Testing the New Keynesian Phillips Curve through Vector Autoregressive models: Results from the Euro area

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February 2004

Abstract

In this paper we set out a test of the New Keynesian Phillips Curve (NKPC) based on Vector Autoregressive (VAR) models. The proposed technique does not rely on the Anderson and Moore (1985) method and can be implemented with any existing econometric software. The idea is to use a VAR involving the inflation rate and the forcing variable(s) as the expectation generating system and find the restrictions that nest the NKPC within the VAR. The model can be estimated and tested through maximum likelihood methods. We show that the presence of feedbacks from the inflation rate to the forcing variable(s) can affect solution properties of the NKPC; when feedbacks are detected the VAR should be regarded as the final form solution of a more general structural model. Possible non-stationary in the variables can be easily taken into account within our framework. Empirical results point that the standard "hybrid" versions of the NKPC are far from being a good first approximation to the dynamics of inflation in the Euro area.

Keywords: Inflation dynamics, New Keynesian Phillips Curve, Forward-looking behavior, VEqCM.

JEL classifications: C22, C32, C52, E31, E52.

1   Introduction

The Phillips curve plays a central role in our understanding of business cycles and the management of monetary policy. In recent years the literature on the

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so-called New Keynesian Phillips Curve (NKPC) has expanded rapidly albeit with no clear-cut consensus on the empirical role of forward-looking components in inflation dynamics[1]. Whereas Fuhrer and Moore (1995) and Fuhrer (1997) present evidence on US inflation that seems to undermine the importance of forward-looking components as relevant causes of inflation, the recent success of the NKPC can be specially attributed to the papers by Galí and Gertler (1999) (henceforth GG) and Galí et al. (2001) (henceforth GGLS), where "hybrid" marginal-cost based versions of the Phillips curve are found to provide "good first approximation" of inflation in the US and Euro area[2].

The NKPC both in the "standard" or "hybrid" formulations, reads as a Linear Rational Expectation (LRE) model where the inflation rate depends on the expected future value of inflation rate, lagged inflation (in the hybrid model) and a measure of demand pressure, usually the output gap or the unemployment rate. It can be derived through different routes within the New Keynesian paradigm (Roberts, 1995). In GG and GGLS real unit labor costs are used as a proxy of real marginal costs; the inclusion of lagged inflation terms in the base "pure forward-looking" version of the model is usually motivated by assuming that a fraction of producers set their prices according to a rule of thumb (GG and Steinsson, 2003), or by referring to models with two (or more) period overlapping wage contracts as in e.g. Fuhrer and Moore (1995)[3]. In this paper we shall refer to the hybrid formulation of the model as the NKPC, except where indicated.

The estimation of the NKPC is carried out either through Generalized Method of Moments (GMM) or Maximum Likelihood (ML) techniques with surprisingly different results. The existing evidence seems to suggest that estimation methods heavily affect the empirical assessment about the NKPC. In general, ML leads to rejections whereas GMM tends to support the model. Comparative drawbacks and merits of ML versus GMM have been widely discussed within the class of RE models[4]. In principle GMM are "ideal" because they are easy to compute and require minimum assumption about exogenous (forcing) variables; however, it is well recognized that GMM-based estimates can be markedly biased in small samples and subject to "weak instruments" or "weak identification" issues (Stock et al., 2002). On the other hand, ML requires a full specification of the model,

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[1] The NKPC can be derived through different routes within the sticky prices paradigm of the New Keynesian economics, see e.g. Roberts (1995) for a survey.

[2] See also Sbordone (2002).

[3] In practise the inclusion of lags of inflation in the baseline model allows to overcome the "jump" dynamics that the non-hybrid specification would entail, making hard a reconciliation among observed inflation patterns and the way actual central banks react to supply shocks. Policy implications are different if one appeals to the standard or hybrid formulation of the NKPC: according to the former monetary policy can drive a positive rate of inflation to zero with virtually no loss of output and employment ("disinflation without recession"). In the latter disinflation experiments can not be accompanied by low sacrifice ratios.

[4] For instance Fuhrer et al. (1995) focus on the expectations-based linear-quadratic inventory model and find that GMM tends to reject the model whereas ML supports it.

including the process generating explanatory variables and its implementation generally results in numerical optimization procedures[5]. A considerable bulk of the recent literature on the NKPC tries to explain discrepancies of results through different (often contrasting) arguments[6].

The use of the NKPC as a model of inflation dynamics seems to disregard (at least apparently) that there exist many possible sources of price growth[7]. The present paper is in line with Hendry's (2001) view that no "single cause" explanation of inflation can be empirically provided for a given industrialized economy. Moreover, when aggregated data are used as for the Euro area, it should be argued that the aggregation process might blur the actual single-agent behavioral relations connecting prices and other macroeconomic variables at the country level.

However, as the Phillips curve traditionally sustains the debate of monetary policy, the issue of properly testing the empirical validity of the NKPC can be still regarded as a relevant question to address. To our knowledge Bårdsen et al. (2002) is one of the papers in the recent literature where a number of relevant issues characterizing the empirical analysis of the NKPC are highlighted. In short, Bårdsen et al. (2002) argue and show that the empirical analysis of the NKPC can be hardly carried out within a single-equation stationary framework. Also Mavroeidis (2004) stresses that the properties of non-modelled variables are crucial for the identification of the parameters of the NKPC, even when these are thought to be exogenously given.

The aim of this paper is to provide a simple test of the empirical validity of the NKPC. We use Vector Autoregressive (VAR) models and set out a simple ML procedure which can be implemented with any existing econometric package. The method is directly inspired by the technique proposed in Fanelli (2002) for estimating and testing forward-looking models stemming from intertemporal optimization schemes. We show that our VAR-based expectations method to test the NKPC leads to conclusions very similar, in spirit, to those in Bårdsen et al. (2002) and Mavroeidis (2004) obtained through a different routes.

The proposed method differs in some aspects we discuss in the paper from the ML procedure exploited in Fuhrer and Moore (1995) and Fuhrer (1997) also

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[5]The debate among "limited-information" vs "full-information" methods in the estimation and testing of LRE has a long tradition in the literature, see e.g. Wickens (1982).

[6]Rudd and Whelan (2002) and Lindé (2003) point the specification bias associated with GG GMM approach through opposite arguments. Galí and Gertler (2003) reply to these criticisms by showing that their GMM results are robust to a variety of estimation procedures. See also, *inter alia*, Ma (2002), Mavroeidis (2002, 2004), Jondeau and Bihan (2003) and Søndergaard (2003).

