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Abstract

The purpose of this paper is to propose theoretical foundations on the impact of transfer scheme, e.g. Community Agricultural Policy, on income inequality within European Countries. First, we show that ex-post inequality (in the after-transfer distribution) may increase if either initial aggregate income or the amount of fiscal contributions are sufficiently high. Second according to welfare ordering, we characterize a multi-factor decomposition of the Atkinson index to gauge the impact of each income source on the inequality profile. Third, we introduce a methodology to construct a cooperative game played by different income factors (as net incomes and/or incoming transfers) explicitly measuring the cost of inequality across the population in terms of welfare loss. We finally rely on Banzhaf and Shapley values to determine the marginal contributions of each factor to overall inequality.

Keywords: Atkinson index, income sources, Shapley, transfer scheme, cooperative game, CAP
JEL classification: D31; D63; I32

1 Introduction

Since the 70's European countries have systematically employed part of their direct taxation and collections of rents to fund specific activities and to redistribute incomes across countries. The prime objective of these policies is to attract firms based on the assumption that this will decrease regional income inequality, increase employment, wages, and productivity in those regions. In the recent debate, it seems however that these transfer schemes have been quite unsuccessful: political parties of all shades, farmers' unions, environmental campaigners and taxpayer groups all lined up to say these policies went either the wrong way, or not far enough. Empirical evidence even suggests that although income per capita has slightly converged between Member States, it has diverged at the regional level during the same period, see Combes and Overman [9].

Among all these programs, Community Agricultural Policy (CAP, hereafter) is the most expensive scheme in the EU accounting for 47% of its annual budget, and perhaps is even the most controversial. Its main goal is to guarantee a fair standard of living for agricultural community (European Union [13], Article 33) contributing to the supply of European citizens with safe and high quality food. It aims at responding to the public demand for a sustainable agricultural policy in Europe by ensuring sufficient competitiveness while helping rural areas to remain attractive and viable. It relies on a system of subsidies and support programs in a mix between direct payments to farmers together with price or market supports[1]. In each state, tax payments imposed to different class of farmers are largely refunded by a revenue subsidy provided per hectare of land.

One of the most debatable questions on this issue is to understand whether or not, subsidies within CAP, as tools of distributional policy, clearly contribute to the reduction of income inequalities in the agricultural sector. For instance, as far as the issue of distribution of subsidies within agricultural community is concerned, empirical evidence explains that the majority of payments go to the largest businesses. Schmid *et al.* [26] state that in 2001, on average for 14 EU countries, 80% of direct payments went to only 20% of businesses. The results vary significantly even in terms of income inequality profiles. On one side, Keeney [17] shows how the introduction of direct payments has contributed to the more balanced income distribution of agricultural households. Hansen and Teuber [18] suggest that the CAP transfers tend to smooth differences in farmers' revenues in the German State of Hesse. Their empirical analysis, based on Shorrocks [29]'s inequality decomposition, show a positive effect of transfers within state associated to a reduction of inequality per capita in the disposable income across regions. On the other side, Allanson [1] and [2] take into account the redistribution impact of effective agricultural policy as the difference between the inequality rate on income including subsidies and the rate of inequality in income after the deduction of subsidies. He outlines how the distribution of support is found to have exacerbated the inequality of farm incomes in Scotland because of horizontal inequities in the incidence of transfers.

To the best of our knowledge, theoretical foundations devoted to characterize the dynamics of farm support programs and its impact on income distribution among citizens still remain unexplored. The aim of this paper is thus first to model potential outgoing and incoming transfer schemes among the European countries in order to observe how inequality in each state is affected by a standard procedure of aggregation of individual contributions. We study a simple environment where each Member State levies taxes from its citizens, in compliance with its fiscal law, to raise money to contribute to a global (European) fund. The aggregated amount must be subsequently reallocated across States after a negotiation process such as a collective bargaining agreement. Note that we do not address the issue of assignment of shares among Member States but we directly focus on the related vector of received subsidies as given[2]. Whenever each State receives funds, it carries out a

[1] There exist more than 400 kinds of subsidies in Europe. They assume so many forms that it is difficult to classify them as subsidies to production, investment or labor. Some of these funds are granted in the form of interest-rate payments, guarantees and participation in venture capital, while structural funds are mainly proposed as non-repayable grants or indirect aid.

[2] The design of optimal decision rules in the EU has been widely addressed, also in view of its consecutive enlargements. A recent paper by Le Breton *et al.* [20] provides a complete discussion on the application of several power

further internal redistribution, thereby assigning a fraction of funds to its citizens entitled to benefit from it. Income profiles are over time modified by such transfers, and this influences the inequality profile within the State. Our analysis involves the use of Atkinson index (see [3]) helpful to compare the inequality levels before and after the transfers process. A relevant problem of redistribution policy can thus be addressed: provided that the transfer scheme is convenient for a State, i.e., since the aggregate contribution does not exceed the discounted value of its share from the bargaining (defined as *profitability condition*), does its inequality level increase or decrease? Conditioned to it, two kinds of effects seem to show up: a) the impact of subsidies on the income of agricultural households generates lower inequality with respect to the one in the initial income distribution; b) when the poorest types do not contribute to the amount of outgoing transfers, i.e. when their tax rates are zero, a reduction in inequality is easier to accomplish due to the transferring process.

As a second step we characterize a multi-factor decomposition of the Atkinson index since we are interested to capture whether the effect on overall inequality is concentrated in specific income items. It should be crucial in terms of public policy to understand how much of the total income inequality produced into the society is explained by a precise source and in particular to verify the effect of transfer on income profile. Most of studies on this issue have focused on the Gini and the Theil indices since they have particular features for decomposing inequality by income sources. Shorrocks [28] proposes one of the pioneering procedure to afford these types of decomposition. He proves that it is possible to derive an infinite number of decompositions without further restrictions; a property which is called natural decomposition and is valid for the main inequality indices. According to this procedure, Lerman and Yitzhaki [21] (LY, hereafter) propose a decomposition based on the covariance formula of the Gini index *à* la Fei *et al.* [15]. They show that Gini coefficient for the entire income distribution is equal to the sum of Gini coefficients computed by exploiting the covariance between each income source and the cumulative distribution function of total income. In this way, they obtain the impact of the marginal change in a given income source on overall inequality. Although it is a natural decomposition, it indicates a clear measurement of the contribution of each source to income inequality. Taking cue from LY [21], we show how the Atkinson measure can be decomposed by income sources and we step further proposing it as a characteristic function of a cooperative game. We believe that the decomposition by income factor components is a particularly helpful methodology to assess the marginal contribution of each factor in terms of public policy. A recent discussion on analogous games, both in terms of inequality and poverty measures, has been published by Charpentier and Mussard [8]. We prove the subadditivity of the game with a measure of the social cost of inequality according to Blackorby et al. [6]. In this scenario, we apply the standard Banzhaf-Coleman and Shapley-Shubik values to evaluate the net income and the incoming transfers' contributions to the overall inequality.

Our third step (still incomplete) is to normatively evaluate a new measure of welfare loss characterized by the difference between the entire costs of inequality generated by the heterogeneity in income distribution among income sources and the cost of inequality borne by the entire society. In this case our aim is to observe the systematic effect of public policy evaluating positivity and monotonicity properties of this game. Other properties are still to be definitively arranged.
The remainder of this paper has the following structure: section 2 outlines the necessary notation, introduces the setup and the features of the main theoretical results achieved by employing the standard Atkinson index. Section 3 introduces the multi-factor Atkinson index of inequality discussing its properties in a cooperative game of welfare loss. In section 4 we first depict some anecdotal evidence on the Common Agricultural Policy in Europe and then we assess a simple stylized application on the proposed measure of inequality in this context. Conclusions follow in section 5.

indices and on the methodology for the assessment of the fair decision rules in the Council of Ministers of the EU across the years, also showing the disproportionateness of power among States in some cases.

2 The basic setup

Consider a framework endowed with the following characteristics:

- There are n heterogeneous populations of agents. Each population can be viewed as a State whose inhabitants are ranked by their income. They are endowed with p distinct amounts of income: we indicate the income profile of the i-th State with the vector $\mathbf{x}_i = (x_{1i}, \ldots, x_{pi})$. Every coordinate of $\mathbf{x}_i$ can be looked upon as a subgroup of the i-th State's total population. We are hypothesizing that all the subgroups are sufficiently homogeneous and do not contain very different individual types. The aggregate income of the i-th State is $X_i = \sum_{h=1}^{p} x_{hi}$.
- The vector $\mathbf{y}_i = A_i \mathbf{x}_i = (y_{1i}, \ldots, y_{pi})$ collects the fractions of all incomes which are provided by the i-th State as its outgoing transfer.
- All the States' outgoing transfers form a unique aggregate capital which will be shared by the States after a bargaining game, therefore we can view such gross capital as the grand coalition value of a cooperative game, i.e.:
$$\sum_{i=1}^{n} \sum_{h=1}^{p} y_{hi} = v(N),$$
where $n \in 2^n$ is the coalition of all States.
- The bargaining game takes place and consequently, the exogenous solution concept determines the allocation of the benefits gained by each State.
- Finally each State reallocates its share to one or more specific groups of its citizens according to the incoming transfer scheme based on its own local rule.

