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Abstract

This paper investigates the impact of heterogeneous wealth on credit allocation from an egalitarian opportunity and an efficiency point of view. Under asymmetric information on both wealth and the responsibility variable there is no trade-off between equality and efficiency, actually wealth inequality delivers both inequality of opportunity and inefficiency. Due to decreasing absolute risk aversion, poor entrepreneurs, other things equal, realize better projects. This notwithstanding, due to the bidimensional hidden information, they may be rationed out or obtain a loan only at the cost of cross subsidizing bad projects realized by rich entrepreneurs. In the first case inefficiency arises in the form of insufficient investment, in the second in the form of inefficient projects being realized. An egalitarian redistribution of endowments may lead to perfect screening, no inefficiencies in the allocation of credit and equality of opportunity.

Keywords: Wealth, Credit, Cross-subsidization, DARA, Equality of Opportunity

JEL classification: D31; D82; G21

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1 Introduction

The credit market is supposed to transfer resources from savers without an entrepreneurial option, to illiquid would-be entrepreneurs, creating surplus in the process. However imperfect information may interfere with a fair and efficient allocation of credit. Collateral may be required for incentive or screening purposes putting the relatively poor entrepreneurs at a disadvantage. We investigate the properties of equilibria in credit markets with heterogeneous wealth entrepreneurs characterized by decreasing absolute risk aversion ($DARA$, hereafter), under the Equality of Opportunity (EOp) benchmark and explore the relationship with correlated inefficiencies in credit allocations.

The ideal of a society in which no one suffers disadvantage on grounds of unequal opportunities is widely upheld as desirable. The requirement extends far beyond the vague injunction to eschew public sphere discrimination as it implies a central normative investigation for deciding on which grounds one might justify responsibility-sensitive policy interventions. The idea of competing on equal terms was formalized in the Equality of Opportunity principle which requires the distinction between unchosen circumstances and individual choices (see among egalitarian philosophers, Rawls (1971) and Sen (1973)). The former are terms imposed on an individual in ways that she could not have influenced or controlled; these terms are just given. The latter, instead, constitute the personal responsibility of individuals. Main exogenous circumstances for instance include the wealth inherited and early environment provided by parents and, in general, all the features of the world in which one finds oneself prior to any opportunity for responsible choice (Roemer, 1998). In the last 20 years the opportunity egalitarian literature has extended on the measurement of inequality of opportunity as a tool to implement an efficiency-enhancing redistribution.[1]

The EOp principle requires that jobs and options to borrow money for investment purposes, such as starting a business, should be open to all applicants (think of young applicants or new borrowers). However a strong evidence demonstrates unequivocally that entry in the credit market is heavily wealth-dependent, and that potential investment is constrained by personal

[1] A variety of measures was adopted with the aim of separating 'inequality of opportunity' and 'responsibility sensitive inequality', and was applied mainly in the context of income inequality but also in taxation, education, health. For a survey, see Pignataro (2012).

and family wealth (Evans and Jovanovich, 1989). Most empirical contributions lie in the field of development economics, where inefficiencies in credit market are likely to be tantamount (for comprehensive surveys, see Benabou, 1996; Banerjee, 2002; Banerjee and Duflo, 2010). Wealth dependence can be the product of many factors, among which endogenous risk preferences (Cressy, 2000) or myopic consumption, but most recent evidence refutes the hypotheses that these factors are the sole relevant (see for instance Kan and Tsai, 2006 and Berg, 2012). The likely explanation is that imperfect information may force the bank to choose on the basis of collateral provision. This in turn may be due to the need to control for opportunistic entrepreneurial behaviour or to screen better applicants in presence of hidden information. We build an imperfect information model that fits well with most stylized facts about credit markets in developing economies as reported by Banerjee and Duflo (2010), in particular with the fact that rich people borrow more (wealth dependence) and pay lower interest rates. We will use a further stylized fact, e.g., that monopoly power does not appear to be a cause for high interest rates, as an assumption.[2]

Our model entails bidimensional heterogeneity among individuals with hidden information and moral hazard. Potential entrepreneurs differ for both circumstances and personal responsibility. Circumstances are perfectly represented by the ex-ante endowed wealth, while the individuals' responsibility variable is codified as effort aversion (an indicator of preferences) affecting the measure of the chosen actions (effort) the individual takes. Effort aversion affects individual willingness to supply effort and therefore measures (inversely), other things equal, the propensity of individual to work hard. We want to investigate the properties of the equilibrium in a framework where both features are unobservable by competitive lenders. We investigate in particular whether moral hazard and adverse selection involve the violation of the equality of opportunity principle.

Our characterizing assumptions are that individuals' wealth is not publicly observable, while agents exhibit decreasing absolute risk aversion (DARA, hereafter)[3]. This first assumption requires some discussion[4]. In most of the credit market literature (except for Stiglitz and Weiss,

[2] The last fact reported, high lending rates, is outside the scope of this paper.

[3] For the empirical evidence in favour of the DARA assumption, see among others, Black (1996); Ogaki and Zhang (2001).

[4] For a ,more complete discussion see Coco and Pignataro (2013).

1992; Coco and Pignataro, 2013) wealth is supposed to be observable while entrepreneurial ability is not. However in other fields (for example tax evasion), the idea that financial income and wealth positions of individuals are common knowledge would be considered rather odd. The idea that a bank official can assess the extent of one individual's wealth is even stranger. In most papers the assumption of common knowledge on wealth possibly proxies for the belief that there are no reasons to conceal one's wealth. In this paper this is not the case because decreasing risk aversion may turn wealth into a bad signal. We assume heterogeneity and asymmetric information also on the responsibility variable. This makes the model more realistic and allows us to discuss the equilibria in terms of the EOp paradigm.

This bidimensional asymmetric information and the ensuing moral hazard complicate considerably the game[5]. In a situation where individuals are risk averse, their willingness to bear risk is an important additional channel through which the distribution of wealth determines the contract form with the efficiency and the equity properties of equilibrium. In particular, $DARA$ gives the personal endowment a new role in providing incentives that can mitigate or exacerbate information problems. More wealth (and less risk aversion) negatively impacts on effort provision (see Newman, 2007). Adverse selection on wealth types is therefore endogenously generated by different optimal levels of effort along the distribution of wealth. As a consequence poor individuals end up as hard-working agents, other things equal. Moreover for each wealth class, preferences on effort aversion affect effort provision as well and the two dimensions interact in a complex manner.

We consider a contract space in terms of collateral and interest rate. Risk aversion and effort aversion (through their consequent effort choice) determine the willingness to post collateral and therefore the existence and the form of equilibrium. Risk aversion influences the willingness to post collateral both directly and through effort choice (moral hazard) in different directions. When the moral hazard channel dominates, no equilibrium exists where poor entrepreneurs can be served. Instead when the direct effect of risk aversion prevails, we discover a unique pure strategy sub-game perfect equilibrium in the screening game. For some preferences, only some poor entrepreneurs are excluded from the market. Different risk classes (rich and poor) may be pooled at a single contract in equilibrium, where cross-subsidization occurs not only between

[5]See Aney *et al.* (2012) for a case of bidimensional heterogeneity with observable wealth.

wealth classes but also between different effort aversion types. The credit market equilibrium is then characterized by inequality of opportunity and inefficiency in contrast with the traditional trade-off. The rich are charged a low rate of interest (relative to their risk) even if they are characterized by high level of effort aversion, while poor borrowers (with low effort aversion) are charged too high an interest rate. We then demonstrate that inequality of opportunity and the perverse redistribution prospect are always associated to inefficiency. When hard working poor individuals are excluded from credit lines, some potential surplus is not realized. On the other side when cross subsidization occurs some surplus-wasting projects are carried out.

Of course the result that poor entrepreneurs may be rationed out or served at worse term contracts has already been discussed in the literature. It is worth to point out however that exclusion or worse term on credit result from the inability to write incentive compatible contracts in the presence of ex-post moral hazard (see for instance Banjeree and Newman, 1993) also in connection with the existence of a quasi-fixed lending cost (for a useful survey Banjeree and Duflo 2010). In this case borrowing is obviously constrained by the amount of assets owned. A discussion of whether ex-ante or ex-post moral hazard is more important in the credit market is widely beyond the scope of this paper, but certainly most of the literature of the credit market uses our setting. One could also contend that the assumption of ex-post moral hazard, that is the possibility of strategic bankruptcy and flight of the borrower is more likely and relevant in a developed and mobile context rather than in developing countries. Also the possibility of endogenous cross-subsidization has appeared in the theoretical literature (e.g. among others, Banerjee and Newman, 1993; Black and de Meza, 1997; de Meza and Webb, 1999, 2000; Ghatak *et al.*, 2007; Martin, 2009; Parker, 2003), but not between different wealth classes. Moreover the interplay between equity, implicit redistribution and efficiency has not been discussed satisfactorily so far. Our results link convincingly inefficiencies, originating in a wrongful allocation of credit, to inequality and inequality of opportunity in a novel way, denying forcefully the existence of the classic trade-off between equity and efficiency objectives

Our modeling strategy follows the literature on ex-ante imperfect information in the credit market (de Meza and Webb, 1987) and in particular the theory of collateral use. Inefficient levels of investments may occur notwithstanding collateral (Bester, 1985; Besanko and Thakor, 1987;

for survey see Coco, 2000) serving as a screening device[6]. Related to our work are also the papers by Stiglitz and Weiss (1992) and Coco (1999). They demonstrate the impossibility of screening by collateral in the credit market with two classes of borrowers with different risk attitudes. Risk preferences and project quality interact through moral hazard in conflicting ways, so that collateral is not a meaningful signal of project quality. Finally, Gruner (2003) considers a setting where rich borrowers crowd out productive poor ones. He suggests that an ex-ante complete redistribution of endowments, by inducing the substitution of rich entrepreneurs with poor ones, may lead to a Pareto-improvement due to a rise in the risk-free interest rate.

The structure of the paper is as follows. Section 2 introduces the baseline model while section 3 discuss the characterization of the loan contracts. The inequality of opportunity equilibrium is instead investigated in section 4. Concluding remarks follow in section 5.

