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# Students' Social Origins and Targeted Grade Inflation

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**Abstract**

Grade inflation or soft grading is a common feature of the educational systems of many countries. In this paper I analyse grade inflation in a setting in which students differ in social background, and the grading policy can be targeted according to student type. I consider a signalling game where firms decide whether to hire students and their salary after observing their grades and social background, a university can inflate grades, when students decide whether to attend university. A targeted grade inflation may have redistributive effects by raising the salary of students with disadvantaged social background, if their grades are less inflated than other students'.

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*- Praise, like gold and diamonds, owes its value only to its scarcity -*
***Samuel Johnson, 1751***

# 1 Introduction

Grade inflation arises when teachers award students with higher grades than they deserve, leading to a high concentration of students with top grades. The presence of grade inflation makes it more difficult to distinguish a student's ability, both in evaluating university applications and in job recruitment, and brings about potential distortions. The presence of grade inflation is nowadays a common feature in several educational systems. In the United States, the evidence of grade inflation has been recently documented by Rojstaczer and Healy (2011), who collected historical data on letter grades awarded by more than 200 four-year colleges and universities. Their results show the drastic rise in the share of A grades awarded over the years.[1] In Canada, Allahar and Côté (2007) show that the 52.6 % of high-school graduates applying to universities in Ontario in 1995 had an A average, and then this rose to 61% in 2004. Also in Ontario in 1995, the 9.4% of high school graduates reported an A+ average and it increased to 14.9% in 2003. In addition, the average grade of university applicants was 80% in 1997, and this percentage has steadily increased since then. In the United Kingdom, graduates who obtain a first-class honours rose from 7.7% of total graduates in year 1996/97 to 14% in year 2008/09. For graduates with an upper-second honour, the percentage rose from 41.1% of total graduates in year 1996/97 to 48% in year 2008/09 (Higher Education Statistic Agency). In Italy, the analysis "Stella" reports one third of graduates achieved the highest grade (110/110) in 2004 and 2005 (Modica, 2008). The established presence of grade inflation across

[1] In earlier contributions, Rosovsky and Hartley (2002) and Johnson (2003) survey the empirical literature on grade inflation in the U.S. The emergence of grade inflation can also be observed in Figlio and Lucas (2004), who analyse the impact of standard grades in educational achievement in the Alachua County, Florida.

countries requires the attention of policy makers. A theoretical understanding of its consequences becomes necessary in order to design an adequate policy intervention.

In this paper, I examine an education system in which students' differ in social background and grading policy can be targeted according to a student type. Like grades, students' social background contributes to determining their outcomes in the job market.[2] A targeted grading policy can be interpreted as a tracking system, in which students differing in social background are separated at the beginning of the education program, due to a different initial preparation. Another situation in which grading policy can be targeted occurs in those disciplines in which students' achievement can be assessed in a very subjective way: in this case a targeted grading policy may be undetected (so that it does not look discriminatory).

I consider a signalling game in a static setting, with a number of firms, a university and students who differ both in ability and social background as players. I assume that students with an advantaged social background are more likely to have high ability, this due to the influences of a better environment to develop skills and by a stronger parental and social pressure about life achievement. A university aims to maximise the job opportunities of its students and may give a high grade to low-ability students. On the other hand, each firm observes the students' grades and social backgrounds, and wants to hire only high-ability students. Moreover, a firm is fully aware of the grading policies adopted by the university.

The results suggest that optimally targeted grade inflation may have redistributive effects by raising the salary of disadvantaged students. This outcome hinges upon the fact that the educational signal is worse for advantaged than disadvantaged students if the formers' grades are more inflated

[2]There is large empirical evidence suggesting that the students' social background influences their job opportunities. For instance, Glyn and Salverda (2000) and Berthoud and Blekesaune (2006) show that a disadvantaged social background negatively affects the chance of finding a job in OECD countries and the United Kingdom, respectively.

than the latters'. In particular this result emerges when the lower level of grade inflation for disadvantaged students more than offsets the higher probability of having high ability of advantaged students, who thus have a higher expected ability than advantaged students. Interestingly, this result may occur with no redistributive intentions in the grading policy.

The remainder of the paper is organised as follows. Section 2 briefly surveys some of the related literature. The model is presented in Section 3, and Section 4 examines the baseline results. Section 5 considers some extensions of the baseline results. Section 6 extends the analysis to the case with university competition, and Section 7 concludes.

## 2 Related literature

The economic literature has only recently taken on interest in grade inflation, with then few but noteworthy contributions. Yang and Yip (2003) present a model where universities have an incentive to inflate grades and they mutually reinforce each other's practice, thus determining a competitive effect in grade inflation. This is due to the fact that each university does not consider the collective reputation of graduates, but is willing to help some of its own low-ability students by inflating their grades, leading to a free-riding problem. Popov and Bernhardt (2013) develop a similar model to Yang and Yip (2003) to identify the increase over time in the quantity of good jobs as a driving force of grade inflation. They also extend the analysis by considering students with different social skills.

Chan *et al.* (2007) develop a signalling model where firms observe the students' grade but are not aware of their ability and the proportion of talented ones in the population of students. This gives rise to an incentive to help some low ability students by giving them good grades. They also show that when the average qualities of students among schools are correlated, soft grades are strategic complements, and thus inflating schools mutually reinforce each other's practices. Ehlers and Schwager (2012) modifies the analysis of Chan *et al.* (2007) by introducing a reputational element. They add a second cohort of graduates that arrives on the labor market when the first cohort has already revealed their true ability and therefore the school's grading policy. The reputation effect may shrink the level of grade inflation.

Bar *et al.* (2012) examine the recent policy of "putting grades in context", according to which American colleges can reveal the distribution of grades in different disciplines to employers, in order to prevent the distortion in information caused by grade inflation. Accordingly, they propose a framework where students can choose different courses and the university can vary grading standards according to the course. They show that, when information on grading policies is provided only to students, some of them become more attracted to leniently graded courses, whereas if the information is provided to both students and employers, some students choose the strictly graded courses and some choose the leniently graded courses.

The main difference between these papers and my analysis is that the differences in students' social background and grade inflation is targeted according to a student type.[3] My analysis is mostly related to Schwager (2012),

[3] The paper is also related to the theoretical literature on educational standards, which examines the criteria adopted by schools in evaluating students. Costrell (1994) considers a policymaker who maximises social welfare under the assumption that utility-maximising students choose whether to meet the standard, thus leading to the fact that earnings are an endogenous function of educational achievement. The welfare analysis shows that more egalitarian policymakers set lower standards. Betts (1998) instead argues that an egalitarian policy maker might prefer higher standards than would a policy maker whose goal was to maximize the sum of earnings. The result is based on the assumption of heterogeneous ability among workers. As a consequence, a rise in educational standards will increase the earnings of both the most-able and the least-able workers. The only workers whose earnings fall are those workers who after the increase fail to continue meeting the standard. Himmler and Schwager (2012) extend the Costrell (1994)'s analysis by assuming that, in addition to the standard, also the social origin affects the wage earned by graduates. For a given standard, students from disadvantaged backgrounds obtain a lower wage than students from other social classes. Schools with a disadvantaged student body set lower standards than other schools, even if the abilities of the disadvantaged students are identical to those of others. Standards are inflated in this way because the wage discount experienced by graduates from unfavourable backgrounds depresses the

who develops a labour matching model with grade inflation and student differing in social background. In his paper, students are matched with firms offering different kinds of jobs, according to the grade and the expected ability. Regardless of social background, it is possible that mediocre students receive a high grade caused by grade inflation. Also, the high-ability students from advantaged backgrounds may benefit from grade inflation since this shields them from the competition on the part of able and disadvantaged students. Compared to this analysis, I share the same assumptions on the distributions of ability with differing social backgrounds, but Schwager (2012) (i) focuses on the matching between workers and firms, whereas I do not consider the matching in the labour market, and (ii) assumes the same degree of grade inflation along different social class, whereas I assume that the university may target its grading policy.[4] More importantly, in Schwager (2012) grade inflation is a parameter, while in the present paper grade inflation is endogenously determined. Given the different framework, in my results disadvantaged students may in fact benefit from the presence of grade inflation.

