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Abstract

The aim of this paper is to study both the private and the social desirability of a mixed bundling strategy that generates a cost savings effect. We confirm that mixed bundling is the dominant strategy for multiproduct firms, although it may give rise to a prisoner's dilemma. Moreover, we show that mixed bundling may maximise social welfare, provided that cost savings are sufficiently high. Finally, we highlight the parametric regions where the social and the private interests coincide, and those where they do not. The recent evolution of broadband telecommunications services provides an ideal framework to apply our theoretical predictions.

Keywords: Mixed bundling, strategic interaction, broadband market, cost savings.
JEL Classification: D43, L13, L41.

"The complexity of communication offers and bundles has made it increasingly difficult to understand and compare service prices and characteristics. A lack of transparent information about services and their prices makes consumer price comparisons more difficult and leads to market inefficiencies."

(Broadband Bundling: Trends and Policy Implications. OECD Digital Economy Papers, p. 4)

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1 Introduction

Mixed bundling, the practise adopted by many multiproduct firms to sell their products both on a stand-alone basis and in bundles, is becoming more and more widespread in many different sectors. This occurs not only in high-tech industries such as telecommunications, video game consoles, and hardware and software, but also in more traditional industries, such as music, travel, and scientific publishing. Record companies, for example, now release both single songs and the entire album. When booking a flight, it has become more common to pay separately for additional services that were once included, such as luggage and meal options. Likewise, in online scientific publication, one has now the option to purchase a specific article without having to pay for the whole journal edition.

One sector in particular has experienced a profound change: the telecommunications industry. Cable companies started out offering competing bundles including telephone service, high speed internet connection, and digital cable. In turn, both traditional and newly born telephone companies have responded by offering discounted bundles comprising phone services, DSL connection and satellite television services. The increasing digitalization of network services completed the merging process between these once distinct service providers. Broadband Internet access has rapidly expanded and the shift towards an all Internet protocol (IP) technology has enabled operators to provide a variety of retail services over a single platform. The practice of bundling fixed telephony, pay television, broadband internet, and, more recently, mobile telephony, has increasingly been adopted by different operators.[1] In addition to purchasing such double, triple or even quadruple bundles, consumers usually also have the option of buying individual "components" from different operators, thereby creating their customized "system". The market is therefore characterized by the massive use of *mixed* bundle strategies that may benefit both producers and consumers, given the potential cost savings generated by such practise.[2]

However, antitrust concerns have been raised, as bundling can be adopted to exclude potential rivals from at least one of the reference markets, thus reducing competition.[3] Moreover, bundling has recently posed other problems for both competition authorities and sectoral regulators, which

[1] Bouckaert *et al.* (2010) analyse broadband penetration rates across 20 OECD countries in the 2003-2008 period. Greenstein and McDevitt (2011) estimated the economic value created by the diffusion of broadband from 1999 to 2006 in the United States. Calzada and Martínez-Santos (2014) show that in 2011 the percentage of bundled plans among total broadband subscriptions was higher than 80% in France (93%), Portugal (88%), and Spain (89%).

[2] In fact, leaving aside strategic considerations, the economic reason that explains why separate goods can be sold in packages usually refers to the presence of economies of scale and scope in production, together with reduced transactions and information costs for consumers. In addition, bundling may also reduce billing and administrative costs. The presence of such synergetic effects explains why bundles are usually sold at a price which is lower than the sum of the prices charged for stand-alone products.

[3] Following the seminal paper by Whinston (1990), the entry-deterrence use of bundling has been investigated by Choi and Stefanadis (2001), Carlton and Waldman (2002), Nalebuff (2004) and Peitz (2008), *inter alii.*

face scenarios in continuous evolution, such as the double and triple-play product markets. In particular, whereas the price of the bundled goods typically decreases, the price of stand-alone components may increase.[4] Consumers accustomed to buying all products from the same producer gain from bundling, while those who prefer to "mix-and-match" may eventually lose out. It follows that a careful analysis of the impact of mixed bundling on both firms' profitability and social welfare would be recommended, independently of the use of such strategy to foreclose the market.