2 The model

Consider a one period competitive credit market populated by entrepreneurs owning projects with risky income streams. Each project requires both (fixed) investment capital K and effort supplied by the entrepreneur. Specifically the uncertain revenue from an investment can take one of the two values, Y in the event of successful state with a certain level of probability $p(e)$ and zero in case of failure with probability $(1 - p(e))$ where $e \in [0, \bar{e}]$ denotes the amount of effort. Returns to effort are positive and diminishing as usual, i.e., $p'(e) > 0$ and $p''(e) < 0$. In more general terms, a higher level of effort e results in a project whose returns first-order stochastically dominates ($FOSD$) the project returns with lower levels of effort (De Meza and Webb, 1987). Utility for the would-be borrowers is a concave increasing function that exhibits $DARA$, i.e., $d(-U''(w)/U'(w))/dw < 0$ and $U(w = 0) = -\infty$. Each agent has a different amount of illiquid wealth w_i, $i \in [R, P]$ for rich and poor respectively, which are both insufficient to achieve full collateralization, $w_i < (1 + r)K$, and is also illiquid at the moment of the realization of the project This implies the need to borrow the whole amount of capital, K, in order to undertake

[6]Empirical evidence suggests that in developing countries, lenders may enforce collateral free loans using third-party guarantees and relationship lending, see Menkhoffa *et al.* (2012) on this issue with an investigation in Thailand.

the investment projects. Moreover let us denote $X = (1 + r)K$ as the total repayment where r is the interest rate required by the bank for an amount of collateral c. Individuals differ also because of a scalar indexed effort aversion μ_j, $j \in [L, H]$ which can be respectively low or high. We assume that a project realized by entrepreneurs with a low effort aversion (L) may have a positive net value, while high effort aversion (H) always induces such a low level of effort that its net return is negative[7]. As a consequence projects of types$-H$ should not be undertaken from a social perspective, as they produce less than the resources employed. Moreover an equilibrium with separation of types$-H$ can be ruled out from the outset. While simplifying considerably the picture this assumption is quite realistic in delivering a world in which some potential entrepreneurs are basically looters (in the words of Akerlof *et al.*, 2003) and could only realize their projects when obtaining pooling contracts with positive net present value projects/entrepreneurs.

Besides being characterized by L (low) or H (high) effort aversion, entrepreneurs differ also for their endowment and they may be either of type R (rich) or P (poor). The two features of the borrowers are distributed independently in the population and therefore λ is the proportion of rich borrowers while $(1 - \lambda)$ is the proportion of poor ones in the market. Further v is the proportion of $H-$borrowers in the market and $(1-v)$ is the proportion of $L-$ones. The borrower's wealth w_i, her own effort aversion μ_j and her consequent effort choice $e(w_i, \mu_j)$ are known to the individual but not observable by a competitive lender. The expected utility of a borrower ij equals the expected net revenue from the project minus the cost of effort:

$$U_{ij}(X, c) = p(e_{ij})U(Y - X_{ij} + w_{ij}) + (1 - p(e_{ij}))U(w_{ij} - c_{ij}) - \mu_j e_{ij} \tag{1}$$

Intuitively, if a lender can observe a borrower's level of effort and can enforce an effort contingent contract, then there is no moral hazard and a first-best outcome will emerge. If such a contract is not possible then moral hazard persists and the lender must infer $e^*(w_i, \mu_j)$, the participating borrowers' optimal level of effort as a function of wealth w and effort aversion μ.

[7] This assumption is introduced to simplify the treatment, but it is not restrictive. As we are going to see in the next sections the contracts designed could be rewritten as contracts in which entrepreneurs at $H-$levels has a positive expected return with a potential separation among wealth classes. We adopt the former characterization because it makes contracts easier to analyse delivering a more tractable framework in the inequality of opportunity perspective.

Bertrand competition in the credit market implies that the payment specified by the contract must be such that a competitive lender just expects to break even and so (2) is equal to zero. Now consider the case of ex-ante asymmetric information. Lenders know the wealth distribution of borrowers, but are not able to distinguish the particular borrower's wealth when a loan application is made. We assume zero risk-free interest rate and an infinitely elastic supply of funds in the deposit market. In such a scenario it is known that the standard optimal form of finance would be equity, but assuming unverifiable ex-post returns makes debt the only feasible form of finance (see de Meza and Webb, 2000). For a single borrower, a bank's expected profit from accepting an application for a contract from a type$-ij$ is given by:

$$\pi_{ij} = p(e_{ij})X_{ij} + (1 - p(e_{ij}))c_{ij} - K \tag{2}$$

In the successful state entrepreneurs pay back the amount borrowed X with probability $p(e_{ij})$ otherwise the banks keep the amount of resources put up as collateral c. Entrepreneurs and banks sign a contract of the general form $\{X, c\}$. We seek subgame perfect Nash equilibria of the following two-stage screening game à la Rothschild and Stiglitz (1976)[8]. In the first stage, banks compete for the pool of customers whose type is unknown to them. They may potentially offer applicant borrowers a menu of loan contracts $\{X_{ij}, c_{ij}\} \in ij\ \{R, P\} \times \{L; H\}$. Then entrepreneurs are able to weigh up the pros and cons of entering the market and, if so, choose their preferred offer (one) among those available. Therefore formally a Nash equilibrium here is a set of contracts such that (1) each bank earn nonnegative profits on each contract and (2) there exists no other (set of) contract that would earn positive expected profits if offered in addition to the original set. We restrict our attention to pure strategy equilibria.

[8]The general structure of our model uses the definition of pure-strategy equilibria proposed by Bester (1985).

3 Characterization of loan contracts

3.1 Agents'preference map

In the standard explanation of separation among classes (Bester, 1985), the whole weight of screening is borne by the amount of collateral required on contracts to sort good and bad risk. In the present multidimensional context, instead, the banks' statistical inference problem is more complicated. A borrower signing a contract which requires to post higher levels of collateral may belong to the rich or the poor class (unless collateral exceeds the poor's wealth) but at the same time she can be relatively highly-averse to supply effort, influencing adversely the performance of the contract.

To explore further the effect of wealth and effort aversion on effort choice we must temporarily analyse a case where the two variables are continuously distributed. We start by looking at the effect of moral hazard. Using eq. (1), the first order condition for the borrower's optimal choice of effort $e^*(w_i, \mu_j)$ is given by:

$$p'(e_{ij})U(Y - X + w_i) - p'(e_{ij})U(w_i - c) = \mu_j \tag{3}$$

Eq. (3) shows that the borrower supplies effort until the expected value of marginal effort equals its marginal cost. The maximization conditions are satisfied since the probability of success $p(e_{ij})$ is concave. Rearranging eq. (3), the optimal choice of effort $e^*_{ij}(w_i, \mu_j)$ is described by:

$$p'(e^*_{ij}) = \frac{\mu_j}{U(Y - X + w_i) - U(w_i - c)} \tag{4}$$

From straightforward comparative statics it follows that $\frac{de^*_{ij}}{dY} > 0$; $\frac{de^*_{ij}}{dc} > 0$; $\frac{de^*_{ij}}{dX} < 0$ as is customary in moral hazard models. On one side a higher amount of collateral reflects higher penalty in case of failure, providing incentives in effort. On the other side a higher repayment negatively impacts the borrower's return in case of success, but not in the case of failure. This reduces incentives to supply effort.

As shown in Newman (2007), it may be argued that more wealth and less risk aversion worsen the moral hazard issue. In particular in our model the adverse selection on wealth types endogenously generated as a function of different choices of effort is combined with the role attributed to effort aversion. A multidimensional moral hazard effect changes as a combined function of $DARA$ and effort aversion.

Consider a case where the terms of the bank contract are fixed for all borrowers. We define for any class (level) of wealth, w_i, the marginal borrower as the individual who is indifferent between exiting and remaining active in the credit market. As a direct consequence the marginal set is defined as the set of individuals indifferent between two options along the wealth distribution. Under these conditions, one can show that there exists a negative relation between effort and wealth, i.e., the marginal effort is lower, the higher is the wealth of individuals:

$$\frac{de_{ij}^*}{dw} < 0 \tag{5}$$

Proof. See the Appendix

while at the same time, a negative correspondence between effort and aversion is established,

$$\frac{de_{ij}^*}{d\mu} < 0 \tag{6}$$

Proof. See the Appendix

which implies that individuals with a higher effort aversion also display a higher probability of default due to moral hazard. Since the marginal individuals capture the lowest share of project expected returns, their choice of effort is farthest from the socially efficient value.

Because of decreasing risk aversion, moral hazard impacts more heavily on wealthier borrowers and of course the effect is heavier for people with larger effort aversion. We can now state the following result:

Lemma 1. *Given a certain effort aversion μ_j, marginal poor entrepreneurs are the first to exit from the market:*

$$\frac{d\mu}{dw}|_{U_{ij}(.)=0} > 0 \tag{7}$$

Proof. See the Appendix

Condition (7) is crucial for at least two reasons. First it constitutes a signal about the possibility of inequality of opportunity in the market. The hard-working agents are excluded from the market due to their initial conditions and independently from their level of effort aversion. Second, concerning the role of private information on wealth, we observe that decreasing absolute risk aversion may turn wealth (and availability to post collateral) into a bad signal. Moreover another effect is in place because of the combination between wealth and effort aversion. In order to catch the idea, let us suppose (as proposed in the model) four classes of entrepreneurs $ij \in \{R, P\} \times \{L; H\}$. In this setting rich borrowers (both types) could benefit from not signaling their wealth because of an implicit cross subsidy they would earn in a pooling with hard working poor entrepreneurs.

3.2 Single-crossing preferences

To investigate the type of equilibria that may arise in the multidimensional framework, it is useful to analyse a diagrammatic representation able to capture the impact of endogenous adverse selection on wealth and effort aversion and its consequent effect in terms of moral hazard. Using (1) and from the Envelope Theorem, we know that the slope of an indifference curve of a borrower in the $(X, c)-$ space is:

$$s_{ij}(X, c) = \frac{dX}{dc} < 0 \tag{8}$$

Proof. See the Appendix

representing the marginal rate of substitution between income in the two states at a certain contract (X, c). The first element in order to establish the possibility of separating equilibria is the slope of the indifference curves with respect to the wealth dimension of borrowers. In this respect we try to single out the direct effect of risk preferences from the impact of moral hazard. We rewrite the slope of the indifference curve in (8) as:

$$s_{ij}(X, c) = \frac{dX}{dc} = M_{ij}(w)R_{ij}(w)$$

where $M_{ij}(w) = -\frac{(1-p(e_{ij}))}{p(e_{ij})}$ while $R_i(w) = \frac{U'(W_{ij}^F)}{U'(W_{ij}^S)}$ where $W_{ij}^S = Y - X_{ij} + w_{ij}$ and $W_{ij}^F = w_{ij} - c_{ij}$.

