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Abstract

The recent years have exhibited a burst in the amount of collaborative activities among firms selling complementary products. This paper aims at providing a rationale for such a large extent of collaboration ties among complementors. To this end, we analyze a game in which the two producers of a certain component have the possibility to form pairwise collaboration ties with each of the two producers of a complementary component. Once ties are formed, each of the four firms decides how much to invest in improving the quality of the match with each possible complementor, under the assumption that collaborating with a complementor makes it cheaper to invest in enhancing match quality with such complementor. Once investment choices have taken place, all firms choose prices for their respective components. Our main finding in this setting is that firms end up forming as many collaboration ties as it is possible, although they would all prefer a scenario where collaboration were forbidden, unlike a social planner.

Key words: Systems Competition, Complementary Products, Interoperability, Collaboration Link, Co-opetition, Exclusivity.

JEL code: L13, M21.

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1 Introduction

The recent decades have witnessed a shift in the competitive paradigm in high-tech industries that is driven to a large extent by the increasing importance of product complementarity. Indeed, cooperation among firms selling complementary products is playing a prominent role in industries such as consumer electronics, semiconductors or telecommunications. More generally, hardware-software industries have exhibited a surge in the extent of cooperation among producers of complementary goods with the aim of improving the interoperability of their respective products (see e.g. Moore 1996, Gawer and Cusumano 2002, Adner 2006, Adner and Kapoor 2010, Gawer and Henderson 2007).[1] Building such *innovation ecosystems* (Adner 2006) with the producers of complementary goods seems to be the key competitive weapon in most high-tech industries, in which the notion of competition has been displaced by that of *co-opetition* (Brandenburger and Nalebuff 1996). A noteworthy feature of collaboration with complementors (i.e., firms selling products that complement each other from the point of view of consumers) is that it is not unusual for firms to collaborate with several complementors that sell substitutes of each other.[2]

A natural question that arises in these settings is whether such extensive collaboration is desirable from the standpoints of firms and consumers. Intuitively, one would be tempted to think that collaboration in improving the interoperability of complementary products is efficient both for the firms involved, and in fact for society as a whole. The purpose of this paper is to show that this intuition may be valid for society, but not for the firms involved, which may be trapped in a prisonner's dilemma. Collaboration may then result in equilibria in which firms are worse off than when firms do not collaborate with complementors. This holds regardless of whether collaboration ties are exclusive or not, under the assumption that collaborating with a complementor makes it easier to enhance interoperability with such complementor.

To formally analyze these issues, we consider a game played by two firms X_1 and X_2 that sell components that (perfectly) complement those sold by firms Y_1 and Y_2 (both of which are also engaged in the game). In this mix-and-match setting (Matutes and Regibeau 1988 and Economides 1989), there are four systems that are contemplated by consumers

[1] The interoperability of the components of which a composite good consists refers to their coherence to work together with each other as a sole system. This is largely related to the absence of conflicts arising from possible incompatibility issues.

[2] To give concrete examples, mobile phone manufacturer Nokia allied first with Intel to develop the MeeGo operating system for smartphones, and later signed an agreement with Microsoft to support the Windows Phone operating system. In addition, the Intel Architecture Lab (IAL) was formed to foster investment in components complementary to Intel's microprocessors by firms that many times competed against each other.

when they make their purchase decisions: X_1Y_1, X_1Y_2, X_2Y_1 and X_2Y_2. The game that we study consists of three stages. In the first stage, each firm decides whether to form a (pairwise) collaboration link with each of its possible complementors (collaboration among firms selling substitute components of a system is not allowed). In the second stage, each firm decides how much to invest in improving the interoperability of its component with each of its complementors.[3] It is assumed that a firm that has formed a collaboration link with a complementor faces lower costs when enhancing interoperability with such complementor. In the third and final stage, each firm decides independently on the price of its component, given past interoperability investments of all the firms involved in the game.

We find in this setting that the (unique) equilibrium collaboration network involves each firm forming (pairwise) collaboration links with its two complementors. If collaboration ties can be formed only in an exclusive manner, then exactly the same forces (subject to the exclusivity restriction) imply that in equilibrium each firm forms a collaboration link with just one of its complementors. In both the exclusive and non-exclusive settings, equilibria exhibit all firms collaborating with at least one complementor, which seems to accord well with the empirical evidence on innovation ecosystems.

These equilibrium outcomes seem quite intuitive, but it is worth noting that intuition may conceal the effect of several forces working at the same time. Thus, two complementors that form a new collaboration link between them benefit from cost sinergies and increase their investments in enhancing the interoperability with each other. This effect conforms to the intuition that one may have on the impact of a new collaboration link. However, two firms that form a new collaboration tie with each other must also bear in mind that the firms not involved in such a tie will strategically react. This strategic effect of collaboration turns out to be positive, and hence reinforces the effect of the cost synergy that arises when two firms start collaborating. Collaboration has a strategic effect in that the firms not involved in the new collaboration tie reduce their investments in each other as well as in the complementor involved in the new collaboration tie. From the viewpoint of the firms that start collaborating, the latter reduction in interoperability investments is harmful, but its impact is lower than the former reduction, which is beneficial, hence the positive strategic effect of collaboration. Factoring all the incentives, we have that it is always desirable to form a new collaboration tie with a complementor with which a firm does not have one. This rat race ends when no more ties are possible, and hence each firm collaborates with as many complementors as it can.

[3] Greater investment in the interoperability of two components is modeled as an enhancement in the (perceived) quality of the system comprising *both* components (e.g., the investment by X_1 in improving interoperability with component Y_2 is specific to Y_2, and has no effect on the interoperability of components X_1 and Y_1).

Although a firm would benefit from its competitor committing not to collaborate with any complementor, it holds that all the firms would be better off if each could make such commitment. Hence, the equilibrium outcome exhibits the features of a prisonner's dilemma *despite all firms are more productive in enhancing interoperability between complementary components*. Being more productive, each firm invests more in interoperability than in the absence of any collaboration amongst complementors. The greater investment leads to higher investment costs, incurred with the aim of vertically differentiating the systems in which a firm participates. Because all other firms act in the same way, firms boost investments but do not manage to vertically differentiate any system, and hence they attain the same profit in the product market as in the absence of collaboration. This growth in investment costs without greater product market profits explains why the equilibrium outcome is inefficient for firms. As for consumers, all of them benefit in equilibrium from the better functionality of every system relative to when firms do not collaborate. This explains why collaboration arising as an equilibrium outcome enhances social welfare relative to the situation in which firms do not collaborate with complementors.

Our result that collaboration in R&D among complementors results in private inefficiencies is in stark contrast with the result that R&D collaboration among firms selling substitute goods may be desirable both for firms and society, as shown in the seminal papers by D'Aspremont and Jacquemin (1988) and Kamien, Muller and Zang (1992). These papers do not consider whether a firm has incentives to collaborate with other firms, a limitation that has been overcome by subsequent work by Bloch (1995) using a coalitions approach, and more recently by Goyal and Moraga-González (2001) using a bilateral link formation approach.[4] Both of these papers show that excessive collaboration may arise in equilibrium. Although we also contend that equilibria displaying collaboration may be inefficient, it is worth noting that the results in Bloch (1995) and Goyal and Moraga-González (2001) are derived for substitute goods, not for complementary goods, as is our focus.

Our paper also contributes to the literature analyzing strategic competition when there exists at least one complementor whose pricing activities interact with those of two firms selling components that constitute substitutes for each other. This literature was pioneered by Economides and Salop (1992) as an extension of early work by Cournot (1838), who analyzed the effect of a merger of two monopolists that produce complementary goods.

The paper by Economides and Salop (1992) examines the effect of cooperation in prices

[4]See Leahy and Neary (1997) for a generalization of the models in D'Aspremont and Jacquemin (1988) and Kamien, Muller and Zang (1992). See also Bloch (2005) for a comprehensive survey that covers strategic network formation games in settings with R&D activities. Finally, it is worth pointing out that Westbrock (2010) builds on Goyal and Moraga-González (2001) and Goyal and Joshi (2003) so as to analyze how asymmetric R&D networks may be socially efficient if collaboration ties are somewhat costly to establish.

