ISSN 2282-6483

Alma Mater Studiorum - Università di Bologna
DEPARTMENT OF ECONOMICS

Safe Assets' Scarcity, Liquidity and Spreads

Gabriella Chiesa

Quaderni - Working Paper DSE N°927

Safe Assets' Scarcity, Liquidity and Spreads

Gabriella Chiesa
Department of Economics
University of Bologna

Abstract

This paper constructs a simple general equilibrium model to analyse the interactions between the financial and the real sector in an environment where liquidity holdings is an input of the credit/investment process. The supply of liquidity is constrained in that income pledgeability limits inside liquidity, and not all sovereign debt is safe/liquid. We pin down the determinants of liquidity/collateral premia and bond spreads, and with reference to the eurozone: (i) the implications of the ECB's policies on liquidity provision and credit, and (ii) the debt management policy that would increase welfare with no need for transfer payments.

JEL Classifications: E44, H63, G18

Keywords: Safe assets, Liquidity, Credit, Sovereign debt spreads

1 Introduction

Siemens, Europe's largest engineering group, is setting up its own bank.

As well as broadening its sources of funding, it would allow the company to deposit cash at the Bundesbank, Germany's central bank.

"Frankly, we'd be happy to get no interest rate just to know our cash is completely safe" , a person close to the company says. (FT September 2, 2010)

Berkshire Hathaway disclosed its liquidity instruments*:*

80% of $10.8 billion on non-US Gov. Debt are from Germany, UK, Canada, Australia and the Netherlands

Comment (Guy LeBas chief fixed income strategist):

"*If a firm is looking at government debt as a source of potential liquidity, then it's extremely important to remain in these bulletproof nations*" (Bloomberg 27.02.012)

Why don't these firms, i.e. the industrial firm Siemens and the financial firm Berkshire Hathaway simply use bank deposits as a liquidity instrument? The key observation is that deposits are bank liabilities and their safety is constrained by bank income pledgeability and/or by deposit insurance. The latter being limited to "small" amounts make bank deposits money/liquidity instruments for households. That is, in Holmstrom and Tirole's parlance, "small" deposits, by virtue of deposit insurance, constitute *outside liquidity.* By contrast, the degree of safety of large/non-insured deposits is constrained by the income flows that a bank can credibly pledge as guarantee for its non-insured liabilities, i.e. the constraints that limit *inside liquidity.* Moreover, the further constraint that limits the money/liquidity property of "large" deposits is the degree of information symmetry and trust required for their "transferability" – the key ingredient for a money/liquidity instrument.

Siemens and Berkshire Hathaway's are just an example of the so called "institutional cash pools", i.e. the large, centrally managed cash balances of global corporations and institutional investors. Institutional cash pools have become increasingly prominent since the 1990s as a by product of globalization (see Pozsar, 2011, and with specific reference to financial/banking corporations Bruno and Shin, 2011). As documented by Pozsar (2011), over 90% of institutional cash pools are subject to written cash investment policies which govern the investment styles and fiduciary responsibilities of their managers. In order of priority, the objectives of these policies are: (i) safety of principal; (ii) liquidity; and (iii) yield. The empirical evidence clearly shows that institutional cash pools' preferred habitat is not deposits, but insured deposit alternatives: Government insured securities (government debt securities) and privately insured money market instruments, such as REPOs and asset backed commercial paper – where collateral provides safety and substitutes for government guarantee (Gorton, 2010; Stein, 2010; Krishnamurthy and Vissing-Jorgensen, 2012; Singh and Stella, 2012)

To sum up, the demand for money/liquidity instruments of corporations, a large sector of the economy and most active player in the real investment process and in the money/credit market, is not satisfied by M2 types of money, but rather directly or indirectly (via REPO arrangements) by securities that meet the requirement of safety of principal and liquidity.

The world's outstanding stock of safe assets has expanded steadily over the period 2001-2007, and declined impressively since 2007 (IMF Global Financial Stability Report, 2012 ch.3). Two asset class lost the safety status: loan/mortgages-securitization products, first, and then the sovereign debt of the "peripheral" countries of the eurozone. The first asset class consists of private claims on real investments (inside liquidity), the second one consists of securities issued by governments (outside liquidity). The downgrading of peripheral debt amounts to a fall of outside liquidity, the evidence is then of a general decline of the amount of liquidity instruments available for satisfying the non-household sector's demand for liquidity.

Moreover, parallel to the observed decline of inside liquidity and the peripheral debt downgrading is the fall in the yields of the *bulletproof nations'* debt (in the eurozone, Germany) and the increase of the yields of peripheral countries' debt (e.g. Italian sovereign debt).

This paper constructs a simple general equilibrium model to analyze the interactions between the financial and the real sector in an environment where liquidity holdings is an input of the credit/investment process. We build on Holmstrom and Tirole (1998, 2011) which provide a model framework in which liquidity conditions affect investment and asset prices. We extend their baseline model to incorporate a menu of outside liquidity instruments (sovereign bonds) that differ in terms of safety – price fluctuations and collateral value. The supply of liquidity is then constrained because income pledgeability limits inside liquidity, and because not all sovereign debt is safe/liquid. We pin down the determinants of liquidity/collateral premia and bond spreads, and with reference to the eurozone: (i) the implications of the ECB policies on liquidity provision and credit, and (ii) the debt management policy that would increase welfare with no need for transfer payments.

We borrow from the literature (and from reality) the idea that firms and financial institutions are best viewed as ongoing entities whose project completion requires renewed injections of resources.[1] Limited pledgeability of project outcomes constrains the amount of outside finance that can be raised and gives rise to the need of hoarding liquid/safe assets to cope with adverse shocks and/or to take future investment opportunities. In such an environment, the value of an asset to a firm is determined by the resources it gives access to when resources are most valuable – i.e., when projects need completion and/or further investment-opportunities materialize. Firms are willing to pay a premium for liquidity: the liquidity benefits amount to the option value of exercising future investment opportunities that would not be taken otherwise.

Firms may insure against liquidity needs by securing credit lines from financial institutions; that is, they can contract with a bank for the right to draw specific amounts of cash by a given date. Thanks to these arrangements, liquidity to corporations is provided by the bank, while the burden of liquidity hoarding is on the bank. The bank needs to hold a sufficient amount of liquid assets in order to fund the take-downs that its clients/firms are entitled to make under a credit line/loan commitment. Liquidity provision is the key activity of banks – the largest share of commercial and industrial loans are take-downs under loan commitments – credit lines (Bhattacharya and Thakor, 1993; Strahan, 2008). During the financial crisis, banks holding assets with low market liquidity (e.g. mortgage-backed securities, and asset backed securities) increased their holdings of liquid assets and lowered their liquidity provision to firms – new commitments to lend shrunk (Cornett et al., 2011).

The intimate relation between banks' liquidity provision and liquid assets holdings makes the availability of safe/liquid assets at the center of the credit/investment process. We focus on outside liquidity and, with some

[1] Corporations' concern for refinancing is emphasized in various contexts by the finance literature – Thakor, Hong and Greenbaum (1981), and Froot, Scharfstein, and Stein (1993), among others.

reference to the eurozone, we allow for two types of government debt, one being perfectly safe has a non-volatile market price, while the other being risky has a volatile market price (Section 2). We derive firms'/banks' composition of liquid asset portfolios and real investment/credit-lines provision, given asset/bond prices, and then solve for the equilibrium values of government bonds' prices, the associated liquidity/collateral premia and bond spreads, aggregate investment/credit and return on capital (Section 3). We find that: i) credit expansion, real investment and return on capital are increasing functions of the amount of liquid assets, the reverse holds for liquidity/collateral premia and bond spreads; ii) the bond spread is largely driven by the liquidity/collateral premium, and the impact of a bond's volatility on bond spread is more relevant when liquidity is tight than when liquidity is abundant; that is, how an asset behaves when liquidity is abundant is less relevant than how it behaves when liquidity is tight; iii) the share of safe/liquid assets is constant (in line with the empirical evidence provided by Gorton, Lewellen and Metrick, 2012). Liquid assets' availability is determined by the amount of sovereign bonds outstanding and crucially by the volatility of their market values. An increase in market-value volatility of a bond induces a substitution away from that bond and the macro effect of depleting the amount of assets that are eligible for satisfying liquidity needs, and for sufficiently high volatility, the bond loses the status of liquid asset (it's excluded from asset holdings for liquidity purposes).

