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Abstract

Daily data on the German market index return are used to consider multiple issues in a forecasting comparison of ARCH-type specifications. First, attention is paid to the impact of different sample sizes, different horizons and fitting of historical versus implied data. Secondly, the issue of volatility transmission is addressed by modelling French and Germany market indexes into a simultaneous conditionally heteroskedasticity framework. Errors obtained by updating the Black and Scholes formula with the different volatility forecasts are compared. The findings support, if no implied volatility is available, the use of the simplest GARCH specification estimated on short recent sample.

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1. INTRODUCTION

Volatility measurement in financial markets is a crucial task for investment decision and risk management. To this purpose a very extensive literature has been flowering and there are some extensive surveys summarizing the fundamental results on the arguments focusing on principal aspects regarding volatility (Andersen, Bollerslev and Diebold 2002b, Poon and Granger 2003). The first one is how to define volatility and its relationships with concepts of risk and standard deviation. In fact volatility is intended as measure of uncertainty or dispersion of returns in such a sense it distinguishes from risk which is more properly associated with small or negative returns, moreover, volatility is correctly measured by the standard deviation only into Gaussian framework, but not all. On the other hand the research entails principal methodologies for volatility modelling and forecasting which are time series models and option-based models. Time series models explicitly specify volatility as a function of the information set containing past returns and eventually other observable or latent state variables while option-based models exploit information contents in option prices and approximate volatility with the so called option implied volatility basing on the rationale that in option prices the market makes volatility, which is intrinsically a latent variable, observable.

In recent years option valuation has been an appealing area for volatility analysis, from both empirical and theoretical point of view. Option are derivatives financial securities used as speculative assets to gain from expected price changes or as hedging instruments to insure risky position in the underlying security. Their diffusion in organized exchange has been largely growing during last years both in North America and in Europe, as documented by the Bank of International Settlements Report (table 1).

The plain vanilla option corresponds to the European call option which gives the investor the right to buy the security, S , at the specified date in the future, T , paying the strike price, K , agreed at current time $t < T$. Since the option payoff could be instantaneously replicated by an underlying and riskless asset portfolio, an option pricing model should be a suitable instrument either to determine option fair value and, after calculating the delta Greek, to rebalance the synthetic portfolio to replicate the payoff for

Table 1: Options Traded on Organised Exchanges

| | Amounts outstanding (in billions of US dollars) | | | Turnover (in billions of US dollars) | |
|---------------|--|-----------|-----------|---|-----------|
| | Dec. 1999 | Dec. 2000 | Dec. 2001 | Year 2000 | Year 2001 |
| North America | 3,377.1 | 3,884.7 | 10,292.4 | 43,996.8 | 107,677.2 |
| Europe | 1,643.6 | 1,894.9 | 3,734.6 | 17,703.4 | 33,655.4 |

Source: BIS Quaterly Review, Sept. 2002.

hedging purposes. Just replicating the portfolio dynamics, with arbitrage arguments, Black and Scholes derived their closed-form model for option valuation (Black and Scholes 1973). The Black and Scholes European call option pricing formula states that option price at time t is a function of the price of the underlying asset, S_t , the strike price, K , the risk free interest rate r , the time to expiration T and the standard deviation of the underlying asset over the period from t to T .

Therefore, option valuation is influenced by the volatility but, unfortunately the actual volatility is unobservable and has to be estimated through a suitable strategy. The observed option prices can be used to construct the market's implicit perception of conditional volatility just inverting Black and Scholes formula. In fact, given that S_t , K , r and T are all observable, from each option price, through backward induction it is possible to derive the σ which market use to price that option itself.

Of course, this operation implies the drawback of involving not obvious extra hypotheses on option market efficiencies and the acknowledgment of the superior Black and Scholes formula (BS), on the other side balanced by the evidence that nowadays implied volatility is a security itself, priced in financial markets, and by the relevant forecasting power empirically tested in the literature (Ederington and Guan 2002). It is widely known that the principal drawback of BS arises from the strong constant volatility restriction in spite of the opposite empirical facts. Many theoretical papers have been written to the aim at allowing for a time varying volatility. The most general setup consists in a two diffusion (or differential in discrete time) equation system, explaining stock and volatility dynamics. Different diffu-

sion processes are assumed basing upon alternative restrictions for movements and correlation between stock and volatility a closed form solution for option price is provided by the affine diffusion model (Heston 1993) in which drift and volatility parameters are assumed linear functions of the stock and volatility. Alternatively it is proposed the log-volatility process which, even if does not deliver a close-form pricing formula, is more in line with standard discrete-time specification of SV as the popular EGARCH representation for financial series volatility. Recent models further generalize adding into the affine framework a jump component to better fit the tail behavior (Andersen, Benzoni and Lund 2002a, Eraker 1998, Pan 2002) or two additional volatility factors, the first to control the volatility persistence and the second to explain the tail behavior (Chernov, Gallant, Ghysels and Tauchen 2003).