[7]For instance, using data from the eighties onwards, Gerlach and Svensson (2003) refer to a backward-looking formulation of the Phillips curve where both the output gap and the real money gap (the difference between the real money stock and the long run equilibrium real money stock) paly a role. They find that both contain considerable information on future inflation in the Euro area.

based on VARs. In our set up a VAR system involving the inflation rate and the explanatory (forcing) variable(s) is used as an approximated solution to the NKPC[8]. The VAR is used as the expectation generating system (or the final form solution to the NKPC) so that applying the Undetermined Coefficient method it is possible to find the cross restrictions between its parameters and those of the NKPC. These restrictions can be used to test the model and to recover ML estimates of structural parameters. We show that under certain conditions the absence of Granger-causality from the inflation rate to the forcing variable can be sufficient for the existence of a unique and stable solution to the NKPC. Nevertheless, the absence of feedbacks from inflation to e.g. wages, the unemployment rate or the output gap is rather implausible in practise. Following the arguments in Timmerman (1994), feedbacks from the decision to the forcing variable(s) in LRE models might signal that relevant economic mechanisms (for instance "the other side of the market") have not been modelled. For the situations where feedbacks from the inflation to the forcing variable(s) are detected, we do not impose any explicit saddlepath restrictions on the parameters of the VAR; rather we argue that in these situations solution properties of the NKPC should be investigated within a structural system involving for instance a structural wage (or unemployment or output gap) equation and so on.

We also show that non-stationarity and the possibility of cointegration can be easily accommodated within our framework by appealing to Vector Equilibrium Correction (VEqC) representations of the VAR.

Our tests of the NKPC based on Euro area data and the 1970-1998 period show that even using different measures of the forcing variable (wage share, output gap, unemployment rate) the empirical evidence is not supportive of the NKPC, at least in the standard "hybrid" formulation currently very popular in the literature. This does not rule out that more dynamically complex forward-looking specifications might be appropriate for describing Euro area inflation. Moreover, using a simple spurious regression argument our results point that when estimating the NKPC through GMM as if variables were stationary may lead to misleading inference. Indeed we find that the persistence of the inflation rate and driving variables over the 1970-1998 period can be well described as that of unit-roots processes. Moreover, feedbacks from the inflation rate to the driving variables are found, suggesting that the single-equation based estimation of the NKPC might be based on a LRE model with no stable solution.

The paper is organized as follows. In Section 2 we introduce the "hybrid" version of the NKPC and in Section 3 we discuss solution properties in the presence of feedbacks from the inflation rate to the explanatory variable. In Section

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[8]Throughout we shall use the terms "explanatory variable", "forcing variable" and "driving variable" interchangeably. Indeed, though the term "forcing variable" should refer to a variable exogenously given within the model, we show that the variables which are commonly selected to play this role in single-equation NKPC specifications are likely to be Ganger-caused by the inflation rate.

4 we define out VAR-based test of the NKPC. We identify three different cases depending on the stationarity- integration/cointegration properties of variables. In Section 5 we summarize the empirical results for the Euro area. Section 6 contains a summary and some insights for further research.

2 The New Keynesian Phillips curve

Following Galí and Gertler (1999), Galí et al. (2001) in its "final" structural form the model can be formulated as

$$\pi_t = \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \lambda x_t \qquad (1)$$

where π_t is the inflation rate at time t, x_t a forcing (driving) variable, usually a measure of representative firm's real marginal costs (percent deviations from its steady state value), i.e. the labor share in output or the output gap, $E_t \pi_{t+1}$ is the expected value at time t of the inflation rate prevailing at time $t+1$ and λ, γ_f and γ_b are structural (positive) parameters. Expectations are conditional on the information set available at time t, i.e. $E_t \pi_{t+1} = E(\pi_{t+1} \mid \mathcal{F}_t)$ where $\{\pi_t,\, x_t\,,\, \pi_{t-1},\, x_{t-1},\, ...\} \subseteq \mathcal{F}_t$.

The equation (1) is derived in Galí and Gertler (1999), Galí et al. (2001) by appealing to the RE staggered-contracting model of Calvo (1983). Within this framework the parameters of (1) are given by

$$\gamma_f = \rho\theta\phi^{-1}$$

$$\gamma_b = \omega\phi^{-1}$$

$$\lambda = (1-\omega)(1-\theta)(1-\rho\theta)\phi^{-1}$$

where $\phi = \theta + \omega[1 - \theta(1-\rho)]$ and $0 < \rho < 1$, $0 < \theta < 1$ and $0 \leq \omega < 1$ are the "deep" parameters measuring respectively the discount factor, the degree of price stickiness and the degree of "backwardness" in price setting. Therefore (1) incorporates two types of firms: firms that behave in the forward-looking manner as in Calvo (1983), and firms that behave according to a simple "rule of thumb" where prices are set according to past evolution in order to incorporate structural inertia and persistence in inflation dynamics. In general $\gamma_f \geq 0$, $\gamma_b \geq 0$ and $\gamma_b + \gamma_f \leq 1$.

On the other hand, a version of (1) with $\gamma_b = 1/2 = \gamma_f$ is derived in Fuhrer and Moore (1995) and Fuhrer (1997) by appealing to a two-period version of the Taylor staggered contracting framework. Within this framework a more general

dynamic specification can be formulated as

$$\pi_t = \varphi \left[\frac{1}{3}(\pi_{t-1} + \pi_{t-2} + \pi_{t-3}) \right]$$
$$+ (1 - \varphi) \left[\frac{1}{3} E_t(\pi_{t+1} + \pi_{t+2} + \pi_{t+3}) \right] + \lambda x_t \qquad (2)$$

where the parameter φ indexes the weight on the past relative to expectations of the future (Fuhrer, 1997).

Turning on the equation (1), observe that with $\gamma_b = 0$ the model collapses to the "standard" formulation of the NKPC. The forward-solution associated to (1) is given by

$$\pi_t = \delta_1 \pi_{t-1} + \frac{\lambda}{\delta_2 \gamma_f} \sum_{j=0}^{\infty} \left(\frac{1}{\delta_2} \right)^j E_t x_{t+j} \qquad (3)$$

where δ_1 and δ_2 are respectively the stable and unstable roots of the characteristic equation

$$\gamma_b z^2 - z - \gamma_f = 0 \qquad (4)$$

see e.g. Pesaran (1987, Section 5.3.4)[9].

From the policy point of view the NKPC (1) implies that a fully credible disinflation implies a positive sacrifice ratio which increases with the fraction of backward-looking firms. On the other hand if $\gamma_b = 0$ the purely forward-looking NKPC entails that a fully credible disinflation has no output costs.

Formally the equations (1) and (3) are specified as "exact" LRE models in the sense of Hansen and Sargent (1991). This means that no term unobservable for the econometrician is included on the right-hand-side of (1) (and (3)). The "exact" formulation of the NKPC is used in e.g. GG and GGLS, whereas specifications where a disturbance term is added on the right hand side of (1) may be found, *inter alia*, in Bårdsen et al. (2002) and in Galí and Gertler (2003). The inclusion of an exogenous disturbance term on the right and side of (1) is usually interpreted as a cost push shock or simply as a pricing error. The inclusion of such term in the model is not irrelevant for solution properties and estimation issues.

3 Solution properties

The NKPC (1) belongs to the class of LRE models with future expectations on the endogenous variables. It is well recognized that LRE models which include forward-looking terms typically imply equations of motion with unstable roots.

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[9]Althought in a different context Fanelli (2002) shows that making inference on (1) or in the forward counterpart (3) may imply no loss of information if the link (4) characterizing the parameters of the two models is taken into explicit accout.