The first step consists in evaluating the initial level of inequality of income profiles employing one of the common inequality indexes, e.g. the Atkinson one, see [3].

Subsequently, we take into account the structure of outgoing and incoming transfers. In order to characterize the i-th vector of outgoing transfers, we construct a taxation matrix A_i. $A_i \in M_p(\mathbb{R})$ is a block matrix whose k main diagonal blocks are diagonal matrices. Each of them represents the tax rate for a certain group of citizens involving at least one subgroup, whereas all the off-diagonal blocks are zero matrices. Suppose that each main diagonal block is a $p \times p$ matrix and that the j-th block is a diagonal $p_j \times p_j$ matrix, where $p_{i1} + \cdots + p_{ik} = p$. The blocks of A_i can also be interpreted as the k different fiscal types of the i-th population. By this representation, we are partitioning the population into k distinct types in accordance with their incomes' characteristics.

If $c_{ij} \geq 0$ is the tax rate for the j-th group of citizens of the i-th State, we have the following construction for the i-th vector of outgoing transfers:

$$A_i \mathbf{x}_i = \left(\begin{array}{cccc|ccc|cccc} c_{i1} & 0 & \cdots & 0 & & & & & & & \\ \cdots & \cdots & \cdots & \cdots & & & & & & & \\ 0 & \cdots & 0 & c_{i1} & & & & & & & \\ \hline & & & & \cdots & \cdots & \cdots & & & & \\ & & & & \cdots & \cdots & \cdots & & & & \\ & & & & \cdots & \cdots & \cdots & & & & \\ \hline & & & & & & & c_{ik} & 0 & \cdots & 0 \\ & & & & & & & \cdots & \cdots & \cdots & \cdots \\ & & & & & & & 0 & \cdots & 0 & c_{ik} \end{array} \right) \begin{pmatrix} x_{1i} \\ x_{2i} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ x_{pi} \end{pmatrix} =$$

$$= \begin{pmatrix} c_{i1}x_{1i} \\ c_{i1}x_{2i} \\ \vdots \\ c_{i1}x_{p_1,i} \\ c_{i2}x_{p_1+1,i} \\ \vdots \\ c_{i2}x_{p_1+p_2,i} \\ \vdots \\ \vdots \\ c_{ik}x_{pi} \end{pmatrix} = \begin{pmatrix} y_{1i} \\ y_{2i} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ y_{pi} \end{pmatrix} = \mathbf{y}_i,$$

which is the i-th vector of contributions provided by the i-th State after taxation. Note that there can be some low incomes that are not subject to taxation, and in that case the corresponding tax rate is zero (for example, if the last block corresponds to the lowest incomes, $c_{ik} = 0$). The total contribution provided by the i-th State is given by the sum of coordinates, i.e.

$$C_i := \sum_{h=1}^{p_1} c_{i1}x_{hi} + \sum_{h=p_1+1}^{p_1+p_2} c_{i1}x_{hi} + \cdots + \sum_{h=p_1+\cdots+p_{k-1}}^{p_1+\cdots+p_k} c_{ik}x_{hi}, \tag{1}$$

then the total amount of all countries' transfers is given by $T := \sum_{i=1}^{n} C_i$.

T is the aggregate contribution that is going to be allocated among the member States according to a collective agreement motivated as a result of a bargaining process. If we define a cooperative game whose solution concept is the allocation of T among players, we can consider T as the worth of the grand coalition of the game, i.e., $v(n) = T$, see [23]. The topic of power assessment in EU's Countries has been extensively studied in recent years, and there is still a lively debate on the most suitable power index to be used (see [5], [19], [20]). We avoid delving into such issue which is beyond the scope of our paper.

Suppose that, after the bargaining game, the distribution of T among the States is described by the vector $D := (d_1, \ldots, d_n) \in \mathbb{R}^n_+$. Subsequently, the i-th State will have to implement an incoming transfer procedure. If d_i is the positive amount that the i-th State must allocate, such quantity will have to be assigned to the specific group of inhabitants which benefit from the funding. We are assuming that the involved group is composed by subgroups which had a specific unique tax rate, say c_{il}, when they contributed to overall outgoing transfer. This means that the l-th block of the matrix A_i corresponds to the only type which receives a fraction of the incoming transfer. Because d_i must be apportioned according to an intrinsic distribution scheme, we can indicate with $\alpha_{il}(s) \in [0,\ 1)$ the share of d_i which is assigned to the s-th subgroup in the l-th block, for $s = 1, \ldots, p_l$. By construction, for all $i = 1, \ldots, n$, we will have that $\alpha_{il}(1) + \cdots + \alpha_{il}(p_l) = 1$.

Furthermore, assume that the incoming transfer takes place at time t_1, and that it is subject to a discount factor (for example, depending on the ongoing inflation rate) $e^{-\delta t_1}$. Then, after the bargaining and transfer processes the final income vector of the i-th State becomes:

$$\mathbf{z}_i = \begin{pmatrix} x_{1i} - c_{i1}x_{1i} \\ \vdots \\ x_{p_1,i} - c_{i1}x_{p_1,i} \\ \vdots \\ x_{p_1+\dots+p_{l-1}+1,i} - c_{il}x_{p_1+\dots+p_{l-1}+1,i} + \alpha_{il}(1)d_i e^{-\delta t_1} \\ \vdots \\ x_{p_1+\dots+p_l,i} - c_{il}x_{p_1+\dots+p_l,i} + \alpha_{il}(p_l)d_i e^{-\delta t_1} \\ \vdots \\ \vdots \\ x_{pi} - c_{ik}x_{pi} \end{pmatrix}. \tag{2}$$

The condition of profitability for the i-th State after the transfers process is given by:

$$X_i \le \sum_{h=1}^{p} z_{hi} \iff C_i \le d_i e^{-\delta t_1}, \tag{3}$$

whose economic meaning is straightforward: the aggregate amount of outgoing transfers must not exceed the discounted value of the share gained in the bargaining game. Also note that (3) implies the positivity of the final aggregate income, i.e. the sum of coordinates of (2).

We are going to evaluate the inequality level of $\mathbf{z}_i$ by calculating its Atkinson index of inequality and then to carry out a comparison between the *ex ante* and *ex post* distributions.

In its standard form, given an income distribution $\mathbf{x}_i = (x_{1i}, \dots, x_{pi})$, the Atkinson index of inequality I_A reads as follows:

$$I_A(\mathbf{x}_i) = 1 - \frac{p^{-\frac{\epsilon}{1-\epsilon}} \left[\sum_{h=1}^{p} x_{hi}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}}{\sum_{h=1}^{p} x_{hi}}, \tag{4}$$

where $\epsilon \in (0,\ 1)$ is an aversion parameter. In particular, we have that:

$$I_A(\mathbf{z}_i) = 1 - \frac{p^{-\frac{\epsilon}{1-\epsilon}} \Big[\sum_{h=1}^{p_1+\dots+p_{l-1}} (x_{hi} - y_{hi})^{1-\epsilon} + \sum_{h=1}^{p_l} \big(x_{p_1+\dots+p_{l-1}+h,i} - y_{p_1+\dots+p_{l-1}+h,i} +}{\sum_{h=1}^{p} (x_{hi} - y_{hi}) + \sum_{h=1}^{p_l} (\alpha_{il}(h) d_i e^{-\delta t_1})}$$

$$\frac{+\alpha_{il}(s) d_i e^{-\delta t_1}\big)^{1-\epsilon} + \sum_{h=p_1+\dots+p_l+1}^{p} (x_{hi} - y_{hi})^{1-\epsilon}\Big]^{\frac{1}{1-\epsilon}}}{\sum_{h=1}^{p} (x_{hi} - y_{hi}) + \sum_{h=1}^{p_l} (\alpha_{il}(h) d_i e^{-\delta t_1})}.$$

Before delving into the assessment of such comparison, we build the following Lemma:

Lemma 1. *Given two vectors* $\mathbf{a} = (a_1, \dots, a_p)$, $\mathbf{b} = (b_1, \dots, b_p) \in \mathbb{R}_+^p$, *for any pair of integer numbers* r *and* s *such that* $1 \le r < s < p$, *and for any* $\alpha \in (0,\ 1)$, *if* $\sum_{h=1}^{p} a_h \le \sum_{h=1}^{p} b_h$ *and if*

$$\sum_{h=1}^{p} a_h^\alpha \le \sum_{h=1}^{r} b_h^\alpha + \sum_{h=r+1}^{r+s} b_h^\alpha + \sum_{h=r+s+1}^{p} b_h^\alpha. \tag{5}$$

then $I_A(\mathbf{a}) - I_A(\mathbf{b}) \ge 0$.