In spite of such sophisticated specifications, whose implementation involve many numerical and statistical issues, the operators in the exchanges continue to use BS to compute option price regularly updating volatility parameter backing out the market information. For this reason, although it is proved that there could be a benefit in using SV instead of BS (Bakshi, Cao and Chen 1997), it is also interesting to investigate alternative approaches able to provide improvements in forecasting volatility investigating the relation between the two areas therefore alternative strategies to forecast volatility are examined to the aim of updating BS formula, just like the practitioners do. Among time series models, in the last decades, many empirical applications have been implemented into ARCH-type framework, following the publication of the seminal paper by Engle (Engle 1982). In fact, since then a vast quantity of extensions of the original autoregressive conditionally heteroskedastic formula have been developed.

The purpose of the paper consists in investigating the forecasting properties of a wide also if far from being exhaustive class of competing ARCH-type volatility models, posing insights in statistical issues such as choice of data, sample periods, horizons, etc. Forecasting power is tested through option pricing. Then, the paper complements the literature in others aspects it extends analysis to multivariate ARCH-type models, enlarging the information set to more securities finally, it provides an application on German option exchange index (DAX) which is the second largest in the world and

is scarcely analyzed due to prevalence of American literature. An advantage for paper purpose arises from the quotation in this market of an index based on implied volatilities of options on German market index (VDAX) computed as linear interpolation of the implied volatilities of the two sub-indices nearest to 45 days to maturity.

In the following section, we introduce the estimated volatility models motivating the experimental design then the empirical results are presented and discussed. Concluding remarks are given in section 3.

2. VOLATILITY MODELS

Univariate Models

The standard GARCH formulation (Bollerslev 1986), for the DAX rate of return r_t is given by

$$\begin{aligned} r_t &= \mu_t + \eta_t \\ \eta_t &\sim N(0, \sigma_t) \\ \sigma_t^2 &= \omega + \alpha \eta_{t-1}^2 + \beta \sigma_{t-1}^2 \end{aligned} \quad (1)$$

where η_t represents the deviation at time t from the conditional expectation function μ_t . The error follows a Gaussian distribution with 0 mean and time varying conditional standard deviation σ_t . The conditional variance is modelled as an increasing function of itself and squared error lagged. The parameter β indicates the persistence in volatility and higher α implies higher conditional kurtosis in GARCH specification (as well as in ARCH one) to guaranty volatility positivity all the parameters must be non-negative. It could be useful to note that it consists in an alternative parsimonious formulation for the conditional variance of a pure autoregressive process dependent on the magnitude of lagged errors irrespective of their signs.

Other specifications into ARCH-type class have been analyzed the Threshold ARCH (Engle and Ng 1993) formulated to admit asymmetrical effect of bad information by means of incorporating a component dependent on negative news in the formula of the conditional variance,

$$\sigma_t^2 = \omega + \alpha \eta_{t-1}^2 + \gamma d_{t-1} \eta_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (2)$$

where $d_{t-1} = 1$ if $\eta_{t-1} < 0$ and 0 otherwise, and the Exponential formulation (Nelson 1991) which allows both asymmetrical information and no restrictions on parameters through modelling of the logarithm of the conditional variance, as follows:

$$\ln \sigma_t^2 = \omega + \alpha \left| \frac{\eta_{t-1}}{\sigma_{t-1}} \right| + \gamma \frac{\eta_{t-1}}{\sigma_{t-1}} + \beta \ln \sigma_{t-1}^2 \quad (3)$$

Specification (1), (2) and (3) have been estimated for daily index returns respectively on two sample periods, the first from January 1995 to December 2000, the second shorter limited from January to December 2000. While globally in the first period the DAX exhibits a quite small but positive rate of returns (0.07%), in the last year 2000 the rate of return falls negative (-0.03%) in correspondence with a stagnation of world wide exchanges. Moreover in the last sub-period the index return distribution exhibits neither skewness, nor kurtosis, nor significant deviation from normality. The annual volatilities computed for the two sub-periods are respectively 21.36% and 23.72%.