The solutions of models similar to (1) are explicitly discussed in e.g. Pesaran (1987), Chap. 6 and 7 for the cases where there are no feedbacks from the decision to the forcing variables (i.e. from π_t to x_t). As shown by Timmermann (1994) in the context of present-value models the presence of feedbacks from the forcing to the decision variables may potentially affect stability and uniqueness of solutions. In this section we discuss the stability of solutions to (1) in the presence of Granger-causality from π_t to x_t. In Section 5 we will show that this assumption is not at odd with the empirical evidence in the Euro area[10].

To discuss solution properties we exploit a Blanchard and Kahn (1980) (henceforth BK) representation of the NKPC. First, to make discussion more general, we add a disturbance term u_t on the right-hand side of (1) such that $E_t u_{t+1} = 0$. By rewriting terms opportunely we get the expression

$$E_t \pi_{t+1} - \gamma_f^{-1} \pi_t + \gamma_f^{-1} \gamma_b \pi_{t-1} + \gamma_f^{-1} \lambda x_t = \gamma_f^{-1} u_t. \tag{5}$$

Let us assume for the moment that the process generating x_t is given by the Autoregressive model (AR(2))

$$x_t = a_{11} x_{t-1} + a_{12} x_{t-2} + \varepsilon_t^x \tag{6}$$

where a_{1j}, $j = 1, 2$ are parameters such that the roots of the characteristic equation

$$1 - a_{11} s - a_{12} s^2 = 0 \tag{7}$$

lie outside the unit circle ($\mid s \mid > 1$) and ε_t^x is a White Noise term. For simplicity we consider a two-lag model in (6) without loss of generality.

It is then possible to represent (6) and (5) jointly in companion form

$$\begin{pmatrix} X_{t+1} \\ E_t P_{t+1} \end{pmatrix} = A \begin{pmatrix} X_t \\ P_t \end{pmatrix} + \gamma Z_t \tag{8}$$

where

$$\begin{pmatrix} X_{t+1} \\ E_t P_{t+1} \end{pmatrix} = \begin{pmatrix} x_{t+1} \\ x_t \\ E_t \pi_{t+1} \\ \pi_t \end{pmatrix} ; A = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ -\gamma_f^{-1}\lambda & 0 & \gamma_f^{-1} & -\gamma_f^{-1}\gamma_b \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

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[10] To my knowledge Petursson (1998) is the only paper where it is not found Granger causality from the inflation rate to the forcing variables in a forward-looking type model of price determination of the Icelandic economy. Petursson (1998) derives his NKPC-type equation from an intertemporal optimizing problem similar to Rotemberg's (1982) model, where the forcing variables are the wage rate and import prices.

$$\gamma = I_4 \ ; \quad Z_t = \begin{pmatrix} \varepsilon^x_{t+1} \\ 0 \\ -\gamma_f^{-1} u_t \\ 0 \end{pmatrix}.$$

System (8) is a representation consistent with Blanchard and Kahn (1980) model; here $m = 1$ represents the number of non-predetermined (forward-looking) variables of the system. The advantage of the BK representation (8) is that properties of solutions (uniqueness and stability) can be easily characterized through the eigenvalues of the A matrix. Following Blanchard and Kahn (1980), Proposition 1, 2 and 3, if the number h of eigenvalues of A outside the unit circle is equal to the number of non-predetermined (forward-looking) variables ($h = 1$), then there exists a unique stable solution. If $h > 1$ then there is no stable solution for the model (8); finally, if all eigenvalues lie inside (or on) the unit circle then there is an infinity of (stable) solutions.

The eigenvalues of A in (8) are given by

$$\frac{1}{2\gamma_f}\left(1 + \sqrt{\left(1 - 4\gamma_f\gamma_b\right)}\right) \ , \ \frac{1}{2\gamma_f}\left(1 - \sqrt{\left(1 - 4\gamma_f\gamma_b\right)}\right)$$
$$\frac{1}{2}a_{11} + \frac{1}{2}\sqrt{\left(a_{11}^2 + 4a_{12}\right)} \ , \ \frac{1}{2}a_{11} - \frac{1}{2}\sqrt{\left(a_{11}^2 + 4a_{12}\right)}$$

where the last two lie inside the unit circle by construction (they correspond to the inverse of the roots of (7)). As concerns the first two eigenvalues, it can be shown that if $\gamma_f + \gamma_b < 1$ one lie inside and the other outside the unit circle, i.e. the NKPC has a unique stable solution (see also Mavroeidis, 2002, footnote 14); if $\gamma_f + \gamma_b = 1$ one of the two eigenvalues is exactly at one whereas the other can be greater or less than one depending on whether γ_f is less or greater than $1/2$ [11].

Suppose now that (6) is replaced by

$$x_t = a_{11}x_{t-1} + a_{12}x_{t-2} + f_{11}\pi_{t-1} + f_{12}\pi_{t-2} + \varepsilon^x_t \tag{9}$$

where, other things remaining unchanged, $f_{1j} \neq 0$ at least for one $j = 1, 2$. Here the inflation rate π_t Granger-causes x_t. It is still possible to represent (9) and (5) jointly as in (8) but with the A matrix given by

$$A_f = \begin{bmatrix} a_{11} & a_{12} & f_{11} & f_{12} \\ 1 & 0 & 0 & 0 \\ -\gamma_f^{-1}\lambda & 0 & \gamma_f^{-1} & -\gamma_f^{-1}\gamma_b \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

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[11] If $\gamma_f + \gamma_b > 1$ no stable solution exists.

where we used the subscript "f" to highlight the presence of feedbacks. Now the eigenvalues of A_f correspond to the roots, ρ, of the following polynomial:

$$\rho^4 \gamma_f + \left(-1 - a_{11}\gamma_f\right)\rho^3 + \left(\gamma_b - \gamma_f a_{12} + \lambda f_{11} + a_{11}\right)\rho^2$$

$$- \left(a_{11}\gamma_b - a_{12} - \lambda f_{12}\right)\rho - a_{12}\gamma_b = 0$$

where it is evident that the feedback parameters f_{1j}, $j = 1, 2$, affect roots properties. It is not guaranteed now that just one root falls outside the unit circle, unless parameters are properly constrained. The point here is: is there any economic reason for such constraints to hold?

Consider as an example the following set of values taken from Table 2 in GGLS: $\gamma_f = 0.69$, $\gamma_b = 0.27$, $\lambda = 0.006$ where x_t is measured as the wage share. These values are in GGLS GMM estimates of the parameters of the NKPC (1) obtained over the quarterly data 1970-1998, but here we treat them as the "true" structural values. Assume further that in (9) $a_{11} = 0.89$, $a_{12} = 0$, $f_{11} = 0$, $f_{12} = 0.21$; then the eigenvalues of A_f are: 1.08, 0.91, 0.35, 0 and a unique and stable solution occurs. However, if *ceteris paribus,* the parameters of (9) are $a_{11} = 0.95$, $a_{12} = 0$, $f_{11} = 0$, $f_{12} = 0.53$, the eigenvalues of A_f are: $1.03 \pm 0.042i$, 0.35, 0 and no stable solution exists.

These simple examples show that in the presence of feedbacks from the inflation to the driving variable it is not clear whether the NKPC can be reconciled with a non-explosive inflation process, unless the parameters of the model are opportunely constrained. These example also highlight that GMM-based estimation of the NKPC (1) (or equivalently (3)) that ignores the properties of the process generating x_t is based on an implicit stability condition.