For all these reasons, the aim of our paper is to provide a theoretical analysis of the strategic incentive that leads multiproduct firms to adopt mixed bundling in presence of cost savings effects. We build on a component model initially developed by Matutes and Regibeau (1988), Economides (1989), and Economides and Salop (1992), and successively used by Choi (2008) and Maruyama and Minamikawa (2009), among others. In particular, we consider two competing multiproduct firms, each producing a specific version of two complementary goods/components, α and β, which are valuable only when consumed together. Therefore, four possible horizontally differentiated composite systems are available for consumers, given that we assume full compatibility between components. We acknowledge that this is a strong assumption, since, in principle, consumers may be interested in only one of the two components. However, especially in the telecommunication industry, recent papers show that double and triple-play bundles are relevant product markets and that the components of each bundle are perceived by consumers as complements which are usually consumed together (Pereira and Ribeiro, 2011; Pereira *et al.*, 2013).[5]

Firms are engaged in a non cooperative two-stage game. In the first stage they have to decide whether to sell the two goods on a stand-alone basis, or to adopt mixed bundling. In the second stage they compete in prices. A system composed by two complementary goods produced by the same firm and sold in a package will be referred to as "bundled system", while a system composed by two complementary goods produced by the two different producers will be referred to as "hybrid system". Based on the previous discussion on IP technology, we assume that bundling allows to reduce the cost of producing/shipping the products. Potential gains for consumers in terms of improved functionality and/or reduced search and assembly costs will not be considered. Hence, in our analysis a crucial role will be played by the *strategic* effect of mixed bundling in the presence of cost savings on the producers' side.

[4] Choi (2008) considers the controversial decision of the European Commission in 2001 to block the proposed merger between General Electric and Honeywell on the basis of the possibility of bundling between GE's jet aircraft engines and Honeywell's avionics products. One of the critical point raised by the European Commission was the possibility that prices for stand-alone components would have increased after the merger. Greenlee *et al.* (2008) demonstrate that bundled loyalty discounts have ambiguous welfare effects precisely because they may favour the increase of the price for stand-alone components.

[5] Pereira *et al.* (2013), for example, use data from Portugal and show that "the double-play product of fixed voice and fixed broadband is a complement of the single-play product of subscription television. Similarly, the single-play product of fixed voice is a complement of the single-play product of subscription television and of the single-play product of fixed broadband" (p. 768).

The results of our paper are as follows. First of all, we confirm the finding (Thanassoulis, 2007; Maruyama and Minamikawa, 2009; *inter alii*) that mixed bundling is a dominant strategy for both firms, and that they apply a discount on the price of the bundle. We also find that, although in most cases the prices of stand-alone components tend to increase when both firms adopt mixed bundling, there are also instances where the opposite may occur. In particular, this happens when marginal cost savings are high enough to even benefit component prices, provided the systems are sufficiently differentiated. The presence of significant cost synergies may therefore also favour those consumers who still prefer to assemble their composite system on a "mix-and-match" basis. This extends previous studies in which cost savings were neglected and the price of individual components always increased when producers opted for mixed bundling (Choi, 2008, *inter alii*).

We then demonstrate that there exist circumstances where mixed bundling entails the highest profit for firms, thus solving the prisoner's dilemma situation that has often been observed. Not surprisingly, this occurs for *(i)* relatively high levels of marginal cost savings, independently of the degree of differentiation across the systems, and *(ii)* when systems are perceived as extremely substitutable, independently of cost savings. Moreover, this also holds *(iii)* when the systems are sufficiently differentiated and at least some synergetic effects generated by bundling are at work. This last case is relatively new in the literature and it deserves particular attention. In fact, our analysis reveals that, by adopting mixed bundling, each firm enjoys a profit gain from the sale of the bundled system, while it suffers a profit loss from the sale of hybrid systems. When cost savings are intermediate, the balance between such gains and losses depends on the degree of product substitutability in a *non-monotonic* way. This is due to the fact that, when systems become progressively less differentiated, the discounted price associated to the bundle first increases when hybrid systems are still taken into account, and then decreases when consumers virtually neglect such systems due to the strategic increase of the prices of stand-alone components.

Finally, when considering social welfare, we demonstrate that mixed bundling is beneficial for society at large when cost savings are sufficiently high. Interestingly, the higher the degree of product substitutability, the higher the cost savings required for mixed bundling to be socially preferred compared to separate pricing. Indeed, as we noticed above, the discounted price associated to the bundled products tend to decrease when the degree of product substitutability is high. At the same time, in this parametric region, the prices for stand-alone components substantially increase with respect to separate selling. As such, consumers lose out under mixed bundling, unless the cost savings effect is relevant enough to significantly contain the price surge of stand-alone components, while at the same time still guaranteeing a convenient discount on the price of the bundled system.

In general, a careful examination of the different parametric regions provides useful indications about the desirability of an intervention to prohibit mixed bundling, and its consequences for producers. In fact, we show that there exist parametric regions where the private and social interests coincide. This consistently happens for relatively high levels of cost savings, given that mixed bundling is both socially and privately profitable. For intermediate cost savings, such an incentive alignment depends not only on the degree of product substitutability, but also on the measure of social welfare that one considers. When systems are sufficiently differentiated, consumers gain while firms may lose with mixed bundling. The opposite holds when systems are highly substitutable. Based on total welfare, the decision to allow bundling or not represents an attempt to mediate between the two welfare components, but may favour only one category and damage the other, depending on the effective cost reduction induced by bundling. Finally, when cost savings are low, the policymaker should prohibit bundling. This would always benefit consumers, whose surplus is higher under separate pricing, but it may hurt firms when the degree of substitutability is extremely high. On the contrary, prohibiting bundling would solve their prisoner's dilemma when systems are sufficiently differentiated.

