Mathematical properties of EOQ models with special cost structure

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Abstract

An existence-uniqueness theorem is proved about a minimum cost order for a class of inventory models whose holding costs grow according to a stock level power law. The outcomes of [1] are then extended to different environments: i.e. when the holding costs change during time generalizing a model available in [11], or with invariable holding costs but adopting a backordering strategy. Application cases are provided assuming several functional behaviors of demand versus the stock level.

KEYWORD: Economic order quantity, Optimal ordering, Non constant demand, Variable holding costs, Planned backorders

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1 Introduction

In previous papers by two of us [1, 2], the economic order quantity (EOQ) determination has been generalized starting from the Giri and Chaudhuri approach, see [3], that the demand of a consumer good depends on the available quantity. In order to see what this means exactly, it will be recalled that EOQ is a set of mathematical models defining the optimal quantity of a single item (or good) which shall be ordered for minimizing the total costs: ordering and inventory holding. These models have been in existence long before the computer, their origin going back in time to Harris [4] even though Wilson [5] is credited for his early in-depth analysis on the subject. Basic underlying assumptions:

1. the monthly (annual or, generally: relevant to unit time) demand for the goods is known and deterministic;
2. no lead time (between order and arrivals) is taken into account;
3. the receipt of the order occurs in a single instant and immediately after ordering it;
4. quantity discounts are not calculated as part of the model;
5. the ordering cost A is a constant.

Several extensions can be made to the basic EOQ model: for instance the deterministic demand of goods can change with the instantaneous stock level or with time; the model can include backordering costs and multiple items. Should they undergo deterioration, the perishability can be assumed: constant or variable with the stock level. Finally, the above determinism could be released, leading to a probabilistic insight, but we will keep out of it. Recently several papers have been issued in the field of EOQ under stock-dependent demand and good perishability. In such a way [6] studies the case of perishability with price and stock-dependent demand [7] operates too with nonlinear holding cost of perishables [8] add inflation and time value of money, while [9] introduces the problem that EOQ car requires an additional storage facility.

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2 Our contribution

Our contribution consists of building, in the frame of EOQ theory, a cost model where arbitrary functions can be employed in order to describe the consumption dynamics and the goods perishability with backordering too. In a first time costs are described, see equation (3.3), without backordering, while equation (4.2) Lemma 4.1 models this effect too, with the perishability not depending on the stock level. Sufficient conditions are then established on the consumption dynamics assumed as autonomous, namely depending on the store level only. Such assumptions affect consequently the blowdown process so that are capable of assuring existence and uniqueness to the batch minimizing the cost, i.e. the EOQ. Such conditions are fulfilled by several models of literature, starting from the Wilson one, [5], including its next improvements and/or evolutions, [3, 10, 11]. Our extension works and generalizes all the EOQ models where the dynamic of the demand is autonomous i.e. is a function of the inventory's level only.

The approach followed hereinafter is of "geometrical" nature in the sense that quadrature closed form relationships are obtained for the reordering time, the global cost function and the minimum cost, namely the optimality condition in such a way that no previous approximation is inserted, very unlike [3] where *at the beginning* a linearization is done by a truncated series expansion or in [12], where the cost function, equation (3) therein is approximated. In our treatment numerical approximations arise *only at the end*, in order to evaluate the economic order quantity Q^* as a solution to a nonlinear, possibly transcendental, equation. We build sound foundations to all the subjects obtaining, in a rather general frame, some sufficient conditions ensuring that the inventory cost function attains a minimum and its uniqueness, namely the EOQ-problem well-posedness. The contribution newness consists of the application of method of [1] to new meaningful cases, namely the time-dependent holding costs, according to [10, 11], or when the store manager follows a backordering strategy.

Backordering has also been recently tackled by several authors, but always under a constant rate of store level change, [13, 14, 15, 16] who propose to detect the optimal batch backordering levels without calculus, but founded upon classic inequalities such that they are between the arithmetic and geometric means powered by the methods in [17, 18]. Anyway in our very general frame where the stock inventory level is ruled by a nonlinear dynamics, the classic approach through the infinitesimal calculus is by no means compulsory.