Besides estimating the expected volatility as an explicit function of observed returns, ARCH models have been estimated on time series of implied variances (IV) obtained by options prices. There is a wide literature on this topic and generally it is found that implied volatilities are more accurate forecasts of future volatility than GARCH models.

In table 2 parameters estimates with corresponding standard errors in parentheses of specifications (1), (2) and (3) on the two sample periods, respectively for historical and implied times series are reported. With respect to historical estimates the overall diagnostics in table 3 indicate that, although more complex models reach higher maximum likelihood, Schwartz and Akaike criteria suggest to prefer the simplest GARCH specification. Coherent findings for each specification can be drawn: volatility mean level is higher (ω) and more persistent (β) in the 2000 sub-period the parameter α is smaller for shorter and more recent period, indicating scarce con-

Table 2: Univariate ARCH-type Estimates

| | Historical | | | | Implied | | | | | |
|------|---------------------|---------------------|--------------------|--------------------|---------------------|---------------------|---------------------|--------------------|--------------------|---------------------|
| | μ | ω | α | β | γ | μ | ω | α | β | γ |
| G 95 | 0.0824 (0.0262) | 0.0260 (0.0115) | 0.0920 (0.0191) | 0.8963 (0.0211) | | 0.0448 (0.0322) | 0.0456 (0.0109) | 0.0240 (0.0017) | 0.960 (0.0041) | |
| G 00 | -0.0418 (0.0869) | 0.0447 (0.0336) | 0.0425 (0.0255) | 0.9321 (0.0359) | | -0.0293 (0.0919) | 0.0443 (0.0303) | 0.0021 (0.0028) | 0.9766 (0.0132) | |
| T 95 | 0.0722 (0.0278) | 0.0282 (0.0056) | 0.071 (0.0152) | 0.8948 (0.0125) | 0.0390 (0.0160) | 0.0376 (0.0335) | 0.0469 (0.0110) | 0.0224 (0.0028) | 0.9603 (0.0041) | 0.0023 (0.0032) |
| T 00 | -0.0684 (0.0856) | 0.0328 (0.0378) | 0.0072 (0.0378) | 0.9339 (0.0360) | 0.0868 (0.0586) | -0.0150 (0.0921) | 0.0370 (0.0302) | 0.0061 (0.0034) | 0.9801 (0.0132) | -0.0089 (0.0043) |
| E 95 | 0.0634 (0.0281) | -0.1362 (0.0273) | 0.1910 (0.0387) | 0.9752 (0.0093) | -0.0433 (0.0278) | 0.0490 (0.0332) | -0.0369 (0.0064) | 0.0337 (0.0049) | 0.9927 (0.0025) | -0.0201 (0.0041) |
| E 00 | -0.1030 (0.0863) | -0.0417 (0.0455) | 0.0633 (0.0588) | 0.9856 (0.0434) | -0.0924 (0.0168) | -0.0328 (0.0917) | -0.0045 (0.0152) | 0.0113 (0.0104) | 0.9860 (0.0109) | -0.0045 (0.0088) |

Table 3: Univariate ARCH-type Diagnostics

| | L* | AIC | SIC | LB(2) | L | AIC | SIC | LB(2) | |
|------|--------|--------|--------|-------|------|--------|--------|--------|-------|
| Gh95 | -1.880 | 0.5981 | 0.6118 | 0.370 | Gh00 | -1.786 | 0.8305 | 0.8853 | 1.998 |
| Th95 | -1.592 | 0.5994 | 0.6165 | 0.490 | Th00 | -1.789 | 0.8349 | 0.9031 | 3.297 |
| Eh95 | -1.591 | 0.5994 | 0.6165 | 0.584 | Eh00 | -1.778 | 0.8406 | 0.9031 | 3.197 |

* L is the mean log-likelihood AIC and SIC are Akaike and Schwartz information criteria, not comparable for the two periods because of their dependence on number of observations! LB(2) is Ljung-Box statistic with 2 lags.

ditional kurtosis. Negative news has positive effect on volatility, in both TARCH and EGARCH specifications.