(i.e., a merger) between the two existing producers of one of the two components of which a system consists. They consider two scenarios, depending on whether or not the two producers of the complementary component are already cooperating in prices. In our work, we do not analyze price cooperation and, in fact, firms always choose prices noncooperatively regardless of the structure of the collaboration network. The network architecture does have an effect on cooperation in R&D activities, though.[5] Our paper is also related to recent work by Casadesus-Masanell, Nalebuff and Yoffie (2008). Their paper provides conditions under which a firm may benefit from having a new competitor enter with a substitute good whenever there exists a complementor for both the firm under consideration and its new competitor. Our framework differs in that it does not focus on the effects of entry on co-opetive settings, as they do, but rather it examines the incentives to form collaboration links and to invest in enhancing interoperability among complementors.

The remainder of the paper is organized as follows. Section 2 introduces the game we consider. Section 3 characterizes the efficiency properties of the unique equilibrium of the game depending on whether or not collaboration is exclusive. Section 4 shows that results are robust to changes in the solution concept and the implications of collaboration ties. Section 5 deals with concluding remarks.

2 The model

We define a system as a pair of perfectly complementary goods such as hardware and software. The two perfect complements giving rise to a system are called components X and Y. It is assumed that there are two firms costlessly producing component X, X_1 and X_2, and two firms costlessly producing component Y, Y_1 and Y_2.[6] As a result, there are $n = 4$ systems: X_1Y_1, X_1Y_2, X_2Y_1 and X_2Y_2. System X_iY_j $(i, j = 1, 2)$ can be bought by any consumer at price $p_{i,j} = p_{X_i} + p_{Y_j}$, where p_{X_i} and p_{Y_j} respectively denote the prices at which components X_i and Y_j are sold. Whenever there is no risk of confusion, we will write p_{ij} instead of $p_{i,j}$ for system X_iY_j. Also, firms X_1 and X_2 are typically referred to as the complementors of firms Y_1 and Y_2, and vice versa.

It is assumed that there exists a unit mass of consumers willing to buy at most one system. System X_iY_j is assumed to create a gross utility of $v_{i,j}$ to any consumer (again,

[5] There is a recent literature on (pure and mixed) bundling by firms that produce two perfectly complementary components in competition with firms that produce just one of these components (see e.g. Denicolò 2000 and Choi 2008). The reason why this stream of research building on Economides and Salop (1992) is not related to our work is that we do not consider bundling, an issue that certainly deserves a separate analysis beyond the scope of our paper.

[6] That production is costless is without loss of generality if the marginal cost of production is constant and the fixed costs of operation are not too large.

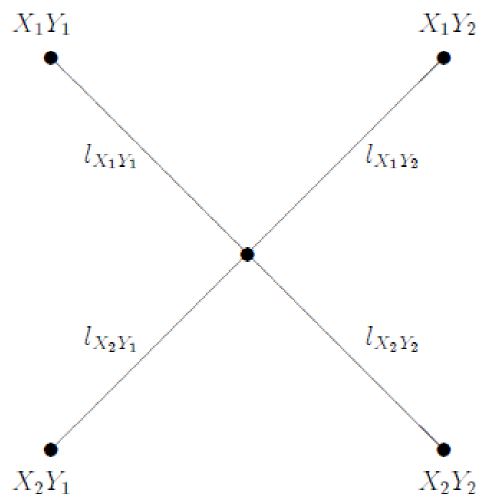
we will typically write v_{ij} instead of $v_{i,j}$). The gross utility v_{ij} is largely the outcome of choices by firms X_i and Y_j. More specifically, for some given scalar $v > 0$, we have that $v_{ij} = v + x_i^j + y_j^i$, where x_i^j is firm X_i's R&D investment in improving the quality of the match with firm Y_j's component and y_j^i is firm Y_j's R&D investment in improving the quality of the match with firm X_i's component.[7] Thus, the investment variables x_i^j and y_j^i affect the vertical attributes of system X_iY_j. Given their system-specificity, they can be viewed as investments in improving the interoperability of components X_i and Y_j, although other interpretations are possible and may be more appealing depending on the context.

Besides (possibly) being vertically differentiated, systems are perceived by consumers as being horizontally differentiated in an exogenous manner. To model consumer preferences over horizontally differentiated systems, we follow Chen and Riordan (2007) in using their "spokes" model of nonlocalized differentiation. Thus, each of the $N = 4$ systems desired by consumers is represented by a point at the origin of a line of length 1/2, a line which is denoted by $l_{X_iY_j}$ for system X_iY_j $(i, j = 1, 2)$. The other end of a line is called its terminal, and it is assumed that the terminals of all lines meet at a point called the center (see Figure 1). All the existing consumers are uniformly distributed along the four lines. A consumer who is located on line $l_{X_iY_j}$ at distance $d_{X_iY_j} \in [0, 1/2]$ from system X_iY_j must incur a transportation/disutility cost of $td_{X_iY_j}$ when buying X_iY_j, where $t \geq 0$ is a unit transportation cost. The same consumer must incur transportation cost $t(1 - d_{X_iY_j})$ when purchasing any other system (since $l_{X_iY_j} = 1/2$ for all $i, j = 1, 2$). It is assumed that X_iY_j is the preferred system for any consumer on $l_{X_iY_j}$, and any other system has probability $1/(N-1) = 1/3$ of constituting the benchmark against which X_iY_j is to be compared by a consumer on $l_{X_iY_j}$. A system that is not deemed as preferred or as a benchmark for a consumer is assumed to yield no utility to such a consumer. This assumption completes the description of the spokes model we use for modeling the horizontal attributes of systems.[8]

[7] See Goyal, Konovalov and Moraga-González (2008) for another setting with relationship-specific actions.

[8] Note that although in the most general version of the spokes model there are $N \geq n$ systems over which preferences are defined, we have let $N = n$ for the sake of simplicity. This means that we have assumed that there is no uncommercialized system that is possibly the object of desire by (some) consumers.

Figure 1 : The Spoke Model



Given these features of firms and consumers, we study a three-stage game. In the first stage, firms X_i $(i = 1, 2)$ simultaneously form pairwise collaboration links with firms Y_j $(j = 1, 2)$. We let $g_{ij} = 1$ if a (costless) collaboration link between X_i and Y_j is formed and $g_{ij} = 0$ otherwise, with the convention that $g_{ji} = g_{ij}$. We denote the network (i.e., the set of collaboration links) by g, that is, $g = \{g_{11}, g_{12}, g_{21}, g_{22}\} \in \{0, 1\}^4$. Note that in principle we allow a firm to form more than one collaboration link with its complementors (e.g., it may be possible that $g_{i1} = g_{i2} = 1$ for some $i \in \{1, 2\}$).

In the second stage of the game we consider, we assume that firm X_i chooses x_i^j at the same time as firm Y_j chooses y_j^i $(i, j = 1, 2)$. Given network g and some parameter $\gamma \in (0, 1)$, investments of x_i^1 and x_i^2 by firm X_i result in an R&D cost equal to $C_{X_i}(x_i^1, x_i^2 \,|g) = \gamma^{g_{i1}}(x_i^1)^2 + \gamma^{g_{i2}}(x_i^2)^2$, whereas investments of y_j^1 and y_j^2 by firm Y_j result in an R&D cost equal to $C_{Y_j}(y_j^1, y_j^2 \,|g) = \gamma^{g_{1j}}(y_j^1)^2 + \gamma^{g_{2j}}(y_j^2)^2$.[9] Hence, collaboration between firms X_i and Y_j yields that it is easier/cheaper for any of them to enhance the quality of the match with the component provided by the complementor. This captures in a simple manner useful but costless information exchanges between firms X_i and Y_j with the aim of improving the interoperability of system X_iY_j. For this reason, the inverse of parameter γ can be understood as representing the extent of information sharing and its economic relevance: lowering the value of γ represents in our model more exchange of technically useful information among collaborators.

