents and voltages are properly exchanged.

[2] It should be noted that considering only the first-order harmonic of the current $i(t)$ does not involve relevant loss of accuracy in the computation of a_o and f_o; in fact, the highly frequency-selective impedance of the resonator is such as to guarantee that the voltage $v(t)$ is almost independent from the higher order harmonics of $i(t)$. Moreover, owing to the highly frequency-selective impedance of the resonator, the frequency-dependence of the nonlinear characteristic F in the neighborhood of the resonance frequency f_u is also practically negligible.

[3] Only perturbations in the ANB are considered since the resonator, which is introduced with the aim of stabilizing the oscillation frequency, is assumed to be much less sensitive than ANB to external factors.

and Δf are related to the variation

$$\Delta F = \Delta F_R + j\Delta F_I = \sum_{n=1}^{N} \left(\frac{\partial F}{\partial p_n}\right)_{a_o, p_o} \Delta p_n \tag{4}$$

of the nonlinear characteristic F around the "unperturbed" operating condition (a_o, f_o, p_o) by the incremental equations

$$G\Delta a = g_o \Delta a + \Delta F_R \tag{5}$$

$$GQ_u\left(\frac{f_o}{f_u} + \frac{f_u}{f_o}\right)a_o\frac{\Delta f}{f_o} + GQ_u\left(\frac{f_o}{f_u} - \frac{f_u}{f_o}\right)\Delta a = b_o\Delta a + \Delta F_I \tag{6}$$

where $b_o = (\partial F_I/\partial a)_{a_o,p_o}$ and $g_o = (\partial F_R/\partial a)_{a_o,p_o}$; these incremental equations are obtained by linearizing the real and imaginary parts of the nonlinear phasor equation (1) in the neighborhood of the "unperturbed" operating condition. Taking into account that in a resonator-stabilized oscillator $f_o/f_u + f_u/f_o \simeq 2$, (5) and (6) can be expressed, according to (1), in the form:

$$\frac{\Delta f}{f_o} = \frac{1}{2Q_u}\left(\frac{b_o/G - \tan\phi_o}{1 - g_o/G}\cos\delta + \sin\delta\right)\frac{1}{\cos\phi_o}\left|\frac{\Delta F}{F_o}\right| \tag{7}$$

where $F_o = F(a_o)$, $\phi_o = \arg(F_o)$ and $\delta = \arg(\Delta F)$.

Equation (7), which relates the frequency deviation Δf to the variation ΔF in the characteristic of the ANB, can be used to compute the frequency sensitivity $\Delta f/\Delta e$ to any external circuit perturbation Δe (e.g., a variation in temperature, bias voltage or load, etc.) once the relation between ΔF and Δe has been determined according to the formula

$$\frac{\Delta F}{\Delta e} = \sum_{n=1}^{N}\left(\frac{\partial F}{\partial p_n}\right)_{a_o,p_o}\frac{\partial p_n}{\partial e}. \tag{8}$$

In practice, however, computing $\Delta F/\Delta e$ by means of (8) can be a nontrivial problem since suitable models (which are necessarily technology-dependent) are needed in order to characterize the sensitivities $\partial p_n/\partial e$ of the parameters of all the circuit elements to different possible sources of perturbation.

As an alternative to such a *detailed* analysis, a simpler criterion for the *overall* evaluation of the frequency stabilizing effect introduced by the resonating element can be defined on the basis of (7). In fact, if the "worst-case" value (i.e., the value that maximizes $|\Delta f/f_o|$) is assumed for the phase $\delta = \arg(\Delta f)$, (7) defines an upper limit on the frequency deviation $|\Delta f/f_o|$ for a given amplitude $|\Delta F/F_o|$ of the perturbation in the characteristic of the ANB; this upper limit is defined by the inequality

$$\left|\frac{\Delta f}{f_o}\right| \leqslant \frac{1}{K_s}\left|\frac{\Delta F}{F_o}\right| \tag{9}$$

where

$$K_s = \frac{2Q_u}{\gamma} \quad \text{and} \quad \gamma = \left|\frac{1}{\cos\phi_o}\right|\sqrt{1 + \left(\frac{b_o/G - \tan\phi_o}{1 - g_o/G}\right)^2}.$$

The parameter K_s can be clearly interpreted as a *frequency-stabilization index* that globally characterizes the "strength" of the frequency-stabilizing effect deriving from the presence of a high-Q resonator, *independently* of the actual sensitivities of the circuit components to external sources of perturbation.