# 3 The framework

For simplicity, I abstract from student effort and from competition across universities, and I focus on the interplay between students, one university and a number of firms interested in hiring graduate workers.[5]

return to learning effort for these students. They are thus less willing to satisfy any given standard than students from an average social background.

[4] In Section 4.3.5 I examine how the equilibrium changes when grade inflation cannot be targeted, by obtaining qualitatively similar results to Schwager (2012).

[5] In Section 6 I illustrate how the baseline model can be developed by introducing competition between universities.

## 3.1 Students

I study an economy with a continuum of students, with measure normalised to one. Students decide whether or not to attend university. University admission does not involve any requirement apart for paying tuition fees. A student who attends university will obtain a graduate degree with certainty, and after the university period she will apply for a graduate job in one of the firms.

Students can have high ($H$) or low ($L$) ability and an advantaged ($a$) or disadvantaged ($d$) social background. Social background is public information, and can be seen as a bivariate measure of family environment, income, neighbourhood, peer effects, ethnic origins and so forth.[6] A student attends university if tuition fees $k \in (0,1)$ are lower than the probability of obtaining a job, $\phi_{ji} \in [0,1]$, where $j \in \{H, L\}$, and $i \in \{a, d\}$.

I denote as $\eta \in (0,1)$ the proportion of advantaged students, and $p_a, p_d \in (0,1)$ as the probability that an advantaged or disadvantaged student has high-ability, respectively. I assume $p_a > p_d$, that is students with advantaged social background are more likely to have high ability. This assumption can be justified as follows. Given the same distribution of innate ability within a population with differing social backgrounds, an advantaged environment can foster development via parental and peer pressure so that, on average, the overall "ability" is likely to be higher for students with an advantaged background. The assumption is in line with past research documenting that family and environmental factors are major predictors of the individuals' ability (Cunha *et al.*, 2006, Carneiro and Heckman, 2003, Joshi and McCulloc, 2000).

[6] Peer effects arise if students learn better in a group of more able students. Relevant empirical studies are, *inter alia*, Summers and Wolfe (1977), Henderson et al. (1978), Epple et al. (2003) and Zimmer and Toma (2000). From a theoretical point of view, Arnott and Rowse (1987), de Bartolome (1990) and Epple and Romano (1998) consider explicitly the peer group effect. In the present analysis the presence of peer-effects is considered within the background.

## 3.2 University

The university prepares students for a final exam, with equal teaching effort irrespective of the student type, and learns the student's ability during the period spent by a student at university, through their tests and assessments results. The final exam can be interpreted either as a grade for a final test or as the average grades among the university examinations. The possible exam outcomes are a high ($A$) or a low ($B$) grade.

The university decides which grade to appoint each student type. I define $g_{ji} \in [0,1]$, $j \in \{H, L\}$, and $i \in \{a, d\}$, as the probability that the university appoints an $A$ grade to a $ji$ student. I refer to "grade inflation" when the university appoints an $L$ student with an $A$ grade. The fact that the university can differentiate its grading according to a student background can be interpreted in several ways. For instance, the university may track students of different social origins, due to a different initial preparation. Another situation in which a targeted grading policy may take place emerges in those university courses in which students' achievement can be assessed very subjectively. In this case a grading policy being targeted according to student types is hard to be detected and easy to implement.

There is not a standard way of modeling university behaviour. In the economic models of university (or school) competition, the number of enrolled students or the overall amount of tuition fees enter in the school/university objective function (Epple and Romano, 1998, Del Rey, 2001, De Fraja and Iossa, 2002, Brunello and Rocco, 2008, Maldonado, 2008, Ferreyra, 2012, *inter alia*). Other models propose a school objective function determined by the average qualification (De Fraja and Landeras, 2006, De Fraja *et al.*, 2010, Albornoz *et al.*, 2011), the average and the variance of the qualification (Ritzen *et al.*, 1979), the quality of school (Epple *et al.*, 2003 and 2006) and the quality or the attendance in the case of a public school (Epple *et al.*, 2002).[7] Here I assume that a university wants the maxium number of

[7] Some recent contributions (Albornoz *et al.*, 2011, Donze and Gunnes, 2011) rely on

students to be hired, so that its objective function is as follows:

$$\Pi^U = \mathbf{1}_K(\phi_{Ha}) b_{Ha}\eta p_a \phi_{Ha}(g_{Ha}, z_{\xi a}) + \mathbf{1}_K(\phi_{Hd}) b_{Hd}(1-\eta) p_d \phi_{Hd}(g_{Hd}, z_{\xi d}) + \mathbf{1}_K(\phi_{La}) b_{La}\eta(1-p_a)\phi_{La}(g_{La}, z_{\xi a}) + \mathbf{1}_K(\phi_{Ld}) b_{Ld}(1-\eta)(1-p_d)\phi_{Ld}(g_{Ld}, z_{\xi d}), \tag{1}$$

where $\mathbf{1}_K(\phi_{ji})$ is the indicator function defined as

$$\mathbf{1}_K(\phi_{ji}) = \begin{cases} 1 & \text{if } \phi_{ji} \in K \\ 0 & \text{if } \phi_{ji} \notin K \end{cases}, \quad K = \{x \in (0,1] \,|x > k\} \;\; j \in \{H, L\}, \; i \in \{a, d\}, \tag{2}$$

and $b_{ji}$ is the benefit that the university obtains from the hiring of a $ji$ student. The indicator function says that the university benefit is zero if tuition fees are higher than the probability of obtaining a job for a student type, $\phi_{ji}(g_{ji}, z_{\xi i}) < k$. This occurs since, in equilibrium, all the students belonging to a specific type make the same decision about university attendance. I denote as $z_{\xi i} \in [0,1]$, $\xi \in \{A, B\}$ the probability that a firm hires a student according to grade and social background (see the next paragraph). The probability of being hired according to a student type is a function of the university and firm behaviour, $\phi_{ji}(g_{ji}, z_{\xi i})$. In particular, this is given by the probability that a student obtains a grade ($A$ or $B$) times the probability that a firm hires a student with that grade.[8] Thus the probability of obtaining a job according to a student type is given by

$$\phi_{ji}(g_{ji}, z_{\xi i}) = \begin{cases} g_{ji} z_{Ai} & \text{if the students scored } A \\ (1-g_{ji}) z_{Bi} & \text{if the student scored } B \end{cases}. \tag{3}$$

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the "goal theory" (Covington, 2000), according to which achievement goals influence the quality, timing and appropriateness of the students' engagement in their own learning. This effort together with innate ability affect the student's accomplishments. As a consequence, parents and teachers play a key role in influencing the students' achievement goals and, in turn, their effort.

[8] As will be clear shortly, firms cannot observe a student's ability and use the grade as a signal of it.

I make the following assumption.

**Assumption 1** $b_{Ha} = b_{Hd} > b_{La} = b_{Ld} > 0.$

In words, the university wants the maximum number of students to be hired and values more the employment of an $H$ student. Accordingly, (i) each student's employment increases the university reputation as an effective institution for obtaining a job and (ii) the university obtains a higher benefit from the hiring of $H$ students which ensures the universities credibility to firms. It is important to stress that, according to Assumption 1, the university has no preferences whatsoever about a student's social background.[9]

## 3.3 Firms

There is an exogenous number of $J \in [0, 1)$ identical firms in the graduate labour market, each firm is willing to hire at most a single graduate. Firms observe the final grade and the social background of students. The public knowledge of social background seems plausible: in the real world, a personnel manager can probably tell the job candidate's social background through some information such as ethnic origins, name, address, language style, manners, clothing, and so on. Also, each firm is fully aware of the university's grading policy. The assumption that a firm knows the university grading policy reflects the situations in the real world in which either (i) an educational institution claims its own grading policy, (ii) the grading policy is public information due to reputation effects or (iii) some policy intervention induces them to reveal the distribution of grades, like "putting grades in context" in the United States (see Bar *et al.*, 2012).