The curvature of the indifference curve with respect to changes in wealth is then:

$$\frac{\partial}{\partial w}\left(s_{ij}(X,c)\right) = M_{ij}(w)R'_{ij}(w) + M'_{ij}(w)R_{ij}(w) \gtrless 0 \tag{9}$$

Proof. See the Appendix

Here $M_{ij}(w)R'_{ij}(w)$ captures the risk preference effect while $M'_{ij}(w)R_{ij}(w)$ captures the impact of moral hazard. Not surprisingly (9) has an ambiguous sign. On one side, the effect of (decreasing) risk aversion makes the indifference curve flatter as wealth increases. On the other side the negative impact of moral hazard makes it steeper. Indeed, for a given project choice, due to decreasing absolute risk aversion, rich individuals require a smaller reduction in the repayment rate to compensate for an increase in collateral (e.g., they are more willing to post collateral). Whenever the impact of moral hazard prevails as in eq. (10), rich individuals put such a lower level of effort, and their probability of success diminishes by so much that their trade-off between collateral and interest rate becomes worse than poor people's one, notwithstanding their lower risk aversion:

$$\frac{\partial e_{ij}}{\partial w} > p(e_{ij})(1-p(e_{ij}))(A(W_{ij}^S) - A(W_{ij}^F)) \tag{10}$$

Proof. See the Appendix

Note that this ambiguity in general means that the single crossing property of indifference curves, which is a necessary condition to ensure the possibility of separation, does not hold as a general rule.

Now let us investigate the impact of effort aversion μ_j on the marginal rate of substitution between repayment X and collateral c. Independently by the amount of endowed wealth, entrepreneurs characterized by $L-$effort aversion display a relative preference for posting more collateral compared to the ones characterized by $H-$effort aversion at any point in the space (X, c) due to their higher success probability. Thus the impact of effort aversion is always the same and this implies that independently by which one of the two effects prevails the slope of indifference curves with high effort aversion should be steeper.

When $\frac{\partial}{\partial w}\left(s_{ij}(X,c)\right) > 0$, for instance, the direct impact of decreasing risk aversion exceeds the effect of moral hazard[9]. Thus intuitively the marginal cost of the repayment is globally lower for richer or lower effort aversion agents, holding the other characteristic constant.

Let us now consider the slope of the isoprofit curve for a bank lending to the borrower of class ij only:

$$\frac{dX}{dc}|_{\bar{\pi}_{ij}} = -\frac{(1-p(e_{ij})) + (dp(e_{ij})/dc)\,(X-c)}{p(e_{ij}) + (dp(e_{ij})/dX)\,(X-c)} \tag{11}$$

where $\bar{\pi}_{ij}$ is the bank's expected profit from the borrower of class i. Since $dp(e_{ij})/dX$ is negative, (11) could in principle be positive. Note that this becomes more likely for high values of X and lower values of c (see Coco, 1999). We may immediately note that, by construction, under this information structure, individuals with a larger wealth (higher risk from the point of view of banks) may prefer contracts that are actuarially fair for poor individuals due to decreasing risk aversion.

4 Inequality of opportunity equilibria

Under perfect information (our benchmark case), both wealth and effort aversion are observable and first-best conditions can be realized for each type of entrepreneurs $ij \in \{R,P\} \times \{L;H\}$. In particular we can observe that a competitive equilibrium credit policy maximizes a borrower's expected utility (for each type of borrower ij) as follows:

$$\max_{\{(X_{ij};c_{ij})\}} p(e_{ij})U(Y - X_{ij} + w_i) + (1-p(e_{ij}))U(w_i - c_{ij}) - \mu_j e_{ij} \tag{12}$$

subject to the lender's zero profit condition on each type of borrower $ij \in \{R,P\} \times \{L;H\}$.

$$p(e_{ij})X_{ij} + (1-p(e_{ij}))c_{ij} = K \tag{13}$$

[9] When $\frac{\partial}{\partial w}\left(s_{ij}(X,c)\right) < 0$, the procedure is analogous at least for type$-RH$ and type$-PL$.

Proposition 1. *Under perfect information and independently by their amount of wealth, each $L-$ borrower accepts the contract $C(X_{iL}^{*},0)$ where $X_{iL}^{*}=\frac{K}{p(e_{iL})}$, and e_{iL} is the first best effort choice for $iL-$types. No collateral is required.*

Proof. See the Appendix

Intuitively, in the first best case, each $ij-$borrower may in principle get the contract $C(X_{ij},0)$ with no collateral provision[10]. However since lenders can observe agent's effort aversion, they decide to exclude from the market all $H-$types due to their negative expected returns neglecting any effort evaluation of their wealth classes.

Let's now turn to the search for equilibria in the framework where entrepreneurial wealth and effort aversion are both private information. For $\forall i\in[R,P]$ and $\forall j\in[L,H]$, the maximization procedure would be:

$$\max_{\{(X_{ij};c_{ij}),(X_{ij};c_{ij})\}}\{\lambda\nu\left[p(e_{RH})U(Y-X_{RH}+w_{RH})+(1-p(e_{RH}))U(w_{RH}-c_{RH})-\mu_{H}e_{RH}\right]$$

$$+\lambda(1-\nu)\left[p(e_{RL})U(Y-X_{RL}+w_{RL})+(1-p(e_{RL}))U(w_{RL}-c_{RL})-\mu_{L}e_{RL}\right]$$

$$+(1-\lambda)v\left[p(e_{PH})U(Y-X_{PH}+w_{PH})+(1-p(e_{PH}))U(w_{PH}-c_{PH})-\mu_{H}e_{PH}\right]$$

$$+(1-\lambda)(1-\nu)\left[p(e_{PL})U(Y-X_{PL}+w_{PL})+(1-p(e_{PL}))U(w_{PL}-c_{PL})-\mu_{L}e_{PL}\right]\} \qquad (14)$$

subject to twelve incentive constraints and the representative lender's zero profit condition on each $ij-$type $\in[R,P]\times[L;H]$ (see the Appendix for a detailed list of incentive constraints).

Note that the natural source of aggregate uncertainty on banking system is here determined by the combination of endogenous adverse selection on wealth and effort aversion, whereby the interaction of preferences and feasible contracts in the dimensional space makes it difficult to define a Nash equilibrium in pure strategies.

Our starting point has to be the fact that we have both positive and adverse selection of contract terms (particularly collateral). Hence an increase in collateral for example can lead to exit of high-effort averse types (poor or rich depending on the initial contract) but also

[10] Of course the contract above must be supported by the threat of charging a higher interest rate if effort supplied and verified ex-post by the bank is not the first best.

more simply of poor entrepreneurs with low effort aversion. What we know for sure is that, given a certain level of effort-aversion, poor entrepreneurs exit first because of decreasing risk aversion. And conversely that, for a given wealth-type, individuals with higher effort aversion exit first. An important issue concerns the relative slope of the indifference curves of different types. As we observed above, the relative slope of poor vs rich types, keeping constant their effort aversion, cannot be ascertained a priori as it depends on the relative strength of risk aversion versus incentive effect. Instead we know that within a wealth class less effort-averse types are necessarily more willing to post collateral at any contract (e.g., flatter indifference curves), thus suggesting that separation within wealth classes would be in principle feasible. But separation of wealth types is actually not feasible given the violation of the single crossing property.

We will start the analysis of possible equilibria looking at portions of the contract space where all types participate. Let us look at Figure 1 where we observe the participation constraint of poor entrepreneurs with high effort-aversion type denoted PC_{PH} and that of the poor entrepreneurs with low effort aversion PC_{PL}. In area A below the PC_{PH}, every type participates at any contract. However because all $H-$types participate, it is quite unlikely that a feasible zero-profit pooling contract could be offered. However suppose there exists a potential global pooling zero-profit contract at C_1 (or any point in area A). This contract is not an equilibrium one, as we know that $PL-$type can always be attracted by an appropriate higher-collateral contract due to flatter indifference curves, and that it will definitely deliver higher profits to the bank[11]. Hence a pure pooling equilibrium offered to all types is not feasible. However we know that a separating equilibrium with a contract for the $H-$type is never feasible by definition (the surplus would be too low due to negative expected return). Hence no contract is feasible in area A.

Now suppose that there is a contract C_2 in area B, above the PC_{PH}, where a bank breaks even with a contract with the 3 remaining types (PL, RH, RL). Again to explore the possibility that this pooling contract is an equilibrium we should analyse the relative slopes of indifference curves of different types. We know that for a competitive lender, $L-$types represent the ones delivering profits at any given contract and that rich borrowers with low effort aversion (I_{RL}) have definitely flatter indifference curves than rich borrowers with high effort aversion (I_{RH}). So

[11] Other types can be attracted as well depending on the relative slope but what really matters is that the additional contract is not preferred by low wealth/high effort aversion types.

it is always possible to offer a contract separating $RL-$types and delivering positive profits. A three-types pooling can be excluded as well by competition.

Since the single crossing property does not hold any more, evaluating the simultaneous effect of risk aversion and incentive effect is required at this stage in order to establish fixed monotonicity condition. Let us start by analysing the case where the risk aversion effect is larger than the moral hazard one $\left(\frac{\partial}{\partial w}\left(s_{ij}(X,c)\right) > 0\right)$. In this case monotonicity conditions among types hold based on their willingness to post collateral:

$$X_{RL} < \min\{X_{PL}; X_{RH}\} < \max\{X_{PL}; X_{RH}\} < X_{PH} \tag{15}$$

while,

$$c_{RL} > \max\{c_{PL}, c_{RH}\} > \min\{c_{PL}, c_{RH}\} > c_{PH} \tag{16}$$

Two cases may arise depending on an evaluation of the willingness to post collateral of intermediate types. The most interesting one suggests that $c_{RH} > c_{PL}$, i.e., intuitively, the indifference curve of $PL-$type is in principle steeper than $RH-$one[12]. In this case a pooling contract can be offered to $RH-$ and $PL-$ types. Thus the contract $\hat{C}$ in figure 2 is offered on the zero profit line ($O_{PL/RH}$) for the $RH-$type and $PL-$type, while freeing the $RL-$type for a higher collateral 'fair'[13] contract (on the separating contract line O_{RL}), like C_{RL}. The following proposition formally clarifies all the previous characteristics of the contract.

Proposition 2. *Under asymmetric information on both wealth and effort aversion, if the risk aversion effect prevails, ($c_{RH} > c_{PL}$), a partial pooling equilibrium in pure strategies $(\hat{X}, \hat{c})$ exists between $PL-$ and $RH-$classes, with a separating contract for $RL-$type and it is characterized by the pair of contracts:*

$$C(X, c) = \{(X_{RL}, c_{RL}); (\hat{X}, \hat{c})\}$$

where $c_{RL} > \hat{c}$ while $X_{RL} < \hat{X}$.

Proof. See the Appendix

[12] e.g.when the risk aversion direct effect prevails
[13] Meaning with no cross-subsidy.

Note that this contract lies necessarily on the PC_{PH} line. Any contract above this participation constraint would not be a stable equilibrium because an additional contract north west of it could always steal the more profitable (PL)-types. Contract $\hat{C}$ in figure 2 instead cannot be broken as moving north west would necessarily bring in also the worse (PH) types and the additional contract would not be profitable.