3 Variable holding costs

The time-depending holding costs were introduced in order to take into account the higher money effort for keeping fresh some perishable goods. Let the stored goods blow down according to law:

$$\begin{cases} \dot{q}(t) = -f(q(t)) \\ q(0) = Q > 0 \end{cases} \tag{3.1}$$

where the function $f : [0, \infty[\to \mathbb{R}$ is assumed positive, so that the solution to (3.1) fulfills $q(t) \leq Q$ for each $t \geq 0$. The autonomous structure of (3.1) allows a closed form solution: defining

$$F(q) := \int_q^Q \frac{1}{f(u)} \, \mathrm{d}u = t \tag{3.2}$$

then, inverting $F(q)$ we find that $q(t) = F^{-1}(t)$ solves (3.1).

We call *reordering time* generated by the batch Q the real positive value $T(Q)$ solution of $q(t) = 0$ where $q(t)$ solves (3.1):

$$T(Q) = F(0) = \int_0^Q \frac{1}{f(u)} \, \mathrm{d}u.$$

If $A > 0$ is the delivery cost, $\hat{h}(t) > 0$ models the holding cost at time t as a *continuous* function, so that $\hat{h}(0) > 0$, if $\hat{k}(q)$ denotes a continuous and positive function of q so that $\hat{k}(q) \to \infty$ for $q \to \infty$ and that $\hat{k}(0) = 0$, then the total cost for reordering an amount $Q > 0$ of goods is:

$$C(Q) = \frac{A}{T(Q)} + \frac{1}{T(Q)} \int_0^{T(Q)} \hat{h}(t) \, \hat{k}(q(t)) \, \mathrm{d}t. \tag{3.3}$$

The Wilson originary treatment, [5] follows putting $f(u) = \delta$, $\hat{h}(t) = h$, $\hat{k}(q) = q$. Notice that several literature models: [3, 10, 11] are all particular cases of what above, being there $f(u) = au + bh^{\beta}$, $\hat{h}(t) = ht^{\alpha}$. In [1] is treated as the case for whichever $f(u)$.

Theorem 3.1. *Suppose that function f in* (3.1) *is such that*

$$\lim_{v\to\infty}\int_0^v \frac{\mathrm{d}u}{f(u)} = \infty \tag{3.4}$$

Moreover we assume that if $f(0)=0$, the integrability in $u=0$ of both functions:

$$\frac{1}{f(u)}, \quad \frac{\hat{k}(u)}{f(u)}. \tag{3.5}$$

Then the cost function of (3.3) *attains its absolute minimum at $Q^*>0$, which is unique.*

Proof. In the integral at the right hand side of (3.3) we do the change $t=F(u)$ minding that $t=0\Rightarrow u=Q$, $t=T(Q)\Rightarrow u=0$, and that $\mathrm{d}t=-(1/f(u))\mathrm{d}u$, and $q(t)=F^{-1}(t)$, we get:

$$\begin{aligned} C(Q) &= \frac{A}{T(Q)}+\frac{1}{T(Q)}\int_0^Q \hat{h}(F(u))\,\hat{k}(F^{-1}(F(u)))\,\frac{\mathrm{d}u}{f(u)} \\ &= \frac{A}{T(Q)}+\frac{1}{T(Q)}\int_0^Q \hat{h}(F(u))\frac{\hat{k}(u)}{f(u)}\mathrm{d}u \end{aligned} \tag{3.6}$$

The good position of (3.6) follows from (3.5). The structure of (3.6) implies that $Q\mapsto C(Q)$, $Q>0$ has exactly one minimizer. First observe that:

$$\lim_{Q\to 0^+} C(Q)=\infty.$$

Then from (3.4) we see that the cost function (3.6) diverges when $Q\to\infty$, as immediately checked through De l'Hospital rule:

$$\lim_{Q\to\infty} C(Q) = \lim_{Q\to\infty}\frac{\dfrac{\hat{h}(F(Q))\,\hat{k}(Q)}{f(Q)}}{\dfrac{1}{f(Q)}} = \lim_{Q\to\infty}\hat{h}(0)\,\hat{k}(Q)=\infty$$