A firm hires with probability $z_{\xi i}$ a student with background $i \in \{a, d\}$ who graduated with grade $\xi \in \{A, B\}$ and offers a single job type (a "graduate" job). Also, I assume that ability of employees determines a firm's profit

[9]In Section 5.2 I consider different assumptions about university's preferences.

entirely. In particular, each high- and low-ability graduate yields a net profit of $\mu > 0$ and $-1$, respectively. The assumption of a negative profit by hiring an $L$ student can be interpreted in many ways: low-ability employees may have a marginal productivity which is lower than salary cost. In addition, a firm may want to lay off an unproductive employee but this action still comes at a cost, e.g. industrial disputes, wasted training costs and time, and so on. Given these assumptions, a firm's profit is given by:

$$\begin{aligned}
\Pi^F \quad = \quad & \mu \left[\mathbf{1}_K \left(\phi_{Ha}\right) \eta p_a \phi_{Ha} \left(g_{Ha}, z_{\xi a}\right) + \mathbf{1}_K \left(\phi_{Hd}\right) \left(1-\eta\right) p_d \phi_{Hd} \left(g_{Hd}, z_{\xi d}\right)\right] \qquad (4) \\
& - \left[\mathbf{1}_K \left(\phi_{La}\right) \eta \left(1-p_a\right) \phi_{La} \left(g_{La}, z_{\xi a}\right) + \mathbf{1}_K \left(\phi_{Ld}\right) \left(1-\eta\right) \left(1-p_d\right) \phi_{Ld} \left(g_{Ld}, z_{\xi d}\right)\right].
\end{aligned}$$

Each firm maximises its own profit over $z_{\xi i}$. Once hired a student, then a firm decides the level of salary, which is function of the expected profit generated by that student, so that $w = w\left(\Pi^F\left(g_{ji}; \xi, p_i\right)\right)$, $w'(\cdot) > 0$. The presence of many firms in the job market ensures that a job candidate with higher expected productivity would receive a higher salary.[10]

Along the paper, I will refer to $J$ as *labour demand*. The fact that $J$ cannot cover all of the students rules out the unrealistic case where the job market is cleared, and has important consequences on the behaviour of the university. As will be clear below, given the limited amount of job placements and Assumption 1, the university will adopt a grading policy such that none of the $L$ students obtain a job at the expenses of an $H$ student. In turn, in equilibrium $z_{\xi i}$ will equal 0 or 1 for all student types.

[10]In equilibrium, the difference in salary according to expected ability will be in such a way that a firm will be indifferent between hiring a job candidate with (positive) higher or lower expected ability.

## 3.4 The game

Figure 1 summarises the timing of the game. Nature draws the student types, then, each student decides whether to attend university. If so, the university grades the student in the final exam. Finally, all the graduates apply for a job in one of the firms, and firms decide whether to hire or not a job candidate and, if so, her salary level.

**Figure 1. The game timing**

| **Stage 1. Nature** | | **Stage 2. Students** | | **Stage 3. University** | | **Stage 4. Firms** |
|---|---|---|---|---|---|---|
| 4 student types: $H$ or $L$ with $a$ or $d$ background. | $\rightarrow$ | decide whether to attend uni by comparing tuition fees with the chance of a job | $\rightarrow$ | chooses the grade to give to each student type. | $\rightarrow$ | choose whether hiring a graduate student and her salary level. |

The equilibrium concept is perfect Bayesian equilibrium, which is a combination of students, university and firms strategies and beliefs where all the agents maximise their payoff. After observing a grade, each firm has a belief, consistent with Bayes' rule, about the student type, conditional on all the information it has: the student's grade, the distribution of ability according to the student's social background and the university strategy.

For each grade, a firm must maximise its expected profit, given its belief and the university strategy. Labour demand requires that the number of hired graduates is at most $J$. Once a graduate is hired, a firm will offer a salary being a function of the expected unitary profit. In turn the university chooses its grading strategy in order to maximise its expected payoff, given the set of students, firms' strategy and labour demand $J$.[11] Finally, each student decides whether to attend university by comparing the probability of obtaining a job (determined by the university and firms strategies) with the exogenous tuition fees.

[11] Notice that the university has complete information about the firm's behaviour, therefore it is not necessary to determine its beliefs.

I am now in a position to define a firm's beliefs about a student's ability.

**Definition 1** *A firm's beliefs on the students' ability which are consistent with the Bayes' rule are*

$$\begin{aligned}\pi\left(H\left|g_{ji}; A, p_i\right.\right) &= \frac{p_i g_{Hi}}{p_i g_{Hi} + (1-p_i)\, g_{Li}}, \\ \pi\left(L\left|g_{ji}; A, p_i\right.\right) &= \frac{g_{Li}\,(1-p_i)}{p_i g_{Hi} + (1-p_i)\, g_{Li}},\end{aligned} \tag{5}$$

*for a student who obtained a grade A, and*

$$\begin{aligned}\pi\left(H\left|g_{ji}; B, p_i\right.\right) &= 0, \\ \pi\left(L\left|g_{ji}; B, p_i\right.\right) &= 1,\end{aligned} \tag{6}$$

*for a student who obtained a grade $B$.*

# 4 Results

## 4.1 The baseline problem

In this section I show the results of the baseline model. First notice that, since a graduate who scored $B$ has low ability with probability 1, then a firm will never hire one of them, so that $z_{Bi} = 0$ for $i \in \{a, d\}$. Indeed while the university may want to inflate the grade of a low-ability student in order to increase the number of graduates who obtain a job, it would never appoint a $B$ to a high-ability student. This simplifies the exposition of the results and allows me to focus on $A$ students. For the same token, it is always better for the university to confer a grade $A$ to an $H$ student, as this unambiguously raises the payoff of both the university and firms. Therefore, in all the possible scenario the probability of an $H$ student to obtain an $A$ is $g_{Hi} = 1$ for $i \in \{a, d\}$.

The expected payoff of hiring an $A$ student with social background $i$ according to the beliefs (5) is

$$\Pi^F\left(g_{Li};A,p_i\right)=\frac{p_i\mu-\left(1-p_i\right)g_{Li}}{p_i+\left(1-p_i\right)g_{Li}}. \tag{7}$$

A firm hires from a population of students if its expected payoff is positive, $\Pi^F\left(g_{Li};A,p_i\right)>0$. Assume for a moment that labour demand is $J=1$. For every $i\in\{a,d\}$ and $p_i\geq\frac{1}{1+\mu}$, a firm's expected payoff (7) is positive for $g_{Li}=1$. For $p_i<\frac{1}{1+\mu}$, a firm's expected payoff (7) is positive for $g_{Li}<\frac{p_i\mu}{(1-p_i)}$, so that the university strategy is $g_{Li}=\frac{p_i\mu}{(1-p_i)}-\varepsilon$, with $\varepsilon>0$ a small number. Hence the threshold point $\frac{1}{1+\mu}$ is a function of the firm's benefit from hiring an $H$ student, $\mu$. The higher the benefit, the higher the firm's expected payoff by hiring an $A$ student. Therefore a firm tends to hire more $A$ students when $\mu$ is high, and in turn, the university more likely inflates the students' grades.

Given Assumption 1, the university strategy ensures that the maximum number of students would be hired and that all the students who scored $A$ are going to obtain a job.

**Lemma 1** *All the students who scored $A$ will obtain a job.*

Lemma 1 implies that a firm's strategy is $z_{Ai}=1$, so that equation (3) becomes $\phi_{ji}=g_{ji}$, for every $i\in\{a,d\}$ and $\xi=A$. Therefore an $L$ students decides whether or not to attend university by comparing $g_{Li}$ with tuition fees $k$ and would attend it if and only if $g_{Li}>k$, whereas all the $H$ students would attend university, as $g_{Hi}=1$.