Literature and Positioning

Three papers are closely related to our analysis: Choi (2008), Maruyama and Minamikawa (2009), and Flores-Fillol and Moner-Colonques (2011). They have in common an initial scenario characterized by two differentiated brands of two complementary components, each offered by an independent producer, so that there are four firms. Moreover, they all focus on the effect of mergers in complementary markets, in which the merged firm can engage in mixed bundling. Choi (2008) analyses the impact of mixed bundling on pricing decisions and derives relevant welfare implications of mergers. In particular, he finds that the merged entity always increases the prices for the stand-alone components. Although considering the possibility of a counter-merger, his paper mainly focuses on the effects on pricing, profits and social welfare induced by *only one merger* in the industry. Conversely, Maruyama and Minamikawa (2009) investigate a symmetric market structure where *both* pairs of producers of complementary goods decide whether or not to merge, and then to adopt mixed bundling. For this reason, their approach is the closest to our model. They show that merging with bundling is the dominant strategy for both pairs, despite giving rise to a prisoner's dilemma when product differentiation is not particularly high. Of particular interest for our analysis, they also discuss the consequences of bundling in terms of social welfare and find that merging with mixed bundling is never optimal. Flores-Fillol and Moner-Colonques (2011) expand the previous models by introducing consumers that derive utility from *individual consumption* of each of the four products. The new result in their analysis is that the price for the product consumed alone may decrease after bundling. As a consequence, merging is not always an equilibrium strategy, as it becomes unprofitable when markets for individual components are sufficiently relevant.

In our paper, we follow Maruyama and Minamikawa (2009) and Choi (2008) and leave out individual consumption. Differently from their analysis, however, we do not endogenise the merging decision. Rather, we focus on the decision to bundle in presence of a cost savings effect, a point which has been neglected by previous contributions.[6] This allows us to shed more light on the strategic interaction characterising competition between operators which provide a variety of services over the same platform, thereby enjoying significant cost synergies.

As we already introduced, the combination of complementary components into composite systems has been initially studied by Matutes and Regibeau (1988 and 1992) and Economides and Salop (1992). They consider fully integrated firms and show that they prefer compatibility over incompatibility. Einhorn (1992) analyses two vertically differentiated firms, each producing both components of an integrated system. He demonstrates that, in equilibrium, they both choose compatibility. Farrell *et al.* (1998) show that firms may prefer incompatibility with cost heterogeneity in presence of at least three different varieties of each component. Denicolò (2000) analyses compatibility and bundling choices when a generalist firm offering both components competes against two specialist firms. He shows that incompatibility or pure bundling may be profitable for the generalist firm when one component is less differentiated than the other. In Dalkir *et al.* (2002), a merger of quality component leaders benefits the integrated entity, as it results in higher profits and increased market share.

More recently, Gans and King (2006) adopted the Hotelling model to analyse the use of bundled discounts with unrelated products by two pairs of independent producers. They show that the bundled discount is a dominant strategy for both pairs of firms. Armstrong and Vickers (2010) built a model with heterogeneous consumers and elastic demand, in which consumers are allowed to buy from more than one supplier. They find that the impact on social welfare of bundled discounts crucially depends on the combination between demand elasticity, consumer heterogeneity and shopping cost levels. Thanassoulis (2007) points out that mixed bundling may favour producers but reduce consumer surplus if buyers incur firm specific costs. Mialon (2014) extends the standard differentiated products model used in Matutes and Regibeau (1992) to analyse firms' decision between a merger and a strategic alliance in bundling their product with other complementary products.[7]

[6] A relevant exception is the paper by Evans and Salinger (2008), where bundling induces two types of cost reduction. Similarly to our paper, the constant marginal cost of joint production is lower than the sum of the constant marginal cost of separate production. In addition, in their analysis, bundling also reduces the fixed cost of production. However, the framework that they use is different from ours, as they consider contestable markets, thus neglecting strategic effects.

