

Mode transformer for hard-surface waveguides

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Abstract

Mode transformation effect inside a corrugated circular waveguide filled with gyrotropic material, e.g., magnetoplasma or ferrite, is considered. The corrugation is in axial direction and the depth of the corrugation is a quarter wave length forming a boundary condition equal to a hard surface (HS). It is assumed that the material is slightly anisotropic and gyrotropic. This can be achieved at certain frequency range with a proper axial magnetic field strength. The eigenwaves are elliptically polarised hybrid-mode fields, which propagate with slightly different propagation factors. This difference in propagation factors causes the mode transformation.

1 INTRODUCTION

The tuned corrugated waveguides are used in microwave applications where special kind of properties for field propagation are needed, for example, in antenna horn feed [1]. In corrugated waveguide there can propagate *TE*, *TM* and elliptically polarised fields, and in hard-surface waveguide also *TEM* fields [2]. In this study the corrugation is in the axial direction, thus, forming the boundary condition for hard-surface waveguide. Instead of corrugation, an alternative way to implement hard-surface is to use dielectric layer loaded with thin conducting strips [3]. Time harmonic fields (depending on t as $e^{j\omega t}$) are considered and the propagating fields depend on z as $e^{-j\beta z}$, where the propagation factor β is a real number. A mode transformer for a HS waveguide has previously been studied in [4], where chiral medium has been used. In this paper practically more interesting application example is studied. The HS waveguide is filled with ferrite material biased with static magnetic field which is in z -direction. Only a small gyrotropy is required. Schematic Figure 1 shows HS waveguide with some notations used.

2 THEORY

The constitutive relations and material parameters for ferrite are

$$\mathbf{D} = \epsilon \mathbf{E}, \quad \mathbf{B} = [\mu_t \bar{\bar{I}}_t + \mu_z \mathbf{u}_z \mathbf{u}_z - j\mu_g \mathbf{u}_z \times \bar{\bar{I}}] \cdot \mathbf{H}. \quad (1)$$

$$\mu_t = \mu_o \left(1 + \frac{\omega_o \omega_m}{\omega_o^2 - \omega^2}\right), \quad \mu_z = \mu_o, \quad \mu_g = \mu_o \frac{\omega \omega_m}{\omega_o^2 - \omega^2} \quad (2)$$

where $\omega_o = \gamma B_o$ is the Larmor precession frequency (B_o is the strength of the static magnetic flux density in z -direction), $\omega_m = \mu_o \gamma M_s$, γ is the gyromagnetic ratio and M_s is the saturation magnetization [5]. For magnetoplasma constitutive relations are of similar form i.e. one can use dyadic permittivity model. In terms of theoretical analysis, these two media, ferrite and magnetoplasma, are quite similar and can be obtained by duality from each other. In this study the focus is on ferrite material.

First step of the analysis is to find the propagating eigenfields depending on z as $e^{-j\beta z}$ and the corresponding propagation factors. The electric and magnetic fields are written with transverse and axial parts as $\mathbf{E} = \mathbf{e} + E_z \mathbf{u}_z$ and $\mathbf{H} = \mathbf{h} + H_z \mathbf{u}_z$ and are inserted into the Maxwell equations $\nabla \times \mathbf{E} = -j\omega \mathbf{B}$ and $\nabla \times \mathbf{H} = j\omega \mathbf{D}$. Using the small gyrotropy assumption i.e., μ_g/μ_t small, and also assuming that β is not very near k_t , $\beta < k_t$, elimination of transverse fields leads to

$$\nabla_t^2 E_z + \frac{\beta k_t}{k_t^2 - \beta^2} \frac{\mu_g}{\mu_t} \nabla_t^2 (j\eta_t H_z) = (\beta^2 - k_t^2) E_z \quad (3)$$

$$\nabla_t^2 (j\eta_t H_z) + \frac{\beta k_t}{k_t^2 - \beta^2} \frac{\mu_g}{\mu_t} \nabla_t^2 E_z = j\eta_t H_z \frac{\mu_z}{\mu_t} (\beta^2 - k_t^2) \quad (4)$$