Equation (9) shows that low-frequency sensitivity to circuit perturbations can be achieved with high-Q resonators only when the oscillator has been carefully designed; in fact, a large value for the term γ, which characterizes the ANB, can greatly reduce the potentially large stabilizing effect associated to a high value of Q_u. From this point of view it can be said that the term γ takes into account the "loading" effects on the resonator deriving from the ANB; thus, observing that the minimum possible value of γ is 1, the parameter $Q_E = Q_u/\gamma$ can be interpreted as the *effective loaded quality factor* of a resonator when embedded in a given oscillator. In fact, Q_E represents the equivalent quality factor of a resonator that would give the same stabilization K_s in a "perfect" circuit where the ANB provides an optimal loading characteristic (i.e., $\gamma = 1$). Although other definitions for the loaded quality factor of a resonator-stabilized oscillator have been reported in the literature [7], these are purely conventional and do not have any precise physical meaning; the one proposed here has the advantage of being strictly related to frequency-stability properties.

It can be noted that, according to (9), optimal stabilization for a given resonator is obtained when the ANB is resistive (both *globally*: $\phi_o = \arg\{F(a_o)\} = 0$, and *incrementally*: $b_o = (\partial F_I/\partial a)_{a_o,p_o} = 0$) in the neighborhood of the large-signal operating condition (a_o, f_o, p_o). Such a situation (i.e., $\gamma = 1$) can easily be achieved in low-frequency oscillators; in the high-frequency range, owing to the presence of many parasitic reactive phenomena, such an ideal situation cannot be obtained so easily. In fact, although the condition $\phi_o = 0$ can be satisfied even at high frequencies, it may be quite difficult to have $b_o = 0$ at the same time without sacrificing other performance requirements. In such cases the frequency-stabilization index K_s provides a simple but effective criterion for the design of resonator-stabilized oscillators with optimal trade off between frequency stabilization and other performance requirements.

The performance index K_s, like the basic stability factors K_i and K_a, can be quite easily computed with litte additional effort with respect to a conventional oscillator analysis. In fact, the parameter ϕ_o in (9) simply coincides with the phase shift of the first-order harmonic of the current $i(t)$ with respect to that of $v(t)$ in steady-state oscillation. In order to compute the parameters g_o and b_o a linearized incremental circuit analysis is needed; this, however, does not involve a relevant increase in the computing effort, since a linearized circuit analysis is required anyway in most of the iterative Newton-like algorithms used for nonlinear circuit analysis. The situation is particularly favorable in the case of Newton-based harmonic-balance algorithms, which directly provide the incremental conversion matrices relating all the harmonic components of electrical variables.

The above considerations show that when good frequency stability is required, the problem of maximizing the stabilization index K_s should also be adequately considered in design procedures for resonator-stabilized oscillators. When circuit design is carried out by means of numerical optimization techniques [1], a contribution related to K_s can easily be included in the global objective function. Even when using special-purpose oscillator design procedures [2], [3], [8], the performance index K_s can be conveniently taken into account in the search for the "best" circuit performance.

IV. A Practical Application of the Frequency-Stabilization Index

The proposed frequency-stabilization index K_s has been used as the basic criterion for the optimization of frequency stability in the design of a microwave MESFET oscillator where frequency stabilization was achieved through the electromagnetic coupling of a dielectric resonator to a pair of transmission lines in the microstrip circuit configuration shown in Fig. 3.

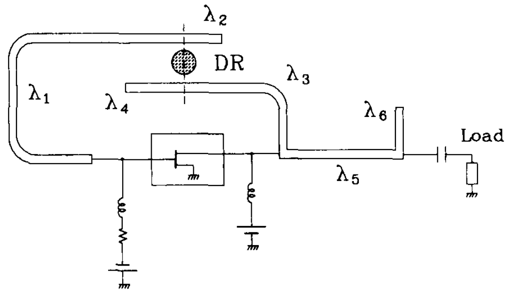

Fig. 3. Schematic circuit diagram of a microwave MESFET oscillator stabilized by means of a dielectric resonator; the feedback and load-matching networks are implemented by using microstrip line stubs.