Estimates of implied variances series, computed by seemingly unrelated regression, provide similar findings, stressing much stronger persistence! asymmetrical effects of bad news are significant only in EGARCH 95 specification.

Diagnostics for checking alternative ARCH specification are usually based on testing autocorrelation of squared residuals (Ljung-Box statistic), or on standard maximum likelihood criteria (Schwartz and Akaike criteria). Alternative methods are based on forecasting performances in this framework it is possible to see how well different models forecast future squared residuals or other proxy of future volatility such as the implied volatility. We chose to assess the usefulness of each specification of the conditional variance on option prices.

According to existing literature and emulating the practitioners, each day call prices are computed by means of BS formula on the basis of the different volatility forecasts obtained, regularly updating the information sets, by ARCH-type models. Option database includes 3009 call prices on DAX index traded from January 1, through June 30, 2001. Performances were controlled, on different maturity and moneyness subclasses, through mean absolute relative pricing error (MARE) and mean relative error (MRE) used as precision and bias measures. Results are reported in tables (4) and (5). As benchmark, the errors of the simplest strategy consisting in substituting the day-back volatility in BS are displayed.

The errors, both relative and relative absolute, are greatly less for implied than for historical data, respectively around 5-6% versus 16-20% and 16% versus 25-32%. We expect a priori that punctual forecasts can not explain the volatility surface produced by option prices, but analysis of errors by maturity and moneyness sub-classes results anyway useful to quantify the pricing errors. Forecasting performance of implied models does not seem strongly sensitive to specification neither to sample period moreover they seem not to be preferred to simple implied volatility substitution except for the greater precision in short maturity. The models estimated on historical data reach the smallest errors, near to implied ones, only for options with less than 2 months to maturity, that is for short maturity options. Specifications estimated over the longest period usually obtain smaller errors than over the shortest one exponential and threshold auto-regressive conditional specifications perform better than simple generalized in short maturity which in its turn forecasts better for the medium and long maturity options and as a whole. Since call price increases as volatility increases, positive systematic errors mean underestimation of real volatilities. The analysis of displacement of errors for moneyness sub-classes exhibits smirkness behavior for each model, confirming that greater volatility is associated with downside risk. The implied forecasts distinguish from historical only, as expected, in the size errors (around 5-6% versus 16-18%) no further insights can be drawn by moneyness analysis.

Multi-step forecasts

For option pricing purpose, a volatility term structure based on multi-step ahead forecasts is theoretically requested usually for an option lasting between day $t + 1$ and T , the forecast of conditional variance at time t is taken equal to the expected variance of daily forecasts from 1 to $T - t$ days ahead, as follows:

$$\frac{1}{T-t} \sum_{k=1}^{T-t} E_t [\sigma_{t+k}^2]$$

where each variance k -steps ahead forecast for $k = 1, \dots, T - t$ is generated, on the basis of estimated parameters, by:

$$E_t [\sigma_{t+k}^2] = \omega \frac{1 - (\alpha + \beta)^k}{1 - (\alpha + \beta)} + (\alpha + \beta)^k \sigma_t^2$$

To this purpose we examine the performance for multi-periods ahead forecasts for GARCH specification and the effect of multi-steps versus one-step forecasts.

Tables 4 and 5 report the result in option prices forecasting for historical models estimated on shorter period multi-steps forecasts are found to reach systematic errors in medium and long maturity options minor than one-step ahead, the opposite happens for historical models estimated on longer period. Multi-steps forecasts obtained by implied models get worst both absolute and relative errors than one-step.

Bivariate Models

Despite of the success of the research on transmission of volatility in others financial application as asset pricing, portfolio selection and risk management, not much is known about its out of sample performance here the forecasting power of volatility transmission is controlled and compared to univariate information.

We choose to model volatility transmission between the German and the French market indexes for which the observed correlations of squared returns are $\rho_{Gt, Ft} = 0.624$ and $\rho_{Gt, Ft-1} = 0.202$, excluding correlations of larger magnitude such as with American S&P500 squared returns ($\rho_{Gt, US_t} = 0.324$ and $\rho_{Gt, US_{t-1}} = 0.350$) since affected by non synchronous trading hours. To assess the importance of French and German markets interdependence and volatility interaction the bivariate specification corresponding to the BEKK model of Engle and Kroner (Engle and Kroner 1995), has been estimated,

$$\begin{aligned} \mathbf{r}_t &= \boldsymbol{\mu}_t + \boldsymbol{\eta}_t \\ \boldsymbol{\eta}_t &\sim N(\mathbf{0}, H_t) \\ H_t &= C'C + A'\boldsymbol{\eta}_{t-1}\boldsymbol{\eta}'_{t-1}A + B'H_{t-1}B \end{aligned}$$

which corresponds to a generalization of the univariate GARCH model. The maximum likelihood estimates of the models are reported in table 6.