We then use the model to analyze the implications of the ECB policies (lending facility and deposit facility) on firms'/banks' liquidity needs, liquidity availability and credit. In our model, the relevant aspects of the lending facility are the eligible collateral (the assets that can be pledged) and the haircuts. Its effectiveness relies on haircuts lower than the market ones, which implicitly amounts to subsidizing the sector that can access the facility (i.e. banks). There are, however, positive externalities on non-financial corporations, via market prices, as well as reduction of bond spreads. The deposit facility brings potentially a further safe/liquid asset in the economy, albeit available only to banks. We find that, by contrast to common wisdom, pursuing credit expansion as well as bond spreads reduction requires increasing the deposit facility rate rather than deposit-rate cuts, and possibly the transferability of these claims – ECB debt certificates (Section 4). The increase in the deposit-facility rate and/or ECB debt certificates succeed in expanding credit and aggregate investment in that they *de facto* expand the availability of safe assets, and thus lower the cost of holding liquidity (the key input of the credit/investment process). The drawback is the increase in the market yields of government debt, which means an increase in sovereign cost of debt. There is however a policy that does not have these drawbacks and does not involve the subsidization which underlines the effectiveness of the ECB's lending policies. It amounts to insulate the safe part of the sovereign risky debt – tranching the debt so as to create a security whose safeness is ensured by sufficient collateral (real assets and tax revenue).[2] Such a policy increases welfare and benefits the issuer by reducing its cost of debt (Section 5).

Related Literature. As anticipated, our paper builds on Holmstrom and Tirole (1998, 2011) which provide a model framework in which liquidity conditions affect investment and asset prices. We extend their baseline model to incorporate a menu of outside liquidity instruments (sovereign bonds) that differ in terms of safety – price fluctuations and collateral value. Our focus is on the joint determination of firms'/banks' composition of liquid asset portfolios, real investment/credit-lines provision, liquidity/collateral premia and bond spreads.

The role of government bonds in facilitating credit/investment has been emphasized by several papers and relies on contractual frictions that limit borrowers' commitment to honor (unsecured) debt obligations. The most closely related paper is Bolton and Jeanne (2011) that analyses the role for government debt securities as collateral for borrowing. The safer is the government debt, the greater investment and credit. The key assumption is the asynchronicity between resource availability and real investment opportunities (as in Woodford, 1990). Non pledgeability of investment returns prohibits unsecured borrowing, real investment undertaking relies on transferring resources into the future by investing resource endowments in government bonds that can be used as collateral for secured borrowing. The safer the government debt, the greater the amount of secured borrowing

[2] The importance of secured/collateralized government debt for a sounder euro area monetary system is emphasized by Nyborg (2011). The euro-nomics group (2011) points out the vital importance of a European safe asset for the long run survival of the euro-zone and calls for the creation of European Safe Bonds, where safeness is provided by pooling the sovereign bonds and then tranching the pooled debt so as to create a security whose safeness is ensured by sufficient collateral.

that can be raised and the investment that can be attained. In a multi country world, safe debt is a public good and selfish governments will supply a socially sub-optimal amount of safe debt. Our paper differs in several respects, by contrast to Bolton and Jeanne, we consider firms and financial institutions as ongoing entities whose investment projects completion requires renewed injections of resources: firms define the investment scale and the bond portfolio holdings in anticipation that further resources will have to be invested in order for projects to generate returns. The larger the amount of assets that are eligible for satisfying liquidity needs, the larger the scale of investment, the lower liquidity premia and bond spreads. An increase in market-value volatility of a bond induces a macro effect of depleting the amount of assets that are eligible for satisfying liquidity needs, and a substitution away from that bond. For sufficiently high volatility, the bond loses the status of liquid asset (it's excluded from asset holdings for liquidity purposes). By contrast, In Bolton and Jeanne a volatile/risky bond always enters in a portfolio, since it allows increasing the investment size in the (risky-debt) no-default state. They analyse various forms of fiscal integration that can mitigate the incentives to under supply safe debt and find that they reduce the welfare of the country that provides the "safe-haven" asset. We focus on debt management and find that insulating the safe part of the sovereign risky debt – tranching the debt so as to create a security whose safeness is ensured by sufficient collateral (real assets and tax revenue) – increases welfare and benefits the issuer by reducing its cost of debt.

2 The Model

There are three periods $t = 0, 1, 2$. Agents (final investors) are risk neutral and evaluate consumption streams according to

$$U(c_0, c_1, c_2) = E(c_0 + c_1 + c_2) \ , \tag{A1}$$

that is, agents' intertemporal marginal rate of substitution (IMRS) is equal to 1. Each agent receives a resource endowment at each date, and this is sufficiently large to ensure that resource scarcity does not limit the investment scale, this will be constrained by contractual frictions (limited pledgeability) and safe assets' scarcity, not by resource scarcity.

Assets

We assume that the storage technology (holding cash under the mattress) is prohibitively costly, purchasing power can be transferred into the future by investing in securities. These consist of securities issued by firms/banks to be discussed below, and of sovereign debt: G government debt and I government debt.

The sovereign debt's unit price at date 0 is denoted q_i, the total amount outstanding is B^i, $i = G, I$.

The G bonds are safe. G's unit value at date 1 is *one* for sure. The I bonds are "risky", I's unit value at date 1 is $\underline{\alpha}$ with probability p, and $\overline{\alpha} > \underline{\alpha}$ with probability $1 - p$.

For simplicity, we assume that in expected value the two government bonds are identical:

$$\underline{\alpha} p + \overline{\alpha} (1 - p) = 1 \tag{A2}$$

The two bonds differ only in the volatility of their interim-date 1 value, this is nil only for the G bonds.

The I bond's volatility may result from the volatility of the feasible tax revenue, possibly due to a high level of I sovereign debt outstanding relatively to the tax base (GDP). The smaller $\underline{\alpha}$, the greater the I bond's volatility (by $(A2)$).

We define the liquidity/collateral premium on government bond i as the excess payment made at date 0 for this bond relatively to its date 1 expected value, that is $q_i - 1$, $i = G, I$.

At an equilibrium:

$$q_I \geq 1, \, q_G \geq 1$$

($q_I < 1$, $q_G < 1$ are ruled out by $(A1)$ and $(A2)$). That is, liquidity premia cannot be negative.

Securities can be used as collateral for borrowing. We shall assume that the fraction that can be raised per unit of collateral value is less than one, the difference $0 < h < 1$ is the "haircut".

Firms/banks

There are N firms/banks. They are risk-neutral and evaluate consumption streams according to $(A1)$. A firm/bank i, $i = 1, ..N$, has initial net worth A_i at date 0 and no endowment in future periods. For simplicity, all firms/banks face the same investment opportunity that requires injections of resources at date 0 and at date 1 and delivers returns at date 2. Specifically, at date 0, i chooses the size of investment I_i; this defines the amount of resources to be invested by i at date 0. At date 1, the funds to be injected amount to θ per unit of investment. At the final date 2, the return y per unit of investment obtains, the possibility of funds diversion and/or bankruptcy costs limits return pledgeability. The per-unit-investment return which is pledgeable is $r < y$; r defines the amount of outside financing that can be raised per unit of investment. The non-pledgeable return, $R \equiv y - r$, is the haircut that market participants apply in extending loans backed by real investment. We assume:

$$y > 1 + \theta \tag{A3}$$

$$\theta > r \equiv y - R \tag{A4}$$

That is, investment projects are positive in net present value (by (A3)), but the limited pledgeability of project returns coupled with the reinvestment need at date 1 requires liquidity holdings of amount $S \equiv \theta - r > 0$ per unit of investment (by $(A4)$). The greater non-pledgeability/haircut R, the greater liquidity needs S.

The above is a reduced form of two possible models. One, simply refers to a firm that faces a constant-to-scale real investment opportunity which requires one unit of resources at date 0, and θ at date 1, per unit of investment. For a given investment size I_i chosen at date 0, the reinvestment needs at date 1 amount to θI_i. Since pledgeable income is $rI_i < \theta I_i$, reinvestment (project completion) will be feasible only if $i's$ liquidity holdings at date 1 does not fall below SI_i.

An alternative, and prominent, case refers to i being a bank endowed with net worth (capital) A_i which faces a continuum of borrowers/firms. Each borrower has an investment opportunity that requires one unit at date 0 and, with probability λ, σ additional units at date 1. The bank at date 0 chooses the size of its credit-lines' portfolio I_i, where a credit line allows a bank's borrower to withdraw one unit at date 0 and σ units at date 1. By pooling borrowers' liquidity needs, at date 1 the bank will face withdrawals of total amount $\lambda\sigma I_i$. Reinterpreting θ as $\theta \equiv \lambda\sigma$, then under the maintained assumption that the income per unit of project that is pledgeable to outsiders is r, the bank will be able to satisfy borrowers' liquidity needs (i.e, the credit lines withdrawals) only if its liquidity holdings at date 1 does not fall below SI_i. If i is a bank, then I_i defines the amount of credit extended by i; that is, the scale of investment originated by i.