[7] In some way our paper is also linked to the literature that analyses strategic competition when there exists at least one producer whose pricing activities interact with those of other firms selling components that represent

Three other papers are related to our idea that the bundling and merging decision may be affected by the degree of product substitutability. Beggs (1994) studies endogenous merger formation between two groups of firms where products within a group are complements, and those across groups are substitutes. His model can be also applied in situations where producers of complementary products engage in pure bundling. He finds that the producers of complementary goods may refrain from merging in order to avoid counter-mergers by rival complementors that would increase price competition. This occurs especially when market pressure is high. Venkatesh and Kamakura (2003) analyse the effect of the degree of substitutability on bundling and optimal pricing. They show that there exists circumstances where a decrease in the degree of substitutability favours the incentive to bundle. Mantovani and Vandekerckhove (2015) focus on merging and (pure) bundling and consider the case where a pair of complementary producers moves before the other. The equilibrium result depends on a combination between market pressure and timing of the game. For intermediate levels of competitive pressure, both pairs of firms bundle when the game is simultaneous, yet they are trapped in a prisoner's dilemma. On the contrary, in the sequential game, the first movers merge, but refrain from bundling, as to not induce rivals to counter-merge.

Finally, recent empirical contributions focus on the impact of bundling in the telecommunication industry. Pereira *et al.* (2013) show that triple-play bundles are a relevant product market and discuss the impact of bundles on competition and regulatory policy. Calzada and Martínez-Santos (2014) and Fageda *et al.* (2014) analyse the determinants of broadband Internet access prices as well as the operators' marketing strategies. The former paper is based on access prices in a group of 15 EU countries between 2008 and 2011, and the authors show the effect of different bundling on prices. They also examine the impact of competition and regulation on broadband retail prices. The latter paper draws on data from the Spanish market between 2005 and 2011 and demonstrates, among other things, that the impact of bundles on prices is crucially affected by the development of the broadband market achieved by that country. Üner *et al.* (2015) focus on the Turkish telecommunication sector, in which top leading telecom operators are not allowed to offer comprehensive bundles. They investigate the preference for bundle of a random sample of 766 residential consumers across 14 large city centers and discover the presence of many opportunities for bundled services. While such empirical literature touches very interesting points related to the proliferation of bundle activities in the telecommunication sector, these papers do not consider the issue related to potential increase in prices for stand-alone products.

substitutes for each other. This literature was pionereed by Economides and Salop (1992) and has recently resurfaced thanks to the work of Casadesus-Masanell, Nalebuff and Yoffie (2008), Hermalin and Katz (2013), and Mantovani and Ruiz-Aliseda (2015), *inter alii*.

The rest of the paper is organized as follows. The next section summarizes the theoretical model. Section 3 provides the solution of the two-stage and it characterizes the parametric region in which mixed bundling constitutes a prisoner's dilemma. In Section 4 we perform the welfare analysis and draw policy suggestions. In Section 5 we confirm the robustness of the results obtained in the baseline model. Section 6 concludes.

2 The theoretical model

We build our theoretical model on the framework introduced by Economides and Salop (1992) to examine the incentive for producers of complementary products to adopt mixed bundling. Consider two products or services, α and β, which are valuable for customers only when used together, on a one-to-one basis. There are two differentiated brands of each component α and β. Unlike Economides and Salop (1992) and the following literature on this topic, we leave out the incentive for producers of complementary products to merge. Rather, we consider two multiproduct firms, denoted by $i, i = 1, 2$, each of them producing a variety of both components α and β. Consumers can buy both components from the same firm, or "mix-and-match" the components produced by different firms. Complementarities arising from higher functionality/compatibility when buying both components from the same producers are not taken into account.[8] Therefore, four equally performing composite systems are available in the market: $\alpha_1\beta_1$, $\alpha_1\beta_2$, $\alpha_2\beta_1$, $\alpha_2\beta_2$.

Formally, the prices of brands α_i and β_i are respectively given by p_{α_i} and p_{β_i}, $i = 1, 2$. For simplicity of notation, q_{11} indicates the demand for $\alpha_1\beta_1$, and so on. The price for each system is given by the sum of the prices of the two components, where $p_{ij} = p_{\alpha_i} + p_{\beta_j}$ indicates the price of system $\alpha_i\beta_j$, with $i, j = 1, 2$. The four systems $\alpha_i\beta_j$ are substitute for one another, thus their demand q_{ij} is decreasing in its own price, p_{ij}, and increasing in the prices of the other systems. Following Choi (2008) and Maruyama and Minamikawa (2009), we assume that the four systems are equally substitutable. Without loss of generality, we also normalize both demand intercept and own-price elasticity to 1. The four demand functions are linear and symmetric, and given by

$$\begin{aligned} q_{11} &= 1 - p_{11} + \gamma(p_{12} + p_{21} + p_{22}), && (1) \\ q_{12} &= 1 - p_{12} + \gamma(p_{11} + p_{21} + p_{22}), && (2) \\ q_{21} &= 1 - p_{21} + \gamma(p_{11} + p_{12} + p_{22}), && (3) \\ q_{22} &= 1 - p_{22} + \gamma(p_{11} + p_{12} + p_{21}). && (4) \end{aligned}$$

where γ represents the symmetric cross-price elasticity of demand, with the restriction $\gamma < 1/3$

[8] Using the definition introduced by Stremersh and Tellis (2002), our paper is therefore about price bundling and not about product bundling, where the integration of separate components in the bundle improves the performance of the system, thereby raising consumers' reservation price

due to the fact that composite systems are assumed to be gross substitutes.[9]

The two multiproduct firms engage in a two-stage game. In the first stage, they simultaneously decide whether to sell the products only on a stand-alone basis or to adopt mixed bundling. In the second stage, upon observing their sale strategies, they set market prices.