In the third and last stage, prices p_{X_i} and p_{Y_j} are set simultaneously in the standard noncooperative manner, and consumers make their purchase decisions given p_{ij} for $i, j = 1, 2$.

The solution concept is the same as in Goyal and Moraga-González (2001). Thus, for

[9] For example, if $g = \{1, 0, 0, 0\}$, then $C_{X_1}(x_1^1, x_1^2 \,|g) = \gamma(x_1^1)^2 + (x_1^2)^2$, $C_{X_2}(x_2^1, x_2^2 \,|g) = (x_2^1)^2 + (x_2^2)^2$, $C_{Y_1}(y_1^1, y_1^2 \,|g) = \gamma(y_1^1)^2 + (y_1^2)^2$ and $C_{Y_2}(y_2^1, y_2^2 \,|g) = (y_2^1)^2 + (y_2^2)^2$.

each possible g, we will look for subgame perfect Nash equilibria, which will give equilibrium payoffs given g. In order to solve for the equilibrium network structure in the first stage, we will use the pairwise stability notion proposed by Jackson and Wolinsky (1996). This concept is very weak, and aims at capturing (possibly) complex communication and negotiation activities that would be hard to capture through noncooperative game theory.

Introducing the concept of pairwise stability requires some notation. In particular, we let $g - g_{ij}$ denote the network that results from suppressing the collaboration link between firms X_i and Y_j in network g. We also let $g + g_{ij}$ denote the network that results from adding a collaboration link between firms X_i and Y_j in network g. Denoting the equilibrium payoffs obtained by firm X_i and Y_j given network g by $\Pi^*_{X_i}(g)$ and $\Pi^*_{Y_j}(g)$, network g would be pairwise stable if the following two conditions held for all $i, j \in \{1, 2\}$: (i) $\Pi^*_{X_i}(g) \geq \Pi^*_{X_i}(g - g_{ij})$ and $\Pi^*_{Y_j}(g) \geq \Pi^*_{Y_j}(g - g_{ij})$ for $g_{ij} = 1$ and; (ii) $\Pi^*_{X_i}(g + g_{ij}) \geq \Pi^*_{X_i}(g)$ implies that $\Pi^*_{Y_j}(g + g_{ij}) < \Pi^*_{Y_j}(g)$. The first condition requires that neither X_i nor Y_j have an incentive to unilaterally break their collaboration relationship (provided it exists). In turn, the second condition requires that, if firms X_i and Y_j are not linked to each other, then a desire by X_i to form a collaboration link with Y_j should not be reciprocal. It is worth noting that the results we derive still hold if the network is required to be pairwise Nash stable, that is, if a firm is allowed to unilaterally break more than one collaboration link at a time.

3 Resolution of the model

3.1 Third stage

As is standard, we solve the last two stages of the game by working backwards. So assume that first-stage and second-stage choices lead to a gross valuation of v_{ij} for system X_iY_j, $i, j = 1, 2$. We first derive the demand functions for each system and then we find out profits attained by each firm as a function of v_{11}, v_{12}, v_{21} and v_{22}. It is assumed throughout that v is large enough so that the market is always fully covered and all firms make positive sales. If collaboration between firms X_i and Y_j drove a system in which none of them participates out of the market, then there would be an additional incentive to form collaboration links. It is in this sense that we make the weakest case for collaboration to take place, and still find that it emerges in equilibrium.

In order to characterize the demand functions of each system, let $l_{X_iY_j} + l_{X_{i'}Y_{j'}} = \{d : d \in l_{X_iY_j} \cup l_{X_{i'}Y_{j'}}\}$ ($i, j, i', j' = 1, 2$, with $i \neq i'$ or $j \neq j'$ or both) denote the set consisting of all the points that belong to either line $l_{X_iY_j}$ or $l_{X_{i'}Y_{j'}}$ or both. In defining $l_{X_iY_j} + l_{X_{i'}Y_{j'}}$, we establish the convention that $i \leq i'$ and $j \leq j'$.[10] To find out the demand for system X_1Y_1, consider a consumer who happens to be on $l_{X_1Y_1} + l_{X_1Y_2}$. This occurs either because X_1Y_1 is her preferred system and X_1Y_2 is the benchmark, or because X_1Y_2 is her preferred system and X_1Y_1 is the benchmark. The consumer will be indifferent between both systems if her distance $d_{11}^{12} \in [0, 1]$ from X_1Y_1 is given by $v_{11} - p_{11} - td_{11}^{12} = v_{12} - p_{12} - t(1 - d_{11}^{12})$,[11] that is, if

$$d_{11}^{12} = \frac{t + v_{11} - v_{12} + p_{12} - p_{11}}{2t}.$$

Because the measure of consumers between the locations of systems X_1Y_1 and X_1Y_2 is $2/N$, we then have that the number of consumers who prefer X_1Y_1 over X_1Y_2 given p_{11} and p_{12} is $2d_{11}^{12}/N$. Similarly, the number of consumer who prefer X_1Y_1 over X_2Y_j ($j = 1, 2$) can be shown to be $2d_{11}^{2j}/N$, where

$$d_{11}^{2j} = \frac{t + v_{11} - v_{2j} + p_{2j} - p_{11}}{2t}.$$

Conditional upon X_1Y_1 being the preferred system or the benchmark one, we have that X_1Y_2, X_2Y_1 and X_2Y_2 have each probability $1/(N - 1) = 1/3$ of being the system with respect to which X_1Y_1 is to be assessed by consumers. It then follows that demand for X_1Y_1 is

$$Q_{11} = \frac{2(d_{11}^{12} + d_{11}^{21} + d_{11}^{22})}{N(N - 1)}.$$

Simple algebra yields that

$$Q_{11} = \frac{3t + 3v_{11} - v_{12} - v_{21} - v_{22} - 3p_{11} + p_{12} + p_{21} + p_{22}}{12t}.$$

Similar steps lead to the following demand for system X_iY_j ($i, j = 1, 2$):

$$Q_{ij} = \frac{3t + 3v_{i,j} - v_{3-i,j} - v_{i,3-j} - v_{3-i,3-j} - 3p_{i,j} + p_{3-i,j} + p_{i,3-j} + p_{3-i,3-j}}{12t}.$$

Recalling that $p_{i,j} = p_{X_i} + p_{Y_j}$ and letting $Q_{X_i} \equiv Q_{i1} + Q_{i2}$ denote X_i's demand, we have that

$$Q_{X_i}(p_{X_i}, p_{X_{3-i}}) = \frac{3t + v_{i,1} + v_{i,2} - v_{3-i,1} - v_{3-i,2} - 2p_{X_i} + 2p_{X_{3-i}}}{6t}.$$

We have made the arguments of Q_{X_i} explicit to highlight that the volume of sales by firm X_i does not depend on how any complementary product is priced. Under full market coverage,

[10] Observe from the definition of $l_{X_iY_j} + l_{X_{i'}Y_{j'}}$ that $i \neq i'$ or $j \neq j'$ or both, so we cannot have both $i = i'$ and $j = j'$.

[11] Recall that the set $l_{X_1Y_1} + l_{X_1Y_2}$ has unit (Lebesgue) measure.

different prices by Y_1 and Y_2 just affect with which component X_i wishes to be matched, but firm X_i's demand solely depends on p_{X_i} and $p_{X_{3-i}}$. One can similarly find out that

$$Q_{Y_j}(p_{Y_j}, p_{Y_{3-j}}) = \frac{3t + v_{1,j} + v_{2,j} - v_{1,3-j} - v_{2,3-j} - 2p_{Y_j} + 2p_{Y_{3-j}}}{6t},$$

where $Q_{Y_j} \equiv Q_{1j} + Q_{2j}$.