Thus $C(Q)$ is bounded from below: so has at least one stationary value. The extremum is seen to be attained at only one value since the first derivative of $C(Q)$ vanishes if and only if the batch Q solves the equation:

$$\hat{h}(0)\,\hat{k}(Q)\int_0^Q \frac{\mathrm{d}u}{f(u)} - \left\{A+\int_0^Q \hat{h}(F(u))\frac{\hat{k}(u)}{f(u)}\,\mathrm{d}u\right\}=0. \tag{3.7}$$

But the function

$$\mathcal{N}(Q) := \hat{h}(0)\,\hat{k}(Q)\int_0^Q \frac{\mathrm{d}u}{f(u)} - \left\{A+\int_0^Q \hat{h}(F(u))\frac{\hat{k}(u)}{f(u)}\,\mathrm{d}u\right\}$$

is the difference of two increasing functions; thus this minimizing batch is unique.

□

Through a similar way it is possible to prove that thesis of Theorem 3.1 holds with slightly different assumptions on f.

Corollary 3.1.1. *The same conclusion of Theorem 3.1 holds if:*

$$\int_0^\infty \frac{\mathrm{d}u}{f(u)}\in\mathbb{R}, \quad \int_0^\infty \hat{h}(F(u))\frac{\hat{k}(u)}{f(u)}\,\mathrm{d}u=\infty$$

and

$$\int_0^\infty \frac{\mathrm{d}u}{f(u)}\in\mathbb{R}, \quad \int_0^\infty \hat{h}(F(u))\frac{\hat{k}(u)}{f(u)}\,\mathrm{d}u\in\mathbb{R}$$

4 Backordering

In order to analyze the backordering, we assume $\hat{h} = \text{const.}$ and $\hat{k}(q) = q$. The quantity Q ordered at each cycle undergoes two different uses: a first share $Q - R$ covers the demand of the previous cycle, and then does not enter the inventory; while R is the residual share which enters the store so that the outstanding amount is again $Q - R$, and so on. As a consequence, the reordering time becomes:

$$T(Q) = F(R-Q) = \int_{R-Q}^{R} \frac{1}{f(u)}\, \mathrm{d}u,$$

where the function $f : [R-Q, \infty[\to \mathbb{R}$ is assumed positive, and the total cost is:

$$C(R,Q) = \frac{A}{T(Q)} + \frac{h}{T(Q)} \int_0^{T(R)} q(t)\, \mathrm{d}t - \frac{b}{T(Q)} \int_{T(R)}^{T(Q)} q(t)\, \mathrm{d}t. \tag{4.1}$$

It is possible to get easier (4.1), by the following Lemma.

Lemma 4.1. *Let $f(q)$ be the law describing the the q-blowdown dynamics: then the total cost is given by:*

$$C(R,Q) = \frac{A + h \displaystyle\int_0^R \frac{u}{f(u)} \mathrm{du} - b \int_{R-Q}^{0} \frac{u}{f(u)} \mathrm{d}u}{\displaystyle\int_{R-Q}^{R} \frac{\mathrm{d}u}{f(u)}}, \tag{4.2}$$

where, if $f(0) = 0$ we assume the integrability of both functions:

$$\frac{1}{f(u)}, \quad \frac{u}{f(u)}.$$

Proof. Putting in (4.1) $t = F(u)$, notice that $t = 0 \Rightarrow u = R$, $t = T(R) \Rightarrow u = 0$, $t = T(Q) \Rightarrow u = R - Q$, and $\mathrm{d}t = -(1/f(u))\mathrm{d}u$, so that, minding that $q(t) = F^{-1}(t)$ one finds:

$$\begin{aligned} C(R,Q) &= \frac{A}{T(Q)} + \frac{h}{T(Q)} \int_{F(R)}^{F(0)} F^{-1}(t)\, \mathrm{d}t - \frac{b}{T(Q)} \int_{F(0)}^{F(R-Q)} F^{-1}(t)\, \mathrm{d}t \\ &= \frac{A}{T(Q)} + \frac{h}{T(Q)} \int_0^R F(q)\, \mathrm{d}t - \frac{b}{T(Q)} \left((R-Q)F(R-Q) + \int_{R-Q}^{0} F(q)\, \mathrm{d}t \right) \end{aligned}$$