Proof. Let $\epsilon = 1-\alpha$ be the aversion parameter. Calculating the difference between the two Atkinson inequality indices of the vectors yields:

$$I_A(\mathbf{a}) - I_A(\mathbf{b}) = -p^{-\frac{1}{\alpha}+1}\left\{\frac{\left[\sum_{h=1}^p a_h^\alpha\right]^{\frac{1}{\alpha}}}{\sum_{h=1}^p a_h} - \frac{\left[\sum_{h=1}^r b_h^\alpha + \sum_{h=r+1}^{r+s} b_h^\alpha + \sum_{h=r+s+1}^p b_h^\alpha\right]^{\frac{1}{\alpha}}}{\sum_{h=1}^p b_h}\right\} =$$

$$= -p^{-\frac{1}{\alpha}+1}\left\{\left[\frac{\sum_{h=1}^p a_h^\alpha}{\left(\sum_{h=1}^p a_h\right)^\alpha}\right]^{\frac{1}{\alpha}} - \left[\frac{\sum_{h=1}^r b_h^\alpha + \sum_{h=r+1}^{r+s} b_h^\alpha + \sum_{h=r+s+1}^p b_h^\alpha}{\left(\sum_{h=1}^p b_h\right)^\alpha}\right]^{\frac{1}{\alpha}}\right\},$$

which is positive if and only if

$$\frac{\sum_{h=1}^p a_h^\alpha}{\left(\sum_{h=1}^p a_h\right)^\alpha} \leq \frac{\sum_{h=1}^r b_h^\alpha + \sum_{h=r+1}^{r+s} b_h^\alpha + \sum_{h=r+s+1}^p b_h^\alpha}{\left(\sum_{h=1}^p b_h\right)^\alpha}. \tag{6}$$

Consider the following quantity:

$$\left(\sum_{h=1}^p a_h\right)^\alpha \left[\sum_{h=1}^r b_h^\alpha + \sum_{h=r+1}^{r+s} b_h^\alpha + \sum_{h=r+s+1}^p b_h^\alpha\right].$$

The assumption $\sum_{h=1}^p a_h \leq \sum_{h=1}^p b_h$ implies that

$$\left(\sum_{h=1}^p a_h\right)^\alpha \left[\sum_{h=1}^r b_h^\alpha + \sum_{h=r+1}^{r+s} b_h^\alpha + \sum_{h=r+s+1}^p b_h^\alpha\right] \leq$$

$$\leq \left(\sum_{h=1}^p b_h\right)^\alpha \left[\sum_{h=1}^r b_h^\alpha + \sum_{h=r+1}^{r+s} b_h^\alpha + \sum_{h=r+s+1}^p b_h^\alpha\right].$$

Hence (6) holds if we have:

$$\sum_{h=1}^p a_h^\alpha \leq \sum_{h=1}^r b_h^\alpha + \sum_{h=r+1}^{r+s} b_h^\alpha + \sum_{h=r+s+1}^p b_h^\alpha. \tag{7}$$

□

Lemma 1 leads us to evaluate the difference between $I_A(\mathbf{x}_i)$ and $I_A(\mathbf{z}_i)$.

Proposition 1. *Under the profitability condition (3), if*

$$\sum_{h=1}^p x_{hi}^{1-\epsilon} \leq \sum_{h=1}^{p_1+\cdots+p_{l-1}} (x_{hi} - y_{hi})^{1-\epsilon} +$$

$$+\sum_{h=1}^{p_l}\left(x_{p_1+\cdots+p_{l-1}+h,i} - y_{p_1+\cdots+p_{l-1}+h,i} + \alpha_{il}(h) d_i e^{-\delta t_1}\right)^{1-\epsilon} + \sum_{h=p_1+\cdots+p_l+1}^{p} (x_{hi} - y_{hi})^{1-\epsilon},$$

then $I_A(\mathbf{x}_i) \geq I_A(\mathbf{z}_i)$.

Proof. It suffices to apply Lemma 1 to the vectors $\mathbf{x}_i$ and $\mathbf{z}_i$, using the parameters $r = p_1 + \cdots + p_{l-1}$ and $s = p_l$. □

Proposition 1 seems to suggest that if either the initial aggregate income or the fiscal contributions are sufficiently high, the level of inequality tends to increase after the transfers process. Adopting the same approach, Proposition 2 tends to characterize the same assessment when the poorest (i.e., endowed with the lowest incomes) types are granted a full tax exemption. We suppose that the tax rate is zero for all subgroups that appear under the l-th block in the matrix A_i, meaning that all types of citizens having an income lower than the farmers' income do not contribute to the outgoing transfers at all. Such assumption can be expressed by making the related tax rates vanish, i.e.:

$$c_{i,l+1} = c_{i,l+2} = \cdots = c_{ik} = 0. \tag{8}$$

Proposition 2. *If (8) holds, under the profitability condition (3), we have that if*

$$\sum_{h=1}^{p} x_{hi}^{1-\epsilon} \leq \sum_{h=1}^{p_1+\cdots+p_{l-1}} (x_{hi} - y_{hi})^{1-\epsilon} +$$

$$+ \sum_{h=1}^{p_l} \left(x_{p_1+\cdots+p_{l-1}+h,i} - y_{p_1+\cdots+p_{l-1}+h,i} + \alpha_{il}(h) d_i e^{-\delta t_1}\right)^{1-\epsilon},$$

then $I_A(\mathbf{x}_i) \geq I_A(\mathbf{z}_i)$.

Proof. We can repeat the process adopted in the proof of Lemma 1 and with the notation employed in Proposition 1, adding the hypothesis that $x_{hi} = y_{hi}$ for all $h = p_1 + \cdots + p_l + 1, \ldots, p$. □

Note that the sufficient hypothesis in Proposition 2 is more restrictive than the one in Proposition 1. In the case of no-tax area, the policy seems to be more effective due to the reduction of inequality in the income profile. Intuitively in the ex-post evaluation, after delivering funds, equality mainly improves since one (or more) types does not contribute to the policy in the first stage while major contributors cross-subsidizes minor ones who finally get the whole amount of money assigned to the State. Improved agricultural production thus could be seen as one of the overall objectives for unequal reduction in the country. Note that for the funds to create the impacts for which they were designed, they not only need continuity in the ranking profile, but also proper structures in terms of profitability condition for the state.

3 Assessment of inequality in a multi-factor setup

3.1 The Atkinson index of inequality in a multi-factor setup

In our view the evaluation of policy cannot be simply related on the ex-post distribution of income profile but a deeper investigation is required. We believe that some information can be captured disentangling the effect of each source; thus for instance, we may understand how income profile performs on the basis of assigned funds by the related policy. A more complex framework must be introduced where each individual gains her income from more than one source. The generic income unit is y_{ij}, which is the j-th component of income of individual i. Since the population of N individuals is subject to m distinct income factors, let $S = N \times m$ be the number of income units in our society.

Each individual is endowed with an income vector, and every income vector collects all income sources of the related individual, that is the i-th individual is associated to the vector $\mathbf{y_i} = (y_{1i}, y_{2i}, \ldots, y_{mi}) \in \mathbb{R}_+^m$. Assuming that at least one of the coordinates of each income vector is strictly positive, we call $Y = \{\mathbf{y}_1, \ldots, \mathbf{y}_m\} \in \mathbb{R}_+^S$ the set of income vectors. In other words, we are supposing that the factors $\mathcal{F}_1, \ldots \mathcal{F}_m$ are employed to generate Y, i.e., there exists a production function $f(\cdot)$ such that $Y = f(\mathcal{F}_1, \ldots \mathcal{F}_m)$.

Note that the circumstances where not all individuals are affected by all income factors is taken into account as well, i.e., some income is allowed to be zero. If the i-th individual is not affected by the j-th factor, we posit $y_{ij} = 0$, always requiring that at least one of the incomes of the same factor is strictly positive, i.e., for each individual i there exists at least one factor $\mathcal{F}_k$ such that $y_{ik} > 0$.

In order to construct a measure of inequality when more than one income factor affects the population, we can separately take into account each factor and evaluate the inequality level caused by each source.