Firms X_1 and X_2 choose p_{X_1} and p_{X_2} to maximize $\pi_{X_1}(p_{X_1}, p_{X_2}) \equiv p_{X_1} Q_{X_1}(p_{X_1}, p_{X_2})$ and $\pi_{X_2}(p_{X_2}, p_{X_1}) \equiv p_{X_2} Q_{X_2}(p_{X_2}, p_{X_1})$, respectively. Using the strict concavity of profit functions, we have that the solution to the following system of equations delivers the equilibrium prices for firms X_1 and X_2:

$$3t + v_{11} + v_{12} - v_{21} - v_{22} - 4p_{X_1} + 2p_{X_2} = 0 \tag{1}$$

and

$$3t + v_{21} + v_{22} - v_{11} - v_{12} - 4p_{X_2} + 2p_{X_1} = 0. \tag{2}$$

The system consisting of equations (1) and (2) has the following solution:

$$p^*_{X_i} = \frac{9t + v_{i,1} + v_{i,2} - v_{3-i,1} - v_{3-i,2}}{6}, \; i = 1, 2.$$

Similarly, one can show that

$$p^*_{Y_j} = \frac{9t + v_{1,j} + v_{2,j} - v_{1,3-j} - v_{2,3-j}}{6}, \; j = 1, 2.$$

We then have that the sales of system X_iY_j are

$$Q^*_{ij} = \frac{9t + 5v_{i,j} + v_{3-i,3-j} - 3v_{i,3-j} - 3v_{3-i,j}}{36t}.$$

The profit that system X_iY_j generates for firm X_i ($i, j = 1, 2$) is $\pi^j_{X_i} \equiv p^*_{X_i} Q^*_{ij}$, so recalling that $v_{ij} = v + x^j_i + y^i_j$, we can write it as a function of second-stage choices:

$$\begin{aligned} \pi^j_{X_i} \;\; = \;\; & \frac{1}{216t}(9t + x^j_i + x^{3-j}_i - x^j_{3-i} - x^{3-j}_{3-i} + y^i_j + y^i_{3-j} - y^{3-i}_j - y^{3-i}_{3-j}) \times \\ & (9t + 5x^j_i + x^{3-j}_{3-i} - 3x^j_{3-i} - 3x^{3-j}_i + 5y^i_j + y^{3-i}_{3-j} - 3y^i_{3-j} - 3y^{3-i}_j). \end{aligned}$$

Similarly, the profit that system X_iY_j generates for firm Y_j can be written as follows:

$$\begin{aligned}\pi^i_{Y_j} &= \frac{1}{216t}(9t + x^j_i + x^j_{3-i} - x^{3-j}_i - x^{3-j}_{3-i} + y^i_j + y^{3-i}_j - y^i_{3-j} - y^{3-i}_{3-j}) \times \\ &\quad (9t + 5x^j_i + x^{3-j}_{3-i} - 3x^j_{3-i} - 3x^{3-j}_i + 5y^i_j + y^{3-i}_{3-j} - 3y^i_{3-j} - 3y^{3-i}_j).\end{aligned}$$

Letting $\pi^*_{X_i} \equiv \pi^1_{X_i} + \pi^2_{X_i}$ and $\pi^*_{Y_j} \equiv \pi^1_{Y_j} + \pi^2_{Y_j}$ respectively denote the overall profits made by firms X_i and Y_j, it is easy to show for $i, j = 1, 2$ that

$$\pi^*_{X_i} = \frac{(9t + x^1_i + x^2_i - x^1_{3-i} - x^2_{3-i} + y^i_1 + y^i_2 - y^{3-i}_1 - y^{3-i}_2)^2}{108t},$$

and

$$\pi^*_{Y_j} = \frac{(9t + x^j_1 + x^j_2 - x^{3-j}_1 - x^{3-j}_2 + y^1_j + y^2_j - y^1_{3-j} - y^2_{3-j})^2}{108t}.$$

The following is worth noting for $i, j = 1, 2$:

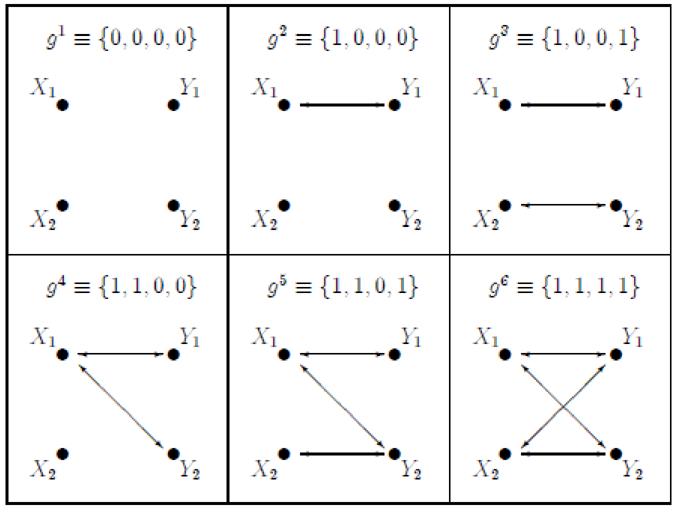
Remark 1 *We have that* $\frac{\partial^2 \pi^*_{X_i}}{\partial x^j_i \partial x^{3-j}_i} = \frac{\partial^2 \pi^*_{X_i}}{\partial x^j_i \partial y^i_j} = \frac{\partial^2 \pi^*_{X_i}}{\partial x^j_i \partial y^i_{3-j}} > 0$ *and* $\frac{\partial^2 \pi^*_{X_i}}{\partial x^j_i \partial x^j_{3-i}} = \frac{\partial^2 \pi^*_{X_i}}{\partial x^j_i \partial x^{3-j}_{3-i}} = \frac{\partial^2 \pi^*_{X_i}}{\partial x^j_i \partial y^{3-i}_j} = \frac{\partial^2 \pi^*_{X_i}}{\partial x^j_i \partial y^{3-i}_{3-j}} < 0$. *Similarly,* $\frac{\partial^2 \pi^*_{Y_j}}{\partial y^i_j \partial y^{3-i}_j} = \frac{\partial^2 \pi^*_{Y_j}}{\partial y^i_j \partial x^j_i} = \frac{\partial^2 \pi^*_{Y_j}}{\partial y^i_j \partial x^j_{3-i}} > 0$ *and* $\frac{\partial^2 \pi^*_{Y_j}}{\partial y^i_j \partial y^i_{3-j}} = \frac{\partial^2 \pi^*_{Y_j}}{\partial y^i_j \partial y^{3-i}_{3-j}} = \frac{\partial^2 \pi^*_{Y_j}}{\partial y^i_j \partial x^{3-j}_i} = \frac{\partial^2 \pi^*_{Y_j}}{\partial y^i_j \partial x^{3-j}_{3-i}} < 0.$

Remark 1 will be heavily used in what follows, so it is worthwhile expressing in words what it means. Essentially, a firm's incentive to invest in enhancing the match quality with any one of its complementors becomes less intense as there is less investment in any of the systems in which the firm participates. This incentive is also weakened as there is more investment in any of the systems in which it does not participate.

3.2 Second and first stages

We now consider the investment subgames for each of the possible network structures arising from the first stage. Up to a relabeling of firms, there are six network structures that should be considered (see Figure 2): $g^1 \equiv \{0, 0, 0, 0\}$, $g^2 \equiv \{1, 0, 0, 0\}$, $g^3 \equiv \{1, 0, 0, 1\}$, $g^4 \equiv \{1, 1, 0, 0\}$, $g^5 \equiv \{1, 1, 0, 1\}$ and $g^6 \equiv \{1, 1, 1, 1\}$. Besides characterizing equilibrium play for each, we also show which one emerges as the unique (pairwise) stable network, thus effectively providing a complete resolution of the network formation game.