where $\eta_t = \sqrt{\mu_t/\epsilon}$ and $k_t = \omega\sqrt{\mu_t\epsilon}$. The eigenfield solution for (3) and (4) is a field, for which it holds $E_z = \alpha j\eta_t H_z$ ($\alpha = \text{constant}$). Thus, longitudinal field functions E_z and H_z differ only by a scalar factor. The special case $\mu_t \approx \mu_z = \mu$, which is achieved at higher frequency range $\omega \gg \omega_o$, leads to $\alpha = \pm 1$, i.e., (3) and (4) can be written as

$$\nabla_t^2 (E_{z\pm}) + \left[\frac{k^2 - \beta_{\pm}^2}{1 \mp \frac{\beta_{\pm} k}{k^2 - \beta_{\pm}^2} \frac{\mu_g}{\mu}} \right] E_{z\pm} = 0 \quad \text{or briefly} \quad (\nabla_t^2 + k_{c\pm}^2) E_{z\pm} = 0 \quad (5)$$

The solutions $E_{z\pm} = \frac{1}{2}(E_z \mp j\eta H_z)$ are so-called wave-fields, often used with structures involving chiral material and elliptically polarised eigenfields [6]. Corresponding transverse fields are elliptically polarised and the sign (\pm) indicates the handedness: $+$ and $-$ correspond to right- and left-handed polarisations, respectively.

The solutions of the Helmholtz equation (5) in cylindrical coordinates are Bessel functions of the first kind

$$E_{z\pm} = A_{n\pm} J_n(k_{c\pm} \rho) e^{jn\varphi} \quad (6)$$

The coefficients $k_{c\pm}$ are determined by the hard-surface boundary condition at $\rho = a$: [2]

$$\mathbf{u}_z \cdot \mathbf{E} = 0, \quad \mathbf{u}_z \cdot \mathbf{H} = 0 \quad \Rightarrow \quad E_{z\pm}|_{\rho=a} = 0. \quad (7)$$

Using this condition with equation (6) leads to the cut-off number solution $k_{c\pm} = k_c = \frac{p_{ns}}{a}$, where p_{ns} are the zeros of the Bessel functions. From the dispersion relation

$$k_c^2 = \frac{k^2 - \beta_{\pm}^2}{1 \mp \frac{\beta_{\pm} k}{k^2 - \beta_{\pm}^2} \frac{\mu_g}{\mu}}, \quad (8)$$

propagation factors can be solved, still assuming $\frac{\mu_g}{\mu}$ small:

$$\beta_{\pm} = \beta_m \pm \beta_d = \frac{\beta_0}{1 - \left(\frac{k}{\beta_0} \frac{\mu_g}{2\mu}\right)^2} \pm \beta_0 \frac{\frac{k}{\beta_0} \frac{\mu_g}{2\mu}}{1 - \left(\frac{k}{\beta_0} \frac{\mu_g}{2\mu}\right)^2}. \quad (9)$$

where $\beta_0 = \sqrt{k^2 - k_c^2}$.

3 MODE TRANSFORMING EFFECT

The polarisation properties of fields are considered in a circular hard-surface waveguide filled with slightly gyrotropic medium. Assuming $\frac{k}{\beta_0} \frac{\mu_g}{2\mu} \ll 1$, the propagation factors can be further approximated from (9) as

$$\beta_{\pm} = \beta_0 \pm \frac{k\mu_g}{2\mu}, \quad (10)$$

Taking all the spatial dimensions into account, the $+$ and $-$ waves are

$$E_{z\pm}(\rho, \varphi, z) = A_{n\pm} J_n(k_c \rho) e^{jn\varphi} e^{-j\beta_{\pm} z} = \frac{1}{2} [E_z(\rho, \varphi, z) \mp j\eta H_z(\rho, \varphi, z)] \quad (11)$$

and the total longitudinal electric and magnetic fields, due to + and - waves, are