We shall refer to $B^G + \underline{\alpha} B^I$ as the volume of assets eligible for satisfying liquidity needs, and assume:

$$B^G + \underline{\alpha} B^I < \frac{S \sum_{i=1}^{N} A_i}{1+S} \tag{A5}$$

This will imply strictly positive liquidity/collateral premia. To simplify the analysis we also assume:

$$R > 1 + S \left[\frac{\sum_{i=1}^{N} A_i - \frac{B^G}{S}}{B^G} \right] \tag{A6}$$

where the expression in squared brackets is greater than 1 (by ($A5$)). Assumption (A6) will imply that at equilibrium all firms/banks are active (invest) – conditionally upon N firms/banks being active, the return on capital exceeds IMRS. Henceforth, $\sum$ denotes the summation form 1 to N.

2.1 Liquidity Demand and Investment Choice

Limited pledgeability of real investment return implies that liquidity must be planned in advance. At date 0, firm/bank i chooses its liquid assets portfolio: the amount of G government bonds, L_i^G, and the amount of I government bonds, L_i^I. The value of I government bond holdings at date 1 will depend on the state realization: in state $\underline{\alpha}$ the value is $\underline{\alpha} L_i$, in state $\overline{\alpha}$ the value is $\overline{\alpha} L_i$. Therefore, for a given bond portfolio $\left(L_i^I, L_i^G\right)$ and real investment size I_i the date 1 state-contingent liquidity held and that needed are as in Table 1

Table 1

State	*Liquidity held*	*Liquidity needs*
$\underline{\alpha}$	$\underline{\alpha} L_i^I + L_i^G$	SI_i
$\overline{\alpha}$	$\overline{\alpha} L_i^I + L_i^G$	SI_i

The percentage of the bond portfolio revenue that is pledgeable is $1-h$, accordingly for investors to be willing to supply funds to bank/firm i the following participation constraint must be satisfied:

$$I_i + q_I L_i^I + q_G L_i^G - A_i \leq$$

$$(1-h)\left[p\left(\underline{\alpha} L_i^I + L_i^G - SI_i\right) + (1-p)\left(\overline{\alpha} L_i^I + L_i^G - SI_i\right)\right] \tag{PC}$$

$$S \equiv \theta - r$$

the LHS of (PC) is the amount of outside financing required for real investment, I_i, and liquid assets holdings $\left(L_i^I, L_i^G\right)$ (i.e., the difference between total expenditure, $I_i + q_I L_i^I + q_G L_i^G$, and inside funds/capital A_i). The RHS is the firm/bank's pledgeable income: the fraction $1-h$ of the expected value of date 1 idle liquidity (i.e., the expected value of the bond portfolio at date 1 net of the amount absorbed by investment-project completion).

The firm/bank's expected profits are given by its non-pledgeable income; that is, the non-pledgeable return (haircut) on real investment, RI_i, plus the expected value of the non-pledgeable date 1 idle liquidity:

$$RI_i + h\left[p(\underline{\alpha} L_i^I + L_i^G - SI_i) + (1-p)(\overline{\alpha} L_i^I + L_i^G - SI_i)\right] \ .$$

These profits will obtain provided the firm/bank at date 1 holds sufficient liquidity so as to meet the reinvestment needs (credit lines withdrawals), that is if

$$\underline{\alpha} L_i^I + L_i^G \geq SI_i \ .$$

We rule out government bond short-selling by imposing

$$L_i^I \geq 0 \ , \ L_i^G \geq 0 \quad , \ \forall i$$

The firm/bank's optimization problem amounts to choose the real investment size I_i, and its government bond portfolio $\left(L_i^I, L_i^G\right)$ so as to maximize its profits subject to the investor participation constraint, the liquidity/reinvestment constraint and the no-short-selling constraints:

$$\underset{I_i,L_i^I,L_i^G}{Max}\left\{\Pi_i \equiv RI_i + h\left[p(\underline{\alpha}L_i^I + L_i^G - SI_i) + (1-p)(\overline{\alpha}L_i^I + L_i^G - SI_i)\right]\right\}$$

$$st$$

$$I_i + q_I L_i^I + q_G L_i^G \leq A_i + (1-h)\left\{p\left[\underline{\alpha}L_i^I + L_i^G - SI_i\right] + (1-p)\left[\overline{\alpha}L_i^I + L_i^G - SI_i\right]\right\} \tag{PC}$$

$$\underline{\alpha}L_i^I + L_i^G \geq SI_i \tag{LC'}$$

$$L_i^I \geq 0 \ , \ L_i^G \geq 0 \tag{NNC}$$

At an optimum, the liquidity constraint (LC') binds,

$$\underline{\alpha}L_i^I + L_i^G = SI_i \quad , \tag{LC}$$

if not then the firm/bank would benefit by reducing the bond portfolio size so as to eliminate idle liquidity. This would free up h units of capital per unit of idle-liquidity reduction and thereby expand real investment and profits. Hence at an optimum, the date 1 state-contingent idle liquidity is as in Table 2.

Table 2

State	*Idle Liquidity*	*prob*
$\underline{\alpha}$	$\underline{\alpha}L_i^I + L_i^G - SI_i \equiv 0$	p
$\overline{\alpha}$	$\overline{\alpha}L_i^I + L_i^G - SI_i$ $\equiv (\overline{\alpha} - \underline{\alpha})\, L_i^I$	$1-p$

Using I bonds for liquidity purposes entails holding $(\overline{\alpha} - \underline{\alpha})$ units of date 1 idle liquidity with probability $1-p$, and zero units with the residual probability, per unit of I bond. The certain equivalent of this lottery is $(1-\underline{\alpha})$ (because of risk neutrality, and by (A2)).

Observation 1: *I bond holdings entails a financial investment that yields* $(1-\underline{\alpha})$ *units of date 1 idle liquidity per bond.*

The greater I bond volatility, i.e., the lower $\underline{\alpha}$, the greater the date 1 idle liquidity per unit of I bond holdings. Date 1 idle liquidity is partially pledgeable – the amount of outside financing that can be raised per unit is $1-h$, the residual fraction (haircut) h is financed with inside capital. I bond holdings then entails an unwarranted financial investment that subtracts resources to real investment undertaking.

Substituting (LC) into the investor participation constraint (which at an optimum holds with equality) and using the identity $(1-p)(\overline{\alpha}-\underline{\alpha}) \equiv (1-\underline{\alpha})$ gives the size of investment I_i:

$$I_i = \frac{A_i + L_i^I Z}{1 + q_G S} \tag{1}$$

$$Z \equiv [1 + \underline{\alpha}(q_G - 1)] - [q_I + h(1-\underline{\alpha})] \tag{2}$$

As we will see, a positive haircut, $h > 0$, implies that at equilibrium $Z < 0$; using the I bonds for liquidity purposes lowers real investment. As observed above, positive holdings of the I bonds entails an investment in date 1 idle liquidity ($(1-\underline{\alpha})$ per unit of I bond). The amount of outside financing that can be raised per unit of date 1 idle liquidity is $1-h$, which implies that the firm/bank invests $h(1-\underline{\alpha})$ units of its own capital for each unit of I bond holdings. Real investment falls accordingly.