Figure 2 : Network Structures



We start by analyzing networks structures when no firm can have more than one collaboration link (i.e., $g = g^1$, or $g = g^2$, or $g = g^3$), which may be due to exclusivity, for instance.[12] We then consider network architectures when firms can have more than one collaboration tie (i.e., $g = g^4$, or $g = g^5$, or $g = g^6$). At this point, it is useful to define the following functions:

$$\Pi_{X_i}(x_i^1, x_i^2, x_{3-i}^1, x_{3-i}^2, y_j^1, y_j^2, y_{3-j}^1, y_{3-j}^2 \,|g) \equiv \pi_{X_i}^* - C_{X_i}(x_i^1, x_i^2 \,|g)$$

and

$$\Pi_{Y_j}(y_j^1, y_j^2, y_{3-j}^1, y_{3-j}^2, x_i^1, x_i^2, x_{3-i}^1, x_{3-i}^2 \,|g) \equiv \pi_{Y_j}^* - C_{Y_j}(y_j^1, y_j^2 \,|g).$$

Given network architecture g, we then have that firm X_i ($i = 1,2$) chooses $x_i^1 \geq 0$ and $x_i^2 \geq 0$ to maximize Π_{X_i}, while firm Y_j ($j = 1,2$) chooses $y_j^1 \geq 0$ and $y_j^2 \geq 0$ to maximize Π_{Y_j}, where we have suppressed the arguments of the functions to avoid clutter. We also recall that all second-stage choices are made simultaneously. Lastly, we note that we will avoid equilibrium inexistence by making t large enough.[13]

3.2.1 Network structures under exclusivity

We first consider network $g = g^1 \equiv \{0,0,0,0\}$. Assuming that $t > 1/54$ to ensure that payoff functions are strictly concave, we have that the unique equilibrium is symmetric, and

[12] The formation of exclusive collaboration links may be due to explicit or implicit contracting requirements, or to highly competitive conditions that preclude several complementors from being willing to collaborate with the same firm.

[13] The inexistence problem is already present in Casadesus-Masanell, Nalebuff and Yoffie (2008), who get around it by introducing vertical differentiation.

it is characterized by each firm investing $x_i^j(g^1) = y_j^i(g^1) = 1/12$ $(i, j = 1, 2)$ in trying to (unilaterally) improve the match with each complementary component. Equilibrium profits for each firm under $g = g^1$ are

$$\Pi^*_{X_i}(g^1) = \Pi^*_{Y_j}(g^1) = \frac{54t - 1}{72},$$

which are positive for $t > 1/54$.

We now turn to the case in which there is just one collaboration link, i.e., $g = g^2 \equiv \{1, 0, 0, 0\}$. If one makes the assumption for $g = g^2$ that $t > 1/(27\gamma)$ to ensure that payoff functions are strictly concave and investment levels are positive, it holds that the unique equilibrium is characterized by the following investments in match quality: $x_1^1(g^2) = y_1^1(g^2) = \dfrac{27t - 1}{6(54t\gamma - 1 - \gamma)}$, $x_1^2(g^2) = y_1^2(g^2) = \dfrac{\gamma\,(27t - 1)}{6(54t\gamma - 1 - \gamma)}$ and $x_2^1(g^2) = x_2^2(g^2) = y_2^1(g^2) = y_2^2(g^2) = \dfrac{27t\gamma - 1}{6(54t\gamma - 1 - \gamma)}$. Equilibrium profits under $g = g^2$ are then

$$\Pi^*_{X_1}(g^2) = \Pi^*_{Y_1}(g^2) = \frac{\gamma(108t\gamma - 1 - \gamma)(27t - 1)^2}{36(54t\gamma - 1 - \gamma)^2}$$

and

$$\Pi^*_{X_2}(g^2) = \Pi^*_{Y_2}(g^2) = \frac{(54t - 1)(27t\gamma - 1)^2}{18(54t\gamma - 1 - \gamma)^2}.$$

All profits are positive for $t > 1/(27\gamma)$, as can be easily demonstrated. It can also be shown that this parametric assumption ensures that equilibrium quantities of each system are positive, as required by the full market coverage assumption we have made. We are now in a position to prove the following result.

Lemma 2 *Network $g = g^1$ cannot arise in equilibrium for $t > 1/(27\gamma)$.*

Proof. Noting that $g^2 = g^1 + g_{11}$, it holds that $\Pi^*_{X_1}(g^2) = \Pi^*_{Y_1}(g^2) > \Pi^*_{Y_1}(g^1) = \Pi^*_{X_1}(g^1)$ for $t > 1/(27\gamma)$, and firms X_1 and Y_1 would mutually benefit from forming a link with each other, so $g = g^1$ cannot be a stable network. ■

The result follows because both firms X_1 and Y_1 would benefit from forming a collaboration tie if the network were $g = g^1$. To understand why this happens, note that, relative to the case in which $g = g^1$, there arise several incentives for firms X_1 and Y_1 if $g = g^2$. On the one hand, the investment cost sinergy leads them to increase x_1^1 and y_1^1, which in turn creates pressure towards increasing x_1^2 and y_1^2 in the light of Remark 1. On the other, the negative impact of higher x_1^2 on firm X_2's marginal payoff is completely offset by the positive impact of higher y_1^2. Taking this into consideration as well as the fact that system X_1Y_1

is stronger, it then holds that firm X_2 prefers to lower x_2^1 in such a way that $x_2^1 + y_1^2$ does not vary with respect to the level under $g = g^1$. In an analogous fashion, total investment in system X_1Y_2 does not vary because firm Y_2 lowers y_2^1 in way that offsets the increase in x_1^2. The strength of systems X_1Y_2 and X_2Y_1 is then unaffected, but firms X_2 and Y_2 end up respectively decreasing x_2^2 and y_2^2 because system X_1Y_1 becomes stronger. Interestingly, firm X_2 reduces x_2^1 and x_2^2 by the same amount (and analogously for firm Y_2 with e_2^1 and e_2^2). The reason why this happens is that firm X_2 equally benefits from investing in the match with Y_1 or Y_2, so we must have that both x_2^1 and x_2^2 are reduced by the same amount because the strict convexity of R&D costs implies that it is more efficient to spread effort over two complementors rather than just one.

In short, g^1 is not a stable network because firms X_1 and Y_1 would mutually benefit from forming a link. This incentive to form a link arises because of the cost synergy that is fostered by their collaboration and the positive strategic effect of such collaboration.[14] Despite firms X_1 and Y_1 benefit from the fact that firms X_2 and Y_2 reduce their investment in each other, firms X_2 and Y_2 exploit the incentive that firms X_1 and Y_1 have to invest more in systems X_1Y_2 and X_2Y_1 by cutting down their respective investments in such systems, which harms X_1 and Y_1 a bit. Overall, the strategic reaction of firms X_2 and Y_2 benefits firms X_1 and Y_1, thus reinforcing the positive direct effect of cost sinergies exploited by X_1 and Y_1.

We conclude this subsection by analyzing what happens if each firm has one, and only one, collaboration link, i.e., $g = g^3 \equiv \{1, 0, 0, 1\}$. Under the assumption that $t > (1+\gamma)/(108\gamma)$, all payoff functions are strictly concave, and the unique equilibrium is symmetric, being characterized by the following investments in match quality: $x_1^1(g^3) = x_2^2(g^3) = y_1^1(g^3) = y_2^2(g^3) = 1/12\gamma$ and $x_1^2(g^3) = x_2^1(g^3) = y_1^2(g^3) = y_2^1(g^3) = 1/12$. Equilibrium profits under $g = g^3$ are

$$\Pi^*_{X_i}(g^3) = \Pi^*_{Y_j}(g^3) = \frac{108t\gamma - 1 - \gamma}{144\gamma},$$

which are positive for $t > (1+\gamma)/(108\gamma)$. This parametric assumption also yields that quantity sold of each system is positive in equilibrium. We then have all the elements to rule out $g = g^2$ as an equilibrium outcome.

[14] We compute the direct (profit) effect of collaboration between firms X_1 and Y_1 through the following thought experiment. Upon collaborating, both of these firms react to the change in their investment costs taking into account the reactions of each other in an optimal manner, but keeping the investments of firms X_2 and Y_2 as in $g = g^1$ (i.e., X_2 and Y_2 do not react to the change in the network architecture). This yields some profit for firms X_1 and Y_1, which after respectively subtracting $\Pi^*_{X_1}(g^1)$ and $\Pi^*_{Y_1}(g^1)$, gives the direct effect of collaboration for each of them. The difference between $\Pi^*_{X_1}(g^2) - \Pi^*_{X_1}(g^1)$ and the direct effect for firm X_1 then gives the strategic (profit) effect of collaboration for this firm, that is, how its profits change because of the reaction of firms X_2 and Y_2 to collaboration between X_1 and Y_1. One can compute the strategic (profit) effect of collaboration for firm Y_1 in an analogous manner.