Oscillator design was carried out by means of a special-purpose design procedure [8] that provides computer generated maps where constant performance loci are plotted in the space of the designable parameters; on such design maps a good compromise between the different performance requirements (i.e., large output power P_{OUT}, large stabilization K_s, with $K_i > 1$ and $K_a < 1$) was found (see Design 1 in Table I). In order to verify the validity of the frequency-stabilization index proposed, other different possible designs of the same circuit structure (i.e., the same topology, MESFET and resonator, with different values of the stub lengths λ) were also considered. All these designs are characterized (see columns 2–5 in Table I) by very similar values for the basic performance parameters P_{OUT}, K_i, K_a; such designs could have been considered practically equivalent if the frequency stabilization index K_s, which assumes quite different values, had not been taken into account.

An accurate circuit analysis was then carried out by means of harmonic balance techniques [9]–[13]; this provided the normalized parametric frequency sensitivities $s_n \doteq \Delta f / f_o / \Delta p_n / p_{o_n}$ to a number of circuit parameters p (i.e., transmission line lengths λ, load reflection coefficient ρ_L, characteristic parameters g_m and C_{GS} of the MESFET model). The results summarized in Table I show a relevant correlation between decreasing values of K_s and generally increasing values for the parametric frequency sensitivities. In fact, although not all the *single* parametric frequency sensitivities s_p have a perfectly monotonous behavior versus K_s, it can be clearly seen that, in all the design examples considered, a *globally* smaller frequency sensitivity is associated to higher values of K_s. This is confirmed by the monotonous behavior versus K_s of the mean-square parametric sensitivity $\sigma^2 = 1/N \sum_{n=1}^{N} s_n^2$, which can be considered as a typical multiparametric sensitivity index; in fact, σ^2 represents a typical choice for the objective function when minimum frequency sensitivity is sought for by means of numerical optimization techniques.

The above considerations and preliminary results show that the proposed frequency-stabilization index K_s provides relevant information on the level of frequency stabilization introduced by a high-Q resonator in an oscillator and that much better frequency stability can be achieved when maximization of K_s is considered as one of the design goals. The main advantages of this criterion, in comparison with other possible approaches for the evaluation of frequency stability (e.g., multiparametric sensitivity or temperature-stability analysis), are the extreme simplicity of the computations involved and its intrinsic independence of the particular characteristics of the actual circuit implementation. In fact, multiparametric sensitivity and, in particular, temperature-stability analysis, not only require relatively complex computations, but are also strongly technology-dependent since suitable modelling of the dependence of each circuit element on the different possible sources of perturbation is needed. The *frequency-stabilization index* K_s provides a meaningful criterion for the evaluation and possible optimization of frequency stability in resonator-stabilized oscillators by simply considering the unloaded quality factor of the resonator and the "loading" effect of the active circuit, independent of the particular sensitivity characteristics of the components used in the circuit implementation.

TABLE I

Basic Performance and Parametric Sensitivity Analysis for Different Designs of a Dielectric-Resonator-Stabilized Oscillator; The Same Circuit Topology and Dielectric Resonator ($Q = 5000$) were Used in all Cases