However job positions are limited because $J<1$. Since the university prefers that an $H$ rather than an $L$ student obtains a job, it will inflate grades at most for the amount of labour demand net to the share of $H$ students, denoted as

$$\Phi\equiv p_a\eta+p_d\left(1-\eta\right), \tag{8}$$

so that the remainder of labour demand is $J-\Phi$. This ensures that none of the $L$ students would obtain a job opportunity at the expenses of an $H$ student. Finally, if the expected profits are positive from hiring both an $a$ and a $d$ students, then a firm compares the two expected profits, and offers a higher salary to the graduate type whose expected productivity is higher:

$$\Pi^F(g_{Ld}; A, p_d) = \frac{p_d\mu - g_{Ld}(1-p_d)}{p_d + g_{Ld}(1-p_d)} \lesseqgtr \frac{p_a\mu - g_{La}(1-p_a)}{p_a + g_{La}(1-p_a)} = \Pi^F(g_{La}; A, p_a). \tag{9}$$

## 4.2 Equilibria with no grade inflation

In this paragraph I show the cases in which no grade inflation takes place. In this setting, this situation may occur for two reasons: (i) there are too few job positions while the university favours $H$ over $L$ students, or (ii) tuition fees are higher than the probability of an $L$ student to obtain a job. The following Lemma shows the first situation.

**Lemma 2** *Suppose Assumption 1 holds and $J \leq \Phi$. Then the university never inflates grades and a firm is indifferent to a student's social background.*

**Proof.** It follows directly from Assumption 1, $b_{Hd} = b_{Ha} > b_{Ld} = b_{Ha}$. Given the available positions, the university objective function is maximised by not inflating grades, in order to favour $H$ over $L$ students. ■

If the number of jobs is lower than the number of $H$ students, then $g_{Hi} < 1$, so that they would attend university only if $g_{Hi} > k$. This situation may be defined as "grade deflation equilibrium", since not all the $H$ students obtain a high grade. Consider now the case in which tuition fees are so high that no $L$ student would attend university.

**Lemma 3** *Suppose Assumption 1 holds, and $\Phi < J < 1$. For $k > g_{Li}$ and $i \in \{a, d\}$, then none of the $L$ students attend university, all the $A$ students have high ability, so that a firm is indifferent about social background.*

In either of these two cases described, a firm maximises its payoff by hiring all the $A$ students possible given $J$, as all the $A$ students have high ability with probability 1.

## 4.3 Equilibria with grade inflation

Consider next the more general case in which labour demand is larger than the number of high ability students and tuitions fees are sufficiently low, according to the following assumption.

**Assumption 2** *$\Phi < J < 1$, $k < g_{La}$ and $k \lesseqgtr g_{Ld}$.*

According to Assumption 2, two possible situations may occur: one in which tuition fees are lower than the probability of obtaining an $A$ for both $L$ $a$ and $d$ students, and another in which tuition fees are lower than the probability of obtaining an $A$ for $L$ $a$ students only. The results are sorted according to the distribution of ability in the populations of advantaged and disadvantaged students, then an interpretation is provided in Section 4.3.4.

### 4.3.1 Large proportion of $H$ students in both populations

First consider the case (Case 1) in which there is a large number of $H$ students in both populations.

**Proposition 1** *Suppose Assumption 1 and 2 hold and $p_a > p_d > \frac{1}{\mu+1}$. Then in equilibrium, $g_{Li} > k$ (= all the students attend university):*

1. *Grade inflation is given with the same proportion to $a$ and $d$ students;*

2. *The probability of obtaining a job is the same for $a$ and $d$ students;*

3. *An $a$ student receives a higher salary than a $d$ student.*

**Proof.** See the appendix. ■

In Proposition 1, the university inflates grades for the same proportion of $a$ and $d$ students. Therefore, given Lemma 1, their probability of obtaining a job is the same. Nonetheless, $a$ students provide a higher expected profit than $d$ students, so that a firm offers them a higher salary.

#### 4.3.2 Small proportion of $H$ $d$ students

In this Section I analyse the case (Case 2) in which there is a small number of $H$ students in the population of $d$ students only.

**Proposition 2** *Suppose Assumption 1 and 2 hold and $p_a > \frac{1}{\mu+1} > p_d$, and define*

$$\widehat{\mu} \equiv \frac{p_a \varepsilon + p_d \left[1 - p_a \left(1 + \varepsilon\right)\right]}{p_a p_d}. \tag{10}$$

*For*

- $g_{La} > g_{Ld} > k$ *(= all the students attend university):*
    1. *Grade inflation is given to more $a$ than $d$ students.*
    2. *The probability of obtaining a job is greater for $a$ rather than $d$ students.*
    3. *A $d$ $(a)$ student receives a higher salary than an $a$ $(d)$ student for $\mu \leq (>) \mu$.*

- $g_{La} > k > g_{Ld}$ *(= all the $a$ students, together with the $H$ $d$ students, attend university):*
    1. *Grade inflation is positive for $a$ students and zero for $d$ students.*
    2. *The probability of obtaining a job is greater for $a$ than $d$ students.*
    3. *A $d$ student receives a higher salary than an $a$ student.*

**Proof.** See the appendix. ■

In the equilibria depicted by Proposition 2, a $d$ student's expected ability is lower than before, so that a firm prefers not to hire one of them if too much grade inflation is provided. In order to maximise the number of hired students, the university inflates more grades of $a$ than $d$ students. In the first part of Proposition 2, grade inflation is high enough so that all the $L$ students would attend university, a firm's preferences towards a student type (reflecting the level of salary) strictly depends on the level of $\mu$. If a firm's benefit from hiring an $H$ student is sufficiently, low then $d$ students are offered a higher salary than $a$ students. This is due to the following reason. The higher $\mu$, the higher a firm's expected profit, so that it is willing to hire with higher probability. As a consequence, the optimal level of grade inflation rises with $\mu$, but a higher grade inflation worsens the education signal and thus leads $a$ students to be preferred to $d$ students.

In the second part of Proposition 2, grade inflation is not sufficiently strong for $d$ students. Thus $L$ students do not attend university as tuition fees are too high, hence a firm would prefer to hire $A$ students from that social group (since all of them have high ability), and offer them a higher salary than an $a$ student.

#### 4.3.3 Small proportion of $H$ students in both populations

Consider next the case (Case 3) in which there is a limited amount of $H$ students in both populations.

**Proposition 3** *Suppose Assumption 1 and 2 hold and* $\frac{1}{\mu+1} > p_a > p_d$*. For*

- $g_{La} > g_{Ld} > k$ *(= all the students attend university):*

    1. *Grade inflation is given to more a than d students.*
    2. *The probability of obtaining a job is greater for a rather than d students.*

*3. A $d$ student receives a higher salary than an $a$ student.*

- $g_{La} > k > g_{Ld}$ *(= all the $a$ students, together with the $H$ and $d$ students, attend university):*

  *1. Grade inflation is positive for $a$ students and zero for $d$ students.*

  *2. The probability of obtaining a job is greater for $L$ and $a$ rather than $d$ students.*

  *3. A $d$ student receives a higher salary than an $a$ student.*

**Proof.** See the appendix. ■

When the share of $H$ students is low in both populations, the university lowers the level of grade inflation for $a$ students, but this remains higher than the level of grade inflation for $d$ students. The main difference with the results of Proposition 2 is that a firm unambiguously prefers hiring $d$ rather than $a$ students, irrespective of $\mu$, and thus it offers a higher salary to them. In other words, if the probability of having high ability is sufficiently low in both population, then in equilibrium the lower grade inflation conferred to $d$ students more than offsets the effect of $p_a > p_d$ in the comparison of firm's expected profits (9). This is due to the fact that $p_a$ is too low to compensate the lower grade inflation to $d$ students, so that their educational signal is better.

#### 4.3.4 Remarks

Figure 2 illustrates Proposition 1, 2 and 3. Function $\widehat{\mu}$ refers to Case 2 when $g_{Li} > k$. In the figure, $p_a > p_d$ holds above the upward-sloping 45 degrees line. This is the key assumption and makes a firm obtain a higher expected profit by hiring advantaged than disadvantaged students, given the same grading policy. However this may not happen if grading is softer for advantaged students, as it would raise the expected quality of the disadvantaged and $A$

students. On the other hand, a low grade inflation may induce $L$ students not to attend university.