Lemma 3 *Network $g = g^2$ cannot arise in equilibrium for $t > (1+\gamma)/(108\gamma)$.*

Proof. Noting that $g^3 = g^2 + g_{22}$, it holds that $\Pi^*_{X_2}(g^3) = \Pi^*_{Y_2}(g^3) > \Pi^*_{Y_2}(g^2) = \Pi^*_{X_2}(g^2)$ for $t > (1+\gamma)/(108\gamma)$, and firms X_2 and Y_2 would mutually benefit from forming a link with each other, so $g = g^2$ cannot be a stable network. ■

Starting from $g = g^2$, let us consider the incentive for firms X_2 and Y_2 to form a tie, an incentive that is somewhat similar to the one that firms X_1 and Y_1 to form a link starting from $g = g^1$. Of course, the sinergistic effect of collaboration leads to higher x_2^2 and y_2^2. In the light of Remark 1, though, the increases in x_2^2 and y_2^2 also create an incentive for firms X_2 and Y_2 to respectively increase x_2^1 and y_2^1. The higher x_2^1 has a negative impact on firm X_1's marginal payoff, whereas the higher y_2^1 has a positive impact on firm X_1's marginal payoff (by Remark 1). Taking into account that both of these effects cancel out and that system X_2Y_2 is stronger, it follows from Remark 1 that firm X_1 prefers to lower x_1^2, and it does it in such a way that $x_1^2 + y_2^1$ remains unchanged with respect to the level under $g = g^2$. Similarly, total investment $x_2^1 + y_1^2$ in system X_2Y_1 does not vary because firm Y_1 lowers y_1^2 so as to offset the increase in x_2^1. Even though systems X_2Y_1 and X_1Y_2 are neither strengthened nor weakened, the fact that system X_2Y_2 is stronger induces firms X_1 and Y_1 to respectively decrease x_1^1 and y_1^1. Again, collaboration by firms X_2 and Y_2 results in a positive strategic effect that reinforces the cost sinergies that arise because of their collaboration, and hence X_2 and Y_2 mutually benefit from forming a link with each other.

In the light of Lemmata 2-3, it is clear that the unique stable network that arises when collaboration is exclusive is $g = g^3$. However, each firm would be better off if collaboration were forbidden or impossible. Gross profits are the same under $g = g^1$ and $g = g^3$ because firms do not change their pricing and end up selling the same (of course, systems X_1Y_1 and X_2Y_2 are bought more under $g = g^3$, but this is at the expense of X_1Y_2 and X_2Y_1). However, total investment costs are greater under $g = g^3$ than under $g = g^1$ (more precisely, $(1+\gamma)/(144\gamma)$ vs. $1/72$), which explains why a firm's payoff decreases when going from $g = g^1$ to $g = g^3$. Firms are worse off, but consumers are much better off under $g = g^3$ than under $g = g^1$, and as a result social welfare increases when going from $g = g^1$ to $g = g^3$. In particular, we have the following result.

Proposition 4 *Let $t > 1/(27\gamma)$ and suppose that collaborating with a complementor precludes a firm from collaborating with the complementor's competitor. Then:*

(i) The unique (up to a relabeling of firms) equilibrium network is $g^* = \{1, 0, 0, 1\}$.

(ii) In equilibrium, firm X_1 chooses to invest $x_1^1(g^*) = 1/(12\gamma)$ in improving the quality of its match with complementor Y_1, whereas it chooses to invest $x_1^2(g^*) = 1/12$ in improving the quality of its match with complementor Y_2. In turn, firm Y_1 chooses to invest $y_1^1(g^*) =$

$1/(12\gamma)$ *in improving the quality of its match with complementor* X_1*, whereas it chooses to invest* $y_1^2(g^*) = 1/12$ *in improving the quality of its match with complementor* X_2*. In addition, each firm earns a payoff of* $(108t\gamma - 1 - \gamma)/(144\gamma)$*.*

(iii) The equilibrium network $g^* = \{1, 0, 0, 1\}$ *results in a payoff for each firm smaller than that achieved when* $g = \{0, 0, 0, 0\}$*, even though* $g^* = \{1, 0, 0, 1\}$ *is socially preferred over* $g = \{0, 0, 0, 0\}$*.*

Proof. Both for $g = g^1$ and $g = g^3$, it holds that $p^*_{X_1} = p^*_{X_2} = p^*_{Y_1} = p^*_{Y_2} = 3t/2$, so $p^*_{11} = p^*_{12} = p^*_{21} = p^*_{21} = 3t$. In addition, the number of consumers purchasing system X_iY_j $(i, j = 1, 2)$ under $g = g^1$ is $Q^*_{ij}(g^1) = 1/4$. However, the number of consumers purchasing systems X_1Y_1 and X_2Y_2 under $g = g^3$ is $Q^*_{11}(g^3) = Q^*_{22}(g^3) = \dfrac{1}{4} + \dfrac{1-\gamma}{36t\gamma}$, whereas the number of consumers purchasing systems X_1Y_2 and X_2Y_1 under $g = g^3$ is $Q^*_{12}(g^3) = Q^*_{21}(g^3) = \dfrac{1}{4} - \dfrac{1-\gamma}{36t\gamma}$. Taking into account that line $l_{X_iY_j}$ $(i, j = 1, 2)$ has a length of 1/2 and that that there exists a unit mass of consumers uniformly spread all over the four existing lines, the aggregate consumer surplus under $g = g^1$ is

$$CS(g^1) = 4\left[\frac{1}{2}\left(v + \frac{1}{12} + \frac{1}{12} - 3t - t\int_0^{\frac{1}{2}} zdz\right)\right],$$

while the aggregate consumer surplus under $g = g^3$ is

$$\begin{aligned} CS(g^3) &= 2\left[\frac{1}{2}\left(v + \frac{1}{12\gamma} + \frac{1}{12\gamma} - 3t - t\int_0^{\frac{1}{2}+\frac{1-\gamma}{18t\gamma}} zdz\right)\right] + \\ & \quad 2\left[\frac{1}{2}\left(v + \frac{1}{12} + \frac{1}{12} - 3t - t\int_0^{\frac{1}{2}-\frac{1-\gamma}{18t\gamma}} zdz\right)\right]. \end{aligned}$$

Because $54t\gamma > 27t\gamma > 1 > \gamma$ implies that

$$CS(g^3) - CS(g^1) = \frac{(1-\gamma)(54t\gamma + \gamma - 1)}{324t\gamma^2} > 0$$

and

$$\sum_{i=1}^{2}[\Pi^*_{X_i}(g^3) - \Pi^*_{X_i}(g^1)] + \sum_{j=1}^{2}[\Pi^*_{Y_j}(g^3) - \Pi^*_{Y_j}(g^1)] = -\frac{1-\gamma}{36\gamma} < 0,$$

it follows from the fact that $45t\gamma > 27t\gamma > 1$ that social surplus increases by $\dfrac{(1-\gamma)(45t\gamma + \gamma - 1)}{324t\gamma^2} > 0$ when going from $g = g^1$ to $g = g^3$ despite firms are worse off than when collaboration is forbidden or impossible. ■

Given that the social welfare comparison is entirely driven by the increase in consumer surplus, it is worthwhile explaining why it happens. Note first that, under $g = g^3$, the investment in enhancing the match quality with a complementor with which a firm does not collaborate is exactly the same as under $g = g^1$. However, the investment in enhancing the match quality with a complementor with which a firm does collaborate increases relative to network $g = g^1$. Taking into account that component prices are the same under $g = g^1$ and $g = g^3$, it follows that some systems are more appealing in their vertical attributes when $g = g^3$, and hence are bought more than when $g = g^1$. However, *no* consumer is worse off under $g = g^3$ than under $g = g^1$. Those consumers who were already consuming one of the enhanced systems are obviously better off given the enhancements. In turn, those new consumers attracted by any of the enhanced systems experience a greater transportation cost, but still prefer purchasing one of the enhanced systems. This revealed preference argument shows that these consumers achieve a greater utility under $g = g^3$ than under $g = g^1$. Finally, those consumers who were already consuming one of the systems whose quality is not enhanced make the same utility under $g = g^3$ than $g = g^1$ given that prices and total investments in these systems do not change.