We take into account a social welfare function for differentiated income components as follows:

$$W(y_{11}, \ldots, y_{Nm}) = \frac{1}{Nm} \sum_{j=1}^{m} \sum_{i=1}^{N} U_{ij}(y_{ij}) =$$

$$= \frac{1}{m} \left(\frac{\sum_{p=1}^{N} U_{p1}(y_{p1})}{N} + \frac{\sum_{p=N+1}^{2N} U_{p2}(y_{p2})}{N} + \cdots + \frac{\sum_{p=N(m-1)}^{mN} U_{pm}(y_{pm})}{N} \right).$$

If we indicate with

$$W_j(y_{1j}, \ldots, y_{Nj}) = \frac{1}{N} \sum_{p=N(j-1)+1}^{jN} U_{pj}(y_{pj}),$$

then obviously

$$W(y_{11}, \ldots, y_{Nm}) = \sum_{j=1}^{m} \frac{W_j(y_{1j}, \ldots, y_{Nj})}{m},$$

consequently the expression becomes the average of m different welfare functions where each one of them is associated to a different factor and received an equal weight on welfare orderings. If we replicate the procedure carried out in the one-factor setting, we have that, for all factors $j = 1, \ldots, m$:

$$W_j(y_{1j}, \ldots, y_{Nj}) = W_j(y_{ej}, \ldots, y_{ej}) = U_j(y_{ej}) \iff \begin{cases} \dfrac{1}{1-\epsilon} y_{ej}^{1-\epsilon} = \dfrac{1}{N} \sum_{p=1}^{N} \dfrac{y_{pj}^{1-\epsilon}}{1-\epsilon} \\ \\ \log\ y_{ej} = \dfrac{1}{N} \sum_{p=1}^{N} \log\ y_{pj} \end{cases} \tag{9}$$

Assuming that a unique inequality aversion parameter is required, we can define the form of the utility function. Calling $Y_{\mathcal{F}_j}$ the income vector associated to factor $\mathcal{F}_j$, by (9) we are able to determine the multi-factor *ede* y_{ej} as in the standard one-factor Atkinson setting, i.e.

$$\begin{cases} y_{ej} = \left[\dfrac{1}{N} \sum_{p=1}^{N} y_{pj}^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} & \text{if } \epsilon \in (0,\ 1) \\ \\ y_{ej} = \left[\prod_{p=1}^{N} y_{pj} \right]^{\frac{1}{N}} & \text{if } \ \epsilon = 1 \end{cases} , \tag{10}$$

for all $j = 1, \ldots, m$. If we call $\mu_j = \frac{1}{N} \sum_{i=1}^{N} y_{ij}$ the mean income related to the j-th factor, we can simply express the Atkinson inequality measure as $I_A(Y_{\mathcal{F}_j}) = 1 - \frac{y_{ej}}{\mu_j}$, and even more generally

$$\mathcal{I}_A(Y_{\mathcal{F}_1, \ldots, \mathcal{F}_k}) = 1 - \frac{1}{k} \sum_{j=1}^{k} \frac{y_{ej}}{\mu_j},$$

for $k = 2, \ldots, m$. We are ready to introduce a new formulation for an Atkinson-based index of inequality $\mathcal{I}_A(Y)$.

Definition 1. *Given the income distribution $Y \in \mathbb{R}^S_+$ and the factors $\mathcal{F}_1, \dots \mathcal{F}_m$, the* ***multi-factor Atkinson index of inequality*** *$\mathcal{I}_A(Y_{\mathcal{F}_1,\dots\mathcal{F}_m})$ is given by:*

$$\mathcal{I}_A(Y_{\mathcal{F}_1,\dots,\mathcal{F}_m}) = 1 - \frac{1}{m}\sum_{j=1}^{m}\frac{y_{ej}}{\mu_j} = \begin{cases} 1 - \dfrac{N^{-\frac{\epsilon}{1-\epsilon}}}{m}\sum_{j=1}^{m}\left(\dfrac{\left[\sum_{p=1}^{N} y_{pj}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}}{\sum_{p=1}^{N} y_{pj}}\right) & \textit{if} \quad \epsilon \in (0,\ 1) \\ \\ 1 - \dfrac{N}{m}\sum_{j=1}^{m}\left(\dfrac{\left[\prod_{p=1}^{N} y_{pj}\right]^{\frac{1}{N}}}{\sum_{p=1}^{N} y_{pj}}\right) & \textit{if} \quad \epsilon = 1 \end{cases}. \tag{11}$$

Note that when $m = 1$, the two indices coincide: $I_A(Y) = \mathcal{I}_A(Y_\mathcal{F})$[3].

Furthermore, if we call $E^j_A(Y)$ the Atkinson index of equality calculated with respect to the j-th factor, it is straightforward to prove the following identity, which connects the one-factor index with the multi-factor index:

$$\mathcal{I}_A(Y) = 1 - \frac{1}{m}\sum_{j=1}^{m} E^j_A(Y). \tag{12}$$

3.2 The Atkinson index as the characteristic function of a cooperative game

We are going to carry out a decomposition of the inequality index (12) based on values or power indices such as the Banzhaf value[4] and the Shapley value[5]. Such approach has been adopted in literature on inequality assessment, in particular we refer to theoretical background decomposition based on Shorrocks [28], [29]; whereas important applications are developed by Shorrocks [30] and Chantreuil and Trannoy [7][6]. This technique leads to establish the marginal contribution of each factor to the aggregate inequality level, by looking upon factors as if they were players of a cooperative game. A complete theoretical construction of inequality games, including a rich overview of their relations with factor decomposition and Shapley value, has been recently provided by [8]. We will confine our attention to the cooperative game structure which is appropriate for our outgoing/incoming transfer setting and deduce some of its properties to delve into its economic significance.

Call $\mathcal{P} = \{\mathcal{F}_1, \dots, \mathcal{F}_m\}$ the set of players. In such game, coalitions of factors are supposed to be defined rigorously as follows:

Definition 2. *$S \subseteq \mathcal{P}$ is a* ***coalition of factors*** *if for all $j \in S$, there exists at least one income y_{dj} different from the arithmetic mean $\dfrac{\sum_{p=1}^{N} y_{pj}}{\mu_j}$.*

Consequently, when we evaluate inequality related to some factors, we rule out all constant factors, i.e., all elements which assign the same income to all individuals or types. The characteristic function

[3]Furthermore, the index introduced by Definition 1 can be viewed as the arithmetic mean between the standard Atkinson indices: it is easy to show that if $\epsilon \in (0, 1)$

$$\mathcal{I}_A(Y_{\mathcal{F}_1,\dots,\mathcal{F}_m}) = \frac{\mathcal{I}_A(Y_{\mathcal{F}_1}) + \cdots + \mathcal{I}_A(Y_{\mathcal{F}_m})}{m}.$$

[4]The Banzhaf value was initially introduced in [4] in 1965 as a power index for voting games and subsequently axiomatized and generalized to arbitrary cooperative games.

[5]The Shapley value is a world famous solution concept in Cooperative Game Theory, initially introduced in [27] in 1953 and then widely employed in Election Games, Bargaining Theory and many other areas. An exhaustive overview of power indices, including axiomatization and applications, is [23].

[6]Pignataro [24] proposes an application of Shapley value in the opportunity egalitarian environment.

of the game is the inequality function $\mathcal{I}_A(Y_S)$, i.e., the multi-factor Atkinson index evaluated at S, which can be any coalition of factors, such that:

$$\mathcal{I}_A(Y_S) = 1 - \frac{1}{|S|}\sum_{j\in S}\frac{y_{ej}}{\mu_j} = 1 - \frac{N^{-\frac{\epsilon}{1-\epsilon}}}{|S|}\sum_{j\in S}\left(\frac{\left[\sum_{p=1}^{N} y_{pj}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}}{\sum_{p=1}^{N} y_{pj}}\right), \tag{13}$$

for all $S \subseteq \mathcal{P}$, $S \neq \emptyset$. In (13), $|S|$ indicates the cardinality of the set $S \subseteq \mathcal{P}$. In order to describe a correct cooperative game structure, the characteristic function must be well-defined, in that it has to vanish when it is evaluated at the empty set. Basically, when no income source is considered, inequality must not be different from 0, but clearly the expression (13) does not vanish in that case. Hence, we can establish it by convention: $\mathcal{I}_A(Y_\emptyset) = 0$, that is the maximum equality is attained when no factors are taken into account. This hypothesis is crucial to construct a cooperative game, but on the other hand it is easy to show that such a game is neither monotone nor superadditive. Furthermore, it is not a simple game, so many classical properties of cooperative games do not hold. In particular, we can prove its subadditivity property:

Proposition 3. *The game* $(\mathcal{I}_A,\ \mathcal{P})$ *is* ***subadditive****, i.e. for all coalitions* $S, T \subset \mathcal{P}$*, such that* $S \cap T = \emptyset$*, we have that:*

$$\mathcal{I}_A(Y_{S\cup T}) < \mathcal{I}_A(Y_S) + \mathcal{I}_A(Y_T). \tag{14}$$