As previously introduced and motivated by the recent evolution of the telecommunications sector, we assume that bundling gives rise to synergetic effects in production captured by a reduction in the marginal cost. In particular, while the provision of each single component entails a symmetric marginal cost equal to c per unit, when the firm sells the two products in a package (or it provides the two retail services over the same platform), it benefits from a cost reduction captured by $\theta \in (0,1)$. Hence, the marginal cost of the bundled system reduces to $c(1+\theta)$. Notice that $c(1-\ \theta)$ can be interpreted as a measure of the cost savings effect: the higher its value (the lower is θ), the higher the (marginal) cost synergy enjoyed by the firm when selling the two goods together.[10] To keep our analysis as simple as possible, we assume that $\theta = 0$, meaning that the production/provision of the bundled goods only implies a marginal cost equal to c, instead of $2c$. This is without loss of generality, as it will be discussed in Section 4.[11] To summarize:

Assumption 1: *The marginal cost of producing/providing the two complementary components of each system is equal to the marginal cost of producing each single component.*

Before proceeding with the formal analysis, notice that one of the consequences of adopting Assumption 1 is that parameter c becomes an "indirect" measure of cost savings; the higher its value, the more the firms save with mixed bundling.

[9] The four demand functions can be initially written as follows

$$\begin{aligned} q_{11} &= \sigma - \eta p_{11} + \gamma p_{12} + \delta p_{21} + \epsilon p_{22} \\ q_{12} &= \sigma - \eta p_{12} + \gamma p_{11} + \delta p_{22} + \epsilon p_{21} \\ q_{21} &= \sigma - \eta p_{21} + \gamma p_{22} + \delta p_{11} + \epsilon p_{12} \\ q_{22} &= \sigma - \eta p_{22} + \gamma p_{21} + \delta p_{12} + \epsilon p_{11} \end{aligned}$$

where $\sigma, \eta, \gamma, \delta > 0$ and $\eta > \gamma + \delta + \epsilon$ because the composite products are gross substitutes. By assuming $\gamma = \delta = \epsilon$, we obtain that $\eta > 3\gamma$. Parameter σ represents market size, while η and γ respectively reflect the own-price and the cross-price elasticity of demand. Finally, normalizing to 1 both demand intercept and the own-price elasticity ($\sigma = \eta = 1$) implies that the cross-price elasticity of demand γ is such that $\gamma < 1/3$, and the demand functions are then simplified as in (1)-(4).

[10] Pereira and Vareda (2013) state: "The digitalization of electronic communication networks and the adoption of packet switching implied that seemingly different services, such as voice, data and subscription television services, became simply a stream of packets, susceptible of being delivered over a single network.[...] Under these circumstances, both telecommunications firms and cable television firms have the ability to offer all of these services, and incentives to do so in bundles.[...] Bundling may emerge due to the type of technological reasons described above, in which case it will typically involve cost reductions." (p. 531).

[11] In Section 5 we demontrate that the main results of the theoretical model still hold if we consider $\theta \in (0,1)$.

3 The solution of the game

The first part of this section considers the second stage of the game and it highlights the impact of mixed bundling on prices and firms' profits. We first consider the case of separate pricing, our benchmark scenario. We then study the asymmetric situation in which only one firm adopts mixed bundling. We complete the analysis with the case in which both firms adopt mixed bundling. In the second part of the section we define the equilibrium of the game by considering the first stage in which both players decide whether to adopt mixed bundling or not. A detailed discussion about the profitability of mixed bundling in presence of cost savings will be provided.