writing F in terms of f we find out:

$$C(R,Q) = \frac{1}{T(Q)} \left(A + h \int_0^R \left(\int_q^R \frac{\mathrm{d}u}{f(u)} \right) \mathrm{d}q - b \left((R-Q) \int_{R-Q}^{R} \frac{\mathrm{du}}{f(u)} + \int_{R-Q}^{0} \left(\int_q^R \frac{\mathrm{du}}{f(u)} \right) \mathrm{d}q \right) \right)$$

exchanging the integrations order and computing the inner one

$$\begin{aligned} C(R,Q) &= \frac{1}{T(Q)} \Bigg[A + h \int_0^R \frac{u}{f(u)} \mathrm{d}u \\ &-b \left((R-Q) \int_{R-Q}^{R} \frac{\mathrm{d}u}{f(u)} + \int_{R-Q}^{0} \frac{u-(R-Q)}{f(u)}\, \mathrm{d}u + \int_0^R \frac{Q-R}{f(u)}\, \mathrm{d}u \right) \Bigg] \\ &= \frac{A + h \displaystyle\int_0^R \frac{u}{f(u)} \mathrm{d}u + b \left((Q-R) \int_{R-Q}^{R} \frac{\mathrm{d}u}{f(u)} - \int_{R-Q}^{0} \frac{u-(R-Q)}{f(u)} \mathrm{d}u - \int_0^R \frac{Q-R}{f(u)} \mathrm{d}u \right)}{\displaystyle\int_{R-Q}^{R} \frac{\mathrm{d}u}{f(u)}} \end{aligned}$$

A numerator straightforward reduction completes the proof. $\square$

Theorem 4.1. *The cost function introduced in* (4.2) *attains its absolute minimum at proper positive values* (Q^*, R^*). *Such a minimizing batch is unique.*

Proof. Recall that

$$\lim_{(R,Q)\to(0,0)} C(R,Q) = \infty$$

and that if $R \to Q$, we go back to the originary model, furthermore, by De l'Hospital rule one finds that:

$$\lim_{Q\to+\infty} C(R,Q) = +\infty$$

Let us change variables passing from $C(R,Q)$ to $C(R,Q-R)$: accordingly, the total cost $C(R,Q-R)$ is:

$$\frac{A + h\displaystyle\int_0^R \frac{u}{f(u)}\mathrm{d}u - b\int_{R-Q}^0 \frac{u}{f(u)}\mathrm{d}u}{\displaystyle\int_{R-Q}^R \frac{\mathrm{d}u}{f(u)}} \tag{4.3}$$

Partial derivatives with respect to R and Q provide:

$$\frac{\partial C}{\partial Q} = \frac{-A + b\,(Q-R)\displaystyle\int_{R-Q}^R \frac{1}{f(u)}\,\mathrm{d}u - h\int_0^R \frac{u}{f(u)}\,\mathrm{d}u + b\int_{R-Q}^0 \frac{u}{f(u)}\,\mathrm{d}u}{f(R-Q)\left(\displaystyle\int_{R-Q}^R \frac{1}{f(u)}\,\mathrm{d}u\right)^2}$$

$$\frac{\partial C}{\partial R} = \frac{-A + h\,R\displaystyle\int_{R-Q}^R \frac{1}{f(u)}\,\mathrm{d}u - h\int_0^R \frac{u}{f(u)}\,\mathrm{d}u + b\int_{R-Q}^0 \frac{u}{f(u)}\,\mathrm{d}u}{f(R)\left(\displaystyle\int_{R-Q}^R \frac{1}{f(u)}\,\mathrm{d}u\right)^2} - \frac{\partial C}{\partial Q}$$