Proof. Employing (13) yields:

$$\begin{aligned}
\mathcal{I}_A(Y_{S\cup T}) - \mathcal{I}_A(Y_S) - \mathcal{I}_A(Y_T) &= 1 - \frac{N^{-\frac{\epsilon}{1-\epsilon}}}{|S+T|}\sum_{j\in S\cup T}\left(\frac{\left[\sum_{p=1}^{N} y_{pj}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}}{\sum_{p=1}^{N} y_{pj}}\right) - 1 + \\
&+\frac{N^{-\frac{\epsilon}{1-\epsilon}}}{|S|}\sum_{j\in S}\left(\frac{\left[\sum_{p=1}^{N} y_{pj}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}}{\sum_{p=1}^{N} y_{pj}}\right) - 1 + \frac{N^{-\frac{\epsilon}{1-\epsilon}}}{|T|}\sum_{j\in T}\left(\frac{\left[\sum_{p=1}^{N} y_{pj}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}}{\sum_{p=1}^{N} y_{pj}}\right) = \\
&= -1 - \frac{N^{-\frac{\epsilon}{1-\epsilon}}}{|S+T|}\left[\sum_{j\in S}\left(\frac{\left[\sum_{p=1}^{N} y_{pj}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}}{\sum_{p=1}^{N} y_{pj}}\right) + \sum_{j\in T}\left(\frac{\left[\sum_{p=1}^{N} y_{pj}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}}{\sum_{p=1}^{N} y_{pj}}\right)\right] + \\
&+\frac{N^{-\frac{\epsilon}{1-\epsilon}}}{|S|}\sum_{j\in S}\left(\frac{\left[\sum_{p=1}^{N} y_{pj}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}}{\sum_{p=1}^{N} y_{pj}}\right) + \frac{N^{-\frac{\epsilon}{1-\epsilon}}}{|T|}\sum_{j\in T}\left(\frac{\left[\sum_{p=1}^{N} y_{pj}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}}{\sum_{p=1}^{N} y_{pj}}\right) = \dots = \\
&= -1 + \frac{|T|}{|S+T|}\left(1 - \mathcal{I}_A(Y_S)\right) + \frac{|S|}{|S+T|}\left(1 - \mathcal{I}_A(Y_T)\right) = \\
&= \frac{-|S+T| + |T| - |T|\mathcal{I}_A(Y_S) + |S| - |S|\mathcal{I}_A(Y_T)}{|S+T|} < 0,
\end{aligned}$$

after exploiting that $|S+T| = |S| + |T|$, thus completing the proof of (14). □

The subadditivity property may suggest a further interpretation of the framework: typically, the subadditive cooperative games are *cost games*, that is games where the value function indicates a measure of cost assigned to each coalition, which can be viewed as the social cost of inequality in our case. In such games, subadditivity enlightens that players or coalitions have an incentive to form coalitions to lower their costs with respect to the individual costs they would have to afford without cooperating (see [11], Chapter 2, for example).

In a general theoretical setup, also with an arbitrarily large number of factors, these values are helpful to determine the marginal contribution of each factor to inequality.

To explain this procedure, we begin by adapting the standard definition of Shapley value and of Banzhaf value to our framework:

Definition 3. *The **Banzhaf value** of the game $(\mathcal{I}_A,\ \mathcal{P})$ is a vector*

$$\beta(\mathcal{I}_A) = (\beta_1(\mathcal{I}_A), \dots, \beta_m(\mathcal{I}_A)) \in \mathbb{R}^m$$

such that:

$$\beta_j(\mathcal{I}_A) = \frac{1}{2^{m-1}} \sum_{S \subseteq \mathcal{P},\ j \in S} (\mathcal{I}_A(S) - \mathcal{I}_A(S \setminus \{j\})), \tag{15}$$

for all $j = 1, \dots, m$.

Definition 4. *The **Shapley value** of the game $(\mathcal{I}_A,\ \mathcal{P})$ is a vector*

$$\Phi(\mathcal{I}_A) = (\phi_1(\mathcal{I}_A), \dots, \phi_m(\mathcal{I}_A)) \in \mathbb{R}^m$$

such that:

$$\phi_j(\mathcal{I}_A) = \sum_{S \subseteq \mathcal{P},\ j \in S} \frac{(m - |S|)!(|S| - 1)!}{m!} (\mathcal{I}_A(S) - \mathcal{I}_A(S \setminus \{j\})), \tag{16}$$

for all $j = 1, \dots, m$.

Both values may be employed to define the marginal contributions of factors to inequality. For example, in a 3-factor case, calling the factors $\mathcal{F}_1$, $\mathcal{F}_2$ and $\mathcal{F}_3$, $\beta_1(\mathcal{I}_A)$ and $\phi_1(\mathcal{I}_A)$ respectively amount to:

$$\beta_1(\mathcal{I}_A) = \frac{1}{4} \left[\mathcal{I}_A(Y_{\{\mathcal{F}_1,\mathcal{F}_2,\mathcal{F}_3\}}) - \mathcal{I}_A(Y_{\mathcal{F}_2,\mathcal{F}_3}) + \mathcal{I}_A(Y_{\{\mathcal{F}_1,\mathcal{F}_2\}}) - \mathcal{I}_A(Y_{\{\mathcal{F}_2\}}) + \right. \\ \left. + \mathcal{I}_A(Y_{\{\mathcal{F}_1,\mathcal{F}_3\}}) - \mathcal{I}_A(Y_{\mathcal{F}_3}) + \mathcal{I}_A(Y_{\mathcal{F}_1}) - \mathcal{I}_A(Y_{\emptyset}) \right],$$

and

$$\phi_1(\mathcal{I}_A) = \frac{1}{6} \left[2(\mathcal{I}_A(Y_{\{\mathcal{F}_1,\mathcal{F}_2,\mathcal{F}_3\}}) - \mathcal{I}_A(Y_{\mathcal{F}_2,\mathcal{F}_3})) + \mathcal{I}_A(Y_{\{\mathcal{F}_1,\mathcal{F}_2\}}) - \mathcal{I}_A(Y_{\{\mathcal{F}_2\}}) + \right. \\ \left. + \mathcal{I}_A(Y_{\{\mathcal{F}_1,\mathcal{F}_3\}}) - \mathcal{I}_A(Y_{\mathcal{F}_3}) + 2(\mathcal{I}_A(Y_{\mathcal{F}_1}) - \mathcal{I}_A(Y_{\emptyset})) \right].$$

As is well-known, when the factors (or players) are 2, the Banzhaf value and the Shapley value coincide.

One of the standard notions of Cooperative Game Theory which seems to be helpful in this framework is the concept of *excess* of a coalition with respect to a given allocation. In our case, an allocation is a vector $(u_1, \dots, u_m) \in \mathbb{R}^m$ such that $\sum_{i=1}^m u_i = \mathcal{I}_A(Y_{\mathcal{F}_1,\dots,\mathcal{F}_m})$. We can apply the standard definition:

Definition 5. *Given an allocation $u \in \mathbb{R}^m$, the **excess** of a coalition S with respect to u is the following number:*

$$e(S, u) = \mathcal{I}_A(Y_S) - \sum_{i \in S} u_i. \tag{17}$$

Provided that the inequality driven by all factors is given by $\mathcal{I}_A(Y_{\mathcal{F}_1,\dots,\mathcal{F}_m})$, an allocation is any vector representing a method for sharing such social cost among factors (the Shapley value is an allocation, whereas the Banzhaf value is not). Therefore, choosing an allocation has the same meaning of attributing the cause of inequality to several factors according to some scheme, so the excess of a set of factors may measure the difference between the real inequality-based cost provoked by a group of factors and the cost which is attributed to those factors by this assignment. If we set

$$e_{min}(u) := \min \{e(S, u) \mid \emptyset \neq S \neq \mathcal{P}\},$$

we can define two further relevant solution concepts (see [14]):

Definition 6. *The set*

$$Core(\mathcal{I}_A) := \{u \in \mathbb{R}^m \mid u_1 + \cdots + u_N = \mathcal{I}_A(Y_{\mathcal{F}_1,\ldots,\mathcal{F}_m}),\ e_{min}(u) \geq 0\}$$

is the **core** *of the game* $(\mathcal{I}_A, \mathcal{P})$*, i.e. the set of all allocations whose excesses are non-negative.*

Definition 7. *Given an allocation* $u \in \mathbb{R}^m$*, such that* $u_i \leq \mathcal{I}_A(Y_{\mathcal{F}_i})$ *for all* $i = 1, \ldots, m$[7]*, if* $\Theta(u)$ *is the* $(2^{|\mathcal{P}|} - 2)$*-dimensional vector of all excesses* $e(S, u)$*, where* $\emptyset \neq S \neq \mathcal{P}$*, arranged in lexicographic order, the* **nucleolus** *of* $(\mathcal{I}_A, \mathcal{P})$ *is the unique* u^* *that lexicographically maximizes* $\Theta(u)$ *over the subset of all such allocations.*

In particular, the nucleolus, introduced by Schmeidler [25] in 1969, can be quite meaningful for our purpose. Namely, because the nucleolus minimizes the maximum difference between the actual cost and the sum of contributions in a feasible distribution of costs among factors, we might interpret it as the distribution of costs which represents the aggregate cost in the most suitable way. Extending this characterization to inequality leads to design the nucleolus as a precise indicator of each factor's marginal contribution. An related idea is developed in the next Subsection.