The firm/bank's expected profits are

$$\Pi_i = RI_i + h(1-\underline{\alpha}) L_i^I$$

that is:

$$\Pi_i = \rho A_i + [\rho Z + h(1-\underline{\alpha})] L_i^I$$
$$\rho \equiv \frac{R}{1+q_G S} \; .$$

ρ is the return per unit of capital devoted to real investment. At equilibrium, $\rho \geq 1$ (by (A1))

Let $\widehat{q}_I(q_G)$ be given by:

$$\frac{\partial \Pi_i \left(L_i^I, q_G, \widehat{q}_I\right)}{\partial L_i^I} = 0$$

$$\Longleftrightarrow \rho Z(q_G, \widehat{q}_I) + h(1-\underline{\alpha}) = 0$$

that is:

$$\widehat{q}_I(q_G) \equiv [1 + \underline{\alpha}(q_G - 1)] - h(1-\underline{\alpha})\left[\frac{\rho - 1}{\rho}\right] \tag{3}$$

Lemma 1: *The I bonds are used for liquidity purposes, $L_i^I > 0$, if and only if $q_I \leq \widehat{q}_I(q_G)$, and for $q_I < \widehat{q}_I(q_G)$, the G bonds are excluded from liquidity holdings.*

This follows because

$$q_I > \widehat{q}_I(q_G) \longrightarrow \frac{\partial \Pi_i}{\partial L_i^I} < 0 : (L_i^I = 0, L_i^G = SI_i)$$

$$q_I < \widehat{q}_I(q_G) \longrightarrow \frac{\partial \Pi_i}{\partial L_i^I} > 0 : (L_i^I = \frac{SI_i}{\underline{\alpha}}, L_i^G = 0)$$

$$q_I = \widehat{q}_I(q_G) \longrightarrow \frac{\partial \Pi_i}{\partial L_i^I} = 0 : (L_i^I > 0, L_i^G = SI_i - \underline{\alpha} L_i^I > 0)$$

Lemma 2: *The threshold $\widehat{q}_I(q_G)$ is decreasing in I bond's volatility and in non-pledgeability/haircut h. And*

$$\widehat{q}_I(q_G) - q_G \equiv -(1-\underline{\alpha})\left[(q_G - 1) + h\left(\frac{\rho - 1}{\rho}\right)\right] \quad < 0$$

At equilibrium, $\widehat{q}_I(q_G) - q_G < 0$ because $\rho \geq 1$, $q_G \geq 1$ (by (A1)-(A2)). For the I bonds to be used as liquidity instruments it must be that $q_I \leq \widehat{q}_I(q_G)$, and $\widehat{q}_I(q_G)$ is decreasing in I bond's volatility (the inverse of $\underline{\alpha}$) and in non-pledgeability/haircut h. It then follows that, for any given level of the G bond price q_G, the greater I bond volatility and/or non-pledgeability/haircut h, the smaller the price q_I such that I bonds are used for liquidity purposes. The key point is that the greater the volatility, the greater date 1 idle liquidity per unit of volatile asset holdings – the greater $1 - \underline{\alpha}$. And with limited pledgeability, *i.e.*,for $h > 0$, the greater the forgone amount of real investment return.

It follows from Lemma 2 that if the volatile I bonds are used as liquidity instruments, i.e., if $q_I \leq \widehat{q}_I(q_G)$, then necessarily they sell at a discount with respect to the safe G bonds. Bond prices are bounded below by 1 – that is, $q_I \geq 1$, $q_G \geq 1$ (by $(A1)$, $(A2)$). Then, for sufficiently high levels of volatility and/or haircut h, such that $\widehat{q}_I(q_G) < 1$, the I bonds will be excluded from asset holdings, $L_i^I = 0$, and held by "buy and hold" investors.

3 Equilibrium: Aggregate Investment, Bond Prices

and Spreads

We first observe that the safe G bonds are used for liquidity purposes:

Lemma 3: *At an equilibrium, necessarily $q_I \geq \widehat{q}_I(q_G)$: the G bonds are demanded for liquidity purpose, $L_G > 0$*

Proof: By contradiction: Suppose $q_I < \widehat{q}_I(q_G)$ and therefore $L_G = 0$, i.e., the G bonds are held entirely by "buy and hold investors". Then $q_G = 1$, $\rho = \frac{R}{1+S}$, and $\widehat{q}_I(q_G) = 1 - h(1-\underline{\alpha})\left[\frac{R-(1+S)}{R}\right] < 1$ (because $R > 1+S$, by $(A3)$), which contradicts $q_I < \widehat{q}_I(q_G)$ since $q_I \geq 1$ (by $(A1)$, $(A2)$) ■

Moreover, the G bonds carry a strictly positive liquidity premium:

Lemma 4 *At equilibrium the liquidity premium on the G bonds is strictly positive – the amount of G bonds outstanding is absorbed entirely by the demand for liquidity.*

Proof: At an equilibrium $q_I \geq \widehat{q}_I(q_G)$ (by Lemma 3), then it amounts to proving that for $q_I \geq \widehat{q}_I(q_G)$, $q_G > 1$. This follows because:

a) if at equilibrium $q_I = \widehat{q}_I(q_G)$, then $q_G = \frac{q_I R - (1-\underline{\alpha})[(1-h)R+h]}{\underline{\alpha}R + h(1-\underline{\alpha})S} > 1$, by $q_I \geq 1$, $\underline{\alpha} < 1$, $h > 0$ and $R > 1+S$ (by A3).

b) if at equilibrium $q_I > \widehat{q}_I(q_G)$, then $L_i = 0$, $I_i = \frac{A_i}{1+q_G S}$. That is, liquidity is provided exclusively by the G bonds and the aggregate demand for G bonds is $\sum L_i^G = S\sum I_i \equiv \frac{S\sum A_i}{1+q_G S}$. Aggregate demand $\sum L_i^G$ is decreasing in q_G, and $\frac{S\sum A_i}{1+S} > B^G$ (by $(A5)$), then the G bonds' market clearing necessarily implies that $q_G > 1$. ■

To sum up. At equilibrium, either:

i) $q_I > \widehat{q}_I(q_G)$, in which case $L_i = 0, \forall i$. Liquidity is provided exclusively by the G bonds and therefore the amount of assets that are eligible for satisfying liquidity needs is limited to the amount of G bonds outstanding B^G, or;

ii) $q_I = \widehat{q}_I(q_G)$. Both bonds provide liquidity and therefore the amount of assets that are eligible for satisfying liquidity needs expands to $B^G + \underline{\alpha}B^I$.

In any case, the amount of G bonds outstanding is absorbed entirely by the demand for liquidity, that is the G-bonds market clears for $\sum L_i^G = B^G$.

The equilibrium, aggregate investment, credit, return on capital, bond prices and spreads, depends on the amount of government bond outstanding, B^G, B^I, and crucially on the volatility of the I bond value (the inverse of $\underline{\alpha}$) and the degree of securities' non-pledgeability/haircut h. This defines the opportunity cost of the financial investment in date 1 idle liquidity that the I bonds entail (Observation 1). Would h be nil, then the opportunity cost of holding date 1 idle liquidity would be nil - the amount of outside financing raised per unit of date 1 idle liquidity would be one, and no capital would be subtracted to real investment undertaking.

Observation 2. *With perfect pledgeability, $h = 0$, date 1 idle liquidiy is costless. The entire amount of I bond outstanding is used for liquidity purposes, and bond prices are: $q_I^* = \widehat{q}_I(q_G^*) \equiv 1 + \underline{\alpha}(q_G^* - 1)$; $q_G^* = \frac{\sum A_i - \frac{(B^G + \underline{\alpha} B^I)}{S}}{B^G + \underline{\alpha} B^I}$. The liquidity premium is $l \equiv q_G^* - 1$, and bond spread*

$$q_G^* - q_I^* = (1 - \underline{\alpha})\, l$$

Aggregate investment and credit are determined by the aggregate quantity of safe assets, $B^G + \underline{\alpha} B^I$,

$$\sum I = \frac{B^G + \underline{\alpha} B^I}{S}$$

Proof: At equilibrium $q_I \geq \widehat{q}_I(q_G)$, $q_G > 1$, $\sum L_i^G = B^G$ (by Lemmas 3-4). And for $h = 0$, $\widehat{q}_I(q_G) \equiv 1 + \underline{\alpha}(q_G - 1) > 1$ (by $q_G > 1$). Then necessarily $L_i > 0$, that is $q_I = \widehat{q}_I(q_G)$, and $\sum L_i^I = B^I$. If not, then the I bonds would be held, at least partially, by "buy and hold" investors. Since these value securities according to the underlying fundamental value, the I bond price would be $q_I = 1$ which contradicts $q_I \geq \widehat{q}_I(q_G)$. Thus, at equilibrium, necessarily: $q_I = \widehat{q}_I(q_G)$, and $\sum L_i^I = B^I$. Since $L_i^G + \underline{\alpha} L_i^I = SI_i$ (by (LC)), bond markets clear for $S \sum I_i = B^G + \underline{\alpha} B^I$, where $I_i = \frac{A_i}{1 + q_G S}$ (because $Z = 0$, by $h = 0$, and $q_I = \widehat{q}_I(q_G)$). Thus, q_G^* : $S \sum \frac{A_i}{1 + q_G S} = B^G + \underline{\alpha} B^I$, that is $q_G^* = \frac{\sum A_i - \frac{(B^G + \underline{\alpha} B^I)}{S}}{B^G + \underline{\alpha} B^I}$ (> 1, by $(A5)$). ■

With perfect pledgeability, $h = 0$, one unit of date 1 idle liquidity allows to raise exactly one unit of date 0 liquidity. An I bond is then exactly equivalent to a bundle of $\underline{\alpha}$ units of G bonds, and $1 - \underline{\alpha}$ units of date 0 liquidity. This bundle costs $\underline{\alpha} q_G^* + (1 - \underline{\alpha})$, and so does an I bond. The amount of assets eligible for satisfying date 1 liquidity needs is $B^G + \underline{\alpha} B^I$ and this determines the amount of aggregate investment/credit. The greater I bond volatility (the lower $\underline{\alpha}$), the lower the aggregate amount of safe/liquid assets, the greater the liquidity premium ($l \equiv q_G^* - 1$) and the greater bond spread ($q_G^* - q_I^*$).