Table 4: Pricing Errors by Maturity

| | MARE | | | | | MRE | | | | |
|-------|----------|--------|--------|--------|--------|----------|--------|--------|--------|--------|
| | Maturity | | | | | Maturity | | | | |
| n | <2M | 2/6M | 6M/1Y | >1Y | tot | <2M | 2/6M | 6M/1Y | >1Y | tot |
| | 247 | 609 | 1444 | 709 | 3009 | 247 | 609 | 1444 | 709 | 3009 |
| Gh95 | 0.1804 | 0.3274 | 0.3072 | 0.4060 | 0.3256 | 0.0692 | 0.2225 | 0.1314 | 0.2300 | 0.1659 |
| Gh00 | 0.1806 | 0.2673 | 0.2297 | 0.3002 | 0.2516 | 0.0924 | 0.2037 | 0.1352 | 0.2415 | 0.1685 |
| Eh95 | 0.1714 | 0.3361 | 0.3274 | 0.3882 | 0.3320 | 0.0520 | 0.2254 | 0.1631 | 0.2538 | 0.1855 |
| Eh00 | 0.1653 | 0.2695 | 0.2624 | 0.3046 | 0.2674 | 0.0577 | 0.1953 | 0.1420 | 0.2190 | 0.1617 |
| Th95 | 0.1765 | 0.3295 | 0.3154 | 0.4020 | 0.3287 | 0.0616 | 0.2233 | 0.1414 | 0.2266 | 0.1694 |
| Th00 | 0.1749 | 0.3059 | 0.2846 | 0.3412 | 0.2948 | 0.0818 | 0.2471 | 0.1857 | 0.2633 | 0.2057 |
| Gt95 | 0.1551 | 0.1722 | 0.1624 | 0.1764 | 0.1690 | 0.0371 | 0.0936 | 0.0441 | 0.0897 | 0.0619 |
| Gt00 | 0.1543 | 0.1683 | 0.1570 | 0.1706 | 0.1642 | 0.0366 | 0.0889 | 0.0391 | 0.0852 | 0.0575 |
| Et95 | 0.1540 | 0.1663 | 0.1557 | 0.1695 | 0.1629 | 0.0340 | 0.0834 | 0.0320 | 0.0775 | 0.0510 |
| Et00 | 0.1541 | 0.1669 | 0.1559 | 0.1697 | 0.1632 | 0.0350 | 0.0854 | 0.0345 | 0.0802 | 0.0533 |
| Tt95 | 0.1551 | 0.1722 | 0.1625 | 0.1764 | 0.1690 | 0.0372 | 0.0938 | 0.0446 | 0.0901 | 0.0623 |
| Tt00 | 0.1544 | 0.1684 | 0.1568 | 0.1706 | 0.1641 | 0.0362 | 0.0885 | 0.0377 | 0.0839 | 0.0564 |
| Gh95m | 0.1784 | 0.2891 | 0.2725 | 0.3167 | 0.2786 | 0.0777 | 0.2362 | 0.2178 | 0.3092 | 0.2316 |
| Gh00m | 0.1726 | 0.2063 | 0.1719 | 0.1690 | 0.1782 | 0.0840 | 0.1324 | 0.0830 | 0.1355 | 0.1054 |
| Gr95m | 0.1657 | 0.2964 | 0.4132 | 0.5095 | 0.3919 | 0.0724 | 0.2642 | 0.3739 | 0.5088 | 0.3258 |
| Gr00m | 0.1580 | 0.2047 | 0.2274 | 0.2575 | 0.2242 | 0.0549 | 0.1506 | 0.1694 | 0.2466 | 0.1744 |
| Bh95m | 0.1769 | 0.2547 | 0.2524 | 0.2892 | 0.2553 | 0.0895 | 0.2026 | 0.1980 | 0.2822 | 0.2099 |
| Bh95 | 0.1801 | 0.2730 | 0.2350 | 0.3256 | 0.2595 | 0.0841 | 0.1933 | 0.1110 | 0.2463 | 0.1573 |
| Bh00m | 0.1825 | 0.3155 | 0.3081 | 0.3825 | 0.3168 | 0.0864 | 0.2610 | 0.2573 | 0.3781 | 0.2725 |
| Bh00 | 0.1843 | 0.3437 | 0.3152 | 0.4154 | 0.3338 | 0.0763 | 0.2283 | 0.1402 | 0.2777 | 0.1852 |
| IV | 0.1769 | 0.1676 | 0.1570 | 0.1710 | 0.1642 | 0.0091 | 0.0861 | 0.0350 | 0.0808 | 0.0538 |