$$E_z = E_{z+} + E_{z-} = J_n(k_c \rho) e^{jn\varphi} [(A_{n+} + A_{n-}) \cos(\frac{k\mu_g}{2\mu} z) - j(A_{n+} - A_{n-}) \sin(\frac{k\mu_g}{2\mu} z)] e^{-j\beta_0 z} \quad (12)$$

$$H_z = \frac{(E_{z+} - E_{z-})}{-j\eta} = \frac{j}{\eta} J_n(k_c \rho) e^{jn\varphi} [(A_{n+} - A_{n-}) \cos(\frac{k\mu_g}{2\mu} z) - j(A_{n+} + A_{n-}) \sin(\frac{k\mu_g}{2\mu} z)] e^{-j\beta_0 z} \quad (13)$$

Let us assume that a TM-mode incident field is coming from the non-gyrotropic HS waveguide section ($z < 0$). Thus, at the medium-interface $z = 0$ (see Figure 1), magnetic field $H_z = 0$. From (13) it follows that $A_{n+} = A_{n-}$, which leads to axial fields along the gyrotropic waveguide section:

$$E_z(\rho, \varphi, z) = 2A_{n+} J_n(k_c \rho) e^{jn\varphi} e^{-j\beta_0 z} \cos(\frac{k\mu_g}{2\mu} z), \quad (14)$$

$$H_z(\rho, \varphi, z) = \frac{2A_{n+}}{\eta} J_n(k_c \rho) e^{jn\varphi} e^{-j\beta_0 z} \sin(\frac{k\mu_g}{2\mu} z). \quad (15)$$

It is seen that just after $z = 0$, along the ferrite filled waveguide section, E_z starts to decrease and H_z starts to increase. At distance $z = d = \frac{\pi\mu}{\mu_g k}$, $E_z = 0$ and $H_z \neq 0$. Thus, TM mode has changed to TE mode, which propagates after the gyrotropic section without change. The field configuration after the transformation section satisfies the boundary condition for magnetic conductor. The transformation between TM and TE modes is a dual transformation. The hard-surface waveguide can support fields which satisfy either perfect electric conductor or perfect magnetic conductor boundary conditions. In this example it was assumed that the different waveguide sections are well matched i.e. containing properly chosen media.

Assume a ferrite material with $\epsilon_r = 8$, $B_0 = 0.03$ T and relatively low saturation magnetization $\mu_0 M_s = 0.1$ T, i.e., $\omega_m = 2\pi \cdot 2.8$ GHz (typical value is $\mu_0 M_s = 0.2$ T, i.e., $\omega_m = 2\pi \cdot 5.6$ GHz [5]). The special case, $\mu_t \approx \mu_z = \mu_0$ and μ_g small, is obtained e.g. at $f = 35$ GHz. Using formulas (2) one obtains $\mu_t/\mu_z = 0.998 \approx 1$ and $\mu_g/\mu_z = 0.080$. Assuming mode with $p_{ns} = p_{01} = 2.405$ and core radius $a = 0.0017$ m, propagation factors are approximately $\beta_{\pm} = \beta_0 \pm \frac{k\mu_g}{2\mu} \approx (1.51 \pm 0.08)10^3$ 1/m. The length of the gyrotropic section, to change from TM to TE, is now $d \approx 0.019$ m.

4 IMPEDANCE TRANSFORMER

The wave impedance transforming effect can be studied by considering the transverse fields. The exact transverse fields in a gyrotropic waveguide are

$$\mathbf{h} = \frac{1}{A^2 + B^2} [j\beta(A\nabla_t H_z - B\mathbf{u}_z \times \nabla_t H_z) + j\omega\epsilon(A\mathbf{u}_z \times \nabla_t E_z + B\nabla_t E_z)] \quad (16)$$

$$\mathbf{e} = \frac{1}{j\omega\epsilon} [-j\beta\mathbf{u}_z \times \mathbf{h} - \mathbf{u}_z \times \nabla_t H_z], \quad A = \beta^2 - k^2, \quad B = jk^2 \frac{\mu_g}{\mu} \quad (17)$$