3.3 Assessment of the social cost of inequality

Given the structure of the cooperative game, we extend the analysis decomposing the Atkinson index in terms of *cost of inequality* or *wasted income*, see Blackorby [6]. As a general rule the *cost of inequality* formula measures the fraction of total income which could be sacrificed with no loss of social welfare if the income sources were to be equally distributed, i.e., *ede* among individuals. By taking into account all factors $\mathcal{F}_1, \ldots \mathcal{F}_m$ employed to generate Y, (11) can be expressed as:

$$\mathcal{I}_A(Y_{\mathcal{F}_1,\ldots,\mathcal{F}_m}) = 1 - \frac{1}{m}\sum_{j=1}^{m}\frac{y_{ej}}{\mu_j} = \sum_{j=1}^{m}\left(\frac{1}{m} - \frac{1}{m}\frac{y_{ej}}{\mu_j}\right) = \frac{1}{m}\sum_{j=1}^{m}\left(\frac{\mu_j - y_{ej}}{\mu_j}\right).$$

Thus the cost of inequality due to the j-th source is $C_A(Y_{\mathcal{F}_j}) = \dfrac{\mu_j - y_{ej}}{\mu_j}$, i.e., the difference between the average income due to the j-th factor minus the j-th *ede* in the society in relative terms. The multi-factor Atkinson index of inequality can then be interpreted as the average cost of inequality induced by factors $\mathcal{F}_1, \ldots, \mathcal{F}_m$ into the society, i.e.:

$$\mathcal{C}_A(Y_{\mathcal{F}_1,\ldots,\mathcal{F}_m}) = \frac{1}{m}\sum_{j=1}^{m} C_A(Y_{\mathcal{F}_j}) = \mathcal{I}_A(Y_{\mathcal{F}_1,\ldots,\mathcal{F}_m}).$$

Because we can formulate the standard Atkinson index of inequality due to the factor coalition S as follows:

$$I_A(Y_S) = 1 - \frac{y_{e,S}}{\mu_S} = \begin{cases} 1 - \dfrac{|S|^{-\frac{\epsilon}{1-\epsilon}} N^{-\frac{\epsilon}{1-\epsilon}} \left[\sum_{j=1}^{|S|}\sum_{i=1}^{N} y_{ij}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}}{\sum_{j=1}^{|S|}\sum_{i=1}^{N} y_{ij}} & \text{if} \quad \epsilon \in (0,\ 1) \\[2ex] 1 - \dfrac{|S|\, N \left[\prod_{j=1}^{|S|}\prod_{i=1}^{N} y_{ij}\right]^{\frac{1}{T}}}{\sum_{j=1}^{|S|}\sum_{i=1}^{N} y_{ij}} & \text{if} \quad \epsilon = 1 \end{cases}, \tag{18}$$

the *ede*-income for the involved distribution is then defined as:

[7] Also known as an *imputation*, see for example [14] or [23].

$$
\begin{cases}
y_{e,S} = \left[\dfrac{1}{|S|\,N} \sum_{j=1}^{|S|} \sum_{i=1}^{N} y_{ij}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} & \text{if } \ \epsilon \in (0,\ 1) \\
y_{e,S} = \left[\prod_{j=1}^{|S|} \prod_{i=1}^{N} y_{ij}\right]^{\frac{1}{S}} & \text{if } \ \epsilon = 1
\end{cases}.
$$

From now on, we consider $\epsilon \in (0,1)$ and we rely on the following notation: $\hat{y}_S$ is the *ede*-income associated to the factor coalition S.

Consequently, we can define $\mathcal{K}_A(Y_S) := \dfrac{\mu_S - \hat{y}_S}{\mu_S}$ and adopt a cooperative game approach to evaluate the cost of inequality. Loosely speaking, we introduce a cost allocation game $(\mathcal{C}_A,\ \mathcal{P})$, where $\mathcal{C}_A(Y_\emptyset)$ represents the least cost of in terms of saving when no factors are taken into account. Since we are interested in measuring the effect of communitarian policy, we may take into account a cooperative game $(\mathcal{L}_A,\ \mathcal{P})$, whose characteristic function is defined as follows:

Definition 8. *We call* ***welfare loss function*** *the following characteristic value:*

$$
\mathcal{L}_A(S) = \sum_{j \in S} \mathcal{C}_A(Y_{\mathcal{F}_j}) - \mathcal{K}_A(Y_S),
$$

for all $S \subseteq \mathcal{P}$.

Note that the above convention on the value of the empty set entails $\mathcal{L}_A(\emptyset) = 0$, so the cooperative game is well-defined. Some properties of the function introduced in Definition 8 can be investigated. The following Propositions collect some results.

Proposition 4. $\mathcal{L}_A(S) = \sum_{j \in S} C_A(Y_{\mathcal{F}_j}) - K_A(Y_S)$ *is positive for all* $S \in \mathcal{P} \setminus \emptyset$.

Proof. If $S \in \mathcal{P} \setminus \emptyset$, we have that:

$$
\mathcal{L}_A(S) = \sum_{j=1}^{|S|} \left(1 - \frac{\hat{y}_{\{j\}}}{\mu_j}\right) - 1 + \frac{\hat{y}_S}{\mu_S} = |S| - \sum_{j=1}^{|S|} \frac{\hat{y}_{\{j\}}}{\mu_j} - 1 + \frac{\hat{y}_S}{\mu_S} =
$$

$$
= |S| - \sum_{j=1}^{|S|} \frac{\hat{y}_{\{j\}}}{\mu_j} - 1 + |S| \frac{\hat{y}_S}{\sum_{j=1}^{|S|} \mu_j} \geq 1 - \sum_{j=1}^{|S|} \frac{\hat{y}_{\{j\}}}{\mu_j} + |S| \frac{\hat{y}_S}{\sum_{j=1}^{|S|} \mu_j} \geq 0,
$$

because $|S| - 1 \geq 1$. □

Proposition 5. *If*

$$
\hat{y}_S \geq \hat{y}_{\mathcal{F}_j} \qquad \textit{and} \qquad \mathcal{C}_A(S) > \mathcal{E}_A(\mathcal{F}_j) \tag{19}
$$

then for all $S \subset \mathcal{P} \setminus \mathcal{F}_j$*, we have that* $\mathcal{L}_A(S \cup \mathcal{F}_j) > \mathcal{L}_A(S)$.

Proof.

$$
\begin{aligned}
\mathcal{L}_A(S \cup \mathcal{J}) - \mathcal{L}_A(S) &= 1 - \frac{\hat{y}_j}{\mu_j} - 1 + \frac{\hat{y}_{S \cup j}}{\mu_{S \cup j}} + 1 - \frac{\hat{y}_S}{\mu_S} = \\
&= 1 + \frac{\hat{y}_{S \cup j}}{\mu_{S \cup j}} - \frac{\hat{y}_j}{\mu_j} - \frac{\hat{y}_S}{\mu_S}
\end{aligned}
$$

Since $\mu_{S \cup j} = \frac{|S| \mu_S + \mu_j}{|S|+1}$, we can rewrite:

$$
1 + \frac{(|S|+1)\,\hat{y}_{S \cup j}}{|S|\,\mu_S + \mu_j} - \frac{\hat{y}_j}{\mu_j} - \frac{\hat{y}_S}{\mu_S} \geq
$$

$$
1 + \frac{\hat{y}_{S \cup j}}{|S|\,\mu_S + \mu_j} - \frac{\hat{y}_j}{\mu_j} - \frac{\hat{y}_S}{\mu_S}
$$

If $\hat{y}_S \geq \hat{y}_j$ then $\hat{y}_{S\cup j} \geq \hat{y}_j$, therefore

$$
\begin{aligned}
1+\frac{\hat{y}_{S\cup j}}{|S|\,\mu_S+\mu_j}-\frac{\hat{y}_j}{\mu_j}-\frac{\hat{y}_S}{\mu_S} &\geq 1+\frac{\hat{y}_j}{|S|\,\mu_S+\mu_j}-\frac{\hat{y}_j}{\mu_j}-\frac{\hat{y}_S}{\mu_S}= \\
=1-\frac{\hat{y}_S}{\mu_S}+\frac{\hat{y}_j\mu_j-\hat{y}_j\left(|S|\,\mu_S+\mu_j\right)}{\left(|S|\,\mu_S+\mu_j\right)\mu_j} &= 1-\frac{\hat{y}_S}{\mu_S}-\frac{|S|\,\mu_S}{\left(|S|\,\mu_S+\mu_j\right)\mu_j}\hat{y}_j \\
=\frac{\mu_j\mu_S\left(|S|\,\mu_S+\mu_j\right)-\hat{y}_S\left(|S|\,\mu_S\mu_j+\mu_j^2\right)-\hat{y}_j\,|S|\,\mu_S}{\mu_j\mu_S\left(|S|\,\mu_S+\mu_j\right)} &= \\
=\frac{|S|\,\mu_j\mu_S^2+\mu_j^2\mu_S-|S|\,\mu_S\mu_j\hat{y}_S-\mu_j^2\hat{y}_S-|S|\,\mu_S^2\hat{y}_j}{\mu_j\mu_S\left(|S|\,\mu_S+\mu_j\right)}
\end{aligned}
$$