3.1 The market stage

The benchmark case: separate pricing

When both firms sell the products on a stand-alone basis, profits are given by:

$$\pi_1 = (p_{\alpha_1} - c)(q_{11} + q_{12}) + (p_{\beta_1} - c)(q_{11} + q_{21}), \tag{5}$$

$$\pi_2 = (p_{\alpha_2} - c)(q_{21} + q_{22}) + (p_{\beta_2} - c)(q_{12} + q_{22}), \tag{6}$$

where c indicates the marginal cost of production, as previously mentioned. Using F.O.C.s, equilibrium prices can be easily obtained:

$$p_{\alpha_i}^{II} = p_{\beta_i}^{II} = \frac{2 + c(3 - 5\gamma)}{7 - 17\gamma}, \ i = 1, 2. \tag{7}$$

Additional superscript II indicates equilibrium variables when both firms opt for independent (or separate) pricing. Equilibrium profits amount to:

$$\pi_i^{II} = \frac{8(3 - 5\gamma)[1 + 2c(3\gamma - 1)]^2}{(7 - 17\gamma)^2}, \ i = 1, 2. \tag{8}$$

Equilibrium prices, quantities, and profits are always positive under the assumption that $\gamma < 1/3$, as it can be easily demonstrated.[12]

The asymmetric case: only one firm adopts mixed bundling

Suppose now that firm 1 resorts to mixed bundling, while firm 2 does not. The only difference with respect to demand system (1)-(4) is given by the presence of a price p_{B_1} instead of p_{11}, where p_{B_1} indicates the unique price for the bundle $\alpha_1\beta_1$. Firm 1 now charges a price for the bundle and two separate prices for each component sold on a stand-alone basis. According to our

[12] Equilibrium quantities are not reported in the text in order to reduce the number of expressions that we display. However, they can be easily obtained, and they are available upon request.

assumption, the production of the bundled systems implies a total cost of c, instead of $2c$. Profit functions become:

$$\pi_1 = (p_{B_1} - c)q_{11} + (p_{\alpha_1} - c)q_{12} + (p_{\beta_1} - c)q_{21} \tag{9}$$

$$\pi_2 = (p_{\alpha_2} - c)(q_{21} + q_{22}) + (p_{\beta_2} - c)(q_{12} + q_{22}). \tag{10}$$

Using F.O.C.s, equilibrium prices can be easily computed:

$$p_{B_1}^{BI} = \frac{11 - 9\gamma + c[11 + \gamma(14\gamma - 27)]}{22 - 74\gamma + 48\gamma^2}, \tag{11}$$

$$p_{\alpha_1}^{BI} = p_{\beta_1}^{BI} = \frac{4 - 3\gamma + c[3 - 2\gamma(3 - \gamma)]}{11 + \gamma(24\gamma - 37)}, \tag{12}$$

$$p_{\alpha_2}^{BI} = p_{\beta_2}^{BI} = \frac{(1 - \gamma)[3 + 5c(1 - 2\gamma)]}{11 + \gamma(24\gamma - 37)}. \tag{13}$$

Additional superscript BI indicates equilibrium variables when firm 1 resorts to mixed bundling, while the other keeps independent pricing. Notice that:

$$p_{B_1}^{BI} < p_{\alpha_2}^{BI} + p_{\beta_2}^{BI} < p_{\alpha_1}^{BI} + p_{\beta_1}^{BI}. \tag{14}$$

As a consequence, the price for the bundle is lower than the sum of the prices for the stand-alone products. The pressure to reduce the prices of the stand alone products is stronger for the non-bundling firm. Moreover, by comparing (12)-(13) with (7) we find that:

$$p_{\alpha_1}^{BI} = p_{\beta_1}^{BI} > p_{\alpha_1}^{II} = p_{\beta_1}^{II}, \tag{15}$$

$$p_{\alpha_2}^{BI} = p_{\beta_2}^{BI} < p_{\alpha_2}^{II} = p_{\beta_2}^{II}. \tag{16}$$

Hence, with respect to separate selling, the prices for the stand-alone products rise only for the firm that bundles, while the rival has to lower its prices as it faces competition from the bundled system. Therefore, we confirm that Choi's result (2008, Proposition 1, p. 562) can also extend to the case where both producers of complementary goods have already merged, but only one of them bundles and benefits from cost savings.

Equilibrium profits amount to:

$$\pi_1^{BI} = \frac{3 \cdot \Phi + 2c(1 - 2\gamma)(29 - 27\gamma)[\gamma(51 - 32\gamma) - 13] + c^2(1 - 2\gamma) \cdot \Delta}{4[11 + \gamma(24\gamma - 37)]^2}, \tag{17}$$

$$\pi_2^{BI} = \frac{2(3 - 5\gamma)\{2c[3 - \gamma(11 - 7\gamma)] - 3(1 - \gamma)\}^2}{[11 + \gamma(24\gamma - 37)]^2}, \tag{18}$$

$$\Phi = [83 - \gamma(290 - 299\gamma + 96\gamma^2)],$$

$$\Delta \equiv 633 - \gamma\{4952 - \gamma[13261 - 2\gamma(6929 - 2464\gamma)]\}.$$

Equilibrium prices, quantities, and profits are positive under the additional condition that $c \leq \min\{\overline{c}, 1\}$, where $\overline{c} \equiv \dfrac{8 - 5\gamma(5 - 3\gamma)}{16 - \gamma[89 - 3\gamma(51 - 26\gamma)]}$.[13] In the remainder of the analysis, the following holds:

Assumption 2 $c \leq \min\{\overline{c}, 1\}$.