Now let us examine the features of this equilibrium. At the pooling contract $(\hat{C})$, $PL-$ types are systematically cross subsidizing $RH-$types, who realize negative NPV projects. This means that the terms of the contract will be penalizing for poor hard working types. In this case rich types get access to credit whatever their effort aversion. Therefore inequality of opportunity follows immediately. Note finally that by assumption $RH-$types carry out their project due to cross subsidization but they are actually burning some surplus.

Whenever instead, the incentive effect prevails $(\frac{\partial}{\partial w}(s_{ij}(X,c)) < 0)$, then the slope of I_{PL} will be flatter than that of I_{RL} and therefore necessarily also of I_{RH}. Monotonicity conditions among types hold in a different way such that:

$$X_{PL} < \min\{X_{PH}; X_{RL}\} < \max\{X_{PH}; X_{RL}\} < X_{RH} \tag{17}$$

while

$$c_{PL} > \max\{c_{PH}, c_{RL}\} > \min\{c_{PH}, c_{RL}\} > c_{RH} \tag{18}$$

We may show that whenever the moral hazard effect is instead higher than the risk aversion impact, there exists no contract that can be part of a stable equilibrium since it is always possible to propose a higher collateral contract which attracts $L-$type borrowers independently by their wealth level.

Proposition 3. *Under asymmetric information on both wealth and effort aversion, when the moral hazard prevails no equilibrium exists among wealth classes and poor entrepreneurs are rationed out from the market*

Proof. See the Appendix

Proposition 3 suggests that it is always possible to find a contract that steals from the candidate three-types pooling the $RL-$type also steals the $PL-$type, making it all the more profitable. So the only possible equilibrium contract set in this area would be a contract separating $RH-$type from the rest. However separating the $RH-$type is impossible because their projects deliver negative return. In this case no set of contracts in area B (area below PC_{PL}) of figure 2 can represent an equilibrium, meaning that $PL-$type will be excluded from the market. A separating contract for the $RL-$type excluding all other types with a sufficiently large amount of collateral will be devised by the bank[14]. As a consequence poor entrepreneurs are systematically excluded from the market due to their inability to distinguish themselves from $RH-$type. Inequality of opportunity is apparent.

Let us finally discuss the equilibria we found in the propositions on the basis of available evidence on credit markets in developing countries. Banerjee and Duflo (2010) list some stylized established facts:

a) Richer people borrow more (wealth dependence);

b) Richer people pay lower interest rates;

c) These divergences in interest rates are not driven by differences in default rates;

d) Lending rates vary widely in the same credit market;

Of course both our equilibria are consistent with fact a), while the first one (the only one where poor entrepreneurs participate) is obviously consistent with fact b). Poor entrepreneurs pay the higher interest rate, while rich ones pay the high or the low one depending on their quality (e.g., effort aversion). The second equilibrium is also clearly consistent with fact c). Poor entrepreneurs pay a higher interest rate, notwithstanding their low default rates, mainly because they subsidize bad quality types. The same equilibrium is also compatible with fact d)[15].

Finally note that inequality of opportunity in equilibrium results in poor entrepreneurs being, on average, necessarily of better 'quality' (effort aversion in our specification) than richer ones. To our knowledge the only test for this hypothesis has been performed by Evans and Jovanovich (1989), who find that 'the correlation of entrepreneurial ability and assets is negative

[14]This is feasible considering that types$-RH$ exit first.

[15]Note that in developed countries the risk premia on loans are usually low and not very variable across loans (see Black and De Meza, 1997).

and statistically significant'.

5 Concluding remarks

The aim of this paper has been to investigate whether equality of opportunity actually holds in a credit market with heterogeneity on unobservable circumstances and responsibility, respectively wealth and effort aversion, and to discuss the relation between violation of EOp and efficient credit allocation. Only if all equally-qualified applicants get credit whatever their endowment situation, the ideal of equality of opportunity holds. In our setting equality of opportunity requires that individuals with the same level of the responsibility (same effort aversion) variable be offered the same opportunities.

We show due to decreasing absolute risk aversion, moral hazard results in rich borrowers supplying less effort to their projects compared to the poor ones. The combination of these features with an endogenous adverse selection on both wealth and effort aversion delivers a complex environment where the search for equilibria is particularly difficult. We demonstrate that any equilibrium entails some forms of inequality of opportunity. In some cases no poor entrepreneur gets credit due to her inability to separate herself from worse (rich) entrepreneurs. Some surplus is lost as a consequence. Under other restrictions instead a partial separating equilibrium with cross-subsidization between classes exists, where not only equality of opportunity is violated, but poor entrepreneurs with a higher level of responsibility (lower effort aversion) cross-subsidize the rich ones with lower responsibility. Access to the credit market is thus paid by hard-working poor entrepreneurs with a perverse redistribution 'tax'. The additional consequence of the subsidy is that negative surplus projects are carried out. In this case efficiency and equity violations occur jointly. This last equilibrium is consistent with most consensus micro-evidence on credit markets particularly in developing countries.

Finally note that wealth heterogeneity is the very factor impeding the perfect screening of types. In absence of wealth heterogeneity, effort aversion is correctly (e.g., inversely) correlated with the willingness to post collateral in order to obtain screening. Therefore when full ex-ante redistribution of wealth is possible, leading to uniform wealth levels in the population of entrepreneurs, willingness to post collateral correctly signals low effort aversion and the project's quality. Perfect screening is in principle possible and only good projects, those carried out by low effort aversion entrepreneurs, are realized in equilibrium. Hence a perfectly egalitarian ex-ante redistribution of resources improves efficiency because it ensures that good projects, and only good projects, are carried out, thus avoiding also waste from realization of negative surplus projects. Contemporaneously, and by definition, in this equilibrium entrepreneurs get credit on the basis solely of the responsibility variable. This intervention would therefore improve final allocations both on distributive grounds, at least under the equality of opportunity benchmark, and on efficiency grounds.

6 Figures

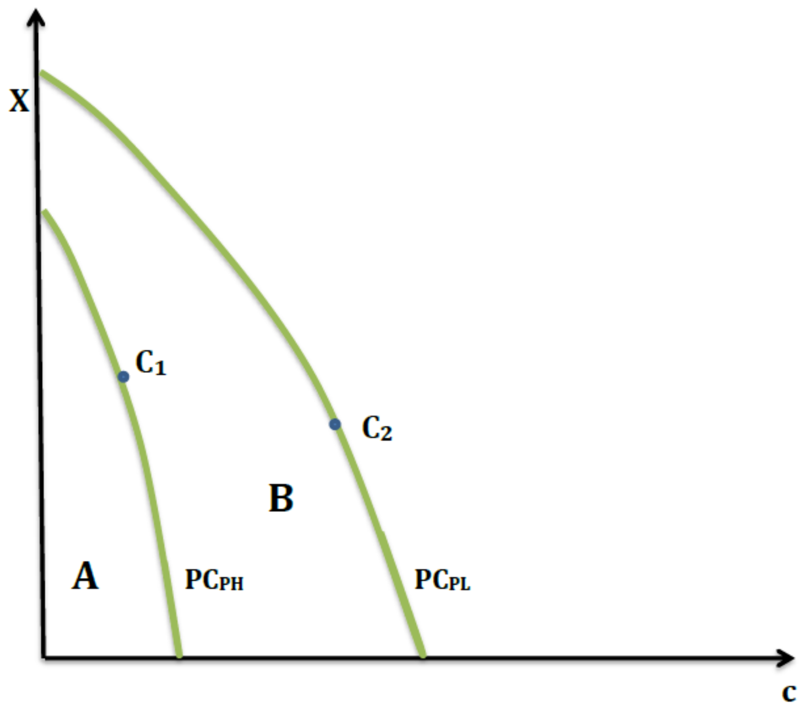

Figure 1: No equilibrium in areas A and B

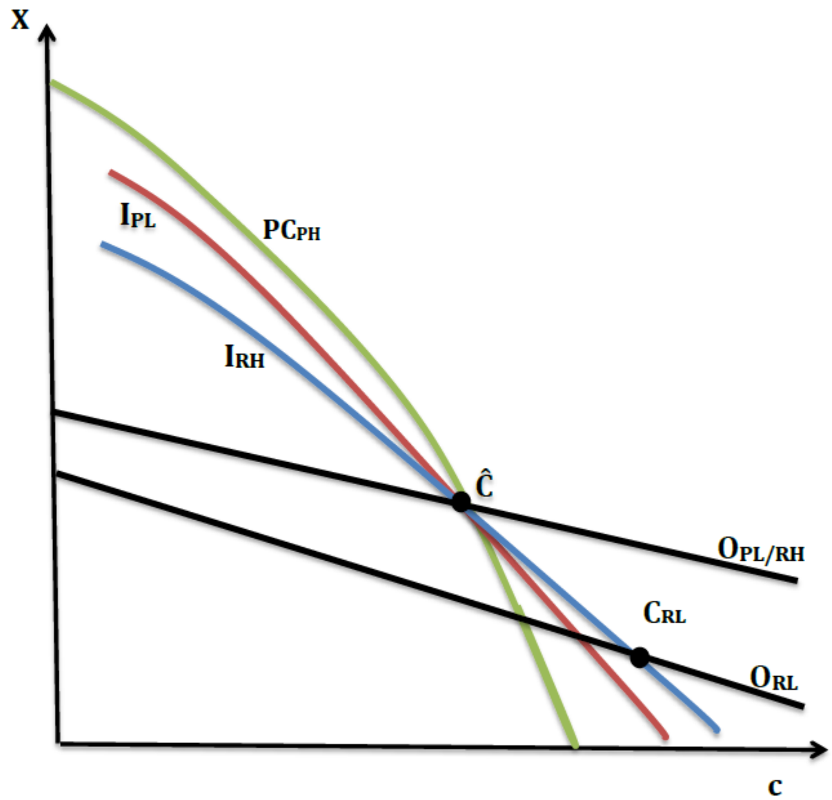

Figure 2: Partial separating equilibrium with cross-subsidization

7 Appendix

Proof of equation (5). Starting by eq. (3) and assuming that $W_{ij}^S = Y - X_{ij} + w_{ij}$ and $W_{ij}^F = w_{ij} - c_{ij}$, we can simply rewrite that:

$$\frac{\partial U}{\partial e_{ij}} = p'(e_{ij}) \left(U(W_{ij}^S) - U(W_{ij}^F) \right) - \mu_j$$

By the Implicit Function theorem and due to decreasing absolute risk aversion, we simply observe that:

$$\left[p''(e_{ij}) \left(U(W_{ij}^S) - U(W_{ij}^F) \right) \right] de + \left[p'(e_{ij}) \left(U'(W_{ij}^S) - U'(W_{ij}^F) \right) \right] dw = 0$$

$$\left[p''(e_{ij}) \left(U(W_{ij}^S) - U(W_{ij}^F) \right) \right] de = - \left[p'(e_{ij}) \left(U'(W_{ij}^S) - U'(W_{ij}^F) \right) \right] dw$$

which implies:

$$\frac{de_{ij}^*}{dw} = - \frac{p'(e_{ij}) \left(U'(W_{ij}^S) - U'(W_{ij}^F) \right)}{p''(e_{ij}) \left(U(W_{ij}^S) - U(W_{ij}^F) \right)} < 0$$