We now derive the equilibrium under the assumption of imperfect pledgeability, $h > 0$ – date 1 idle liquidity is costly, it subtracts capital to real investment undertaking (profits shrink). For sufficiently high volatility, the I bond will be excluded from asset holdings for liquidity purposes.

Suppose that at equilibrium $q_I > \widehat{q}_I(q_G)$, i.e., $L_i^I = 0$, $L_i^G = SI_i$ (by (LC)), and all firms/banks are active (invest), then the aggregate demand for the G bonds is $\sum L_i^G = S \sum I_i$, and aggregate investment is

$$\sum I_i = \sum \frac{A_i}{1 + q_G S}$$

bond prices are:

$$q_I^* = 1 \quad (\text{by } L_i = 0, \forall i, \text{ and by } (A1), (A2)) \tag{4}$$

$$q_G^* : \sum L_i^G \equiv S \sum I_i = B^G$$

that is

$$q_G^* = \frac{\sum A_i - \frac{B^G}{S}}{B^G} \qquad (> 1 \text{ by } (A5)) \tag{5}$$

Firm/bank i's profits:

$$\Pi_i = RI_i \equiv \left[\frac{R}{1 + q_G^* S} \right] A_i \ , \tag{6}$$

The return on capital $\rho \equiv \frac{R}{1+q_G^* S}$ exceeds IMRS, i.e. $\rho > 1$ (by $(A6)$).[3]

This is an equilibrium if $q_I^* = 1 > \widehat{q}_I(q_G^*)$, that is if

$$q_G^* < Q$$

$$Q \equiv \frac{\underline{\alpha} + h(1 - \underline{\alpha})\left(1 - \frac{1}{R}\right)}{\underline{\alpha} + h(1 - \underline{\alpha})\frac{S}{R}} \tag{7}$$

$Q > 1$, because $\underline{\alpha} < 1, h > 0$, $R > 1+S$, is increasing in I bond volatility (decreasing in $\underline{\alpha}$) and in non-pledgeability/haircut h.

[3]If (A6) fais to hold, then, at equilibrium, n^* firms/banks are active, where $n^* < N$ is the smallest integer such that $\frac{R-1}{S} \geq q_G^* \equiv \frac{\sum_{i=1}^{n^*} A_i - \frac{B^G}{S}}{B^G}$ (i.e. $\rho \geq 1$).

Using (5), the condition for an equilibrium where liquidity is provided exclusively by the G bonds is

$$Q > \frac{\sum A_i - \frac{B^G}{S}}{B^G} \tag{C$_1$}$$

The more volatile the I bond value (the lower $\underline{\alpha}$) and/or the greater non-pledgeability/haircut h, the greater Q. The larger the amount of G bonds outstanding, the smaller the RHS of (C$_1$),the more likely then an equilibrium where the I bonds are not used for liquidity purposes. This leads to:

Proposition 1: *If the I bond value's volatility and/or non-pledgeability/haircut h are sufficiently high so that condition (C_1) holds, then the assets that are eligible for satisfying liquidity needs are defined exclusively by G government bonds: $q_G^* > 1$, $q_I^* = 1$. The liquidity premium is $l \equiv q_G^* - 1$, bond spread*

$$q_G^* - q_I^* = l$$

Aggregate investment and credit are restrained by the amount of G bonds outstanding:

$$\sum I_i = \frac{B^G}{S}$$

Liquidity seeking institutions require a price discount for the unwarranted investment in date 1 idle liquidity which an I bond entails. The higher volatility (the lower $\underline{\alpha}$), the greater the date 1 idle liquidity per unit of I bond, and the higher the haircut the higher the opportunity cost of the idle liquidity. The greater then the price discount required for using the I bonds for liquidity purposes. If volatility and/or the haircut are sufficiently high so that condition (C_1) holds, then the price at which the I bond would be used as liquidity instrument, the threshold $\widehat{q}_I(q_G)$, falls below the I bond's underlying fundamental value (which is 1). The I bonds will be held entirely by "buy and hold" investors and liquidity needs will be met exclusively by the G bonds. Aggregate investment and credit will then be restrained by the amount of G bonds outstanding. The smaller this is, the greater the liquidity/collateral premium and bond spread, the smaller aggregate investment and firms'/banks' profits (by (5) – (6)).

If the volatility of the I bond value and/or non-pledgeability/haircut h lower sufficiently so that condition (C$_1$) is violated, then at equilibrium $q_I = \widehat{q}_I(q_G)$, i.e. $L_i^I > 0, \forall i$. The I bond will be used for liquidity holdings and the availability of the assets eligible for satisfying liquidity needs will expand to $B^G + \underline{\alpha} B^I$. While the entire amount of G bonds outstanding will be used for liquidity purposes, $\sum L_i^G = B^G$ (by Lemma 4), the same will not necessarily hold for the I bonds. We show below that if (C_1) fails to hold, at equilibrium either $\sum L_i^I = B^I$, and $q_I > 1$ (the I bonds carry a liquidity premium), or $0 < \sum L_i^I < B^I$, and $q_I = 1$.

Suppose the entire amount of the I bonds outstanding is used for liquidity purposes, then, since $L_i^G = SI_i - \underline{\alpha} L_i^I$ (by (LC)), bond markets clear for:

$$\sum L_i^G \equiv S \sum I_i - \underline{\alpha} \sum L_i^I = B^G$$

$$\sum L_i^I = B^I$$

where:

$$\sum I_i = \frac{ZB^I + \sum A_i}{1 + q_G S} \quad \text{(by (1) and } \sum L_i^I = B^I\text{)} \tag{8}$$

and since $q_I = \widehat{q}_I(q_G)$,

$$Z = -(1 - \underline{\alpha}) h \left(\frac{1 + q_G S}{R} \right) . \tag{9}$$

The bond market clearing condition is then $S \sum I_i = B^G + \underline{\alpha} B^I$, that is:

$$S \left[\frac{ZB^I + \sum A_i}{1 + q_G S} \right] = B^G + \underline{\alpha} B^I .$$

This gives bond prices:

$$q_G^* = \frac{\sum A_i - \frac{(B^G + \underline{\alpha} B^I)}{S} - \frac{h}{R}(1 - \underline{\alpha}) B^I}{B^G + \underline{\alpha} B^I + \frac{hS}{R}(1 - \underline{\alpha}) B^I} , \tag{10}$$

$$\begin{gathered} q_I^* = \widehat{q}_I(q_G^*) \\ \equiv \\ 1 + \underline{\alpha}\left(q_G^* - 1\right) - h\left(1 - \underline{\alpha}\right)\left(\frac{\rho - 1}{\rho}\right) \\ \rho \equiv \frac{R}{1 + q_G^* S} \quad . \end{gathered} \tag{11}$$

Firm/bank $i's$ profits:

$$\begin{aligned}\Pi_i &= RI_i + h\left(1-\underline{\alpha}\right)L_i^I \\ &\equiv \left[\tfrac{R}{1+q_G^* S}\right] A_i\end{aligned}$$

(by (1), and (9)). The return on capital $\rho \equiv \frac{R}{1+q_G^* S}$ exceeds IMRS (by (10) and (A6)).