Imposing partial derivatives to vanish:

$$\frac{\partial C}{\partial Q} = \frac{-A + b\,(Q-R)\displaystyle\int_{R-Q}^R \frac{1}{f(u)}\,\mathrm{d}u - h\int_0^R \frac{u}{f(u)}\,\mathrm{d}u + b\int_{R-Q}^0 \frac{u}{f(u)}\,\mathrm{d}u}{f(R-Q)\left(\displaystyle\int_{R-Q}^R \frac{1}{f(u)}\,\mathrm{d}u\right)^2} = 0$$

$$\frac{\partial C}{\partial R} = \frac{-A + h\,R\displaystyle\int_{R-Q}^R \frac{1}{f(u)}\,\mathrm{d}u - h\int_0^R \frac{u}{f(u)}\,\mathrm{d}u + b\int_{R-Q}^0 \frac{u}{f(u)}\,\mathrm{d}u}{f(R)\left(\displaystyle\int_{R-Q}^R \frac{1}{f(u)}\,\mathrm{d}u\right)^2} = 0$$

We assume $f > 0$ for each u, then both the denominators are strictly positive; setting the numerators to be zero, first order conditions will provide the critical point system:

$$\begin{aligned} g(R,Q-R) &= -A + h\,R\int_{R-Q}^R \frac{1}{f(u)}\,\mathrm{d}u - h\int_0^R \frac{u}{f(u)}\,\mathrm{d}u + b\int_{R-Q}^0 \frac{u}{f(u)}\,\mathrm{d}u = 0 \\ m(R,Q-R) &= -A + b\,(Q-R)\int_{R-Q}^R \frac{1}{f(u)}\,\mathrm{d}u - h\int_0^R \frac{u}{f(u)}\,\mathrm{d}u + b\int_{R-Q}^0 \frac{u}{f(u)}\,\mathrm{d}u = 0. \end{aligned} \tag{4.4}$$

To solve (4.4), subtracting side by side, one finds:

$$\begin{aligned} Q &= \frac{hR}{b} + R \\ m(R,\frac{hR}{b}) &= -A + hR\int_{\frac{hR}{b}}^R \frac{1}{f(u)}\,\mathrm{d}u - h\int_0^R \frac{u}{f(u)}\,\mathrm{d}u + b\int_{\frac{hR}{b}}^0 \frac{u}{f(u)}\,\mathrm{d}u = 0 \end{aligned}$$

$m(R, hR/b)$ is an increasing function being:

$$\frac{\mathrm{d}}{\mathrm{d}R} m(R, hR/b) = h\int_{\frac{hR}{b}}^R \frac{1}{f(u)}\,\mathrm{d}u > 0,$$

so observing that $m(0,0) < 0$, then $m(R, hR/b)$ has a unique real root, and we have one and only one critical point for the cost function (4.2). Let us show it is a minimum. The Hessian determinant at the critical point is:

$$H = \frac{bh}{f(R)f(-\dfrac{hR}{b})\left(\displaystyle\int_{-\frac{hR}{b}}^{R}\frac{1}{f(u)}\,\mathrm{d}u\right)^2} \tag{4.5}$$

In fact being:

$$H = \frac{\partial^2}{\partial R^2}C(R,Q)\frac{\partial^2}{\partial Q^2}C(R,Q) - \left(\frac{\partial^2}{\partial R\partial Q}C(R,Q)\right)^2$$

minding that $g(R, Q-R) = m(R, Q-R) = 0$ we have

$$\frac{\partial^2 C}{\partial Q^2} = \frac{\dfrac{\partial m}{\partial Q}}{Z^2 f(R-Q)}, \quad \frac{\partial^2 C}{\partial R^2} = \frac{\dfrac{\partial g}{\partial R}}{Z^2 f(R)} - \frac{\dfrac{\partial m}{\partial R}}{Z^2 f(R-Q)}, \quad \frac{\partial^2 C}{\partial Q\partial R} = \frac{\dfrac{\partial m}{\partial R}}{Z^2 f(R-Q)}$$

where we put:

$$Z = \int_{R-Q}^{R}\frac{1}{f(u)}\,\mathrm{d}u.$$

Eventually, recalling that

$$Q = \frac{hR}{b} + R, \quad \frac{\partial m}{\partial R} = \frac{\partial g}{\partial Q} = 0$$

we find (4.5) proving the stationary point to be a minimum.