Finally, it is worth noting that in the presence of feedbacks from π_t to x_t a unique and stable solution may occur even if $\gamma_f + \gamma_b > 1$; if for example: $\gamma_f = 0.75$, $\gamma_b = 0.30$, $\lambda = 0.5$, $a_{11} = 0.89$, $a_{12} = 0$, $f_{11} = 0.03$, $f_{12} = -0.09$, the eigenvalues of A_f are: 1.12, $0.55 \pm .26i$, 0.

As already observed, feedbacks might signal the presence of non modelled relationships. For instance, assume that x_t in (1) represents the output gap and that the model generating x_t can be described as

$$x_t = \varsigma E_t x_{t+1} + (1 - \varsigma)x_{t-1} - \phi(r_t - E_t \pi_{t+1}) + \varkappa_t \qquad (10)$$

consistently with a demand equation (or IS) equation derived from a representative agent intertemporal utility maximizer with external habit persistence (Fuhrer, 2000). In (10) ς and ϕ are structural parameters, r_t is a short term nominal interest rate and $\varkappa_t$ can be regarded as a demand shock. The model can be closed by specifying the monetary policy rule for r_t as in e.g. Clarida et al. (2000); to the purposes of our analysis it is sufficient to observe that by

deriving $E_t\pi_{t+1}$ from (1) and substituting into (10), π_t Granger causes x_t. In this case solution properties of the NKPC should be investigate within a complete LRE structural model comprising (1), (10) and the policy rule as in e.g. Moreno (2003).

4   Testing the NKPC

In this section we discuss a simple test of the NKPC. We refer, for simplicity, to the "exact" specification of the model, nevertheless we show that the approach can be extended with minor modifications to non-exact specifications.

The idea motivating our test is that if the unique and stable solution of the NKPC can be approximated as a VAR involving π_t and x_t, then it is possible to find the cross restrictions between the parameters of the two model by the Undetermined Coefficient method. Then estimation issues can be easily addressed within the restricted VAR[12]. In addition, possible cointegration properties of variables can be suitably captured by referring to a VEqCM representation of the VAR.

This approach for testing "exact" LRE through VARs was originally proposed in Baillie (1989) and then exploited in Johansen and Swensen (1999) for cointegrated LRE models. It differs in some respects from the ML procedure exploited in Fuhrer and Moore (1995) and Fuhrer (1997) also based on VARs.

First, the procedure used by Fuhrer and Moore (1995) and Fuhrer (1997) is based on the Anderson and Moore (1985) solution technique. It first converts the joint process involving the structural model (the NKPC) and the process generating the forcing variables into a companion form, then uses an eigensystem calculation to derive the unique stable solution of the forward-looking LRE model in the form of a VAR embodying "saddlepath" parametric restrictions[13]. It proves to be computationally efficient and straightforward to implement and allows the presence of feedbacks from the inflation rate to the forcing variable(s). However, in the computation of the saddlepath solution to the model the procedure does not provide any insight or economic justification for the "stabilizing" (equilibrium) forces at work. Hence if the are feedbacks from the inflation rate to the forcing variable(s) the parameters of the model are suitably restricted to generate a unique stationary solution without providing any economic justification of the reasons why such restrictions should apply. In our set up the VAR is used as an approximated solution to the NKPC[14]. By means of the Undetermined Coefficient method it is possible to find the cross restrictions between the

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[12]The identification of the model can be investigated by following the same route as Fanelli (2002). It can be proved that given (1) a necessary condition for the identification of the structural parameters if that the number of lags in the VAR is greater or equal to 2. See also Mavroeidis (2002) for a similar result.

[13]See also Fuhrer et al. (1995) for an extensive summary.

[14]Throughout we shall use the terms "explanatory variable", "forcing variable" and "driving

parameters of the VAR and those of the NKPC and these restrictions can be suitably used to test the model and to recover ML estimates of structural parameters. We show that under certain conditions the absence of Granger-causality from the inflation rate to the forcing variable can be sufficient for the existence of a unique and stable solution to the NKPC. Nevertheless, the absence of feedbacks from inflation to e.g. wages, the unemployment rate or the output gap is rather implausible in practise. Following the arguments in Timmerman (1994), feedbacks from the decision to the forcing variable(s) in LRE models might signal that relevant economic mechanisms (for instance "the other side of the market") have not been modelled. For the situations where feedbacks from the inflation to the forcing variable(s) are detected, we do not impose any explicit saddlepath restrictions on the parameters of the VAR; rather we argue that in these situations solution properties of the NKPC should be investigated within a structural system involving for instance a structural wage (or unemployment or output gap) equation and so on.

Second, in structural LRE models, decision rules depend on present value calculations that are sensitive to the degree of persistence of the driving process. However, the econometric analysis of the NKPC is generally carried out as if variables were stationary, without any concern on the statistical properties of variables within the selected sample[15]. A well recognized fact in dynamic modelling is that knowledge about the presence and location of unit roots is crucial in determining the appropriate choice of asymptotic distribution for coefficients and test statistics. We show that non-stationarity and the possibility of cointegration can be easily accommodated within our framework by appealing to Vector Equilibrium Correction (VEqC) representations of the VAR.

We consider three cases: the case where π_t and x_t are generated by a stationary I(0) process, the case where π_t and x_t are generated by an I(1) cointegrated processes in the sense of Johansen (1996) and the case where they are generated by an I(1) non cointegrated process. For expositional convenience in the discussion

variable" interchangeably. Indeed, though the term "forcing variable" should refer to a variable exogenously given within the model, we show that the variables which are commonly selected to play this role in single-equation NKPC specifications are likely to be Ganger-caused by the inflation rate.

[15]Modelling the inflation process as stationary over a period where it is not may lead to bias downward its persistence and hence to misunderstand the related policy interventions. The paper of Fuhrer and Moore (1995) represent an example where efforts are made to characterize the linkages characterizing the inflation rate and its driving variable(s) at both low and higher frequencies. By specifying a VAR including the inflation rate, a short term interest rate and a measure of the output gap for the US economy, Fuhrer and Moore (1995), p. 135 conclude that: *"While we cannot reject the hypothesis that the data contain one or two unit roots, we chose a stationary representation of the data for two reasons. [...]. By viewing inflation as an I(0) process instead of an I(1) process we bias downward our estimate of inflation persistence, and we strengthen the argument that the standard contracting model cannot adequately explain inflation persistence"*.

that follows we shall consider bi-variate VARs including two or three lags; it is clear, however, that the proposed method can be easily extended to more general situations.