First, note that

$$\mu_j^2\mu_S-\mu_j^2\hat{y}_S>0 \quad \text{since} \quad \mu_S-\hat{y}_S \quad \text{by definition}$$

Second, note that

$$|S|\,\mu_j\mu_S^2-|S|\,\mu_S\mu_j\hat{y}_S-|S|\,\mu_S^2\hat{y}_j=\mu_j\mu_S-\mu_j\hat{y}_S-\mu_S\hat{y}_j$$

this is positive when

$$
\begin{aligned}
\mu_j(\mu_S-\hat{y}_S) &> \mu_S\hat{y}_j \text{ so that} \\
\frac{\mu_S-\hat{y}_S}{\mu_S} &> \frac{\hat{y}_j}{\mu_j}
\end{aligned}
$$

we can express this condition as

$$\mathcal{C}_A(S)>\mathcal{E}_A(\mathcal{F}_j)$$

Thus $\mathcal{L}_A(S\cup\mathcal{J})-\mathcal{L}_A(S)$ is monotonically increasing whenever the cost of inequality associated to the coalition S is higher than the equality originated by the evaluation of income sources j.

□

In particular, the latest Proposition intends to capture the marginal effect of the CAP-driven factor. The welfare loss function is constructed similarly to the excess function. In a sense, to the extent that a procedure for minimizing it may be carried out, we would find a correct evaluation of marginal contributions. We expect that relevant developments can be made in applying the same procedure that yields the coordinates of the nucleolus, provided that some minor differences occur. What might provide interesting information on the distance between the standard and the multi-factorial approach to the evaluation of inequality is the determination of the conditions under which a multi-factor index of inequality has the same form of a nucleolus in this setup.

4 Income profiles designed on CAP

4.1 European Common Agricultural Policy

We first describe how the transfers' scheme within European Countries contributes to the EU budget and how getting funds from the CAP (direct aid, export refunds and several kinds of subsidies) implies potential redistribution among farmers. CAP program started in 1962 and was reformed many times. The general idea towards supporting agriculture remains along years in order to ensure a level playing field for farmers competing in the internal and on the global market with a common set of objectives, principles and rules. Without a common policy, Member States would proceed with their own national policies with variable scope and with different degrees of public intervention. Instead the CAP ensures common rules in a single market promoting also a common trade policy since potential re-nationalization of agricultural policy should have a high risk of distorting the common market principle, with diverging support levels and types of policy mechanisms. The CAP budget nowadays is entirely devoted on income support for farmers, projects in rural development and protection of the environment and support of the market when weather shocks occur (for a simple overview, see [10]). In 2006 CAP accounted for 46.7% of the total budget, whereas for 2014-2020 such quota is projected to be considerably cut, probably to around 30%. The CAP plan has also been affected by wide-ranging criticism (see among others, [16] and [22]) concerning oversupply, inappropriate environmental development, creation of artificially high food prices and lack of equity among Member States. The CAP costs European taxpayers on average over € 40 billion per year or in other terms almost 40% of the total EU budget. That is a significant amount of money in particular if we take into account that farming accounts for less than 2% of the EU's workforce. Yet the share of the budget devoted to CAP spending has fallen sharply: 20 years ago, it was 70%. Moreover, the CAP has been reformed significantly, most recently in 2003, when a deal was struck to complete the switch of most CAP subsidies from price supports to direct income payments. After this reform has been fully implemented, some 90% of EU farm support will be classified as "non trade-distorting". Perhaps it is helpful to summarize some basic notions on the EU budget (for more details, see [31]). EU's resources are of three kinds: Traditional Own Resources, consisting of duties that are charged on imports of products coming from non-EU States, which account for 11% of the total budget; the resource based on VAT, applied to each Member State's own harmonised VAT revenue, accounting for approximately 12% of the budget; the resource based on GNI, which is the largest source of revenue and accounts for around 76% of EU's budget. As can be noted, these three sources are not completely tax-based, whereas our assumptions characterize the transfers of each State to the EU only in terms of fiscal contributions. To simplify computations, we consider that each Member State's total contribution to the EU budget is shared among the different types of its population in compliance with the related income profiles, and assuming progressive taxation in all Countries, consequently with the tax rates.

4.2 Multi-factor approach to CAP

Now let us consider the framework outlined in section 2: in the i-th State, k different fiscal types and 3 different income factors are considered. We denote as *Net Income* (NI), the difference $\mathbf{x}_i - \mathbf{y}_i$, while, *Incoming Transfers* (IT) corresponds to $\mathbf{z}_i - (\mathbf{x}_i - \mathbf{y}_i)$, plus a further factor which is not influenced by the transfers scheme. Assuming that a share of individuals' incomes are not subject to taxation, the third factor at hand indicates the possible tax-free rent (TR) earned by the different types, say $\xi_{hi} \geq 0$ is the rent earned by the h-th subgroup of individuals in the i-th State. Since we are going to aggregate income information relying on types, we will assign the arithmetic means of incomes or rents of the involved subgroups to each type. We will subsequently evaluate inequality with the help of the above Atkinson index formulas, with $\epsilon \in (0,\ 1)$ as a suitable aversion parameter. We can fill the following table to describe factors and types:

	Tax-free Rent	Net Income	Incoming Transfer
Type 1	$\frac{\sum_{h=1}^{p_1}\xi_{hi}}{p_1}$	$\frac{\sum_{h=1}^{p_1}(x_{hi}-c_{h1}x_{hi})}{p_1}$	**0**
Type 2	$\frac{\sum_{h=p_1+1}^{p_1+p_2}\xi_{hi}}{p_2}$	$\frac{\sum_{h=p_1+1}^{p_1+p_2}(x_{hi}-c_{h1}x_{hi})}{p_2}$	**0**
$\cdots$	$\cdots$	$\cdots$	**0**
Type l	$\frac{\sum_{h=p_1+\cdots+p_{l-1}+1}^{p_1+\cdots+p_l}\xi_{hi}}{p_l}$	$\frac{\sum_{h=p_1+\cdots+p_{l-1}+1}^{p_1+\cdots+p_l}(x_{hi}-c_{h1}x_{hi})}{p_l}$	$\frac{\sum_{h=1}^{p_l}\left(\alpha_{il}(h)d_ie^{-\delta t_1}\right)}{p_l}$
$\cdots$	$\cdots$	$\cdots$	**0**
Type k	$\frac{\sum_{h=p_1+\cdots+p_{k-1}+1}^{p_1+\cdots+p_k}\xi_{hi}}{p_k}$	$\frac{\sum_{h=p_1+\cdots+p_{k-1}+1}^{p_1+\cdots+p_k}(x_{hi}-c_{h1}x_{hi})}{p_k}$	**0**

The related arithmetic means computed over columns are:

$$\mu_{TR} = \frac{1}{k}\left(\frac{\sum_{h=1}^{p_1}\xi_{hi}}{p_1} + \cdots + \frac{\sum_{h=p_1+\cdots+p_{k-1}+1}^{p_1+\cdots+p_k}\xi_{hi}}{p_k}\right),$$

$$\mu_{NI} = \frac{1}{k}\left(\frac{\sum_{h=1}^{p_1}(x_{hi}-c_{h1}x_{hi})}{p_1} + \cdots + \frac{\sum_{h=p_1+\cdots+p_{k-1}+1}^{p_1+\cdots+p_k}(x_{hi}-c_{h1}x_{hi})}{p_k}\right),$$

$$\mu_{IT} = \frac{1}{kp_l}\sum_{h=1}^{p_l}\left(\alpha_{il}(h)d_ie^{-\delta t_1}\right).$$

In order to employ them in (11), we must keep in mind that the fiscal types are k although each type collects individuals with more income levels, so that the following multi-factor Atkinson index of inequality is obtained:

$$\mathcal{I}_A(Y_{TR,\ NI,\ IT}) = 1 - \frac{1}{3}\left(\frac{\hat{y}_{TR}}{\mu_{TR}} + \frac{\hat{y}_{NI}}{\mu_{NI}} + \frac{\hat{y}_{IT}}{\mu_{IT}}\right),$$

where $\hat{y}_{TR}$, $\hat{y}_{NI}$ and $\hat{y}_{IT}$ respectively correspond to the multi-factor *ede* determined in (10), i.e.