□

Proof of equation (6). With a similar procedure shown in eq. (5), we may write:

$$\left[p''(e_{ij}) \left(U(W_{ij}^S) - U(W_{ij}^F) \right) \right] de - d\mu = 0$$

$$\left[p''(e_{ij}) \left(U(W_{ij}^S) - U(W_{ij}^F) \right) \right] de = d\mu$$

indicating,

$$\frac{de_{ij}^*}{d\mu} = \frac{1}{p''(e_{ij}) \left(U(W_{ij}^S) - U(W_{ij}^F) \right)} < 0$$

□

Proof of Lemma 1. In the marginal set, individuals have utility equal to zero. By the envelope theorem and differentiating equation with respect to μ and w, we observe that:

$$[-e_{ij}]\,d\mu + \left[p(e_{ij})U'(W_{ij}^S) + (1 - p(e_{ij})U'(W_{ij}^F)\right] dw =$$

$$[-e_{ij}]\,d\mu = -\left[p(e_{ij})U'(W_{ij}^S) + (1 - p(e_{ij})U'(W_{ij}^F)\right] dw$$

which implies that:

$$\frac{d\mu}{dw}|_{U_{ij}(.)=0} = -\frac{\left[p(e_{ij})U'(W_i^S) + (1 - p(e_{ij})U'(W_i^F)\right]}{[-e_{ij}]} > 0$$

□

Proof of equation (8). Starting by eq. (1):

$$U_{ij} = p(e_{ij})U(W_{ij}^S) + (1 - p(e_{ij}))U(W_{ij}^F) - \mu_j e_{ij}$$

By envelope theorem and differentiating with respect to X and c, it follows that:

$$\left[-p(e_{ij})U'(W_{ij}^S)\right] dX - \left[(1 - p(e_{ij}))U'(W_{ij}^F)\right] dc = 0$$

$$\left[-p(e_{ij})U'(W_{ij}^S)\right] dX = \left[(1 - p(e_{ij}))U'(W_{ij}^F)\right] dc$$

which implies that:

$$s_{ij}(X, c) = \frac{dX}{dc} = -\frac{(1 - p(e_{ij}))U'(W_{ij}^F)}{p(e_{ij})U'(W_{ij}^S)} < 0$$

□

Proof of equation (9). We can again rewrite the slope of the indifference curve as:

$$s_{ij}(X, c) = \frac{dX}{dc} = M_{ij}(w)R_{ij}(w)$$

where $M_{ij}(w) = -\frac{(1-p(e_{ij}))}{p(e_{ij})}$ while $R_{ij}(w) = \frac{U'(W_{ij}^F)}{U'(W_{ij}^S)}$. The curvature of the indifference curve with respect to change in wealth is then:

$$\frac{\partial}{\partial w}\left(\frac{dX}{dc}\right) = M_{ij}(w)R'_{ij}(w) + M'_{ij}(w)R_{ij}(w)$$

where $M_{ij}(w)R'_{ij}(w)$ captures the effect of risk preference while $M'_{ij}(w)R_{ij}(w)$ explains the moral hazard effect. First, let us solve $M_{ij}(w)R'_{ij}(w)$:

$$\begin{aligned}
M_{ij}(w)R'_{ij}(w) &= -\frac{(1-p(e_{ij}))}{p(e_{ij})}\left[\frac{U''(W_{ij}^F)U'(W_{ij}^S) - U''(W_{ij}^S)U'(W_{ij}^F)}{\left(U'(W_{ij}^S)\right)^2}\right] = \\
&= -\frac{(1-p(e_{ij}))}{p(e_{ij})}\left[\frac{U''(W_{ij}^F)}{U'(W_{ij}^S)} - \frac{U''(W_{ij}^S)U'(W_{ij}^F)}{\left(U'(W_{ij}^S)\right)^2}\right] = \\
&= -\frac{(1-p(e_{ij}))}{p(e_{ij})}\frac{1}{U'(W_{ij}^S)}\left[U''(W_{ij}^F) - \frac{U''(W_{ij}^S)U'(W_{ij}^F)}{U'(W_{ij}^S)}\right] = \\
&= -\frac{(1-p(e_{ij}))}{p(e_{ij})}\frac{U'(W_{ij}^F)}{U'(W_{ij}^S)}\left[\frac{U''(W_{ij}^F)}{U'(W_{ij}^F)} - \frac{U''(W_{ij}^S)}{U'(W_{ij}^S)}\right]
\end{aligned}$$

Let us define $A(W)$ as the coefficient of decreasing absolute risk aversion, we can then rewrite

$M_{ij}(w)R'_{ij}(w)$ as:

$$\begin{aligned}
M_{ij}(w)R'_{i}(w) &= -\frac{(1-p(e_{ij}))}{p(e_{ij})}\frac{U'(W_{ij}^F)}{U'(W_{ij}^S)}\left(A(W_{ij}^S) - A(W_{ij}^F)\right) \\
&= \frac{dX}{dc}\left(A(W_{ij}^S) - A(W_{ij}^F)\right) > 0
\end{aligned}$$

Since $W_{ij}^S > W_{ij}^F$ and considering decreasing absolute risk aversion i.e. risk aversion decreases with wealth, $A(W_{ij}^F) > A\left(W_{ij}^S\right)$ and that by construction that $\frac{dX}{dc}$ is negative, we can surely say that the effect of risk preferences $M_{ij}(w)R'_{ij}(w)$ is positive. Then we can solve $M'_{ij}(w)R_{ij}(w)$:

$$\begin{aligned}
M'_{ij}(w)R_{ij}(w) &= \left[-\frac{-p'(e_{ij})\frac{\partial e_{ij}}{\partial w}p(e_{ij})-(1-p(e_{ij}))p'(e_{ij})\frac{\partial e_{ij}}{\partial w}}{(p(e_{ij}))^2}\right]\frac{U'(W^F_{ij})}{U'(W^S_{ij})} = \\
&= \left[\frac{p'(e_{ij})\frac{\partial e_{ij}}{\partial w}}{(p(e_{ij}))}+\frac{(1-p(e_{ij}))p'(e_{ij})\frac{\partial e_{ij}}{\partial w}}{(p(e_{ij}))^2}\right]\frac{U'(W^F_{ij})}{U'(W^S_{ij})} = \\
&= \frac{p'(e_{ij})}{p(e_{ij})}\frac{\partial e_{ij}}{\partial w}\left[1+\frac{(1-p(e_{ij}))}{p(e_{ij})}\right]\frac{U'(W^F_{ij})}{U'(W^S_{ij})} = \\
&= \frac{p'(e_{ij})}{p(e_{ij})}\frac{\partial e_{ij}}{\partial w}\left[\frac{U'(W^F_{ij})}{U'(W^S_{ij})}-\frac{dX}{dc}\right] = \\
&= -\frac{p'(e_{ij})}{p(e_{ij})}\frac{\partial e_{ij}}{\partial w}\left[\frac{dX}{dc}-\frac{U'(W^F_{ij})}{U'(W^S_{ij})}\right] < 0
\end{aligned}$$

Therefore,

$$\frac{\partial}{\partial w}\left(\frac{dX}{dc}\right) = \frac{dX}{dc}\left(A(W^S_i)-A(W^F_i)\right) - \frac{p'(e_{ij})}{p(e_{ij})}\frac{\partial e_{ij}}{\partial w}\left(\frac{dX}{dc}-\frac{U'(W^F_i)}{U'(W^S_i)}\right) \lesseqgtr 0$$

As shown, the sign of eq. (9) is uncertain due to the combination of the positive effect of risk aversion $\left(\frac{dX}{dc}\left(A(W^S_{ij})-A(W^F_{ij})\right)\right)$ and the negative moral hazard impact $-\frac{p'(e_{ij})}{p(e_{ij})}\frac{\partial e_{ij}}{\partial w}\left(\frac{dX}{dc}-\frac{U'(W^F_{ij})}{U'(W^S_{ij})}\right)$. $\square$

Proof of equation (10). After some algebraic manipulations,

$$\begin{aligned}
\frac{\partial}{\partial w}\left(\frac{dX}{dc}\right) &= s_{ij}\left(A(W_{ij}^S)-A(W_{ij}^F)\right)-\frac{\partial e_{ij}}{\partial w}\frac{p'(e_{ij})}{p(e_{ij})}\left(s_{ij}-\frac{U'(W_{ij}^F)}{U'(W_{ij}^S)}\right)= \\
&= s_{ij}(1-p(e_{ij}))\left(A(W_{ij}^S)-A(W_{ij}^F)\right)-\frac{\partial e_{ij}}{\partial w}p'(e_{ij})\frac{(1-p(e_{ij}))}{p(e_{ij})}\left(s_{ij}-\frac{U'(W_{ij}^F)}{U'(W_{ij}^S)}\right)= \\
&= s_{ij}(1-p(e_{ij}))\left(A(W_{ij}^S)-A(W_{ij}^F)\right)-\frac{\partial e_{ij}}{\partial w}p'(e_{ij})\left(\left(\frac{(1-p(e_{ij}))}{p(e_{ij})}\right)s_{ij}+s_{ij}\right)= \\
&= s_{ij}(1-p(e_{ij}))\left(A(W_{ij}^S)-A(W_{ij}^F)\right)-\frac{\partial e_{ij}}{\partial w}p'(e_{ij})\frac{s_{ij}}{p(e_{ij})} \\
&= s_{ij}\left((1-p(e_{ij}))\left(A(W_{ij}^S)-A(W_{ij}^F)\right)-\frac{\partial e_{ij}}{\partial w}\frac{1}{p(e_{ij})\left(U(W_{ij}^S)-U(W_{ij}^F)\right)}\right)= \\
&= \frac{s_{ij}}{p(e_{ij})\left(U(W_{ij}^S)-U(W_{ij}^F)\right)}\left(p(e_{ij})(1-p(e_{ij}))\left(A(W_{ij}^S)-A(W_{ij}^F)\right)-\frac{\partial e_{ij}}{\partial w}\right)
\end{aligned}$$

The impact of moral hazard prevails if and only if:

$$\frac{\partial e_{ij}}{\partial w}>p(e_{ij})(1-p(e_{ij}))\left(A(W_{ij}^S)-A(W_{ij}^F)\right)$$