□

5 Sample problems

We provide now some applications of above to known models of the literature extended to backorders, getting in any case a transcendental (or algebraic) R- resolvent equation. The following conditions are assumed:

$$h > 0,\ Q > 0,\ A > 0\ R > 0,\ 0 < p < 1,\ b > 0,\ \delta > 0$$

Wilson model

$$f(u) = \delta \Rightarrow C(R,Q) = \frac{\delta}{Q}\left(A + \frac{b\,(Q-R)^2}{2\,\delta} + \frac{h\,R^2}{2\,\delta}\right)$$

Such a case has a theoretical interest due to its final (not transcendental and) exactly solvable resolvent: the minimizing batch is found to be:

$$Q^* = \sqrt{\frac{2A\delta(b+h)}{bh}}; \qquad R^*\sqrt{\frac{2Ab\delta}{h(b+h)}}$$

In such a way the minimized cost will be:

$$C^* = b\left(\sqrt{\frac{b+h}{b}} - \sqrt{\frac{b}{b+h}}\right)\sqrt{2A\delta h}$$

Goh's model $p = 1/2$

$$f(u) = \sqrt{|u|} \Rightarrow C(R,Q) = \frac{3\,A + 2\,h\,R^{\frac{3}{2}} + 2\,b\,(Q-R)^{\frac{3}{2}}}{6\left(\sqrt{R} + \sqrt{Q-R}\right)}$$

By $Q - R = \frac{hR}{b}$ and $g(R, Q-R) = 0$ we get:

$$-3\,A + 4\,h\left(1 + \sqrt{\frac{h}{b}}\right)R^{\frac{3}{2}} = 0$$

Goh's model p general

If $f(u) = |u|^p$ then

$$C(R,Q) = \frac{(1-p)\ \left(b\,R^p\,(Q-R)^2 + h\,R^2\,(Q-R)^p + A\,d\ (2-p)\ (R\,(Q-R))^p\right)}{(2-p)\ (R^p(Q-R) + R(Q-R)^p)}$$

By $Q - R = \frac{hR}{b}$ e $g(R, Q-R) = 0$ we get:

$$\frac{\left(\frac{R^2}{b}\right)^p\ \left\{h\ [b^p\,h - b\,h^p\ (p-3)]\ R^2 - A\,b\ (p-2)\ (h\,R)^p\right\}}{b} = 0$$

collecting R^p

$$R^p\ \left\{-\left[A\,b\,h^p\ (-2+p)\right] + h\ \left[b^p\,h - b\,h^p\ (-3+p)\right]\ R^{2-p}\right\} = 0$$

the solution is straightforward solving to R^{2-p}.

Exponentials

$$f(u) = e^{-u} \Rightarrow C(R,Q) = \frac{e^{Q-R}\ \left\{A + h\ \left[1 + e^R\ (R-1)\right]\right\} + b\ \left(e^{Q-R} - Q + R - 1\right)}{e^Q - 1}$$

If $Q - R = \frac{hR}{b}$ e $g(R, Q-R) = 0$ we get the transcendental R-equation:

$$b - e^{\frac{h\,R}{b}}\ \left(A + b + h - e^R\,h\right) = 0$$

It is then provided a simulation for$A = 1$, $b = 1/3$, $h = 1/4$. Figure 1 shows the iso-cost curves, highlighting the minimizer (Q^*, R^*) numerically detected.

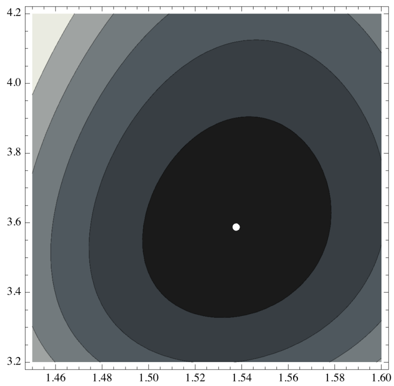

Figure 1: Level curves relevant to the cost function

Figure 2 shows the crossing of the loci of roots of the single first partial derivatives. And finally in Figure 3, a 3-D plot of the global cost function. We have a similar behavior for $f(u) = e^u$.