This is an equilibrium if bond prices satisfy $q_G^* \geq 1$, $q_I^* \geq 1$ which amounts to

$$q_G^* \geq Q \quad (>1, \text{ by } (7))$$

and holds iff:

$$Q \leq \frac{\sum A_i - \frac{\left(B^G+\underline{\alpha}B^I\right)}{S} - \frac{h}{R}(1-\underline{\alpha})B^I}{B^G+\underline{\alpha}B^I+\frac{hS}{R}(1-\underline{\alpha})B^I} \tag{C$_2$}$$

This leads to:

Proposition 2 *If the volatility of the I bond value and/or non-pledgeability/haircut h are sufficiently small so that condition (C_1) is violated, then the I bonds are used for liquidity purposes. And if the aggregate quantity of safe assets $B^G + \underline{\alpha}B^I$ is sufficiently small so that condition (C_2) holds, then the outstanding volumes of both G and I bonds are absorbed entirely by the demand for liquidity. The liquidity premium is $l \equiv q_G^* - 1$, and bond spread*

$$q_G^* - q_I^* = (1-\underline{\alpha})\left\{l\left(1-\frac{hS}{R}\right) + h\left[1-\frac{S+1}{R}\right]\right\}$$

Aggregate investment and credit are determined by the aggregate quantity of safe assets, $B^G + \underline{\alpha}B^I$:

$$\sum I = \frac{B^G + \underline{\alpha}B^I}{S}$$

The smaller the quantity of safe assets, $B^G + \underline{\alpha}B^I$, the greater the liquidity premium and bond spread. Moreover, the bond spread, $q_G^* - q_I^*$, is increasing in non-pledgeability/haircut h and in the volatility of the I bonds. The key is that an increase in volatility (a reduction of $\underline{\alpha}$) produces two effects: i) a depletion of the aggregate amount of safe/liquid assets, $B^G + \underline{\alpha}B^I$, and; ii) an increase in the unwarranted investment in date 1 idle liquidity that each unit of I bond holdings entails. Effect i) pushes up the market price for liquidity and thereby the G bond price, q_G^*. Effect ii) increases the price discount that liquidity seeking institutions require for I bond holdings. The higher the haircut h, the higher the opportunity cost of idle liquidity and the greater the discount (by Lemma 2). The greater I bond volatility and/or the haircut h, the greater then bond spread.

If both (C_1) and (C_2) fail to hold, i.e. if

$$\frac{\sum A_i - \frac{\left(B^G+\underline{\alpha}B^I\right)}{S} - \frac{h}{R}(1-\underline{\alpha})B^I}{B^G+\underline{\alpha}B^I+\frac{hS}{R}(1-\underline{\alpha})B^I} < Q \leq \frac{\sum A_i - \frac{B^G}{S}}{B^G} \tag{C$_3$}$$

then at equilibrium the I bonds are used only partially for liquidity purposes, $0 < \sum L_i^I < B^I$, which implies that $q_I^* = 1$, and since $q_I^* = \widehat{q}_I\left(q_G^*\right)$,

$$q_G^* = Q \tag{12}$$

Aggregate investment:

$$\sum I = \frac{\sum\left(A + ZL^I\right)}{1+QS}$$

$$Z \equiv -\left(1-\underline{\alpha}\right)h\left(\frac{1+QS}{R}\right) \quad \text{(by } q_I^* = \widehat{q}_I\left(q_G^*\right), \text{ and (12))}$$

which implies that $\frac{\partial \Pi_i(..)}{\partial L_i^I} = 0$, and $i's$ profits are

$$\begin{aligned}\Pi_i &= RI_i + h\left(1-\underline{\alpha}\right)L_i^I \\ &\equiv \left[\tfrac{R}{1+QS}\right]A_i\end{aligned}$$

The return on capital $\rho \equiv \frac{R}{1+QS}$ exceeds IMRS (by $Q \leq \frac{\sum A_i - \frac{B^G}{S}}{B^G}$ and (A6)).

From the aggregate demand for liquidity $\sum L_i^G + \underline{\alpha}\sum L_i^I \equiv S\sum I_i$ and the G bond market clearing condition, $\sum L_i^G = B^G$ (by Lemma 4), we obtain the amount of I bonds used for liquidity purposes:

$$\underline{\alpha}\sum L_i^I = S\sum I_i - B^G$$

where $\sum L_i^I < B^I$ (by (C_3)).

Proposition 3: *If the volatility of the I bond value and/or non-pledgeability/haircut* h *are sufficiently small so that condition* (C_1) *is violated, and if the aggregate quantity of safe assets* $B^G + \underline{\alpha} B^I$ *is sufficiently large so that condition* (C_2) *fails to holds, i.e., if condition* (C_3) *holds, then I bonds are used only partially for liquidity purposes,* $0 < \sum L_I < B_I$. *Bond prices are* $q_I^* = 1$, $q_G^* = Q > 1$. *The bond spread equals the liquidity premium carried by the G bonds,* $q_G^* - q_I^* = l$. *Aggregate investment and credit are smaller than that that would attain if the entire supply of I bonds would be used for liquidity holdings,* $\sum I < \frac{B^G + \underline{\alpha} B^I}{S}$.

Table 3 below summarizes our results

Q	$\sum L^I$	q_G^*	s^*	q_I^*	Investm/Credit	ρ^*
$Q \leq \underline{\chi}$	B^I	$\underline{\chi}$	$\underline{s}$	$\underline{\chi} - \underline{s} > 1$	$\frac{B^G + \underline{\alpha} B^I}{S}$	$\overline{\rho}$
$\underline{\chi} < Q \leq \overline{\chi}$	$B^I > \sum L^I > 0$	Q	$\underline{s} < s^* < \overline{s}$	1	$\frac{B^G + \underline{\alpha} B^I}{S} > \sum I > \frac{B^G}{S}$	$\overline{\rho} > \rho^* > \underline{\rho}$
$Q > \overline{\chi}$	0	$\overline{\chi}$	$\overline{s} \equiv \overline{\chi} - 1$	1	$\frac{B^G}{S}$	$\underline{\rho}$

where: s^*, ρ^* are the equilibrium values of bond spread and return on capital, respectively; Q is given by (7), that is

$$Q \equiv \frac{\underline{\alpha} + h\left(1 - \underline{\alpha}\right)\left(1 - \frac{1}{R}\right)}{\underline{\alpha} + h\left(1 - \underline{\alpha}\right)\frac{S}{R}}$$

$Q > 1$ and is increasing in I bond volatility (decreasing in $\underline{\alpha}$) and in non-pledgeability/haircut h. And,

$$\underline{\chi} \equiv \frac{\sum A_i - \frac{\left(B^G + \underline{\alpha} B^I\right)}{S} - \frac{h}{R}(1 - \underline{\alpha})B^I}{B^G + \underline{\alpha} B^I + \frac{hS}{R}(1 - \underline{\alpha})B^I} ;$$

$$\begin{aligned} \overline{\chi} &\equiv \frac{\sum A_i - \frac{B^G}{S}}{B^G} ; \\ \underline{s} &\equiv (1 - \underline{\alpha})\left\{(q_G^* - 1)\left(1 - \frac{hS}{R}\right) + h\left[1 - \frac{S+1}{R}\right]\right\} . \end{aligned}$$

$$\overline{\rho} \equiv \frac{R}{1 + \underline{\chi} S} ; \underline{\rho} \equiv \frac{R}{1 + \overline{\chi} S}$$

The third and the first row of Table 1 summarize the results at Proposition 1 and Proposition 2, respectively; the second row those at Proposition 3. Specifically, the second and the third row summarize the equilibria attained when I bond volatility and/or the haircut h are sufficiently great so that condition (C_2) fails to hold: The amount of I bonds used as liquidity instruments is lower than the outstanding amount B^I, that is $\sum L_i^I < B^I$, and aggregate investment falls below the level that would attain if the entire amount of I bonds outstanding would be used for liquidity purposes, i.e. $\sum I < \frac{B^G + \underline{\alpha} B^I}{S}$. The first row summarizes the equilibrium attained when I bond volatility and/or the haircut h are sufficiently small so that condition (C_2) holds, $\sum L_i^I = B^I$, and $\sum I = \frac{B^G + \underline{\alpha} B^I}{S}$.

Credit expansion, real investment and return on capital are increasing functions of the amount of liquid assets, the reverse holds for liquidity premia and bond spreads. The impact of I bond volatility on bond spread is more relevant when liquidity is tight than when liquidity is abundant. Indeed, the higher the liquidity premium, $q_G^* - 1$, the greater the effect of I bond volatility (measured by $1 - \underline{\alpha}$) on bond spread. Thus "*while a liquidity premium is a form of risk premium, its structure is different from that seen in standard asset-pricing models. In particular, how the asset behaves when liquidity is abundant is less relevant than how it behaves when liquidity is tight*" (Holmstrom and Tirole, 1996). Moreover, in line with the empirical evidence provided by Gorton, Lewellen and Metrick (2012), the share of safe/liquid assets is constant. This results from the proportionality of aggregate investment and safe/liquid assets, that of firm/bank profits and investment, and the fact that firm/bank equity is the sum of the initial value of equity A_i and profits.