Sdymm has to be read as follows: S indicates the specification (G=GARCH, T=TARCH, E=EGARCH, B=BEKK); d indicates the data (h=historical, i=implied); yy indicates the sample period (95=1995:2000, 00=2000); m indicates multistep forecasts.

Table 5: Pricing Errors by Moneyness

| | MARE | | | | | MRE | | | | |
|-------|-----------|-----------|-----------|-----------|--------|-----------|-----------|-----------|-----------|--------|
| | Moneyness | | | | | Moneyness | | | | |
| n | <0.94 | 0.94-0.97 | 0.97-1.03 | 1.03-1.06 | >1.06 | <0.94 | 0.94-0.97 | 0.97-1.03 | 1.03-1.06 | >1.06 |
| | 1519 | 387 | 380 | 330 | 393 | 1519 | 387 | 380 | 330 | 393 |
| Gh95 | 0.4540 | 0.2453 | 0.1854 | 0.1761 | 0.1212 | 0.2882 | 0.0957 | 0.0333 | 0.0145 | 0.0216 |
| Gh00 | 0.3469 | 0.1793 | 0.1474 | 0.1511 | 0.1049 | 0.2707 | 0.1134 | 0.0615 | 0.0354 | 0.0396 |
| Eh95 | 0.4703 | 0.2532 | 0.1791 | 0.1662 | 0.1120 | 0.3216 | 0.1229 | 0.0357 | 0.0069 | 0.0146 |
| Eh00 | 0.3743 | 0.1982 | 0.1472 | 0.1481 | 0.1028 | 0.2762 | 0.1096 | 0.0369 | 0.0080 | 0.0196 |
| Th95 | 0.4597 | 0.2508 | 0.1847 | 0.1736 | 0.1190 | 0.2962 | 0.1028 | 0.0306 | 0.0084 | 0.0164 |
| Th00 | 0.4163 | 0.2185 | 0.1594 | 0.1555 | 0.1078 | 0.3384 | 0.1449 | 0.0630 | 0.0257 | 0.0292 |
| Gt95 | 0.2186 | 0.1175 | 0.1123 | 0.1305 | 0.0989 | 0.1204 | 0.0332 | -0.0017 | -0.0117 | 0.0081 |
| Gt00 | 0.2102 | 0.1158 | 0.1115 | 0.1294 | 0.0981 | 0.1129 | 0.0301 | -0.0028 | -0.0122 | 0.0082 |
| Et95 | 0.2070 | 0.1157 | 0.1121 | 0.1304 | 0.0988 | 0.1038 | 0.0241 | -0.0069 | -0.0145 | 0.0068 |
| Et00 | 0.2078 | 0.1157 | 0.1118 | 0.1300 | 0.0985 | 0.1069 | 0.0262 | -0.0054 | -0.0136 | 0.0074 |
| Tt95 | 0.2187 | 0.1174 | 0.1123 | 0.1304 | 0.0988 | 0.1209 | 0.0335 | -0.0015 | -0.0116 | 0.0081 |
| Tt00 | 0.2098 | 0.1161 | 0.1161 | 0.1298 | 0.0982 | 0.1112 | 0.0292 | -0.0035 | -0.0124 | 0.0082 |
| Gh95m | 0.4020 | 0.2009 | 0.1456 | 0.1377 | 0.0959 | 0.3586 | 0.1817 | 0.0899 | 0.0450 | 0.0458 |
| Gh00m | 0.2323 | 0.1411 | 0.1193 | 0.1287 | 0.0940 | 0.1559 | 0.0934 | 0.0470 | 0.0234 | 0.0329 |
| Gr95m | 0.5887 | 0.2805 | 0.1791 | 0.1458 | 0.1028 | 0.5517 | 0.2755 | 0.1490 | 0.0766 | 0.0706 |
| Gr00m | 0.3193 | 0.1610 | 0.1205 | 0.1210 | 0.0896 | 0.2697 | 0.1405 | 0.0669 | 0.0298 | 0.0386 |
| Bh95m | 0.3620 | 0.1915 | 0.1399 | 0.1320 | 0.0940 | 0.3155 | 0.1721 | 0.0928 | 0.0488 | 0.0514 |
| Bh95 | 0.3590 | 0.1892 | 0.1542 | 0.1571 | 0.1092 | 0.2555 | 0.0944 | 0.0497 | 0.0299 | 0.0393 |
| Bh00m | 0.4650 | 0.2248 | 0.1577 | 0.1424 | 0.0995 | 0.4231 | 0.2073 | 0.1066 | 0.0544 | 0.0545 |
| Bh00 | 0.4719 | 0.2507 | 0.1859 | 0.1738 | 0.1212 | 0.3130 | 0.1075 | 0.0432 | 0.0198 | 0.0292 |
| IV | 0.2127 | 0.1185 | 0.1122 | 0.1301 | 0.0986 | 0.1034 | 0.0262 | -0.0054 | -0.0138 | 0.0072 |