3.2.2 Network structures under non-exclusivity

We now deal with network structures in which at least one firm has more than one collaboration link. As with the previously considered network structures, in equilibrium, firms try to collaborate with as many complementors as it is possible, which will rule out $g = g^3$ as a plausible equilibrium outcome. The underlying economic forces are quite similar to those behind Proposition 4, and it holds that collaboration between two firms involves positive direct and strategic effects. Thus, start from a network architecture in which firms X_i and Y_j are not linked and let us consider what happens if these firms begin collaborating with each other. The cost sinergies that arise from collaboration between such firms result in higher x_i^j and y_j^i. In turn, x_i^{3-j} and y_j^{3-i} also augment (by Remark 1). The negative effect of x_i^{3-j} on firm X_{3-i}'s marginal payoff is offset by the positive effect of y_j^{3-i}, so the fact that X_iY_j becomes stronger leads firm X_{3-i} to reduce x_{3-i}^j in such a way that $x_{3-i}^j + y_j^{3-i}$ does not change. Similarly, y_{3-j}^i is reduced by firm Y_{3-j} in such a way that $x_i^{3-j} + y_{3-j}^i$ remains invariant, so neither system X_iY_{3-j} nor system $X_{3-i}Y_j$ become weaker or stronger. The fact that system X_iY_j is strengthened then implies that x_{3-i}^{3-j} and y_{3-j}^{3-1} are reduced. The outcome of these economic forces leads to the following result.

Proposition 5 *Let $t > \widehat{t} \equiv (3 - 2\gamma + \sqrt{4\gamma^2 - 6\gamma + 3})/(54\gamma)$ and suppose that collaborating with a complementor does not preclude a firm from collaborating with the complementor's* *competitor. Then:*

(i) The unique equilibrium network structure is the complete network, namely $g^{**} = \{1, 1, 1, 1\}$.

(ii) In equilibrium, firm X_i *chooses to invest* $x_i^j(g^{**}) = 1/(12\gamma)$ *in improving the quality of its match with complementor* Y_j*, whereas firm* Y_j *chooses to invest* $y_j^i(g^{**}) = 1/(12\gamma)$ *in improving the quality of its match with complementor* X_i *(*$i, j = 1, 2$*). In addition, each firm earns a payoff of* $(54t\gamma - 1)/(72\gamma)$.

(iii) The equilibrium network $g^{**} = \{1, 1, 1, 1\}$ *results in a payoff for each firm smaller than that achieved when* $g = \{0, 0, 0, 0\}$*, even though* $g^{**} = \{1, 1, 1, 1\}$ *is socially preferred over* $g = \{0, 0, 0, 0\}$.

Proof. We start by noting that neither $g = g^1$ nor $g = g^2$ can arise as stable networks in the light of Proposition 4. We proceed to show that neither $g = g^3$ nor $g = g^4$ nor $g = g^5$ can arise as equilibrium network configurations, which requires that we compute payoffs for each of them (note that this has already been done for $g = g^3$).

So we first compute equilibrium payoffs under $g = g^4 \equiv \{1, 1, 0, 0\}$ under the assumption that $t > (3 - \gamma)/(54\gamma)$ so as to ensure the strict concavity of payoffs and the non-negativity of equilibrium profits, investment levels and quantities sold. Then we have that $x_1^1(g^4) = x_1^2(g^4) = \dfrac{54t\gamma + 1 - 3\gamma}{12\gamma(54t\gamma - 1 - \gamma)}$, $x_2^1(g^4) = x_2^2(g^4) = \dfrac{54t\gamma - 3 + \gamma}{12\gamma(54t\gamma - 1 - \gamma)}$, $y_1^2(g^4) = y_2^2(g^4) = \dfrac{1}{12}$ and $y_1^1(g^4) = y_2^1(g^4) = \dfrac{1}{12\gamma}$. As for equilibrium profits for $g = g^4$, they are

$$\Pi^*_{X_1}(g^4) = \frac{(54t\gamma - 1)(54t\gamma + 1 - 3\gamma)^2}{72\gamma(54t\gamma - 1 - \gamma)^2},$$

$$\Pi^*_{X_2}(g^4) = \frac{(54t\gamma - 1)(54t\gamma - 3 + \gamma)^2}{72\gamma(54t\gamma - 1 - \gamma)^2}$$

and

$$\Pi^*_{Y_j}(g^4) = \frac{108t\gamma - 1 - \gamma}{144\gamma}, \ j = 1, 2.$$

We analyze now the cases in which $g = g^5 \equiv \{1, 1, 0, 1\}$ under the assumption that $t > \widehat{t} \equiv (3 - 2\gamma + \sqrt{4\gamma^2 - 6\gamma + 3})/(54\gamma)$, which guarantees that payoffs are strictly concave and that equilibrium profits, investment levels and quantities sold are all non-negative. Solving for an equilibrium then yields $x_1^1(g^5) = x_1^2(g^5) = y_2^1(g^5) = y_2^2(g^5) = \dfrac{27t - 1}{6(54t\gamma - 1 - \gamma)}$, $x_2^1(g^5) = y_1^2(g^5) = \dfrac{27t\gamma - 1}{6(54t\gamma - 1 - \gamma)}$, and $x_2^2(g^5) = y_1^1(g^5) = \dfrac{27t\gamma - 1}{6\gamma(54t\gamma - 1 - \gamma)}$. As for profits in equilibrium under $g = g^5$, they are

$$\Pi^*_{X_1}(g^5) = \Pi^*_{Y_2}(g^5) = \frac{\gamma(54t\gamma - 1)(27t - 1)^2}{18(54t\gamma - 1 - \gamma)^2}$$

and

$$\Pi^*_{X_2}(g^5) = \Pi^*_{Y_1}(g^5) = \frac{(108t\gamma - 1 - \gamma)(27t\gamma - 1)^2}{36\gamma(54t\gamma - 1 - \gamma)^2}.$$

Noting that $g^5 = g^3 + g_{12}$, it holds that $\Pi^*_{X_1}(g^5) = \Pi^*_{Y_2}(g^5) > \Pi^*_{Y_2}(g^3) = \Pi^*_{X_1}(g^3)$ for $t > \widehat{t}$, and we have that firms X_1 and Y_2 would mutually benefit from forming a link with each other, so $g = g^3$ cannot be a stable network. Network $g = g^4$ can also be discarded as an equilibrium outcome. To show this, note that $g^5 = g^4 + g_{22}$, so the fact that $\Pi^*_{X_2}(g^5) > \Pi^*_{X_2}(g^4)$ and $\Pi^*_{Y_2}(g^5) > \Pi^*_{Y_2}(g^4)$ for $t > \widehat{t}$ implies that firms X_2 and Y_2 would mutually benefit from forming a link with each other.

We deal now with the final network that needs to be considered, namely $g = g^6 \equiv \{1, 1, 1, 1\}$. Under the assumption that $t > 1/(54\gamma)$ (which ensures payoff concavity and non-negativity of the relevant variables), the unique equilibrium is symmetric and involves the following investment levels: $x^j_i(g^6) = y^i_j(g^6) = 1/(12\gamma)$ $(i, j = 1, 2)$. Equilibrium profits under $g = g^6$ are

$$\Pi^*_{X_i}(g^6) = \Pi^*_{Y_j}(g^6) = \frac{54t\gamma - 1}{72\gamma},$$

which are positive for $t > 1/(54\gamma)$.