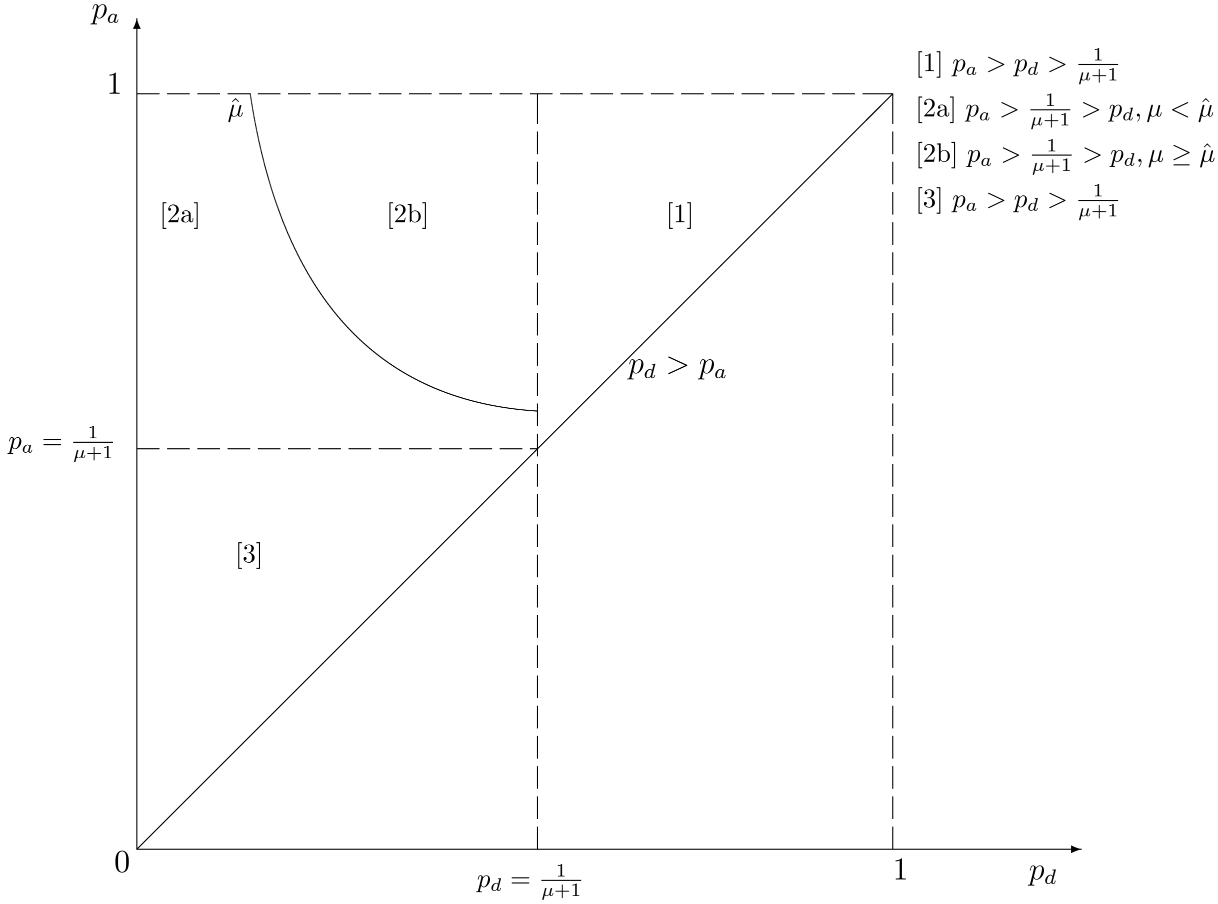
The results show that a targeted grading policy may have the unintended effect of raising the salary of disadvantaged students more than the advantaged ones. Of course there are equilibria in which advantaged students are preferred to disadvantaged students, as the empirical evidence points out and depicted for instance by Case 1, but two effects influence the possible equilibrium. First, a firm's benefit obtained by hiring a high-ability student has ambiguous effects on the results. On the one hand, if $\mu$ is high it is more likely that Case 2 or 3 occur, in which the redistributive effect emerges. On the other hand, in Case 2 the lower a firm's benefit obtained by hiring a high-ability student, the higher the preference for hiring a disadvantaged student.

**Corollary 1** *A rise in the firm's benefit for hiring an $H$ student increases the probability that Case 2 or 3 takes place. In Case 2, it is more likely to be in the parameter range in which a students are offered a higher salary than $d$ students.*

Second, the effect of a low grade inflation in the university attendance. If grade inflation is too low for disadvantaged and low-ability students, then they would not attend university, by increasing the expected quality of the disadvantaged graduates.

**Corollary 2** *Since, by and large, $d$ students need less grade inflation in order to compete with a students, the university attendance of $d$ students will be lower.*

**Figure 2. Equilibria with positive grade inflation**




### 4.3.5 Untargeted grade inflation

Finally compare the results obtained with the situation in which grade inflation cannot be targeted. This may happen in a class where both advantaged and disadvantaged students are mixed together. In this context, a systematic difference in grading policy according to students' social backgrounds would be assessed as discriminating. With untargeted grade inflation, the grading policy is the same for both populations, thus the result in Case 1 remains the same as above. In Cases 2 and 3, $a$ and $d$ students have the same probability of being hired, and it is more likely that $a$ students are preferred by a firm. The reason of due to the combination of same grading policy for each population together with $p_a > p_d$. Given the same grading policy but a higher probability of high ability for $a$ students, the expected profit from hiring one of them is undoubtedly higher. The results can be summarised as follows.

**Proposition 4** *Suppose Assumption 1 and 2 hold, and grade inflation is untargeted, $g_{La} = g_{Ld}$ (= all the students attend university):*

- ***Large proportion of high-ability students in both populations (Cases 1).*** *For $p_a > p_d > \frac{1}{\mu+1}$, then the equilibrium is the same as in Proposition 1.*
- ***Large proportion of low-ability students in the $d$ populations (Cases 2 and 3).*** *For $\frac{1}{\mu+1} > p_d$,*

  1. *The probability of obtaining a job is higher for $a$ than $d$ students.*
  2. *An $a$ student receives a higher salary than a $d$ student.*

**Proof.** See the appendix. ■

Proposition 4 shows that the redistributive effect emerged in Proposition 2 and 3 is driven by the presence of targeted grade inflation, and it disappears by ruling out this assumption.

# 5 Extensions

## 5.1 Tuition fees differentiation

In this paragraph I consider the possibility that tuition fees may be different according to social background. Begin by denoting $k_d$, $k_a$ the tuition fees for $d$ and $a$ students, respectively.

First, assume that tuitions fees are not valued in their absolute, monetary amount but according to a student's household income. Of course if tuition fees are a small fraction of the household's income, then it is relatively less costly to pay them than the case in which tuition fees are a substantial amount of income. Therefore, assuming (consistently with the definition of social background) that an $a$ student's household has a higher income that a $d$ student's household, in turn $k_d > k_a$. This strengthen the result shown by Corollary 2, since it is more likely that a situation in which $g_{Ld} < k_d$ and $g_{La} > k_a$ takes place, leading to a higher preference of a firm for $d$ and $A$ students and a stronger redistributive effect.

Second, consider the presence of a need-based financial aid covering, totally or partially, tuition fees for students with low socioeconomic status. These types of policies are currently present in many countries. In particular in the United States, in which university tuition fees are considerably higher compared to other countries, the financial aid is regulated by the "Higher Education Act of 2008" and the specific type of help is discretionary to the educational institution. Out of the United States, many national governments provide student financial assistance subsidies for students attending a university, such as in Canada, the United Kingdom, Germany, the Netherlands and Scandinavian countries. Given the presence of need-based policies, assume $k_a > k_d \geq 0$. From Proposition 2 and 3 can be seen that, counterintuitively, these kind of policies may in fact decrease the expected salary of disadvantaged students, as they would worsen the education signal of these student types. The foregoing discussion can be summarised as follows.