Design #		1	2	3	4	5
Basic performance parameters	P_{out}	78.6 mW	79.8 mW	84.3 mW	83.4 mW	85.2 mW
	K_i	1.7	1.8	1.5	1.5	1.4
	K_a	$-1.4\cdot10^{-1}$	$-1.5\cdot10^{-1}$	$-8.6\cdot10^{-2}$	$-1.1\cdot10^{-1}$	$-4.7\cdot10^{-2}$
Frequency stabilization index	K_s	$9.1\cdot10^{3}$	$2.9\cdot10^{3}$	$1.5\cdot10^{3}$	$8.3\cdot10^{2}$	$5.3\cdot10^{2}$
Parametric frequency sensitivities	s_{λ_1}	$7.0\cdot10^{-2}$	$2.7\cdot10^{-1}$	$5.4\cdot10^{-1}$	$6.6\cdot10^{-1}$	$7.8\cdot10^{-1}$
	s_{λ_2}	$2.1\cdot10^{-2}$	$1.3\cdot10^{-1}$	$8.0\cdot10^{-2}$	$2.2\cdot10^{-1}$	$1.0\cdot10^{-1}$
	s_{λ_3}	$2.3\cdot10^{-2}$	$2.5\cdot10^{-1}$	$1.7\cdot10^{-1}$	$1.6\cdot10^{-1}$	2.0
	s_{λ_4}	0.0	$6.6\cdot10^{-3}$	$1.4\cdot10^{-2}$	3.7	8.9
	s_{λ_5}	$2.7\cdot10^{-4}$	$1.2\cdot10^{-3}$	$1.3\cdot10^{-3}$	$7.1\cdot10^{-2}$	$2.5\cdot10^{-1}$
	s_{λ_6}	$1.5\cdot10^{-2}$	$6.4\cdot10^{-2}$	$3.6\cdot10^{-1}$	$9.1\cdot10^{-2}$	2.3
	$s_{Re(\rho_L)}$	$5.6\cdot10^{-4}$	$8.3\cdot10^{-3}$	$1.8\cdot10^{-2}$	$1.7\cdot10^{-2}$	$2.2\cdot10^{-2}$
	$s_{Im(\rho_L)}$	$1.4\cdot10^{-3}$	$3.5\cdot10^{-4}$	$6.4\cdot10^{-3}$	$2.9\cdot10^{-2}$	$3.3\cdot10^{-2}$
	$s_{C_{GS}}$	$4.4\cdot10^{-3}$	$6.9\cdot10^{-3}$	$1.2\cdot10^{-2}$	$3.2\cdot10^{-3}$	$1.0\cdot10^{-3}$
	s_{g_m}	$2.2\cdot10^{-2}$	$4.6\cdot10^{-2}$	$2.1\cdot10^{-1}$	$4.0\cdot10^{-1}$	$4.7\cdot10^{-1}$
$\sigma^2 = \frac{1}{N}\sum_{n=1}^{N} s_n^2$		$6.6\cdot10^{-4}$	$1.6\cdot10^{-2}$	$5.0\cdot10^{-2}$	1.4	8.9

V. Conclusion

A criterion for the evaluation and possible optimization of frequency stability in resonator-stabilized oscillators has been proposed. The criterion is based on a *frequency-stabilization index* which allows for a quantitative evaluation of the frequency-stabilizing effect introduced by a high-Q resonator in an oscillator. The proposed index, which has the advantage of being independent from the actual sensitivity characteristics of the circuit components, is quite easy to compute and can be conveniently used in oscillator design either when using special-purpose design procedures or numerical optimization techniques. Also, an *effective loaded quality factor*, which unlike other conventional definitions is strictly related to frequency stabilization, has been defined in the paper.

The validity of the proposed criterion has been tested in the design of microwave oscillators stabilized by means of dielectric resonators. In particular an accurate circuit sensitivity analysis has been carried out for several different oscillator designs. The results have shown that the proposed frequency-stabilization index provides a significant performance characterization for resonator-stabilized oscillators, and that much better frequency stability can be achieved by simply considering maximization of this index as one of the main design objectives.

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Energy and Power in Nonreciprocally Coupled Coils

V. BELEVITCH

***Abstract*—In a recent paper [1], Lin has criticized some proofs of the equality of the mutual inductances $L_{12} = L_{21}$ for coupled coils. In so doing, he has presented some energy and power expressions for the case $L_{12} \neq L_{21}$. Although all the considerations of Lin are correct, his transient analysis for the case $L_{12} \neq L_{21}$ is not sufficiently general and might suggest misleading conclusions. In this note, we generalize Lin's treatment and present a physical interpretation of the inductance 2-port with $L_{12} \neq L_{21}$, based on a novel active nonreciprocal equivalent circuit.**