Using these profits, we can rule out $g = g^5$ as an equilibrium network. Noting that $g^6 = g^5 + g_{21}$, it holds that $\Pi^*_{X_2}(g^6) > \Pi^*_{X_2}(g^5)$ and $\Pi^*_{Y_1}(g^6) > \Pi^*_{Y_1}(g^5)$ for $t > \widehat{t}$. It then follows that firms X_2 and Y_1 would mutually benefit from forming a link with each other, and hence $g = g^5$ cannot be a stable network.

The fact that $\Pi^*_{X_i}(g^6) > \Pi^*_{X_i}(g^5)$ for $i = 1, 2$ and $\Pi^*_{Y_j}(g^6) > \Pi^*_{Y_j}(g^5)$ for $j = 1, 2$ implies that $g = g^6$ is indeed an equilibrium network for $t > \widehat{t}$, which proves parts (i) and (ii).

In order to examine the efficiency properties of the equilibrium network in the absence of exclusivity constraints and thus prove (iii), note that it holds both for $g = g^1$ and $g = g^6$ that $p^*_{X_1} = p^*_{X_2} = p^*_{Y_1} = p^*_{Y_2} = 3t/2$, so $p^*_{11} = p^*_{12} = p^*_{21} = p^*_{21} = 3t$ and $Q^*_{ij} = 1/4$ for $i, j = 1, 2$. Therefore, the aggregate consumer surplus under $g = g^1$ is

$$CS(g^1) = 4\left[\frac{1}{2}\left(v + \frac{1}{12} + \frac{1}{12} - 3t - t\int_0^{\frac{1}{2}} zdz\right)\right],$$

while the aggregate consumer surplus under $g = g^6$ is

$$CS(g^6) = 4\left[\frac{1}{2}\left(v + \frac{1}{12\gamma} + \frac{1}{12\gamma} - 3t - t\int_0^{\frac{1}{2}} zdz\right)\right].$$

It then holds that

$$CS(g^6) - CS(g^1) = (1-\gamma)/(3\gamma) > 0.$$

Because

$$\sum_{i=1}^{2}[\Pi^*_{X_i}(g^6) - \Pi^*_{X_i}(g^1)] + \sum_{j=1}^{2}[\Pi^*_{Y_j}(g^6) - \Pi^*_{Y_j}(g^1)] = -\frac{1-\gamma}{18\gamma} < 0,$$

we have that social welfare increases by $5(1-\gamma)/(18\gamma) > 0$ when going from $g = g^1$ to $g = g^6$ despite firms are worse off than when collaboration is forbidden or impossible. ■

Hence, collaboration cannot improve upon the case in which each firm acts uncoordinatedly. In equilibrium, firms engage in a futile fight to vertically differentiate the systems in which they participate by collaborating with as many complementors as possible and by boosting investments accordingly. The larger investments result in an increase in investment costs, and the greater investment costs end up being just a wasteful rent dissipation, since nothing is gained in return *despite the (possibly substantial) downward shift in cost functions.*[15] All consumers greatly benefit from the greater functionality of every existing system, though. This explains why society would be worse off without collaborative activities involving information sharing among complementors.

When all firms simultaneously start collaborating with their complementors, the greater investments of each complementor in the systems in which it participates do not affect a firm's profit because these effects cancel out. So the existence of complementors is irrelevant for the result that firms prefer the empty network over the complete one. In particular, an envelope argument shows the result is simply driven by the positive direct effect on a firm's profit of having lower investment costs and the negative strategic effect of having the competitor invest more in all the systems in which it participates. The latter effect dominates the former for any admissible value of $\gamma < 1$, which explains why any firm is better off when no collaboration at all takes place. In addition, the strategic effect becomes more important than the direct effect as γ decreases. As a result, a firm's preference for the empty network over the complete one is accentuated as γ is lowered, that is, as information sharing among complementors makes it cheaper to enhance the quality of the match with a complementor.

[15] Thus, if firms invested as much as under $g = g^1$, their profits would indeed augment because the cost function shifts downwards. But precisely this downward shift in the cost function of a firm leads all of them to futilely invest more, thus dissipating the potential gains from the cost function shift.

Also, using Propositions 4 and 5 yields that firms are better off under exclusivity than under non-exclusivity.

Proposition 6 *Let* $t > \widehat{t} \equiv (3 - 2\gamma + \sqrt{4\gamma^2 - 6\gamma + 3})/(54\gamma)$. *Then equilibrium payoffs decrease as* γ *decreases. Furthermore, the equilibrium network under formation of non-exclusive collaboration ties results in a payoff for each firm smaller than that achieved under exclusive collaboration ties.*

4 Robustness of the results

4.1 Stronger equilibrium concepts

We recall that we have used the notion of pairwise stability as our solution concept for the strategic network formation game under consideration. In the context of our game, the main drawback of this solution concept has to do with the possibility that a firm with several links may want to sever more than one link at a time.[16] However, this criticism does not apply to the game just analyzed. Indeed, the fact that $\Pi^*_{X_2}(g^6) > \Pi^*_{X_2}(g^4)$ implies that the complete network is stable even if the pairwise stability solution concept is augmented to allow for the deletion of several links at a time (the complete network is then said to be pairwise Nash stable).

4.2 Collaboration fostering partly cooperative investments

The result that collaboration is inefficient from the viewpoints of firms also holds if free-riding by a firm on a complementor's investment effort is partly mitigated if both firms being collaborating. To this end, let $\gamma = 1$ and suppose that $g_{ij} = 1$ implies now that both firms X_i and Y_j care in some sense about each other's payoff when making investment choices.[17] Specifically, given network architecture g and some parameter $\lambda \in (0, 1)$, suppose that firm X_i $(i = 1, 2)$ chooses in the second stage $x_i^1 \geq 0$ and $x_i^2 \geq 0$ to maximize either

$$\pi^*_{X_i} - (x_i^1)^2 - (x_i^2)^2 + \lambda \sum_{j=1}^{2} g_{ij}[\pi^*_{Y_j} - (y_j^1)^2 - (y_j^2)^2]$$

[16] See Jackson (2008, pp. 156 and 371-376) for a thorough discussion of the virtues and limitations of pairwise stability as a solution concept.

[17] There are many formal or informal arrangements that may lead two complementors that collaborate with each other to make their investments in improving their match quality in a (somewhat) cooperative manner. Reasons range from research alliances (or collusive R&D cartels) to relational capital concerns in ongoing relationships between firms that need each other to some extent because of their complementarity.

or

$$\pi^*_{X_i} - (x_i^1)^2 - (x_i^2)^2 + \lambda \sum_{j=1}^{2} g_{ij}[\pi^i_{Y_j} - (y_j^i)^2],$$

and similarly for firm Y_j ($j = 1, 2$). Suppose also that, except in the second stage, firms solely pay attention to their own payoffs when making decisions of whether to form a collaboration tie or which price to set. It can be shown in these cases that our results go through, with the exception that the equilibrium networks are not only inefficient for the firms but also for consumers (and hence society).[18] In the absence of pecuniary effects of collaboration, one may then expect inefficiencies to arise both at the firm and the social levels. Collaboration with pecuniary effects resolves the social inefficiences but not the private ones, which remain, as Propositions 4 and 5 show.

5 Conclusion

The locus of strategic interaction in many high-tech industries has broadened from the traditional competitive approach based on value capture towards one in which cooperative aspects with regards to value creation also play a critical role, as Brandenburger and Nalebuff (1996) emphasize. Not surprisingly, such "co-opetitive" settings display rich innovation ecosystems in which complementors collaborate with each other in R&D actitivities. This paper has shown that such rich innovation ecosystems may be an equilibrium phenomenon with disturbing properties for their members. In particular, we have shown that they may be an inefficient outcome for competing firms that can collaborate with complementors. They may also be inefficient for society. These results hold under a variety of scenarios (e.g., regardless of whether or not collaboration exhibits exclusive features).

In this paper, we have abstracted away from dynamics to clarify our points, but there are many issues that have to do with dynamic variables. For example, collaboration may refer to the timing at which complementary products are brought to the market. Exploring this kind of issues seems promising enough to warrant further work on this completely unexplored area.

[18] See Mantovani and Ruiz-Aliseda (2011) for a full analysis of these situations, both under exclusivity and the lack thereof.

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