$\square$

Proof of Proposition 1. Looking at the Lagrangian maximization,

$$\begin{aligned}
L &= p(e_{ij})U(Y-X_{ij}+w_{ij})+(1-p(e_{ij}))U(w_{ij}-c_{ij})-\mu_j e_{ij} \\
&\quad +\phi\left(p(e_{ij})X_{ij}+(1-p(e_{ij}))c_{ij}-K\right)
\end{aligned}$$

and, substituting for X_{ij}, into the zero profit constraint such that $X_{ij}=\frac{K-(1-p(e_{ij}))c_{ij}}{p(e_{ij})}$, the following optimization problem must be solved:

$$\begin{aligned}
&p(e_{ij})U(Y-\left[\frac{K-(1-p(e_{ij}))c_{ij}}{p(e_{ij})}\right]+w_i)+(1-p(e_{ij}))U(w_i-c_{ij})-\mu_j e_{ij} \\
&+\phi\left(p(e_{ij})\left[\frac{K-(1-p(e_{ij}))c_{ij}}{p(e_{ij})}\right]+(1-p(e_{ij}))c_{ij}-K\right)
\end{aligned}$$

Differentiating the Lagrangian with respect to c_{ij}:

$$\frac{\partial L}{\partial c_{ij}} = U'(W_{ij}^S) - U'(W_{ij}^F) < 0$$

Thus, the required collateral c_{ij} is zero and the repayment $X_{ij} = \frac{K}{p(e_{ij})}$ for each type. $\square$

Proof of Proposition 2. As a first step, we indicate a complete list of the incentive constraints for each entrepreneur-ij, where $i \in [R, P]$ and $j \in [L, H]$. For the $RH-$type,

$$p(e_{RH})U(Y - X_{RH} + w_{RH}) + (1 - p(e_{RH}))U(w_{RH} - c_{RH}) - \mu_H e_{RH} \geq \\ p(e_{RH})U(Y - X_{RL} + w_{RH}) + (1 - p(e_{RH}))U(w_{RH} - c_{RL}) - \mu_H e_{RH} \tag{19}$$

$$p(e_{RH})U(Y - X_{RH} + w_{RH}) + (1 - p(e_{RH}))U(w_{RH} - c_{RH}) - \mu_H e_{RH} \geq \\ p(e_{RH})U(Y - X_{PH} + w_{RH}) + (1 - p(e_{RH}))U(w_{RH} - c_{PH}) - \mu_H e_{RH} \tag{20}$$

$$p(e_{RH})U(Y - X_{RH} + w_{RH}) + (1 - p(e_{RH}))U(w_{RH} - c_{RH}) - \mu_H e_{RH} \geq \\ p(e_{RH})U(Y - X_{PL} + w_{RH}) + (1 - p(e_{RH}))U(w_{RH} - c_{PL}) - \mu_H e_{RH} \tag{21}$$

For the $RL-$type,

$$p(e_{RL})U(Y - X_{RL} + w_{RL}) + (1 - p(e_{RL}))U(w_{RL} - c_{RL}) - \mu_L e_{RL} \geq \\ p(e_{RL})U(Y - X_{RH} + w_{RL}) + (1 - p(e_{RL}))U(w_{RL} - c_{RH}) - \mu_L e_{RL} \tag{22}$$

$$p(e_{RL})U(Y - X_{RL} + w_{RL}) + (1 - p(e_{RL}))U(w_{RL} - c_{RL}) - \mu_L e_{RL} \geq \\ p(e_{RL})U(Y - X_{PH} + w_{RL}) + (1 - p(e_{RL}))U(w_{RL} - c_{PH}) - \mu_L e_{RL} \tag{23}$$

$$p(e_{RL})U(Y - X_{RL} + w_{RL}) + (1 - p(e_{RL}))U(w_{RL} - c_{RL}) - \mu_L e_{RL} \geq \tag{24}$$
$$p(e_{RL})U(Y - X_{PL} + w_{RL}) + (1 - p(e_{RL}))U(w_{RL} - c_{PL}) - \mu_L e_{RL}$$

For the $PH-$type,

$$p(e_{PH})U(Y - X_{PH} + w_{PH}) + (1 - p(e_{PH}))U(w_{PH} - c_{PH}) - \mu_H e_{PH} \geq \tag{25}$$
$$p(e_{PH})U(Y - X_{RH} + w_{PH}) + (1 - p(e_{PH}))U(w_{PH} - c_{RH}) - \mu_H e_{PH}$$

$$p(e_{PH})U(Y - X_{PH} + w_{PH}) + (1 - p(e_{PH}))U(w_{PH} - c_{PH}) - \mu_H e_{PH} \geq \tag{26}$$
$$p(e_{PH})U(Y - X_{RL} + w_{PH}) + (1 - p(e_{PH}))U(w_{PH} - c_{RL}) - \mu_H e_{PH}$$

$$p(e_{PH})U(Y - X_{PH} + w_{PH}) + (1 - p(e_{PH}))U(w_{PH} - c_{PH}) - \mu_H e_{PH} \geq \tag{27}$$
$$p(e_{PH})U(Y - X_{PL} + w_{PH}) + (1 - p(e_{PH}))U(w_{PH} - c_{PL}) - \mu_H e_{PH}$$

For the $PL-$type,

$$p(e_{PL})U(Y - X_{PL} + w_{PL}) + (1 - p(e_{PL}))U(w_{PL} - c_{PL}) - \mu_L e_{PL} \geq \tag{28}$$
$$p(e_{PL})U(Y - X_{RH} + w_{PL}) + (1 - p(e_{PL}))U(w_{PL} - c_{RH}) - \mu_L e_{PL}$$

$$p(e_{PL})U(Y - X_{PL} + w_{PL}) + (1 - p(e_{PL}))U(w_{PL} - c_{PL}) - \mu_L e_{PL} \geq \tag{29}$$
$$p(e_{PL})U(Y - X_{RL} + w_{PL}) + (1 - p(e_{PL}))U(w_{PL} - c_{RL}) - \mu_L e_{PL}$$

$$p(e_{PL})U(Y - X_{PL} + w_{PL}) + (1 - p(e_{PL}))U(w_{PL} - c_{PL}) - \mu_L e_{PL} \geq \tag{30}$$
$$p(e_{PL})U(Y - X_{PH} + w_{PL}) + (1 - p(e_{PL}))U(w_{PL} - c_{PH}) - \mu_L e_{PL}$$

Now we need to study the incentive compatibility constraints for each type and then summing them two by two in order to obtain a partial ranking in terms of collateral and repayment required. First, adding $IC_{RH/PH}$ with $IC_{PH/RH}$ and $IC_{PL/RL}$ with $IC_{RL/PL}$ implies that $X_{Rj} > X_{Pj}$ while $c_{Rj} < c_{Pj}$ $\forall j \in [L, H]$ as confirmed by the previous analysis of unidimensional case on observable effort aversion. This means that by taking into account effort aversion the repayment rate required by the bank for rich types must be necessarily higher than the one imposed to poor types based on their collateral provision c_{Pj} since they are the most efficient types given effort aversion. Moreover going on, we add $IC_{PL/PH}$ with $IC_{PH/PL}$, we obtain that $X_{iH} > X_{iL}$ while $c_{iH} < c_{iL}$. Obviously, this result does not take into account the effect of negative expected return of $H-$type in terms of equilibrium but just suggests that given individual's wealth, higher repayment should be required to individuals with higher effort aversion with respect to ones with low effort aversion since the latter are more willing to post collateral due to their higher probability of success. Thus the examined incentive constraints among types allows us to show some monotonicity conditions on the credit policy proposed by the banks in terms of willingness to post collateral. First evaluating $IC_{RL/PL}$ with $IC_{PL/PH}$ we simply obtains that the global downward incentive constraint $IC_{RL/PH}$ is satisfied when the two local constraints are satisfied. Moreover, $IC_{RL/PL}$ plus $IC_{PL/RH}$ implies $IC_{RL/RH}$, while $IC_{PL/RH}$ plus $IC_{RH/PH}$ implies $IC_{PL/PH}$. Finally, $IC_{RL/RH}$ plus $IC_{RH/PL}$ implies $IC_{RL/PL}$ while $IC_{RH/PL}$ plus $IC_{PL/PH}$ implies $IC_{RH/PH}$. We study the case when risk aversion effect is higher than the moral hazard impact. In this case monotonicity conditions among types (15) and (16) hold based on their willingness to post collateral. The most interesting case is realized when $c_{RH} > c_{PL}$, i.e., intuitively, the indifference curve of $PL-$type is in principle steeper than $RH-$one. Thus for our maximization procedure, we remain with the only local constraint $IC_{PH/RH}$, $IC_{RH/PL}$, $IC_{PL/RL}$

plus the lender's zero profit condition of each type:

$$
\begin{aligned}
L = & \{\lambda\nu \left[p(e_{RH})U(Y - X_{RH} + w_{RH}) + (1 - p(e_{RH}))U(w_{RH} - c_{RH}) - \mu_H e_{RH}\right] \\
& +\lambda(1-\nu)\left[p(e_{RL})U(Y - X_{RL} + w_{RL}) + (1 - p(e_{RL}))U(w_{RL} - c_{RL}) - \mu_L e_{RL}\right] \\
& +(1-\lambda)v\left[p(e_{PH})U(Y - X_{PH} + w_{PH}) + (1 - p(e_{PH}))U(w_{PH} - c_{PH}) - \mu_H e_{PH}\right] \\
& +(1-\lambda)(1-\nu)\left[p(e_{PL})U(Y - X_{PL} + w_{PL}) + (1 - p(e_{PL}))U(w_{PL} - c_{PL}) - \mu_L e_{PL}\right]\} \\
& +\phi\{p(e_{PH})U(Y - X_{PH} + w_{PH}) + (1 - p(e_{PH}))U(w_{PH} - c_{PH}) - \mu_H e_{PH} \\
& -p(e_{PH})U(Y - X_{RH} + w_{PH}) - (1 - p(e_{PH}))U(w_{PH} - c_{RH}) + \mu_H e_{PH}\} \\
& +\gamma\{p(e_{RH})U(Y - X_{RH} + w_{RH}) + (1 - p(e_{RH}))U(w_{RH} - c_{RH}) - \mu_H e_{RH} \\
& -p(e_{RH})U(Y - X_{PL} + w_{RH}) - (1 - p(e_{RH}))U(w_{RH} - c_{PL}) + \mu_H e_{RH}\} \\
& +\tau\{p(e_{PL})U(Y - X_{PL} + w_{PL}) + (1 - p(e_{PL}))U(w_{PL} - c_{PL}) - \mu_L e_{PL} \\
& -p(e_{PL})U(Y - X_{RL} + w_{PL}) - (1 - p(e_{PL}))U(w_{PL} - c_{RL}) + \mu_L e_{PL}\}
\end{aligned}
$$