In other words, we assume that a positive demand always exists for the four systems, even in the asymmetric case where one multiproduct firm chooses not to bundle while its rival does.[14]

Both firms adopt mixed bundling

We now consider the last case, in which both firms resort to mixed bundling. The demand system can be easily obtained by respectively replacing p_{11} with p_{B_1} and p_{22} with p_{B_2}, which indicates the price for the bundle $\alpha_2\beta_2$ charged by firm 2. Profit functions write:

$$\pi_1 = (p_{B_1} - c)q_{11} + (p_{\alpha_1} - c)q_{12} + (p_{\beta_1} - c)q_{21}, \quad (19)$$

$$\pi_2 = (p_{B_2} - c)q_{22} + (p_{\alpha_2} - c)q_{21} + (p_{\beta_2} - c)q_{12}. \quad (20)$$

Taking F.O.C.s, equilibrium prices are:

$$p_{B_i}^{BB} = \frac{1 + c(1 - 2\gamma)}{2 - 5\gamma}, \quad (21)$$

$$p_{\alpha_i}^{BB} = p_{\beta_i}^{BB} = \frac{2[1 + c(1 - 2\gamma)]}{6 - 15\gamma}, \ i = 1, 2. \quad (22)$$

Additional superscript BB indicates that both firms make use of the mixed bundling strategy. The next result is straightforward:

$$p_{B_i}^{BB} < p_{\alpha_i}^{BB} + p_{\beta_i}^{BB}, \ i = 1, 2. \quad (23)$$

As in the previous case, the price for the bundle is lower than the sum of the prices for the independent products sold by the same firm. We postpone further discussion on the effect on prices to the next subsection.

Equilibrium profits are given by:

$$\pi_i^{BB} = \frac{17 - 32\gamma + c(1 - 2\gamma)\left\{146\gamma - 50 + c[41 + \gamma(319\gamma - 228)]\right\}}{9(2 - 5\gamma)^2}, \ i = 1, 2. \quad (24)$$

Equilibrium prices, quantities and profits are always positive under the basic assumptions of our model, *i.e.* $\gamma < 1/3$ and $c \leq \min\{\overline{c}, 1\}$.

[13] In particular, the condition $c \leq \overline{c}$ derives from $q_{12}^{BI} = q_{21}^{BI} \geq 0$; moreover, we have to take into account that $c < 1$. It is easy to find that $\overline{c} < 1$ when $\gamma < 0.2$.

[14] Indeed, without this assumption, only two systems would be present in the market, *i.e.* $\alpha_1\beta_1$ and $\alpha_2\beta_2$. Moreover, for even higher values of c, the non-bundling firm may exit the market, allowing its opponent to monopolize the market.

3.2 To bundle or not to bundle?

We can now solve the two-stage game and obtain subgame perfect Nash equilibria. The following result holds:

Proposition 1 *The game presents a unique Nash Equilibrium in dominant strategies in which both producers adopt mixed bundling. Such an equilibrium is Pareto efficient when* $\gamma > 0.308$ *and when* $c \in (c^N, \bar{c})$ *if* $\gamma \leq 0.308$,*whereas it gives rise to a prisoner's dilemma when* $c \in (0, c^N]$ *if* $\gamma \leq 0.308$.

Proof. See Appendix, where we also report the precise value of c^N, which is non-negative when $\gamma \lesssim 0.308$. ∎

Each firm would always bundle, if the rival continued to sell its products on a stand-alone basis. Likewise, each firm would respond by bundling as well, if the rival bundles. Hence, under the assumptions of the model, firms always adopt mixed bundling, thus showing that the results found by Maruyama and Minamikawa (2009, Proposition 1, p. 66) extend to our scenario. We also compare equilibrium prices (7) with (21)-(23) and first confirm that the price charged for the bundle is always lower than the sum of the prices of stand-alone components. Then, when γ is sufficiently high ($\gamma > 0.285$), we find that each multiproduct firm increases the prices of stand-alone components. However, when $\gamma \leq 0.285$, the following result holds:

$$p_{B_i}^{BB} < p_{\alpha_i}^{II} + p_{\beta_i}^{II} < p_{\alpha_i}^{BB} + p_{\beta_i}^{BB} \text{ if } c \in \left(0, c^P\right), \ i = 1, 2. \tag{25}$$

$$p_{B_i}^{BB} < p_{\alpha_i}^{BB} + p_{\beta_i}^{BB} < p_{\alpha_i}^{II} + p_{\beta_i}^{II} \text{ if } c \in [c^P, \bar{c}), \ i = 1, 2. \tag{26}$$

Corollary 1 *With respect to separate pricing, when both firms adopt mixed bundling, the prices of stand-alone components rise when* $c < c^P$, *whereas they decrease in* $c \in [c^P, \bar{c})$, *where* $c^P \equiv 2(1-2\gamma)/[4-\gamma(13-7\gamma)] < \bar{c}$. *Notice that* $c^P < 1$ *only when* $\gamma \lesssim 0.285$. *Hence, component prices always increase when* $\gamma > 0.285$.