4.1 Case1: I(0) variables

We consider the vector $Y_t = (\pi_t , x_t')'$ where x_t can be a single scalar or a vector of explanatory variables, and the following process

$$Y_t = A_1 Y_{t-1} + \ldots + A_k Y_{t-k} + \mu + \varepsilon_t \tag{11}$$

where $A_1, \ldots, A_k$ are $(p \times p)$ matrices of parameters, μ is a $(p \times 1)$ constant, k is the lag length, $Y_{-p}, \ldots, Y_{-1}, Y_0$, are given and $\varepsilon_t = Y_t - E_{t-1} Y_t$ is a $(p \times 1)$ martingale difference process with respect to the informations set $\{Y_t, Y_{t-1}, \ldots, Y_1\} \subseteq \mathcal{F}_t$. p, the dimension of the vector Y_t, will be equal to two if just one driving variable is considered, or greater than two if more than one driving variable is included in the analysis. We further assume that $\varepsilon_t \sim N(0, \Omega)$ and that the parameters $(A_1, \ldots, A_k, \mu, \Omega)$ are time invariant. Finally, the roots of the characteristic equation

$$\det(A(z)) = \det(I_p - A_1 z - A_2 z^2 - \ldots - A_k z^k) = 0 \tag{12}$$

are such that $\mid z \mid > 1$ so that the VAR is (asymptotically) stable.

Consider the case where x_t is a scalar and $k = 2$; the two equations of (11) read as

$$\pi_t = a_{11} \pi_{t-1} + a_{12} x_{t-1} + a_{13} \pi_{t-2} + a_{14} x_{t-2} + \mu_\pi + \varepsilon_{\pi t} \tag{13}$$

$$x_t = a_{21} \pi_{t-1} + a_{22} x_{t-1} + a_{23} \pi_{t-2} + a_{33} x_{t-2} + \mu_x + \varepsilon_{xt} \tag{14}$$

therefore

$$E_t \pi_{t+1} = a_{11} \pi_t + a_{12} x_t + a_{13} \pi_t + a_{14} x_{t-1} + \mu_\pi. \tag{15}$$

From (1):

$$E_t \pi_{t+1} = \frac{1}{\gamma_f} \pi_t - \frac{\gamma_b}{\gamma_f} \pi_{t-1} - \frac{\lambda}{\gamma_f} x_t \tag{16}$$

so that equating (15) and (16) and abstracting from the constant, the following set of constraints must hold:

$$a_{11} = \frac{1}{\gamma_f} \quad , \quad a_{13} = -\frac{\gamma_b}{\gamma_f} \quad , \quad a_{12} = -\frac{\lambda}{\gamma_f} \quad , \quad a_{14} = 0. \tag{17}$$

The hypothesis of absence of Granger non-causality from π_t to x_t (which guarantees, if $\gamma_f + \gamma_b < 1$, the existence of a unique stable solution, see Section 3) corresponds to

$$a_{21} = 0 \quad , \ a_{23} = 0. \tag{18}$$

It is evident that (17)-(18) define a set of restrictions that can be easily tested (separately or jointly) in the context of the stationary VAR (11). For instance, under the zero constraints in (17)-(18) (or in (17) alone), the ML estimator of the parameters of the VAR corresponds to the Generalized Least Squares (GLS) estimator, and Wald-type or Likelihood Ratio (LR) tests have standard χ^2-distribution with degree of freedom equal to the number of restrictions being tested (Lütkepohl, 1993)[16].

Abstracting from the Granger-causality between π_t and x_t, a simple test of the NKPC (1) can be carried out by checking whether the zero forward-looking restrictions in (17) are fulfilled or not; this can be interpreted as a test on the "necessary conditions" for the NKPC to hold. If the zero forward-looking restrictions are not rejected, indirect ML estimates of the structural parameters γ_f, γ_b, λ can be obtained from the ML (GLS) estimates of the VAR obtained under the zero restrictions alone. Indeed by inverting the relations in (17) one gets:

$$\widehat{\gamma}_f = \widehat{a}_{11}^{-1} \quad , \quad \widehat{\gamma}_b = -\widehat{a}_{13}\,\widehat{a}_{11}^{-1} \quad , \quad \widehat{\lambda} = -\widehat{a}_{12}\,\widehat{a}_{11}^{-1}$$

where $\widehat{a}_{11}$, $\widehat{a}_{12}$ and $\widehat{a}_{13}$ are the ML (GLS) estimates of the non-zero parameters of the VAR. Alternatively, FIML estimates of the structural parameters can be directly achieved by applying conventional numerical optimization procedures for estimating the VAR (11) subject to the constraints implied by (17) (or by (17)-(18)).

Before moving to the other cases we briefly discuss how the proposed method can be implemented when the focus is on "non-exact" versions of the NKPC. In these situations the expression in (5) reads as

$$E_t\pi_{t+1} = \frac{1}{\gamma_f}\pi_t - \frac{\gamma_b}{\gamma_f}\pi_{t-1} - \frac{\lambda}{\gamma_f}x_t - \frac{1}{\gamma_f}u_t$$

so that applying the law of iterated expectations and using $E_{t-1}u_t = 0$ it follows

$$E_{t-1}\pi_{t+1} = \frac{1}{\gamma_f}E_{t-1}\pi_t - \frac{\gamma_b}{\gamma_f}\pi_{t-1} - \frac{\lambda}{\gamma_f}E_{t-1}x_t. \tag{19}$$

From the equations (13)-(14) it is possible to compute expressions for $E_{t-1}\pi_{t+1}$, $E_{t-1}\pi_t$ and $E_{t-1}x_t$, which substituted into (19) allow to find the restrictions between the parameters of the VAR and those of the NKPC.

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[16]Observe that in (17)-(18) the number of zero restrictions is 3, one of which corresponds to (17) alone.

4.2 Case2: I(1) cointegrated variables

The VAR (11) can be written in the Vector Equilibrium Correction (VEqC) form

$$\Delta Y_t = \Pi Y_{t-1} + \Phi_1 \Delta Y_{t-1} + \ldots + \Phi_k \Delta Y_{t-k+1} + \mu + \varepsilon_t \tag{20}$$

where $\Pi = -(I_p - \sum_{i=1}^{k} A_i)$ is the long run impact matrix and $\Phi_j = -\sum_{i=j+1}^{k} A_i$, $j = 1, ..., k-1$. Assume now that the roots of the characteristic equation (12) are such that $\mid z \mid > 1$ or $z = 1$. The rank of the Π matrix determines the cointegration properties of the system (Johansen, 1996); if $rank(\Pi) = r$, $0 < r < p$, then the I(1) system is cointegrated and $\Pi = \alpha\beta'$, where α and β are two $p \times r$ full rank matrices, where $\beta'Y_t$ are the cointegrating (equilibrium) relations of the system and the elements of α measure the adjustment of each variable to deviations from equilibrium.