Substituting for X_{ij}, from the zero profit condition for each type ij for $\forall i \in [R, P]$ and $\forall j \in [L, H]$, such that $X_{ij} = \frac{K-(1-p(e_{ij}))c_{ij}}{p(e_{ij})}$:

$$
\begin{aligned}
L = & \{\lambda\nu \left[p(e_{RH})U\left(Y - \left[\frac{K-(1-p(e_{RH}))c_{RH}}{p(e_{RH})}\right] + w_{RH}\right) + (1 - p(e_{RH}))U(w_{RH} - c_{RH}) - \mu_H e_{RH}\right] \\
& +\lambda(1-\nu)\left[p(e_{RL})U\left(Y - \left[\frac{K-(1-p(e_{RL}))c_{RL}}{p(e_{RL})}\right] + w_{RL}\right) + (1 - p(e_{RL}))U(w_{RL} - c_{RL}) - \mu_L e_{RL}\right] \\
& +(1-\lambda)v\left[p(e_{PH})U\left(Y - \left[\frac{K-(1-p(e_{PH}))c_{PH}}{p(e_{PH})}\right] + w_{PH}\right) + (1 - p(e_{PH}))U(w_{PH} - c_{PH}) - \mu_H e_{PH}\right] \\
& +(1-\lambda)(1-\nu)\left[p(e_{PL})U\left(Y - \left[\frac{K-(1-p(e_{PL}))c_{PL}}{p(e_{PL})}\right] + w_{PL}\right) + (1 - p(e_{PL}))U(w_{PL} - c_{PL}) - \mu_L e_{PL}\right]\} \\
& +\phi\{p(e_{PH})U\left(Y - \left[\frac{K-(1-p(e_{PH}))c_{PH}}{p(e_{PH})}\right] + w_{PH}\right) + (1 - p(e_{PH}))U(w_{PH} - c_{PH}) - \mu_H e_{PH} \\
& -p(e_{PH})U\left(Y - \left[\frac{K-(1-p(e_{RH}))c_{RH}}{p(e_{RH})}\right] + w_{PH}\right) - (1 - p(e_{PH}))U(w_{PH} - c_{RH}) + \mu_H e_{PH}\} \\
& +\gamma\{p(e_{RH})U\left(Y - \left[\frac{K-(1-p(e_{RH}))c_{RH}}{p(e_{RH})}\right] + w_{RH}\right) + (1 - p(e_{RH}))U(w_{RH} - c_{RH}) - \mu_H e_{RH}
\end{aligned}
$$

$$-p(e_{RH})U\left(Y-\left[\frac{K-(1-p(e_{PL}))c_{PL}}{p(e_{PL})}\right]+w_{RH}\right)-(1-p(e_{RH}))U(w_{RH}-c_{PL})+\mu_H e_{RH}\}$$
$$+\tau\{p(e_{PL})U\left(Y-\left[\frac{K-(1-p(e_{PL}))c_{PL}}{p(e_{PL})}\right]+w_{PL}\right)+(1-p(e_{PL}))U(w_{PL}-c_{PL})-\mu_L e_{PL}$$
$$-p(e_{PL})U\left(Y-\left[\frac{K-(1-p(e_{RL}))c_{RL}}{p(e_{RL})}\right]+w_{PL}\right)-(1-p(e_{PL}))U(w_{PL}-c_{RL})+\mu_L e_{PL}\}$$

Differentiating the Lagrangian with respect to c_{PH}, we obtain

$$\frac{\partial L}{\partial c_{PH}}=(1-\lambda)vp(e_{PH})U'(W_{PH}^S)\frac{(1-p(e_{PH}))}{p(e_{PH})}-(1-\lambda)v(1-p(e_{PH}))U'(W_{PH}^F)$$
$$+\phi p(e_{PH})U'(W_{PH}^S)\frac{(1-p(e_{PH}))}{p(e_{PH})}-\phi(1-p(e_{PH}))U'(W_{PH}^F)$$

Differentiating the Lagrangian with respect to c_{RH}, it follows that:

$$\frac{\partial L}{\partial c_{RH}}=\lambda vp(e_{RH})U'(W_{RH}^S)\frac{(1-p(e_{RH}))}{p(e_{RH})}-\lambda v(1-p(e_{RH}))U'(W_{RH}^F)$$
$$-\phi p(e_{PH})U'\left(Y-\left[\frac{K-(1-p(e_{RH}))c_{RH}}{p(e_{RH})}\right]+w_{PH}\right)\frac{(1-p(e_{RH}))}{p(e_{RH})}+\phi(1-p(e_{PH}))U'(w_{PH}-c_{RH})$$
$$+\gamma p(e_{RH})U'(W_{RH}^S)\frac{(1-p(e_{RH}))}{p(e_{RH})}-\gamma(1-p(e_{RH}))U'(W_{RH}^F)=0$$

Differentiating with respect to c_{PL}, we instead obtain:

$$\frac{\partial L}{\partial c_{PL}}=(1-\lambda)(1-\nu)p(e_{PL})U'(W_{PL}^S)\frac{(1-p(e_{PL}))}{p(e_{PL})}-(1-\lambda)(1-\nu)(1-p(e_{PL}))U'(W_{PL}^F)$$
$$-\gamma p(e_{RH})U'\left(Y-\left[\frac{K-(1-p(e_{PL}))c_{PL}}{p(e_{PL})}\right]+w_{RH}\right)\frac{(1-p(e_{PL}))}{p(e_{PL})}+\gamma(1-p(e_{RH}))U'(w_{RH}-c_{PL})$$
$$+\tau p(e_{PL})U'(W_{PL}^S)\frac{(1-p(e_{PL}))}{p(e_{PL})}-\tau(1-p(e_{PL}))U'(W_{PL}^F)=0$$

Finally, differentiating with respect to c_{RL},

$$\begin{aligned}\frac{\partial L}{\partial c_{RL}} &= \lambda(1-\nu)p(e_{RL})U'(W_{RL}^S)\frac{(1-p(e_{RL}))}{p(e_{RL})}-\lambda(1-\nu)(1-p(e_{RL}))U'(W_{RL}^F)\\ &\quad -\tau p(e_{PL})U'\left(Y-\left[\frac{K-(1-p(e_{PL}))c_{PL}}{p(e_{PL})}\right]+w_{RH}\right)\frac{(1-p(e_{RL}))}{p(e_{RL})}+\tau(1-p(e_{PL}))U'(w_{PL}-c_{RL})\end{aligned}$$

Thus,

- $\frac{\partial L}{\partial c_{PH}} = ((1-\lambda)v + \phi)\left[U'(W_{PH}^S) - U'(W_{PH}^F)\right] < 0$ $PH-$type are excluded from the market or must get a contract such that $c_{PH} = 0$

- $\frac{\partial L}{\partial c_{RH}} = (\lambda v + \gamma)\, U'(W_{RH}^S)(1-p(e_{RH})) - \phi\frac{p(e_{PH})}{p(e_{RH})}U'\left(Y - \left[\frac{K-(1-p(e_{RH}))c_{RH}}{p(e_{RH})}\right] + w_{PH}\right)(1-p(e_{RH})) - (\lambda v + \gamma)\, U'(W_{RH}^F)(1-p(e_{RH})) + \phi(1-p(e_{PH}))U'(w_{PH} - c_{RH}) < 0$

since first, $(\lambda v + \gamma)\left[U'(W_{RH}^S) - U'(W_{RH}^F)\right] < 0$ second,

- $-\phi\frac{p(e_{PH})}{p(e_{RH})}U'\left(Y - \left[\frac{K-(1-p(e_{RH}))c_{RH}}{p(e_{RH})}\right] + w_{PH}\right)(1-p(e_{RH})) + \phi(1-p(e_{PH}))U'(w_{PH} - c_{RH}) < 0$ since

  - $\frac{p(e_{PH})}{p(e_{RH})} > 1$; $p(e_{PH}) > p(e_{RH})$; $(1-p(e_{RH})) > (1-p(e_{PH}))$ and $Y - \frac{K}{p(e_{RH})} + 2c_{RH} - p(e_{RH})c_{RH} > 0$

Thus, even in this case $RH-$type are excluded from the market or otherwise may implement a contract if and only if $c_{PH} = 0$

- $\frac{\partial L}{\partial c_{PL}} = ((1-\lambda)(1-\nu) + \tau)\, U'(W_{PL}^S)(1-p(e_{PL})) - \gamma\frac{p(e_{RH})}{p(e_{PL})}U'\left(Y - \left[\frac{K-(1-p(e_{PL}))c_{PL}}{p(e_{PL})}\right] + w_{RH}\right)(1-p(e_{PL}))$

- $-((1-\lambda)(1-\nu) + \tau)(1-p(e_{PL}))U'(W_{PL}^F) + \gamma(1-p(e_{RH}))U'(w_{RH} - c_{PL}) < 0$

since first $((1-\lambda)(1-\nu) + \tau)\left[U'(W_{PL}^S) - U'(W_{PL}^F)\right](1-p(e_{PL})) < 0$ and
second, $-\gamma\frac{p(e_{RH})}{p(e_{PL})}U'\left(Y - \left[\frac{K-(1-p(e_{PL}))c_{PL}}{p(e_{PL})}\right] + w_{RH}\right)(1-p(e_{PL})) + \gamma(1-p(e_{RH}))U'(w_{RH} - c_{PL}) < 0$ since the risk aversion effect prevails to the moral hazard one.

$\frac{(1-p(e_{PL}))p(e_{RH})}{(1-p(e_{RH}))p(e_{PL})} > 1, Y - \frac{K}{p(e_{PL})} + 2c_{PL} - p(e_{PL})c_{PL} > 0$

Thus $c_{PL} = 0$ is the only possibility to get the contract.