**Corollary 3** *Suppose $k_d > (<)\, k_a$, then it is more likely to be in the parameter range in which $d\,(a)$ students are offered a higher salary than $a\,(d)$ students.*

Notice that the relative tuition fees and need-based financial aid to cover fees can be considered at the same time. Of course in this situation, the result depends on which of the two effects prevails.

## 5.2 Different university objectives

So far, the analysis has been carried out with the assumption that the university did not have any redistributive aim whatsoever in the students' job opportunities. Nonetheless, it is noteworthy to consider the cases where a university has different objectives. In what follows I analyse the behaviour either of a redistributive or of an elitist university. In both the situations, the university still favours $H$ over $L$ students, since the change in objective function is about students social background, and the university wants to keep a good reputation towards firms. As a consequence Lemma 1 still holds, and below I focus in the equilibria with grade inflation, i.e., in which Assumption 2 applies.

### 5.2.1 Redistributive University

First I examine the situation in which the university is interested in helping the job opportunities of disadvantaged students. The model can take into account this by assuming:

**Assumption 3** $b_{Hd} > b_{Ha} > b_{Ld} > b_{La}$.

In order to favour disadvantaged students, the university can design its grading policy by keeping a sufficiently high grade inflation for $a$ students that makes $d$ students to be preferred in the job market. The results are summarised in the following proposition.

**Proposition 5** *Suppose Assumption 2 and 3 hold.*

- ***Large proportion of high-ability students in the*** $a$ ***populations (Cases 1 and 2).*** *For* $p_a > \frac{1}{\mu+1}$ *:*

    1. *For* $g_{La} > g_{Ld} > k$*, all the students attend university, and for* $g_{La} > k > g_{Ld}$*, all the* $a$ *students, together with the* $H$ $d$ *students, attend university;*

2. *Grade inflation is given to more a than d students, the probability of obtaining a job is greater for a than d students, and a d student receives a higher salary than an a student.*

- ***Large proportion of high-ability students in the advantaged populations (Case 3).*** *For $p_a < \frac{1}{\mu+1}$,then the equilibrium is the same as in Proposition 3.*

**Proof.** See the appendix. ■

### 5.2.2 Elitist university

Consider next a university willing to favour advantaged rather than disadvantaged students. This may happen in those countries, like the United States, in which wealthy alumni would give donations to their college. Indeed evidence shows that students who receive loans or financial aid are less likely in the future to make a gift in the future (Meer and Rosen, 2011). The behaviour of an elitist university can be implemented as follows:

**Assumption 4** $b_{Ha} > b_{Hd} > b_{La} > b_{Ld}$.

In this case, the university simply tailors grade inflation in order to favour $a$ students: with a large population of $H$ students in both populations it is sufficient to provide the same proportion of grade inflation to each social group. In the other cases, $a$ students are favoured by adopting a grading policy with higher grade inflation for $a$ than $d$ students but sufficiently low to make a firm prefer hiring $a$ students. Therefore in the presence of an elitist university, a higher proportion of grade inflation leads to a higher salary.

**Proposition 6** *Suppose Assumption 2 and 4 hold.*

- ***Large proportion of high-ability students in both populations (Cases 1).*** *For $p_a > p_d > \frac{1}{\mu+1}$, then the equilibrium is the same as in Proposition 1.*

- ***Large proportion of low-ability students in the*** $d$ ***populations (Cases 2 and 3).*** *For* $\frac{1}{\mu+1} > p_d$*:*

    1. *For* $g_{La} > g_{Ld} > k$*, all the students attend university, and for* $g_{La} > k > g_{Ld}$*, all the* $a$ *students, together with the* $H$ $d$ *students, attend university;*

    2. *Grade inflation is given to more* $a$ *than* $d$ *students, the probability of obtaining a job is greater for* $a$ *than* $d$ *students, and an* $a$ *student receives a higher salary than a* $d$ *student.*

**Proof.** See the appendix. ■

# 6 University competition

In this section, I consider the effects of competition among universities on my findings about grade inflation. I show that the essential results carry over to settings with competition.

## 6.1 Symmetric universities

First assume a competitive university market in which $n$ universities operate with no entry of other competitors. A university has a payoff according to Assumption 1, i.e., it prefers that an $H$ rather than an $L$ student obtains a job, and it is indifferent to student social background. In a competitive setting, Assumption 1 implies that a university prefers to enroll $H$ rather than $L$ students, as they will give to the university a higher payoff in the case they are employed.

Each university can admit a number of students of at most $h_u \in (0, 1)$, identical for each university, i.e.

**Assumption 5** $\sum_u h_u = 1$, $h_u = h_s > 0$, *for all* $u, s = 1, 2, .., n$, $u \neq s$.

This is an important assumption, since a university grading strategy cannot increase its own capacity but only helps to fulfill the maximum number of admitted students. In other words, the maximum supply of enrollment is fixed. Each university decides whether or not to inflate grades and, if so, the amount of grade inflation to provide.

Begin with the case in which labour demand is lower than the number of $H$ students, $J < \Phi$. The result in this case is similar to Lemma 2. Given the limited number of expected positions and the fact that a university prefers to enroll $H$ rather than $L$ students, the university will favour the former over the latter by not inflating grades.

**Lemma 4** *Suppose Assumption 1 and 5 hold and $J \leq \Phi$. Then each university never inflates grades and a firm is indifferent to a student's social background.*

Consider next $J > \Phi$. In this case a university may consider inflating grades in order to increase the number of its students by enrolling some of the $L$ students. There are two possible cases. In the first, assume $\Phi < h_u$. If all but one universities inflate grades, then the non-inflating university will attract all the $H$ students, and a firm would only hire those graduates. If $n-1$ of the universities do not inflate grades while one university deviates, then all $H$ students would attend one of the $n-1$ universities, in which there is always room for them since $\Phi < h_u$, so that the payoff of the university that inflates grades will be zero. Therefore even in this case, none of the university inflates grades.

**Lemma 5** *Suppose Assumption 1 holds and $\Phi < h_u$. Then each university never inflates grades and a firm is indifferent to a student's social background.*

In the second case, assume $h_u < \Phi < J$. If $n-1$ universities inflate grades and one university deviates, it will attract the $H$ students according to its own size $h_u$, and since a university prefers to enroll $H$ over $L$ students, it will increase its own payoff. Hence even in this case a situation in which all the universities inflate grades is never an equilibrium.

Conversely, if $n-1$ universities do not inflate grades and a university deviates, then if all the $H$ students find a placement in the other universities, then the university that inflated grades obtains a zero payoff. Therefore this is not an equilibrium, since none of the universities are willing to inflate grades. On the other hand, if some of these $H$ students cannot find a placement in the other universities, they attend the university who inflated grades. Indeed, an $H$ student will score $A$ with certainty for Lemma 1, since even with competition, a university who inflates grades still will do that in such a way to favour her over an $L$ student. In this case, the university compares the payoff of deviation with the payoff of no grade inflation:

$$\frac{\Phi}{n} b_H > \left[\Phi - (n-1) h_u\right] b_H + \left[g_{La} \eta (1-p_a) + g_{Ld} (1-\eta)(1-p_d)\right] b_L, \quad (11)$$

so that two situations may occur:

**Proposition 7** *Suppose Assumption 1 holds and $h_u < \Phi < J$.*

1. *If inequality (11) holds, in equilibrium all universities will not inflate grades;*

2. *If inequality (11) does not hold, in equilibrium one university inflates grades and the $n-1$ universities will not inflate grades.*

Note that, in point 2 of Proposition 7, the deviation strategy would increase the payoff of all the other non-inflating-grades universities, since they would fill their students' capacity with $H$ students. Therefore none of the other universities has an incentive in deviating, which ensures stability to the equilibrium. More important, for the case analysed in point 2 of Proposition 7, the equilibria depicted above in the analysis with no university competition hold for the population of students net of the $H$ students attending the university that gives grade inflation. Hence the results analysed in the non-competitive case may apply as a benchmark for some market with symmetric competition.