Again, consider the case where x_t is a scalar, $k = 3$ and π_t and x_t are linked by the cointegrating relation: $\beta'Y_t = \pi_t - \beta_{12}x_t \sim I(0)$. The two equations of (20) read as

$$\Delta \pi_t = \alpha_{11}(\pi_{t-1} - \beta_{12}x_{t-1}) + \phi_{11}\Delta\pi_{t-1} + \phi_{12}\Delta x_{t-1} + \phi_{13}\Delta\pi_{t-2} + \phi_{14}\Delta x_{t-2} + \mu_\pi + \varepsilon_{\pi t}$$
$$\Delta x_t = \alpha_{21}(\pi_{t-1} - \beta_{12}x_{t-1}) + \phi_{21}\Delta\pi_{t-1} + \phi_{22}\Delta x_{t-1} + \phi_{23}\Delta\pi_{t-2} + \phi_{24}\Delta x_{t-2} + \mu_x + \varepsilon_{xt}$$

therefore

$$E_t\Delta\pi_{t+1} = \alpha_{11}(\pi_t - \beta_{12}x_t) + \phi_{11}\Delta\pi_t + \phi_{12}\Delta x_t + \phi_{13}\Delta\pi_{t-1} + \phi_{14}\Delta x_{t-1} + \mu_\pi. \tag{21}$$

By simple algebra the NKPC1 (1) can be expressed in the error-correcting form

$$E_t\Delta\pi_{t+1} = \left(\frac{1 - \gamma_f - \gamma_b}{\gamma_f}\right)(\pi_t - \omega\ x_t) + \frac{\gamma_b}{\gamma_f}\Delta\pi_t$$

where $\omega = \frac{\lambda}{1-\gamma_f-\gamma_b}$, provided $\gamma_f + \gamma_b \neq 1$. Equating the last expression with (21) the restrictions between the two models are given by

$$\beta_{12} = \frac{\lambda}{1 - \gamma_f - \gamma_b}; \tag{22}$$

$$\alpha_{11} = \left(\frac{1 - \gamma_f - \gamma_b}{\gamma_f}\right); \tag{23}$$

$$\phi_{11} = \frac{\gamma_b}{\gamma_f}\ , \phi_{12} = 0\ ,\ \phi_{13} = 0\ ,\ \phi_{14} = 0 \tag{24}$$

whereas in order to rule out feedbacks from π_t to x_t:

$$\alpha_{21} = 0\ ,\ \phi_{21} = 0. \tag{25}$$

Also in this case, provided the zero constraints implied by the forward-looking hypothesis are not rejected, it is possible to invert the relations in (22)-(24) to recover indirect ML estimates of the structural parameters from those of the cointegrated VEqC. Indeed, by solving (22), (23) and (24) with respect to the structural parameters:

$$\widehat{\lambda} = \widehat{\beta}_{12}\left(\frac{\widehat{\alpha}_{11}}{\widehat{\alpha}_{11} + \widehat{\phi}_{11} + 1}\right)$$

$$\widehat{\gamma}_f = \frac{\widehat{\phi}_{11}}{\widehat{\alpha}_{11} + \widehat{\phi}_{11} + 1}$$

$$\widehat{\gamma}_b = \frac{1}{\widehat{\alpha}_{11} + \widehat{\phi}_{11} + 1}$$

where $\widehat{\beta}_{12}$ is the super-consistent and efficient ML estimate of the cointegrating parameter and $\widehat{\alpha}_{11}$ and $\widehat{\phi}_{11}$ are the ML (GLS) estimates of the short run parameters of the VEqC[17].

4.3 Case 3: I(1) not cointegrated variables

Assume again that the roots of the characteristic equation (12) are such that $\mid z \mid > 1$ or $z = 1$ but, *ceteris paribus*, in (20) $rank(\Pi) = 0$, i.e. variables are I(1) but not cointegrated. The two equations of the VEqC correspond to those of the following VAR in first differences:

$$\Delta\pi_t = \phi_{11}\Delta\pi_{t-1} + \phi_{12}\Delta x_{t-1} + \phi_{13}\Delta\pi_{t-2} + \phi_{14}\Delta x_{t-2} + \mu_\pi + \varepsilon_{\pi t} \quad (26)$$

$$\Delta x_t = \phi_{21}\Delta\pi_{t-1} + \phi_{22}\Delta x_{t-1} + \phi_{23}\Delta\pi_{t-2} + \phi_{24}\Delta x_{t-2} + \mu_x + \varepsilon_{x t} \quad (27)$$

therefore

$$E_t\Delta\pi_{t+1} = \phi_{11}\Delta\pi_t + \phi_{12}\Delta x_t + \phi_{13}\Delta\pi_{t-1} + \phi_{14}\Delta x_{t-1} + \mu_\pi.$$

By differentiating (1) we get what one could an "accelerationist"-type NKPC:

$$\Delta\pi_t = \gamma_f E_t\Delta\pi_{t+1} + \gamma_b\Delta\pi_{t-1} + \lambda\Delta x_t \quad (28)$$

which on turn implies that

$$E_t\Delta\pi_{t+1} = \frac{1}{\gamma_f}\Delta\pi_t - \frac{\gamma_b}{\gamma_f}\Delta\pi_{t-1} - \frac{\lambda}{\gamma_f}\Delta x_t.$$

---

[17]It is worth noting that if $\gamma_f + \gamma_b < 1$, then under the forward-looking constraints (22)-(24) the parameter α_{11}, which measures the adjustment of the acceleration rate ($\Delta\pi_t$) to the disequilibria, must be positive for a unique and stable solution to occur, as $\gamma_f + \gamma_b < 1$ implies $\alpha_{11} > 0$. Thus when in the cointegrated VEqC it is found that (25) holds but the adjustment parameter is significantly negative, this indicicates that the NKPC (1) can not hold empirically. However, as observed in Section 3 a unique and stable solution might even occur with $\gamma_f + \gamma_b > 1$ for particular parametric configurations in which (25) is violated.

In this case the restrictions are given by

$$\phi_{11} = \frac{1}{\gamma_f} \quad , \quad \phi_{12} = -\frac{\lambda}{\gamma_f} \quad , \quad \phi_{13} = -\frac{\gamma_b}{\gamma_f} \quad , \quad \phi_{14} = 0. \tag{29}$$

whereas feedbacks from $\Delta\pi_t$ to Δx_t are ruled out if

$$\phi_{21} = 0 \quad , \quad \phi_{23} = 0. \tag{30}$$

The estimation of the structural parameters and a test of the NKPC can be carried out exactly as in Case 1 with the difference that the model involve variables in first differences. Standard techniques apply.

5   Results for the Euro area

We consider quarterly data on the Euro area taken from Fagan et al. (2001). Several VARs of the form $Y_t = (\pi_t,\, x_t)'$ are specified with x_t a scalar measured respectively as: (a) the wage share; (b) the output gap; (c) the unemployment rate. We also consider three-dimensional VARs of the form $Y_t = (\pi_t,\, x_t,\, i_t)'$ with x_t measured as in (a), (b) and (c) above and with i_t the short term nominal interest rate. As argued in Fuhrer and Moore (1995), the short-term nominal rate is closely linked to real output and is thus essential to forming expectations of output and closely related variables as the unemployment rate. The output gap in (b) is defined in two different ways: as deviation of real GDP from potential output measured as a constant-returns-to-scale Cobb-Douglas production function and neutral technical progress, and as deviation of real GDP from a quadratic trend. Mnemonics and series definitions are listed in Table 1.

Each VAR was estimated over the 1970:1 - 1998:2 period ($T = 114$ observations) with the sample including initial values (hence only $T - k$ quarterly observations are really exploited in estimation, with k being the lag length)[18]. In all VARs we included a constant and a deterministic seasonal dummy taking value 1 at the fourth quarter of 1974 in correspondence of the inflationary pick due to the oil shock and zero elsewhere. We selected $k = 5$ lags in all estimated models and obtained well-behaved Gaussian-distributed residuals. Simple comparison between the dynamic structure implied by model (1) and that of a VAR with 5 lags suggests that further dynamics should be perhaps incorporated in the forward-looking model to be consistent with the data.