Rational (first)

$$f(u) = \frac{k}{n+u} \Rightarrow C(R,Q) = \frac{6\,A\,k + h\,R^2\ (3\,n + 2\,R) + b\ (3\,n - 2\,Q + 2R)\ (Q-R)^2}{3Q\ (2\,n + 2R - Q)}$$

By $Q - R = \frac{hR}{b}$ and $g(R, Q-R) = 0$ one obtains:

$$\frac{-6\,A\,b^2\,k + h\ (b+h)\ R^2\ (-(h\,R) + b\ (3\,n + R))}{b^2} = 0$$

which is providing promptly the batch.

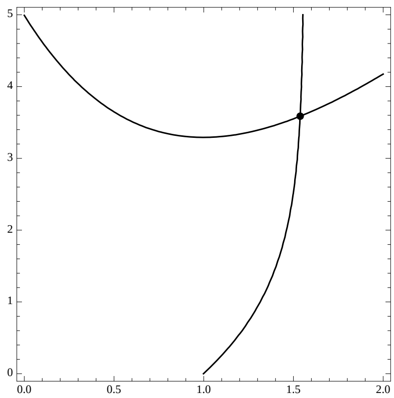

Figure 2: Crossing of the loci of roots of the single partial derivatives

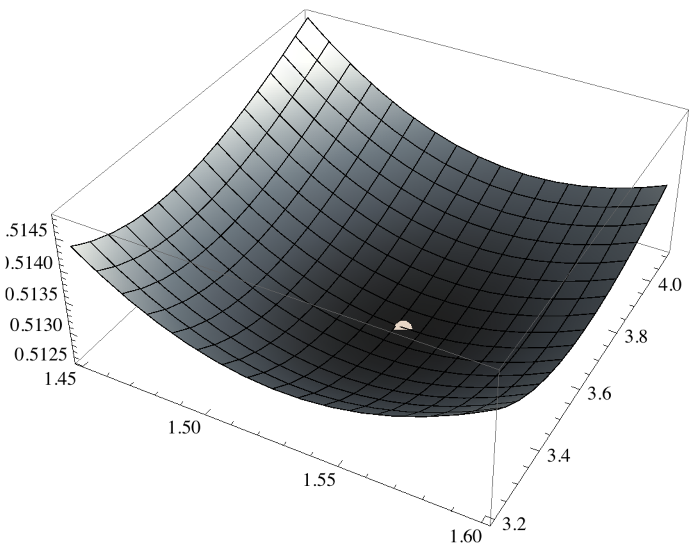

Figure 3: a 3-D plot of $C(Q,R)$ for $f(u) = e^{-u}$

Rational (second)

$$f(u) = \frac{k}{n+u^2} \Rightarrow C(R,Q) = \frac{3\,\left(4\,A\,k + h\,\left(2\,n\,R^2 + R^4\right) + b\,(Q-R)^2\,\left(2\,n + (Q-R)^2\right)\right)}{4\,\left(R^3 + (Q-R)^3 + 3\,n\,Q\right)}$$

By $Q - R = \frac{hR}{b}$ and $g(R, Q-R) = 0$ one finds:

$$-12\,A\,k + \frac{6\,h\,(b+h)\,n\,R^2}{b} + \frac{h\,\left(b^3+h^3\right)\,R^4}{b^3} = 0$$

biquadratic equation.

6 Conclusions

We proved two existence-uniqueness theorem 3.1 and 4.1 about a minimum cost batch for a class of EOQ models with perishable inventory and nonlinear cost and with sole backordering, leading to a set of sufficient conditions which require to check the convergence of some improper integrals, and form the article's main theoretical effort. As application, several cases have been treated of demand $f(q)$ as a continuous function of the stock level q. Being one of the sufficient conditions met in any case, the economic order quantity is unique, and the relevant computations lead to transcendental equations. In some cases the plot of the global cost function is provided, and, even if the optimality condition can be written in closed (but transcendental) form, whose solution shall mostly be faced numerically. Mind that the re-ordering time, the global cost function and the minimum cost (optimum) condition are detected without any previous approximation, being a numerical treatment required only at the end, in order to solve the transcendental equation for the economic batch.

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