Finally, consider the cases examined above in which no grade inflation is given. Accordingly, only $H$ students score $A$, and in turn none of the $L$ students would attend university. Therefore, from an empirical perspective it is not possible to detect the difference between the equilibrium with full grade inflation occurs and the equilibrium with no grade inflation, since all the students attending university will indeed score $A$. The implication is striking: in a competitive university market, an increase in grade point average may not be due to a softer grading policy, it might be in fact the result of competition that skims away low-ability students.

## 6.2 Duopoly with asymmetric universities

The result depicted in Proposition 7 strictly depends on the assumption of equal size of all universities. To understand why, consider this second competitive model. Suppose for simplicity a university duopoly in which one university (denoted by 1) is larger and the number of $H$ students is higher than the size of the smaller university (denoted by 2), i.e.:

**Assumption 6** $h_1 > h_2 > 0$, $h_1 + h_2 = 1$.

In cases $J < \Phi$, $J > \Phi > h_1$, and $J > h_2 > \Phi$, both universities never inflate grades and a firm is indifferent to a student's social background. The argumentations of these results are the same described in the symmetric market.

Consider now $J > h_1 > \Phi > h_2$. If University 1 does not inflate grades, the University 2's dominant strategy is not to inflate grades. Indeed University 1 may enroll all the $H$ students and thus leaving University 2 with $L$ students only. However, if University 2 does not inflate grades, then University 1 compares the payoff of inflating grades with the payoff of no grade inflation, i.e., inequality (11) when $n = 2$:

$$\frac{\Phi}{2} b_H > (\Phi - h_2)\, b_H + [g_{La}\eta\,(1 - p_a) + g_{Ld}\,(1 - \eta)\,(1 - p_d)]\, b_L. \tag{12}$$

Again, two equilibria may occur:

**Proposition 8** *Suppose Assumption 1 and 6 hold and $J > h_1 > \Phi > h_2$.*

1. *If inequality (12) holds, in equilibrium both universities will not inflate grades;*

2. *If inequality (12) does not hold, in equilibrium University* 1 *inflates grades and the University* 2 *does not inflate grades.*

In particular for the case illustrated in point 2 of Proposition 8, the equilibria depicted above in the analysis with no university competition hold for the population of students net of the $H$ students attending University 2. Similarly to point 2 of Proposition (7), the results analysed in the non-competitive case may apply as a benchmark in some market with asymmetric university competition.

# 7 Concluding remarks

This paper has examined the effects of targeted grade inflation when students differ in social background. A university may differentiate grading policy according to a student type, and in equilibrium it may inflate grades with the effect of increasing the salary offered to disadvantaged students. This result is stronger the higher the tuition fees and the higher the redistributive intentions of a university, although it holds even when the university has no redistributive aim. The results hold to some extent in markets with university competition.

Albeit this investigation has a positive flavour, there are implications on the policy side. For instance, a government aiming to reduce the differences in job opportunities among students differing in social origins not necessarily should hammer away at grade inflation, when policy grading can be targeted.

A possible development may be to evaluate the relationship between grade inflation and university reputation (see Ehlers and Schwager, 2012) when students differ in social background. In this direction, it would be necessary to modify the framework in a either repeated or dynamic setting, and to relax the assumption of perfect information about the university strategy.

An interesting follow up paper could analyse how grade inflation affects students' effort incentives. For instance, does more grade inflation create perverse incentives for the students to put forth effort? The relation between grade inflation and students' effort is left for future work.

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# Appendix

## Proof of Proposition 1

Consider $J = 1$. For $p_a > p_d > \frac{1}{\mu+1}$, the equilibrium would be

$$g_{La} = g_{Ld} = 1, \tag{13}$$

thus all the students attend university and obtain a job. Plugging (13) into (9), hiring an $a$ student always gives a higher expected profit.

However, when $p_a\eta + p_d(1-\eta) < J < 1$, this strategy may lead $L$ students to obtain a job at the expense of $H$ students. Therefore the university inflates grades to a number of students that covers labour demand net to the share of $H$, i.e.:

$$g_{La} = g_{Ld} = \frac{J - \Phi}{\eta(1-p_a) + (1-p_d)(1-\eta)}, \tag{14}$$

so that an $L$ and $a$ student has the same chance of obtaining a job than an $L$ and $d$ student. Since $k < g_{Li}$ by Assumption 2, all the students attend university, and firms compare the expected payoffs of a $d$ and an $a$ student. Plugging (14) into (9), hiring an $a$ student gives a higher expected payoff always, so that a firm offers a higher salary to them.

## Proof of Proposition 2

For $p_a > \frac{1}{\mu+1} > p_d$ and $J = 1$, the university strategy is

$$g_{La} = 1, \; g_{Ld} = \frac{p_d\mu}{1-p_d} - \varepsilon. \tag{15}$$

For $k < g_{Ld}$, all the students attend university, and each firm compares the expected profits from hiring a $d$ or an $a$ student. Plugging (15) into (9), hiring a $d$ student gives a higher expected profit for all

$$\mu < \widehat{\mu} \equiv \frac{p_a\varepsilon + p_d\left[1 - p_a\left(1+\varepsilon\right)\right]}{p_a p_d}.$$

Conversely for $k > g_{Ld}$, all the $L$ and $a$ students attend university and none of the $L$ and $d$ students attend it. Notice that, if the university increases grade inflation above $k$, then none of the $d$ students would obtain a job, so that this is the best strategy given the circumstances.

However, when $\Phi < J < 1$, this strategy may lead $L$ students to obtain a job at the expense of $H$ students. In order to avoid this, the university provides grade inflation for the proportion of the expected available positions for low ability students, i.e.:

$$\frac{J - \Phi}{\left(1-p_a\right)\eta + \left(1-p_d\right)\left(1-\eta\right)}.$$

Thus the result would be

$$\begin{aligned} g_{La} &= \frac{J - \Phi}{\left(1-p_a\right)\eta + \left(1-p_d\right)\left(1-\eta\right)}, \\ g_{Ld} &= \left(\frac{p_d\mu}{1-p_d} - \varepsilon\right)\frac{J - \Phi}{\left(1-p_a\right)\eta + \left(1-p_d\right)\left(1-\eta\right)}. \end{aligned} \tag{16}$$

For $g_{La} > g_{Ld} > k$, then all the students attend university, the probability of obtaining a job is higher for $a$ rather than $d$ students, and a firm compares the expected profits from hiring a $d$ or an $a$ student. Plugging (16) into (9), hiring a $d$ student gives a higher expected profit for all $\mu < \widehat{\mu}$, so that for $\mu \leq (>)\,\widehat{\mu}$ a firm would offer a higher salary to $d$ ($a$) rather than $a$ ($d$) students. On the other hand for $g_{La} > k > g_{Ld}$, then none of the $L$ and $d$ students attend university, all the $a$ students attend university, and a firm would choose to hire $d$ rather than $a$ students.

## Proof of Proposition 3

For $\frac{1}{\mu+1} > p_a > p_d$ and $J = 1$, the result would be

$$g_{La} = \frac{p_a \mu}{1 - p_a} - \varepsilon, \; g_{Ld} = \frac{p_d \mu}{1 - p_d} - \varepsilon. \tag{17}$$

For $g_{La} > g_{Ld} > k$, then all the students attend university, and each firm compares the expected profits from hiring a $d$ or an $a$ student. Plugging (15) into (9), hiring a $d$ student gives a higher expected profit always. Conversely for $g_{La} > k > g_{Ld}$, all the $L$ and $a$ students attend university and none of the $L$ and $d$ students attend it.