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[18] Actually, because of data availability the VAR involving the output gap measured as deviation of real GDP from potential output measured as a constant-returns-to-scale Cobb-Douglas production function and neutral technical progress is estimated over a shorter sample, see Table 1.

On the basis of the results of the Trace cointegation test[19] the zero forward-looking restrictions implied by the NKPC were tested as described in Case 1, 2 and 3 of Section 4 (adapting opportunely the restrictions to the case of a VAR with 5 lags); we disentangled the test for the zero forward-looking restrictions from the test for the absence of Granger-causality from the inflation rate the explanatory variable(s). As observed in Section 4 testing the zero forward-looking restrictions implied by the NKPC amounts to test a set of necessary conditions for the model to hold. Rejection of this subset of forward-looking constraints imply a rejection of the whole model. However, in all cases before switching to the VEqC representation of the VAR we first computed a LR test for the forward-looking restrictions implied by the NKPC in the VAR in levels, i.e. treating variables as if they were I(0) (Case 1)[20]. For the situations where a cointegrating relations was found we reported the estimated cointegrating relation with corresponding adjustment coefficients. Overall results are summarized in the tables from 2 to 9.

Results points that whatever is the driving variable(s) used in the analysis, the system $Y_t = (\pi_t, \; x'_t)'$ is perceived as I(1) over the 1970-1998 period. For instance, the results in Table 2 suggest that the inflation rate and the wage share are I(1) and not cointegrated. A "spurious regression" argument can be then advocated for GMM based estimates of the NKPC over the 1970-1998 period when variable are treated as stationary; this issue is completely ignored in GGLS and many other existing papers.

A cointegrating relation between the inflation rate and a single explanatory variable is found in the situations where x_t is proxied by: (i) the unemployment rate; (ii) deviations of real GDP from potential output measured as a constant-returns-to-scale Cobb Douglas production function and neutral technical progress. In particular, the empirical evidence seems to be consistent with the predictions of recently reappraised theories explaining the long run inflation-unemployment trade-off (Karanassaou et al., 2003).

As expected feedbacks from the inflation rate to the explanatory variable(s) x_t are generally found (the only exception is the case of the unemployment rate, see Table 4) suggesting that the NKPC should be probably investigated in the context of a structural system of equations.

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[19]Given the absence of deterministic linear trend in the variables, in the test for cointegration rank the constant was restricted to belong to the cointegration space in all VARs. Critical values are taken from Johansen (1996), Table 15.2.

[20]Sims et al. (1991) highlight the drawbacks associated with the use of standard asymptotic theory when the variables in the VAR are actually I(1). For this reason in the tables from 2 to 9 below we do not report p-values associated with the tests of zero forward-looking restrictions in the VAR in levels.

6   Summary and suggestions for further developments

It might be argued that the possibility of disentangling empirically forward vs backward looking behavior on the basis of aggregate macro-data represents a debated question (Hendry, 1998, Cuthbertson, 1988, Ericsson and Hendry 1999). This issue is probably true but should represent an incentive for further developments on the subject.

In this paper we have proposed a simple technique for testing the NKPC through VAR models. The basic idea is that forward-looking agents compute expectations by means of VARs involving the inflation rate and the driving variable(s). This hypothesis allows to nest the NKPC within a VAR. A number of econometric issues such as stability of parameters and the presence of feedbacks can easily investigated within the VAR. The proposed method leads to ML estimates and gives the possibility of taking into explicit non-stationarity and cointegration. The procedure can be implemented with any existing econometric software.

Referring to their estimates of the hybrid further-inflation-lags version of the NKPC, GGLS, p. 1258 observe that: *"Thus, it appears that the structural marginal cost based model can account for the inflation dynamics with relatively little reliance on arbitrary lags of inflation, as compared to the traditional Phillips curve [...]"*. Our empirical evidence on the NKPC for the Euro area appears in sharp contrast with this claim for the following reasons.

First, feedbacks from the inflation rate to the explanatory variable are generally found suggesting that GMM-based single equation estimates of the NKPC might be carried out on models that might give rise to explosive solutions. The presence of such feedbacks reflects the omission of important structural relationships characterizing variables. We argue that in these circumstances the NKPC should be investigated within a more general structural model where feedbacks are the consequences of behavioral relationships.

Second, the persistence of variables over the investigated span seems to be consistent with that of unit-roots processes. A long run link among the inflation rate and part of the driving variables represent a concrete possibility. Even ignoring the problem of feedbacks, single-equation estimates of the NKPC should be formulated accordingly.

Third, the restrictions implied by the NKPC on the parameters of the VAR are sharply rejected in a "full information" framework. More than one lag of inflation is generally required if one wishes to reconcile the forward-looking model of inflation dynamics with the empirical evidence. This result suggests that specifications of the form (2) (or suitable variants of it) are probably more suited for the forward-looking model. We postpone the empirical investigation of specifications similar to (2) to further research.

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TABLES

Mnemonic	Definition
p_t	log of the implicit GDP deflator
π_t	inflation rate on a quartely basis: $p_t - p_{t-4}$
ws_t	log of deviations of real unit labor costs from its mean[a]
$\widetilde{y}_{1t}$	deviation of real GDP from potential output[b]
$\widetilde{y}_{2t}$	deviation of real GDP from quadratic trend
u_t	unemployment rate
i_t	short term nominal interest rate

Table 1: Quarterly data on Euro area 1970:1 - 1998:2, see Fagan et al. (2001). a = computed as in GGL; b = potential output is assumed to be given by a constant-returns-to-scale Cobb-Douglas production function and neutral technical progress; this series starts at 1971:4

VAR: $Y_t = (\pi_t \ , \ ws_t)'$, lag length 5		
H$_0$: zero forward-looking restrictions (VAR in levels): LR: $\chi^2(7)$=22.96		
Cointegration rank test		
H$_0 : r \leq j$	Trace	5% c.v.
j=0	11.08	19.96
j=1	1.70	9.24
Test of hypotheses on the VAR in differences		
H$_0$: No Granger causality from $\Delta\pi_t$ to Δws_t : LR: $\chi^2(4)$=18.57 [0.00]		
H$_0$: zero forward-looking restrictions: LR: $\chi^2(5)$=22.15 [0.00]		

Table 2: Test of the NKPC (1) in the Euro area over the 1970:1 - 1998:2 period. See Table 1 for variable definitions.

VAR: $Y_t = (\pi_t\ ,\ ws_t\ ,\ i_t)'$, lag length 5		
H$_0$: zero forward-looking restrictions (VAR in levels): LR: $\chi^2(11)$=57.15		
Cointegration rank test		
H$_0 : r \leq j$	Trace	5% c.v.
j=0	47.19	34.91
j=1	17.26	19.96
j=2	4.48	9.24
Estimated cointegrating relation and adjustment coefficients		

$$\widehat{\beta}'Y_t = \pi_t - \underset{(0.05)}{0.81}ws_t + \underset{(0.077)}{0.176}i_t - \underset{(0.12)}{2.08}$$

$$\widehat{\alpha}' = (\underset{(0.04)}{0.08}\ ,\ \underset{(0.06)}{0.26}\ ,\ \underset{(0.08)}{-0.11})'$$

Test of hypotheses on the VEqC

H$_0$: No Granger-causality from $\Delta\pi_t$ to Δws_t , Δi_t: LR: $\chi^2(10)$=32.82 [0.00]

H$_0$: zero forward-looking restrictions: LR: $\chi^2(11)$=75.44 [0.00]

Table 3: Test of the NKPC (1) in the Euro area over the 1970:1 - 1998:2 period. See Table 1 for variable definitions.