- $\frac{\partial L}{\partial c_{RL}} = \lambda(1-\nu)p(e_{RL})U'(W_{RL}^S)\frac{(1-p(e_{RL}))}{p(e_{RL})} - \lambda(1-\nu)(1-p(e_{RL}))U'(W_{RL}^F)$

$-\tau p(e_{PL})U'\left(Y - \left[\frac{K-(1-p(e_{PL}))c_{PL}}{p(e_{PL})}\right] + w_{RH}\right)\frac{(1-p(e_{RL}))}{p(e_{RL})} + \tau(1-p(e_{PL}))U'(w_{PL} - c_{RL}) < 0$

since $\lambda(1-\nu)[U'(W_{RL}^S) - U'(W_{RL}^F)](1-p(e_{RL})) < 0$

and

$-\tau p(e_{PL})U'\left(Y - \left[\frac{K-(1-p(e_{PL}))c_{PL}}{p(e_{PL})}\right] + w_{RH}\right)\frac{(1-p(e_{RL}))}{p(e_{RL})} + \tau(1-p(e_{PL}))U'(w_{PL} - c_{RL}) < 0$

Finally $c_{RL} = 0$ is the only acceptable policy. We can thus observe that there is no possibility to use collateral as a sorting device among types and consequently there is no possibility to get separating policies in order to favour or guarantee access to credit for the most efficient types, independently of their amount of wealth. Moreover, we may show that potential pooling contracts among all four types $(RL, PL;, RH, PH)$ or three types $(RL, PL;, RH)$ above the $PH-$participation constraint are excluded from the market. Let us observe if a pooling equilibrium $(\hat{X}, \hat{c})$ is possible among wealth classes mixing effort aversions $(PL; RH)$. We examine the following incentive constraint $IC_{RH-PL/RL}$,

$$\begin{aligned} p(e_{RL})U(Y-\hat{X}+w_{RL})+(1-p(e_{RL}))U(w_{RL}-\hat{c})\mu_L e_{RL} \geq \\ p(e_{RL})U(Y-X_{RL}+w_{RL})+(1-p(e_{RL}))U(w_{RL}-c_{RL})-\mu_L e_{RL} \end{aligned} \tag{31}$$

and the $PH-$participation constraint:

$$p(e_{PH})U(Y-X_{PH}+w_{PH})+(1-p(e_{PH}))U(w_{PH}-c_{PH})-\mu_H e_{PH} \geq 0 \tag{32}$$

The optimization problem in this case would be the following:

$$\begin{aligned} L = & \{\lambda(1-\nu)\left[p(e_{RL})U(Y-X_{RL}+w_{RL})+(1-p(e_{RL}))U(w_{RL}-c_{RL})-\mu_L e_{RL}\right] \\ & +(1-\lambda)v\left[p(e_{PH})U(Y-X_{PH}+w_{PH})+(1-p(e_{PH}))U(w_{PH}-c_{PH})-\mu_H e_{PH}\right] \\ & +\hat{\lambda}\hat{v}[p(e_{RH})U(Y-\hat{X}+w_{RH})+(1-p(e_{RH}))U(w_{RH}-\hat{c})-\mu_H e_{RH} \\ & +p(e_{PL})U(Y-\hat{X}+w_{PL})+(1-p(e_{PL}))U(w_{PL}-\hat{c})-\mu_L e_{PL}]\} \\ & +\phi[p(e_{RL})U(Y-\hat{X}+w_{RL})+(1-p(e_{RL}))U(w_{RL}-\hat{c})-\mu_L e_{RL} \\ & -p(e_{RL})U(Y-X_{RL}+w_{RL})-(1-p(e_{RL}))U(w_{RL}-c_{RL})+\mu_L e_{RL}] \\ & +\gamma[p(e_{PH})U(Y-X_{PH}+w_{PH})+(1-p(e_{PH}))U(w_{PH}-c_{PH})-\mu_H e_{PH}] \end{aligned}$$

where $\hat{\lambda}\hat{v} = \lambda v + (1-\lambda)(1-v)$.Substituting for X_{ij}, from the zero profit condition for each type ij for $\forall i \in [R, P]$ and $\forall j \in [L, H]$, such that $X_{ij} = \frac{K-(1-p(e_{ij}))c_{ij}}{p(e_{ij})}$ and looking at he repayment rate in case of pooling contract, $\hat{X}$,

$$\hat{X} = \frac{K - [(1-\lambda)(1-v)(1-p(e_{PL})) + \lambda v(1-p(e_{RH}))]\hat{c}}{[(1-\lambda)(1-v)p(e_{PL}) + \lambda v p(e_{RH})]}$$

we can rewrite the maximization procedure as follows:

$$\begin{aligned}
L \;=\; & \{\lambda(1-\nu)\left[p(e_{RL})U\left(Y - \left[\frac{K-(1-p(e_{RL}))c_{RL}}{p(e_{RL})}\right] + w_{RL}\right) + (1-p(e_{RL}))U(w_{RL}-c_{RL}) - \mu_L e_{RL}\right] \\
& +(1-\lambda)v\left[p(e_{PH})U\left(Y - \left[\frac{K-(1-p(e_{PH}))c_{PH}}{p(e_{PH})}\right] + w_{PH}\right) + (1-p(e_{PH}))U(w_{PH}-c_{PH}) - \mu_H e_{PH}\right] \\
& +\hat{\lambda}\hat{v}[p(e_{RH})U\left(Y - \left[\frac{K - [(1-\lambda)(1-v)(1-p(e_{PL})) + \lambda v(1-p(e_{RH}))]\hat{c}}{[(1-\lambda)(1-v)p(e_{PL}) + \lambda v p(e_{RH})]}\right] + w_{RH}\right) \\
& +(1-p(e_{RH}))U(w_{RH}-\hat{c}) - \mu_H e_{RH} \\
& +p(e_{PL})U\left(Y - \left[\frac{K - [(1-\lambda)(1-v)(1-p(e_{PL})) + \lambda v(1-p(e_{RH}))]\hat{c}}{[(1-\lambda)(1-v)p(e_{PL}) + \lambda v p(e_{RH})]}\right] + w_{PL}\right) \\
& +(1-p(e_{PL}))U(w_{PL}-\hat{c}) - \mu_L e_{PL}]\} \\
& +\phi[p(e_{RL})U\left(Y - \left[\frac{K - [(1-\lambda)(1-v)(1-p(e_{PL})) + \lambda v(1-p(e_{RH}))]\hat{c}}{[(1-\lambda)(1-v)p(e_{PL}) + \lambda v p(e_{RH})]}\right] + w_{RL}\right) \\
& +(1-p(e_{RL}))U(w_{RL}-\hat{c}) - \mu_L e_{RL} \\
& -p(e_{RL})U\left(Y - \left[\frac{K-(1-p(e_{RL}))c_{RL}}{p(e_{RL})}\right] + w_{RL}\right) - (1-p(e_{RL}))U(w_{RL}-c_{RL}) + \mu_L e_{RL}] \\
& +\gamma[p(e_{PH})U(Y - \left[\frac{K-(1-p(e_{PH}))c_{PH}}{p(e_{PH})}\right] + w_{PH}) + (1-p(e_{PH}))U(w_{PH}-c_{PH}) - \mu_H e_{PH}]
\end{aligned}$$

Differentiating the Lagrangian with respect to c_{RL}, it follows that:

$$\begin{aligned}
\frac{\partial L}{\partial c_{RL}} \;=\; & \lambda(1-\nu)p(e_{RL})U'(W_{RL}^S)\frac{(1-p(e_{RL}))}{p(e_{RL})} - \lambda(1-\nu)(1-p(e_{RL}))U'(W_{RL}^F) \\
& -\phi p(e_{RL})U'(W_{RL}^S)\frac{(1-p(e_{RL}))}{p(e_{RL})} + \phi(1-p(e_{RL}))U'(W_{RL}^F) > 0
\end{aligned}$$

thus revealing that a contract with a positive amount of c_{RL} is in principle possible based on the willingness to post collateral of the $RL-$type Moreover, differentiating the Lagrangian with respect to $\hat{c}$, it follows that:

$$\begin{aligned}\frac{\partial L}{\partial \hat{c}} &= \hat{\lambda}\hat{v}p(e_{RH})U'(.)\,\frac{(1-\lambda)(1-v)(1-p(e_{PL}))+\lambda v(1-p(e_{RH}))}{[(1-\lambda)(1-v)p(e_{PL})+\lambda vp(e_{RH})]} - \hat{\lambda}\hat{v}\,(1-p(e_{RH}))\,U'(w_{RH}-\hat{c}) \\ &+\hat{\lambda}\hat{v}p(e_{PL})U'(.)\,\frac{(1-\lambda)(1-v)(1-p(e_{PL}))+\lambda v(1-p(e_{RH}))}{[(1-\lambda)(1-v)p(e_{PL})+\lambda vp(e_{RH})]} - \hat{\lambda}\hat{v}(1-p(e_{PL}))U'(w_{PL}-\hat{c}) \\ &+\phi p(e_{RL})U(.)\,\frac{(1-\lambda)(1-v)(1-p(e_{PL}))+\lambda v(1-p(e_{RH}))}{[(1-\lambda)(1-v)p(e_{PL})+\lambda vp(e_{RH})]} - (1-p(e_{RL}))U'(w_{RL}-\hat{c})\end{aligned}$$

which is positive since $Y-\left(\frac{K-[(1-\lambda)(1-v)(1-p(e_{PL}))+\lambda v(1-p(e_{RH}))]}{[(1-\lambda)(1-v)p(e_{PL})+\lambda vp(e_{RH})]}\right)\hat{c}+w_{ij} > w_{ij}-\hat{c}$. Differentiating the Lagrangian with respect to c_{PH}, it follows that:

$$\begin{aligned}\frac{\partial L}{\partial c_{PH}} &= (1-\lambda)vp(e_{PH})U'(W^S_{PH})\frac{(1-p(e_{PH}))}{p(e_{PH})} - (1-\lambda)v(1-p(e_{PH}))U'(W^F_{PH}) \\ &+\gamma p(e_{PH})U'(W^S_{PH})\frac{(1-p(e_{PH}))}{p(e_{PH})} - \gamma(1-p(e_{PH}))U'(W^F_{PH}) < 0\end{aligned}$$

which implies that there is no equilibrium contract for PH-type such that their participation constraint is satisfied. It is worth to note that the only possibility to get a pooling equilibrium is possible just in the case where the willingness to post collateral of $PL-$type is higher than the one of the $RH-$type and an unfair cross-subsidization is realized among wealth classes. Thus a partial pooling equilibrium $(\hat{X},\hat{c})$ among wealth classes $(PL;RH)$ is possible according to a separating strategy involving $RL-$type with a contract $c_{RL} > \hat{c}$ and $\hat{X} > X_{RL}$ since intuitively they have higher willingness to post collateral. □

Proof of Proposition 3. The proof of the first and second part of this proposition is identical to those of Proposition 4. In order to verify that no (separating or pooling) equilibrium exists among types, monotonicity conditions are in place such that:

$$X_{PL} < \min\{X_{PH}; X_{RL}\} < \max\{X_{PH}; X_{RL}\} < X_{RH}$$

while

$$c_{PL} > \max\{c_{PH}, c_{RL}\} > \min\{c_{PH}, c_{RL}\} > c_{RH}$$

In this case it is easy to observe that in accordance with the willingness to post collateral of intermediate types, respectively $PH-$ and $RL-$ types ($c_{PH} \gtrless c_{RL}$), lenders may always offer a contract at the zero profit line which attracts only the $L-$type independently by their available wealth in order to sort out $RH-$class from the others. However, due to the negative expected return on $H-$type, separating contract is not feasible. A higher amount of collateral would be required to screen entrepreneurs although the participation constraint of $PL-$types is not more satisfied. Thus in this case the only possibility is for the $RL-$types to get a contract implicitly suggesting that poor entrepreneurs are completely rationed out from the market independently by their effort aversion level. $\square$

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