Table 6: Bivariate BEKK Estimates

| | c_{11} | c_{12} | c_{21} | c_{22} | α_{11} | α_{12} | α_{21} | α_{22} | β_{11} | β_{12} | β_{21} | β_{22} |
|------|-------------------|-------------------|-------------------|------------------|------------------|------------------|------------------|-------------------|------------------|------------------|------------------|------------------|
| Bh95 | 0.163 (0.163) | -0.101 (2.353) | -0.033 (0.715) | 0.531 (0.451) | 0.203 (0.050) | 0.325 (0.044) | 0.140 (0.067) | -0.024 (0.059) | 0.929 (0.066) | 0.213 (0.091) | 0.022 (0.083) | 0.701 (0.114) |
| Bh00 | -0.003 (0.124) | -0.063 (1.429) | 0.031 (0.208) | 0.596 (0.275) | 0.023 (0.144) | 0.359 (0.127) | 0.207 (0.111) | -0.083 (0.131) | 0.913 (0.118) | 0.154 (0.146) | 0.077 (0.155) | 0.731 (0.196) |

The model estimated over the 1995-2000 period present all the coefficients (excluding α_{22}) in matrices A and B statistically significant, suggesting that time varying volatilities across the two series are highly correlated over time. There is transmission with respect to both innovations and past volatilities. Coefficients estimated on year 2000 are no more globally significant, but transmission of information is again detected in fact, both coefficients explaining cross effects of innovations (α_{12} , α_{21}), and own volatilities (β_{11} , β_{22}) remain significant. Therefore there is evidence of transmission of volatilities across European market to the purpose of confirming the finding, it could be interesting to investigate transmission forecasting power on option pricing. Bivariate GARCH forecasts display less bias if one-step ahead than multi-steps but are relatively less accurate moreover they work better if based on longer sample estimates.

3. CONCLUDING REMARKS

Assets volatility measurement is central in financial markets for investment decision and risk management. After the seminal paper by Engle many competing ARCH-type models have been implemented to filter financial volatility and the goodness of fit is well documented. There is notwithstanding less evidence on forecasting performance. In recent years option valuation has been an appealing area for assessing competing volatility forecasting models. Through option valuation, the paper investigates the forecasting properties of a quite large class of competing ARCH-type volatility models forecasting power is tested by means of comparison of er-

rors obtained by updating the Black and Scholes formula with the different volatility forecasts. We fit the univariate models to different in size sample, both to historical and to option-based data, producing multi-step versus one step ahead forecasts and investigating out-of-sample performance of volatility transmission. Some findings can be drawn: implied fitted models perform better than historical ones, but they do not overcome implied volatility itself. Among historical models the best performance is reached by shorter period estimated GARCH model with multi-steps forecasts. Surprisingly, multistep forecasts get worst than one-step if computed on parameters estimated on longer sample. Volatility transmission significantly explains comovements of index returns but seems not to have relevant forecasting power in option valuation.

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