VAR: $Y_t = (\pi_t\ ,\ u_t)'$, lag length 5		
H$_0$: zero forward-looking restrictions (VAR in levels): LR: $\chi^2(7)$=31.97		
Cointegration rank test		
H$_0 : r \leq j$	Trace	5% c.v.
j=0	29.41	19.96
j=1	3.82	9.24
Estimated cointegrating relation and adjustment coefficients		

$$\widehat{\beta}'Y_t = \pi_t + \underset{(0.16)}{1.38}u_t - \underset{(0.01)}{0.17}$$

$$\widehat{\alpha}' = (\underset{(0.01)}{-0.07}\ ,\ \underset{(0.006)}{0.005})'$$

Test of hypotheses on the VEqC

H$_0$: No Granger causality from $\Delta\pi_t$ to Δu_t: LR: $\chi^2(5)$=9.27   [0.10]

H$_0$: zero forward-looking restrictions: LR: $\chi^2(7)$=34.60   [0.00]

Table 4: Test of the NKPC (1) in the Euro area over the 1970:1 - 1998:2 period. See Table 1 for variable definitions.

VAR: $Y_t = (\pi_t \ , \ u_t \ , \ i_t)'$, lag length 5		
H$_0$: zero forward-looking restrictions (VAR in levels): LR: $\chi^2(11)$=49.21		

Cointegration rank test		
H$_0 : r \leq j$	Trace	5% c.v.
j=0	47.68	34.91
j=1	16.90	19.96
j=2	5.57	9.24

Estimated cointegrating relation and adjustment coefficients

$$\widehat{\beta}'Y_t = \pi_t + \underset{(0.14)}{1.21}u_t - \underset{(0.01)}{0.15} \quad (\text{LR: } \chi^2(1)\text{=1.77} \ [0.18])^a$$

$$\widehat{\alpha}' = (\underset{(0.04)}{-0.07} \ , \ \underset{(0.007)}{0.005} \ , \ \underset{(0.031)}{-0.098})'$$

Test of hypotheses on the VEqC

H$_0$: No Granger-causality from $\Delta\pi_t$ to Δu_t , Δi_t: LR: $\chi^2(10)$=21.81 [0.02]

H$_0$: zero forward-looking restrictions: LR: $\chi^2(11)$=55.72 [0.00]

Table 5: Test of the NKPC (1) in the Euro area over the 1970:1 - 1998:2 period. See Table 1 for variable definitions. Notes: a = test for overidentifying restrictions on the cointegrating relation.

VAR: $Y_t = (\pi_t \ , \ \widetilde{y}_{1t})'$, lag length 5		
H$_0$: zero forward-looking restrictions (VAR in levels): LR: $\chi^2(7)$=38.34		

Cointegration rank test		
H$_0 : r \leq j$	Trace	5% c.v.
j=0	35.95	19.96
j=1	0.39	9.24

Estimated cointegrating relation and adjustment coefficients

$$\widehat{\beta}'Y_t = \pi_t - \underset{(0.013)}{0.09} \ \widetilde{y}_{1t} - \underset{(0.013)}{0.09}$$

$$\widehat{\alpha}' = (\underset{(0.004)}{-0.025}, \ \underset{(0.72)}{1.31})'$$

Test of hypotheses on the VEqC

H$_0$: No Granger causality from $\Delta\pi_t$ to $\Delta\widetilde{y}_{1t}$: LR: $\chi^2(5)$= 25.32 [0.00]

H$_0$: zero forward-looking restrictions: LR: $\chi^2(7)$=40.51 [0.00]

Table 6: Test of the NKPC (1) in the Euro area over the 1970:1 - 1998:2 period. See Table 1 for variable definitions.

VAR: $Y_t = (\pi_t , \widetilde{y}_{1t} , i_t)'$, lag length 5		
H_0: zero forward-looking restrictions (VAR in levels): LR: $\chi^2(11)=45.58$		
Cointegration rank test		
$H_0 : r \leq j$	Trace	5% c.v.
j=0	36.28	34.91
j=1	11.61	19.96
j=2	5.23	9.24
Estimated cointegrating relation and adjustment coefficients		

$$\widehat{\beta}' Y_t = \pi_t - \underset{(0.02)}{0.11}\widetilde{y}_{1t} - \underset{(0.02)}{0.10} \quad (\text{LR: } \chi^2(1)=0.0.6 \ [0.80])^a$$

$$\widehat{\alpha}' = (\underset{(0.004)}{-0.016} , \underset{(0.65)}{0.62} , \underset{(0.79)}{-2.36})'$$

Test of hypotheses on the VEqC		
H_0: No Granger-causality from $\Delta\pi_t$ to $\Delta\widetilde{y}_{1t}$, Δi_t: LR: $\chi^2(10)=41.67$ [0.00]		
H_0: zero forward-looking restrictions: LR: $\chi^2(11)=43.36$ [0.00]		

Table 7: Test of the NKPC (1) in the Euro area over the 1970:1 - 1998:2 period. See Table 1 for variable definitions. Notes: a = test for overidentifying restrictions on the cointegrating relation.

VAR: $Y_t = (\pi_t , \widetilde{y}_{2t})'$, lag length 5		
H_0: zero forward-looking restrictions (VAR in levels): LR: $\chi^2(7)=39.98$		
Cointegration rank test		
$H_0 : r \leq j$	Trace	5% c.v.
j=0	17.81	19.96
j=1	0.35	9.24
Test of hypotheses on the VAR in differences		
H_0: No Granger causality from $\Delta\pi_t$ to $\Delta\widetilde{y}_{2t}$: LR: $\chi^2(4)=20.62$ [0.00]		
H_0: zero forward-looking restrictions: LR: $\chi^2(5)=20.27$ [0.00]		

Table 8: Test of the NKPC (1) in the Euro area over the 1970:1 - 1998:2 period. See Table 1 for variable definitions.

VAR: $Y_t = \left(\pi_t \ , \ \widetilde{y}_{2t} \ , \ i_t\right)'$, lag length 5		
H_0: zero forward-looking restrictions (VAR in levels): LR: $\chi^2(11)=49.96$		
Cointegration rank test		
$H_0 : r \leq j$	Trace	5% c.v.
j=0	24.99	34.91
j=1	12.25	19.96
j=2	4.29	9.24
Test of hypotheses on the VAR in differences		
H_0: No Granger-causality from $\Delta\pi_t$ to $\Delta\widetilde{y}_{2t}$, Δi_t: LR: $\chi^2(8)=27.04$ [0.00]		
H_0: zero forward-looking restrictions: LR: $\chi^2(8)=25.86$ [0.001]		

Table 9: Test of the NKPC (1) in the Euro area over the 1970:1 - 1998:2 period. See Table 1 for variable definitions.