$$\hat{y}_{TR} := \frac{1}{k^{\frac{1}{1-\epsilon}}}\left[\left(\frac{\sum_{h=1}^{p_1}\xi_{hi}}{p_1}\right)^{1-\epsilon} + \cdots + \left(\frac{\sum_{h=p_1+\cdots+p_{k-1}+1}^{p_1+\cdots+p_k}\xi_{hi}}{p_k}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}, \tag{20}$$

$$\hat{y}_{NI} := \frac{1}{k^{\frac{1}{1-\epsilon}}}\left[\left(\frac{\sum_{h=1}^{p_1}(x_{hi}-c_{h1}x_{hi})}{p_1}\right)^{1-\epsilon} + \cdots + \left(\frac{\sum_{h=p_1+\cdots+p_{k-1}+1}^{p_1+\cdots+p_k}(x_{hi}-c_{h1}x_{hi})}{p_k}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}, \tag{21}$$

$$\hat{y}_{IT} := \frac{1}{k^{\frac{1}{1-\epsilon}}}\left[\left(\sum_{h=1}^{p_l}\left(\alpha_{il}(h)d_ie^{-\delta t_1}\right)\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}. \tag{22}$$

Consequently we will achieve the multi-factor Atkinson index as follows:

$$\mathcal{I}_A(Y_{TR,\ NI,\ IT}) = 1 - \frac{1}{3}\left(\frac{k^{-\frac{\epsilon}{1-\epsilon}}\left[\left(\frac{\sum_{h=1}^{p_1}\xi_{hi}}{p_1}\right)^{1-\epsilon} + \cdots + \left(\frac{\sum_{h=p_1+\cdots+p_{k-1}+1}^{p_1+\cdots+p_k}\xi_{hi}}{p_k}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}}{\frac{\sum_{h=1}^{p_1}\xi_{hi}}{p_1} + \cdots + \frac{\sum_{h=p_1+\cdots+p_{k-1}+1}^{p_1+\cdots+p_k}\xi_{hi}}{p_k}} + \right.$$

$$+ \frac{k^{-\frac{\epsilon}{1-\epsilon}}\left[\left(\frac{\sum_{h=1}^{p_1}(x_{hi} - c_{h1}x_{hi})}{p_1}\right)^{1-\epsilon} + \cdots + \left(\frac{\sum_{h=p_1+\cdots+p_{k-1}+1}^{p_1+\cdots+p_k}(x_{hi} - c_{h1}x_{hi})}{p_k}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}}{\frac{\sum_{h=1}^{p_1}(x_{hi} - c_{h1}x_{hi})}{p_1} + \cdots + \frac{\sum_{h=p_1+\cdots+p_{k-1}+1}^{p_1+\cdots+p_k}(x_{hi} - c_{h1}x_{hi})}{p_k}} +$$

$$\left. + \frac{p_l k^{-\frac{\epsilon}{1-\epsilon}}\left[\left(\sum_{h=1}^{p_l}\left(\alpha_{il}(h)d_i e^{-\delta t_1}\right)\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}}{\sum_{h=1}^{p_l}\left(\alpha_{il}(h)d_i e^{-\delta t_1}\right)}\right). \tag{23}$$

In our case, the set of players is $\mathcal{P} = \{TR,\ NI,\ IT\}$ and it is easy to sort out the factors' marginal contributions to inequality, in particular the coordinates of the Banzhaf value are:

$$\beta_{TR}(\mathcal{I}_A) = \frac{1}{4}\left(1 - \frac{7\hat{y}_{TR}}{3\mu_{TR}} + \frac{2\hat{y}_{NI}}{3\mu_{NI}} + \frac{2\hat{y}_{IT}}{3\mu_{IT}}\right), \tag{24}$$

$$\beta_{NI}(\mathcal{I}_A) = \frac{1}{4}\left(1 - \frac{7\hat{y}_{NI}}{3\mu_{NI}} + \frac{2\hat{y}_{TR}}{3\mu_{TR}} + \frac{2\hat{y}_{IT}}{3\mu_{IT}}\right), \tag{25}$$

$$\beta_{IT}(\mathcal{I}_A) = \frac{1}{4}\left(1 - \frac{7\hat{y}_{IT}}{3\mu_{IT}} + \frac{2\hat{y}_{TR}}{3\mu_{TR}} + \frac{2\hat{y}_{NI}}{3\mu_{NI}}\right), \tag{26}$$

It is straightforward to compute the coordinates of the Shapley value of the game as well:

$$\phi_{TR}(\mathcal{I}_A) = \frac{1}{6}\left(2 - \frac{11\hat{y}_{TR}}{3\mu_{TR}} + \frac{5\hat{y}_{NI}}{6\mu_{NI}} + \frac{5\hat{y}_{IT}}{6\mu_{IT}}\right), \tag{27}$$

$$\phi_{NI}(\mathcal{I}_A) = \frac{1}{6}\left(1 - \frac{11\hat{y}_{NI}}{3\mu_{NI}} + \frac{5\hat{y}_{TR}}{6\mu_{TR}} + \frac{5\hat{y}_{IT}}{6\mu_{IT}}\right), \tag{28}$$

$$\phi_{IT}(\mathcal{I}_A) = \frac{1}{4}\left(1 - \frac{11\hat{y}_{IT}}{3\mu_{IT}} + \frac{5\hat{y}_{TR}}{6\mu_{TR}} + \frac{5\hat{y}_{NI}}{6\mu_{NI}}\right). \tag{29}$$

Finally, we are going to evaluate the welfare loss function $\mathcal{L}_A(\cdot)$ in this case. The values that it takes are the following:

$$\mathcal{L}_A(\{TR\}) = \mathcal{L}_A(\{NI\})) = \mathcal{L}_A(\{IT\})) = 0;$$

$$\mathcal{L}_A(\{TR, NI\}) = 1 - \frac{\hat{y}_{TR}}{\mu_{TR}} - \frac{\hat{y}_{NI}}{\mu_{NI}} + \frac{y_{e,\{TR,NI\}}}{\mu_{\{TR,NI\}}};$$

$$\mathcal{L}_A(\{TR, IT\}) = 1 - \frac{\hat{y}_{TR}}{\mu_{TR}} - \frac{\hat{y}_{IT}}{\mu_{IT}} + \frac{y_{e,\{TR,IT\}}}{\mu_{\{TR,IT\}}};$$

$$\mathcal{L}_A(\{NI, IT\}) = 1 - \frac{\hat{y}_{NI}}{\mu_{NI}} - \frac{\hat{y}_{IT}}{\mu_{IT}} + \frac{y_{e,\{NI,IT\}}}{\mu_{\{NI,IT\}}};$$

$$\mathcal{L}_A(\{TR, NI, IT\}) = 2 - \frac{\hat{y}_{TR}}{\mu_{TR}} - \frac{\hat{y}_{NI}}{\mu_{NI}} - \frac{\hat{y}_{IT}}{\mu_{IT}} + \frac{y_{e,\{TR,NI,IT\}}}{\mu_{\{TR,NI,IT\}}}.$$

Also in this case, the calculation of the Banzhaf value and of the Shapley value of this game can be easily carried out.

5 Concluding remarks

The success of European policies in achieving its objectives primarily depends on changes determined on income distributions of people involved. We replicate a typical environment of this policy in order to observe its own effect on inequality terms. Our exercise tends to construct a preliminary characterization of a simple outgoing/incoming transfer scheme to see on one side, the simple effect of a targeted policy on the income profile of a society and then to show on how a multi-factor analysis on this issue may normatively provide different suggestions in terms of welfare loss. To the best of our knowledge, a theoretical setting about the impact of such transfers on inequality distribution of a State is still missing in the ongoing literature. Thus we first investigate the properties of the Atkinson index in this framework and employ them to propose a comparison between levels of inequality before and after the transfers process. Conditioned on the profitability condition, we may infer that the effect of transfers on inequality may be ambiguous. In case of no-tax area, the reduction in inequality is easier to be accomplished in the ex-post (after transfer) distribution. We also believe that higher interpretation of inequality on a targeted policy can be provided by taking into account each income profile among different sources. Indeed a multi-factor methodology is more useful since it allow to distinguish the marginal contribution of each factor. Our exercise was first to decompose the Atkinson index by income sources *à* la LY [21] and then to propose it as a characteristic function of cooperative game (see Chantreuil and Trannoy [7] and Shorrocks [30]). We propose a measure of the welfare loss characterized by the difference between the aggregate cost of inequality captured among income sources and the cost of inequality in the entire income profile. Positivity and monotonicity are demonstrated. Other properties are still to be completed.

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