However, when $\Phi < J < 1$, this strategy may lead $L$ students to obtain a job at the expense of $H$ students. Hence the university provides grade inflation in the proportion of the available position for low ability students, i.e.:

$$\frac{J - \Phi}{(1 - p_a)\,\eta + (1 - p_d)\,(1 - \eta)}.$$

Thus the result would be

$$\begin{aligned} g_{La} &= \left( \frac{p_a \mu}{1 - p_a} - \varepsilon \right) \frac{J - \Phi}{(1 - p_a)\,\eta + (1 - p_d)\,(1 - \eta)}, \\ g_{Ld} &= \left( \frac{p_d \mu}{1 - p_d} - \varepsilon \right) \frac{J - \Phi}{(1 - p_a)\,\eta + (1 - p_d)\,(1 - \eta)}. \end{aligned} \tag{18}$$

For $g_{La} > g_{Ld} > k$, all the students attend university, the probability of obtaining a job is higher for $a$ rather than $d$ students, and a firm compares the expected profits from hiring a $d$ or an $a$ student. Plugging (18) into (9), hiring a $d$ student gives a higher expected profit always, so that she is offered a higher salary than an $a$ student. For $g_{La} > k > g_{Ld}$, none of the $L$ and $d$ students attend university, all the $a$ students attend university, and a firm would choose to hire $d$ rather than $a$ students.

## Proof of Proposition 4

Case 1 does not change compared to the targeted situation, as in Proposition 1 grade inflation is the same in both populations.

Consider next Case 2 and 3. First notice that, if the university cannot target grade inflation, it would choose its grading policy according to the distribution of ability in the $d$ population. Otherwise, if the university applies the optimal level of grade inflation for $a$ students, none of the $d$ students would be hired, as a firm's expected profit from hiring them would be negative, given the excessive grade inflation. Thus by adopting this policy, $a$ and $L$ students would obtain a job over $d$ and $H$ students, going against Assumption 1. As a consequence, both in Cases 2 and 3 the level of grade inflation is the same as for $d$ students in Propositions 2 and 3, i.e.:

$$g_{La} = g_{Ld} = \left( \frac{p_d \mu}{1 - p_d} - \varepsilon \right) \frac{J - \Phi}{(1 - p_a)\,\eta + (1 - p_d)\,(1 - \eta)}. \tag{19}$$

Hence the probability of an $L$ student of being hired is the same along different populations. Since $k < g_{Li}$ by Assumption 2, all the students attend university, and a firm compares the expected profits from hiring a $d$ or an $a$ student. Plugging (19) into (9), hiring an $a$ student gives a higher expected profit always.

## Proof of Proposition 5

In Cases 1 ($p_a > p_d > \frac{1}{\mu+1}$) and 2 ($p_a > \frac{1}{\mu+1} > p_d$) and $J = 1$, the university maximises its objective function by fully inflating grades to $a$ students ($g_{La} = 1$) and keeping the level of grade inflation of $d$ students in such a way than a firm's expected profit is higher by hiring one of them:

$$\frac{p_d \mu - g_{Ld}\,(1 - p_d)}{p_d \mu + g_{Ld}\,(1 - p_d)} > p_a \mu - (1 - p_a)\,. \tag{20}$$

Solving for $g_{Ld}$ yields $g_{Ld} < \frac{(1-p_a)p_d}{(1-p_d)p_a}$, so that the optimal level of grade inflation for $d$ students is

$$g_{Ld} = \frac{(1-p_a)\,p_d}{(1-p_d)\,p_a} - \varepsilon. \tag{21}$$

However, when $\Phi < J < 1$, the university provides grade inflation to a number of students that covers labour demand net to the share of $H$, i.e.:

$$\begin{aligned} g_{La} &= \frac{J-\Phi}{\eta\,(1-p_a)}, \\ g_{Ld} &= \left(\frac{(1-p_a)\,p_d}{(1-p_d)\,p_a} - \varepsilon\right)\frac{J-\Phi}{(1-\eta)\,(1-p_d)}. \end{aligned} \tag{22}$$

For $g_{La} > k > g_{Ld}$, then none of the $L$ and $d$ students attend university, all the $a$ students attend university, and a firm offers a higher salary to a $d$ rather than a $a$ student. For $g_{La} > g_{Ld} > k$, then all the students attend university, and a firm obtains a higher expected payoff by hiring a $d$ student. In Case 3 the equilibrium is the same as in Proposition 3, because with both university's objective function, the $L$ and $d$ students are favoured as much as possible.

## Proof of Proposition 6

In Case 1 the university follows the strategy adopted in Proposition 1, because with both university's objective function, the $L$ and $a$ students are favoured as much as possible. In Cases 2 and 3, and $J = 1$, the result would be

$$g_{La} = 1,\ g_{Ld} = \frac{p_d\mu}{1-p_d} - \varepsilon, \tag{23}$$

ensuring a firm's expected profit is positive by hiring students from both populations, but such that

$$\frac{p_a\mu - g_{La}\,(1-p_a)}{p_a + g_{La}\,(1-p_a)} > \frac{p_d\mu - g_{Ld}\,(1-p_d)}{p_d + g_{Ld}\,(1-p_d)}, \tag{24}$$

that is

$$p_a\mu - (1-p_a) > \frac{p_d\mu - \left(\frac{p_d\mu}{1-p_d} - \varepsilon\right)(1-p_d)}{p_d + \left(\frac{p_d\mu}{1-p_d} - \varepsilon\right)(1-p_d)}, \tag{25}$$

for

$$\varepsilon < \overline{\varepsilon} \equiv \frac{p_d\left[\mu p_a - (1-p_a)\right]}{p_a(1-p_d)}.$$

In order to favour $a$ students in the labour market, the university sets $\varepsilon < \overline{\varepsilon}$.

When $\Phi < J < 1$, the university provides grade inflation in the proportion of the available position for low ability students, i.e.:

$$\frac{J-\Phi}{(1-p_a)\eta + (1-p_d)(1-\eta)}.$$

Thus the result would be

$$\begin{aligned} g_{La} &= \frac{J-\Phi}{(1-p_a)\eta + (1-p_d)(1-\eta)}, \\ g_{Ld} &= \left(\frac{p_d\mu}{1-p_d} - \varepsilon\right)\frac{J-\Phi}{(1-p_a)\eta + (1-p_d)(1-\eta)}, \end{aligned} \tag{26}$$

where $\varepsilon < \overline{\varepsilon}$.

In Case 3 and $J = 1$, the result would be

$$g_{La} = \frac{p_a\mu}{1-p_a} - \varepsilon_a,\ \ g_{Ld} = \frac{p_d\mu}{1-p_d} - \varepsilon_d, \tag{27}$$

ensuring a firm's expected profit is positive by hiring students from both populations, but such that

$$\frac{p_a\mu - g_{La}(1-p_a)}{p_a + g_{La}(1-p_a)} > \frac{p_d\mu - g_{Ld}(1-p_d)}{p_d + g_{Ld}(1-p_d)}. \tag{28}$$

In order to obtain this, the university needs to provide a slightly less amount of grade inflation to $a$ students, and it does that by setting $\varepsilon_a < \varepsilon_d$. Indeed

(28) for

$$\varepsilon_a > \frac{p_a (1 - p_d) \varepsilon_d}{p_d (1 - p_a)}, \tag{29}$$

which in turn holds for $\varepsilon_a < \varepsilon_d$. When $\Phi < J < 1$, the university provides grade inflation in the proportion of the expected position for $L$ students, i.e.:

$$\frac{J - \Phi}{(1 - p_a)\eta + (1 - p_d)(1 - \eta)}.$$

Thus the result would be

$$\begin{aligned} g_{La} &= \left(\frac{p_d \mu}{1 - p_d} - \varepsilon_a\right) \frac{J - \Phi}{(1 - p_a)\eta + (1 - p_d)(1 - \eta)}, \\ g_{Ld} &= \left(\frac{p_d \mu}{1 - p_d} - \varepsilon_d\right) \frac{J - \Phi}{(1 - p_a)\eta + (1 - p_d)(1 - \eta)}, \end{aligned} \tag{30}$$

where $\varepsilon_a > \frac{p_a(1-p_d)\varepsilon_d}{p_d(1-p_a)}$.

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