Liquid assets' availability is determined by the amount of sovereign bonds outstanding and crucially by the volatility of their market values. An increase in volatility of I bonds (a reduction of $\underline{\alpha}$) produces the macro effect of depleting the aggregate amount of safe/liquid assets, $B^G + \underline{\alpha} B^I$, with a corresponding increase of the liquidity/collateral premium $q_G^* - 1$, and an increase in the opportunity cost of using the I bonds for liquidity purposes (the threshold $\widehat{q}_I(q_G)$ lowers). As volatility increases, equilibrium shifts from the first row to the second row and for sufficiently high volatility, the I bonds lose the status of liquid asset – the equilibrium is defined by the third row. Parallel effects are produced by an increase in non-pledgeability/haircut h –

as h increases the opportunity cost of the unwarranted investment in date 1 idle liquidity that volatile assets entail, increases, and eventually the I bonds are excluded from asset holdings for liquidity purposes.

4 Central Bank Liquidity Provision

Although the model is extremely simplified some observations can be made with regard to the role of the Central Bank (ECB) in managing liquidity needs, liquidity availability and credit.

Central Bank Lending Facility

In our model liquidity needs result from imperfect pledgeability of project/loan returns. While the per unit outcome is y, only $r < y$ can be pledged, that is, $R \equiv y - r$ is the "haircut" that market participants apply to projects/loans. The smaller r, the greater the haircut and the bigger liquidity needs $S = \theta - r$ per unit of investment/credit. The ECB's policy of extending credit to the banks accepting as collateral the bank loans matters only in so far the haircut R' it applies falls below the market haircut R. Suppose this is the case, then banks REPO their loans with the ECB and their liquidity needs per unit of assets (loans) drops from S to $S' = S - \Delta$, $\Delta \equiv R - R'$. Bank's credit and return on capital expand, liquidity premia and bond spread shrink. The larger the banking sector with respect to the non-financial sector (i.e., the larger the share of aggregate capital $\sum A_i$ held by banks), the greater the reduction of liquidity premia and bond spread and the greater the increase in aggregate investment. A parallel effect is produced by the ECB extending loans collateralized by sovereign debt securities with an haircut h' lower than the market haircut h. This lowers the opportunity cost of using the I bonds for liquidity purposes (the threshold $\widehat{q}_I(q_G)$ increases): the amount of assets that are eligible for satisfying liquidity needs expand. However, if the lending facility is restricted to the banks, as it is the case, the substitution effect towards the I bonds is limited to the banks: the banking sector ends up holding the largest share of the I bonds outstanding.

Central Bank Deposit Facility

The deposit facility brings potentially a third safe/liquid asset into the picture, albeit available only to banks. The rate at which deposits are rewarded defines the date 0 price of one unit of liquidity at the future date 1, q_{DF}. The higher the deposit rate, the lower q_{DF}. This asset is perfectly safe and hence, as far as banks are concerned, it constitutes a perfect substitute for the G bonds. Whether this asset plays a role it depends on the price q_{DF} relative to the G bond price.

Suppose the deposit-facility rate is sufficiently low so that $q_{DF} > q_G^*$, where q_G^* is the market price of the G bonds in the absence of this third asset (q_G^* is defined by Table 3, depending on parameter values). Then this asset plays no role – banks' liquidity needs are met by holding a portfolio of sovereign debt securities. Now suppose the deposit rate increases sufficiently so that the price is $q_{DF}^{'} < q_G^*$, then banks find it profitable to substitute bond holdings with deposits at the central bank: Liquid assets' availability increases and with it credit and aggregate investment, while liquidity premia and bond spread shrink. Indeed, credit and aggregate investment are maximized, liquidity premia and bond spread are minimized for $q_{DF}^{'} \leq q_G^*$, where q_G^* is the G bonds' equilibrium price defined in Table 1, once $\sum A_i$ is replaced by the aggregate capital holdings of the non-banking sector, $\sum A_i^{NB}$ – the sector that cannot access the deposit facility, unless the deposits at the central bank are made transferable (like "debt certificates").

Thus, by contrast to common wisdom, pursuing credit expansion as well as bond-spread reduction requires an increase in the deposit facility rate rather than deposit-rate cuts, and possibly the transferability of these claims (ECB debt certificates). The key point is that liquidity holdings is an input of the credit/investment process, and liquidity holdings is constrained by the availability of safe/liquid assets. Deposit-facility rate cuts amount to depleting the availability of safe assets. An indeed, the scarcity of safe assets may give scope for the issue of ECB debt certificates, as the financial community points out (Kaminska, 2012).

The increase in the deposit-facility rate and/or ECB debt certificates succeed in expanding credit and aggregate investment in that they *de facto* expand the availability of safe assets, and thus lower the cost of liquidity (the key input of the credit/investment process). The drawback is the increase in the market yields of government debt, which means an increase in governments' cost of debt. We examine below a policy that does not have these drawbacks and does not involve the subsidization which underlines the effectiveness of the ECB's lending policies.

5 Debt Management: Secured Debt

The I bond can be viewed as a bundle of two securities: a safe one that pays $\underline{\alpha}$ for sure, and a risky one that pays 0 with prob. p, and $\overline{\alpha} - \underline{\alpha}$ with the residual prob. $1 - p$. For a liquidity seeking institution the safe component is highly valuable, the risky component constitutes an unwarranted financial investment that subtracts resources to real investment undertaking. Unbundling the security package improves welfare.

It amounts to tranching the debt so as to create a security whose safeness is ensured by sufficient collateral (real assets and tax revenue). Specifically, the former I debt is replaced by $\underline{\alpha}B^I$ safe securities that pay one unit with strict priority, and B^I "risky" securities that make the holders residual claimants – i.e., pay the holder $(\overline{\alpha} - \underline{\alpha})$ with prob. $1 - p$, and 0 with prob. p. These are tailored for "buy and hold" investors, and will be priced $q_r = (1 - p)(\overline{\alpha} - \underline{\alpha})$, that is:

$$q_r = 1 - \underline{\alpha} \qquad \text{(by } (A2))$$

The safe security is a liquidity instrument, perfect substitute of the safe G bond, and as such priced $q_I^s = q_G^*$. The aggregate amount of safe assets is $B^G + \underline{\alpha}B^I$, and bond markets clear for $S \sum I_i = B^G + \underline{\alpha}B^I$, where $I_i = \frac{A_i}{1+q_G S}$. Thus,

$$q_G^* : S \sum \frac{A_i}{1 + q_G S} = B^G + \underline{\alpha}B^I,$$

that is,

$$q_G^* = \frac{\sum A_i - \frac{\left(B^G + \underline{\alpha}B^I\right)}{S}}{B^G + \underline{\alpha}B^I} \quad (>1, \text{ by (A5)}) \tag{13}$$

The revenue per unit of the former security bundle (the former I bond) increases to $q_r + \underline{\alpha}q_G^*$, that is to q_I^u

$$q_I^u = 1 + \underline{\alpha}\left(q_G^* - 1\right) \tag{14}$$

unambiguously greater than the revenue per unit of the security bundle (by (14), (11)).

The spread between the liquidity instruments is nil, and aggregate investment/credit is determined by the supply of safe/liquid assets, $B_G + \underline{\alpha} B_I$,

$$\sum I_i = \frac{B^G + \underline{\alpha} B^I}{S} \quad .$$

That is, aggregate investment increases whenever condition (C_2) fails to hold. And q_G^* increases if (C_2) holds (by comparing (13) with (10)). The key is that unbundling the I bond security package so as to insulate the safe component eliminates the unwarranted financial investment in date 1 idle liquidity that an I bond bundle entails. The greater volatility, the greater date 1 idle liquidity, and with non-perfect pledgeability (positive haircut h), the greater the amount of resources subtracted to real investment, the greater then the opportunity cost of using the I bonds. With sufficiently high volatility, when condition (C_2) fails to hold, in the equilibrium (described by the second and the third row of Table 3), the amount of the former I bonds used as liquidity instruments falls below the outstanding amount B^I, that is $\sum L_i^I < B^I$, and aggregate investment is lower than that that would attain if the entire amount of I bonds outstanding would be used for liquidity purposes, i.e. $\sum I_i < \frac{B^G + \underline{\alpha} B^I}{S}$. Thus, when condition (C_2) fails to hold unbundling expands aggregate investment/credit. If condition (C_2) holds, unbundling has no effect on the equilibrium level of aggregate investment/credit – equilibrium aggregate investment is $\frac{B^G + \underline{\alpha} B^I}{S}$ both with bundling and unbundling (see the first row of Table 3). However, with unbundling the equilibrium price for liquidity, q_G^*, increases - which means that sovereign G's cost of debt lowers. Indeed, under condition (C_2), in the bundling regime (with the former I bonds) $q_I = \widehat{q}_I(q_G)$ and for any given price of liquidity, q_G, aggregate investment/credit is $\sum I_i = \frac{ZB^I + \sum A_i}{1 + q_G S}$, where $Z = -(1 - \underline{\alpha}) h \left(\frac{1 + q_G S}{R} \right) < 0$ (by (8) and (9)). Therefore, for any given price of liquidity, q_G, aggregate investment/credit is greater with unbundling than with bundling. Since the demand for liquid assets is proportional to investment/credit, the price q_G^* that clears the bond market is greater with unbundling than with bundling. This is illustrated in Figure 1. The B curve depicts the demand for safe assets with bundling under condition (C_2), this is $S \sum I_i$, where $\sum I_i \equiv \frac{-(1-\underline{\alpha}) h \left(\frac{1 + q_G S}{R} \right) B^I + \sum A_i}{1 + q_G S}$. The U curve represents the demand for safe assets with unbundling, that is $S \sum I_i$, where