I. Electromagnetic Theory

In electromagnetic field theory, the coefficients of the inductance matrix

$$\begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \tag{1}$$

of a pair of linear time-invariant coupled coils are defined by the relations

$$\Phi_1 = L_{11} i_1 + L_{12} i_2, \qquad \Phi_2 = L_{21} i_1 + L_{22} i_2 \tag{2}$$

between the instantaneous currents and the fluxes. The magnetic energy is then proved to be [2]

$$T = \frac{1}{2}(\Phi_1 i_1 + \Phi_2 i_2) = \frac{1}{2}(L_{11} i_1^2 + 2M i_1 i_2 + L_{22} i_2^2) \tag{3}$$

where

$$M = \frac{1}{2}(L_{12} + L_{21}). \tag{4}$$

Since (3) only involves the combination (4) of the mutual inductances, the reciprocity $L_{12} = L_{21}$ cannot be deduced from the non-negativeness of (3) alone, since this only imposes the conditions

$$L_{11} \geqslant 0, \qquad L_{22} \geqslant 0, \qquad L_{11} L_{22} - M^2 \geqslant 0. \tag{5}$$

From Lenz' law and (2), the 2-port equations result as

$$v_1 = L_{11}\frac{di_1}{dt} + L_{12}\frac{di_2}{dt}, \qquad v_2 = L_{21}\frac{di_1}{dt} + L_{22}\frac{di_2}{dt} \tag{6}$$

so that the instantaneous power entering the network is

$$w = v_1 i_1 + v_2 i_2 = \frac{dT}{dt} + L\left(i_1\frac{di_2}{dt} - i_2\frac{di_1}{dt}\right) \tag{7}$$

where we have used (3) and (6) and have set

$$L = \frac{1}{2}(L_{12} - L_{21}). \tag{8}$$

In passing from a state a to a state b, the energy supplied to the 2-port, as deduced from (7), is

$$\int_a^b w\, dt = T_b - T_a + L\int_a^b (i_1\, di_2 - i_2\, di_1). \tag{9}$$

If the currents are identical in the final and initial states, one has $T_b = T_a$ in (9). By contrast, for $L \neq 0$, i.e., $L_{12} \neq L_{21}$, the last integral in (9) is generally nonzero for arbitrary evolutions of i_1 and i_2 between identical initial and final states, since it represents twice the area of a loop in the (i_1, i_2)-plane, and this is in contradiction with energy conservation. The above proof of $L_{12} = L_{21}$ was given by Tellegen [3] and is equivalent to Lin's proof in the appendix of his paper [1]. Note that when the subscripts 1 and 2 (which are arbitrary labels) are permuted, (9) remains invariant since L changes sign but so does the integral, because the loop in the (i_1, i_2)-plane is run in the opposite direction.

II. Circuit Theory

In circuit theory, the one-port R, L, C elements are defined as abstract black-boxes characterized by their equations. At that abstract level, the concepts of electrical and magnetic energies are not available, the only energetic concept related with the external behavior being the power supplied or absorbed through the ports. This only permits one to define the distinguished class of passive one-ports. One then proves that these are characterized by the property that, for exponential states $i = Ie^{st}$, $v = Ve^{st}$ with $s = \sigma + j\omega$, the real part of the complex power $W = VI^*$ is non-negative for $\sigma \geqslant 0$. This is sufficient for proving that one has $R \geqslant 0$, $C \geqslant 0$, $L \geqslant 0$ for a passive element, without resorting to the concepts of electric or magnetic energy.

Similarly, an abstract inductance 2-port is defined by its equations (6). If such a 2-port is followed in cascade by an ideal transformer of ratio $1/n$, and if the input and output ports of the resulting 2-port are connected in series, one obtains an inductance $L_{11} + 2nM + n^2 L_{22}$. From the positiveness of that inductance for all n, one can deduce (5), but not $L_{12} = L_{21}$.

However, in a harmonic state ($\sigma = 0$), the complex power absorbed through the ports is

$$W = V_1 I_1^* + V_2 I_2^* = 2j\omega T + W_d \tag{10}$$

where we have used the notations (4) and (8) and have set

$$T = \frac{1}{2}[L_{11} I_1 I_1^* + M(I_1^* I_2 + I_1 I_2^*) + L_{22} I_2 I_2^*] \tag{11}$$

$$W_d = j\omega L(I_1^* I_2 - I_1 I_2^*). \tag{12}$$

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