If $\frac{\mu_g}{\mu}$ is very small and β is not near k , $\beta < k$, the gyrotropic factor $B \ll A$, i.e., $B \approx 0$. If also $\frac{k}{\beta_0} \frac{\mu_g}{2\mu}$ is small enough, the approximation $\beta = \beta_{\pm} \approx \beta_0$ can be done with the factors affecting field amplitudes. These approximations lead to expressions

$$\mathbf{e} = -j \frac{\beta_0}{k_c^2} \nabla_t E_z + j \frac{k\eta}{k_c^2} \mathbf{u}_z \times \nabla_t H_z, \quad \mathbf{h} = -j \frac{\beta_0}{k_c^2} \nabla_t H_z - j \frac{k}{k_c^2 \eta} \mathbf{u}_z \times \nabla_t E_z \quad (18)$$

The wave impedance $\overline{\overline{Z}}$ is the relation between the transverse fields as

$$\mathbf{e} = -\overline{\overline{Z}} \cdot (\mathbf{u}_z \times \mathbf{h}) \quad (19)$$

Following the example in the previous section, at $z = 0$, there exists only axial electric field component E_z and the wave impedance dyadic is $\overline{\overline{Z}} = \eta \frac{\beta_0}{k} \overline{\overline{I}}_t$. This is a *TM* impedance. After the distance $d = \frac{\pi\mu}{\mu_g k}$, there exists only axial magnetic field, and the corresponding impedance dyadic is $\overline{\overline{Z}} = \eta \frac{k}{\beta_0} \overline{\overline{I}}_t$ which is

a TE impedance. At other points inside the waveguide section, $0 < z < d$, there is a hybrid mode, a combination of TE and TM fields. The impedance dyadic is

$$\overline{\overline{Z}}(z) = \eta \frac{\beta_0 k \overline{\overline{I}}_t - \frac{k_c^2}{2} \sin\left(\frac{k\mu_g z}{\mu}\right) \mathbf{u}_z \times \overline{\overline{I}}}{k^2 \cos^2\left(\frac{k\mu_g z}{2\mu}\right) + \beta_0^2 \sin^2\left(\frac{k\mu_g z}{2\mu}\right)} \quad (20)$$

In principle, with a proper length of a ferrite filled HS waveguide a hybrid mode field can be changed to TE or TM field. Also the impedance is transformed correspondingly. This kind of waveguide element can be used for matching between different kind of waveguides.

5 CONCLUSION

In a non-gyrotropic HS waveguide TE and TM modes are propagating. Using a gyrotropic HS waveguide section of a proper length, TM -mode can be transformed to TE , and vice versa. Also, the impedances are changed and the gyrotropic HS waveguide works as an impedance transformer. One application is a matching element between a circular metal-wall waveguide and an open dielectric waveguide/antenna. In this study circular waveguide was considered. Similar transformer can be implemented by using rectangular waveguide as well.

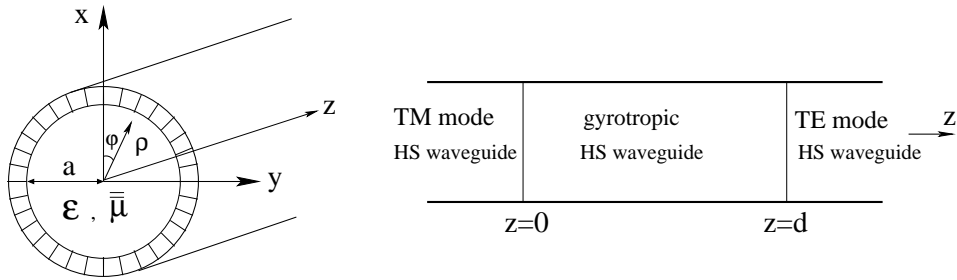


Figure 1. HS waveguide filled with gyrotropic material and TM/TE -transformer.

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