$\sum I_i \equiv \frac{\sum A_i}{1+q_G S}$. Clearly, the market clearing price for liquidity, q_G^*, is greater with unbundling than with bundling – insulating the safe component of the I bond increases the price for liquidity and lowers the cost of debt for both sovereigns I and G.

Proposition 4: *A debt management policy that insulates the safe component of I sovereign debt benefits the issuer, it reduces the issuer's cost of debt, and produces positive externalities: it expands aggregate investment/credit whenever condition (C_2) fails to hold, and lowers sovereign G's cost of debt if (C_2) holds.*

Having assumed that the G bond is safe, pooling the G bond with the safe component of I sovereign bond would still result in a safe security – the equivalent of the *European Safe Bond* advocated by the euro-nomics group for the multi-country euro-zone. Since the total amount of safe assets would still be $B^G + \underline{\alpha} B^I$, the equilibrium would be exactly that derived above (Proposition 4). *European Safe Bonds*, where safety relies on tranching and collateral, are welfare improving. Interestingly, they benefit the *bullet-proof* nations too.

6 Conclusions

We have analyzed the interactions between the financial and the real sector in an environment where liquidity holdings is an input of the credit/investment process. The supply of liquidity is constrained in that income pledgeability limits inside liquidity, and not all sovereign debt is safe/liquid. We have pined down the determinants of liquidity/collateral premia and bond spreads, and with reference to the eurozone: (i) the implications of the ECB policies on liquidity provision and credit, and (ii) the debt management policy that would increase welfare with no need for transfer payments. This amounts to insulate the safe part of the sovereign risky debt – tranching the debt so as to create a security whose safeness is ensured by sufficient collateral (real assets and tax revenue). Such a policy increases welfare and benefits the issuer by reducing its cost of debt.

Having assumed two sovereign debts, G and I, pooling the safe G bond with the safe part of I sovereign debt would result in the *European Safe Bond* advocated by the euro-nomics group for the multi-country euro-zone. Our analysis suggests that *European Safe Bonds*, where safety relies on tranching and collateral, are welfare improving. Interestingly, they benefit the *bullet-proof* nations too.

References

Bhattacharya, S. and A.V. Thakor (1993), "Contemporary Banking Theory", Journal of Financial Intermediation 3, 2-50.

Bolton, P., and O. Jeanne (2011), “Sovereign Default Risk in Financially Integrated Economies”, with Olivier Jeanne, IMF Economic Review Volume 59, Issue 2, 162-194.

Bruno, V., and H.S. Shin (2012), "Capital Flows, Cross-Border Banking and Global Liquidity", working paper Princeton University.

Cornett, M.M., J.J. McNutt, P.E. Strahan, and H. Tehranian (2011), "Liquidity Risk Management and Credit Supply in the Financial Crisis", Journal of Financial Economics 101, 297-312.

Euro-nomics group (2011), "European Safe Bonds", http://euro-nomics.com/esb/

Froot, K, D. Scharfstein, and J. Stein (1993), "Risk Management: Coordinating Corporate Investment and Financing Policies", Journal of Finance 48, 1629-1658.

Holmstrom, B., and J.Tirole (1996), "Modeling Aggregate Liquidity", American Economic Review 86, 187-191.

Holmstrom, B., and J.Tirole (1998), "Private and Public Supply of Liquidity", Journal of Political economy 106, 1-40.

Holmstrom, B., and J.Tirole (2011), *Inside and Outside Liquidity*, MIT Press.

Gorton, G. (2010), *Slapped by the Invisible Hand: The Panic of 2007*, Oxford University Press.

Gorton, G., S. Lewellen and A. Metrick (2012), "The Safe Asset Share", American Economic Review, forthcoming.

International Monetary Fund, *Global Financial Stability Report: The Quest for Lasting Stability*, April 2012

Kaminska, I. (2012), "The Case for ECB debt certificates", FT Alphaville February 20.

Krishnamurthy, A. and A. Vissing-Jorgensen (2012), "The Aggregate Demand for Treasury Debt", Journal of Political Economy 120, 233-267.

Nyborg, K.J. (2011), "The Euro Area Sovereign Debt Crisis: Secure the Debt and Modify Haircuts", Swiss Finance Institute, Occasional Paper Series 11-01

Pozsar, Z. (2011), "Institutional Cash Pools and the Triffin Dilemma of the US Banking System", IMF WP/11/190.

Singh, M. and P. Stella (2012), "Money and Collateral", IMF WP/12/95

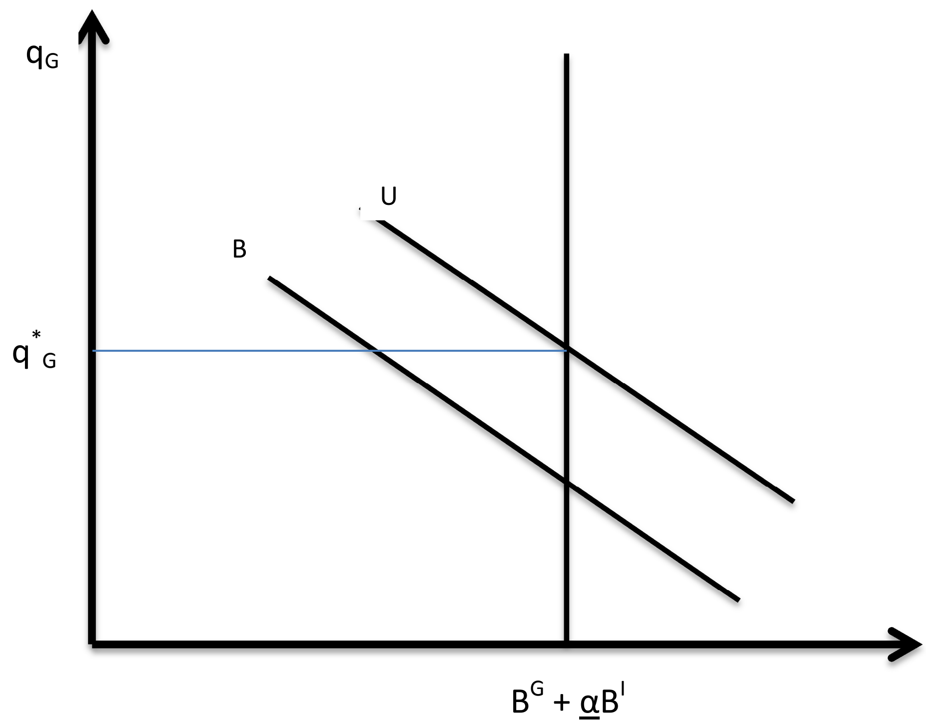
Stein, J. (2010), "Monetary Policy as Financial Stability Regulation", NBER WP 16883.

Strahan, P. (2008), "Liquidity Production in 21st Century Banking", NBER WP 13798.

Thakor, A.V., H. Hong, and S. Greenbaum (1981), "Bank Loan Commitments and Interest Rate Volatility", Journal of Banking and Finance 51, 497-510.

Woodford, M. (1990), "Public Debt and Private Liquidity", American Economic Review 80, 382-388.

Figure 1



Alma Mater Studiorum - Università di Bologna
DEPARTMENT OF ECONOMICS

Strada Maggiore 45
40125 Bologna - Italy
Tel. +39 051 2092604
Fax +39 051 2092664
http://www.dse.unibo.it