It follows that firms offer not only a price discount for the bundle, but they may also charge a lower price for stand-alone components. This is a new result as compared to previous papers. In particular, stand-alone components are cheaper under bundling when $c \in [c^P, \bar{c})$, provided the degree of substitutability between systems is not too high ($\gamma \lesssim 0.285$). Moreover, the more intense the degree of product substitutability, the higher the cost savings required for the hybrid systems to be cheaper under bundling. In other words, we find that c^P is increasing in γ. This reveals a contrasting effect of γ and c on the component price differential. Consider, for example, $p_{\alpha_i}^{BB} - p_{\alpha_i}^{II}$. Obviously, both stand-alone component prices $p_{\alpha_i}^{BB}$ and $p_{\alpha_i}^{II}$ tend to increase in both γ and c. However, they differently react to changes in these parameters. Indeed, on the one hand we have:

$$\frac{\partial(p^{BB}_{\alpha_i} - p^{II}_{\alpha_i})}{\partial\gamma} > 0. \tag{27}$$

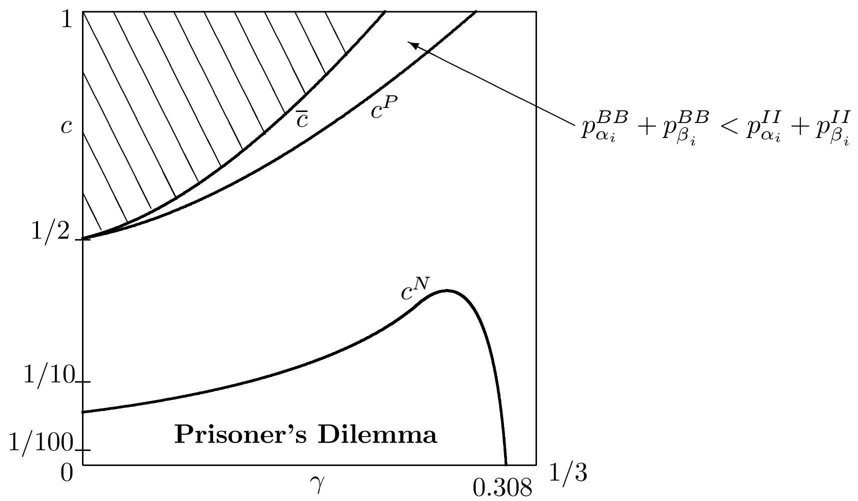
This means that $p^{BB}_{\alpha_i}$ increases more than $p^{II}_{\alpha_i}$ when product substitutability augments, given that firms are interested in reducing the competitive pressure in the market by making hybrid systems less attractive. On the other hand,

$$\frac{\partial(p^{BB}_{\alpha_i} - p^{II}_{\alpha_i})}{\partial c} < 0, \tag{28}$$

and $p^{BB}_{\alpha_i}$ rises less than $p^{II}_{\alpha_i}$ when c increases, as firms enjoy cost reductions in the bundled systems which are also "transferred" to the stand-alone components. For this reason, when $c \in [c^P, \overline{c})$, we obtain that $p^{BB}_{\alpha_i} < p^{II}_{\alpha_i}$. However, when $\gamma > 0.285$, the substitutability effect always dominates the cost savings one, and $p^{BB}_{\alpha_i} > p^{II}_{\alpha_i}$. Systems are perceived as extremely substitutable, and producers have to charge very high prices for individual components in order to render hybrid systems unattractive.

The results of Proposition 1 and Corollary 1 are represented in Figure 1, where the dashed area can be disregarded, following Assumption 2.[15]

Figure 1 : The solution of the game



In primis, the above figure reveals two findings that are not surprising with respect to the possible solution to the prisoner's dilemma. In particular, both firms gain with bundling when: *(i)* the synergetic effect induced by such practise is extremely relevant, independently of the degree

[15]Note that the bottom half of the figure has been rescaled in order to focalize on the important aspects that we want to highlight.

of product substitutability captured by γ, and *(ii)* systems are very substitutable ($\gamma \geq 0.308$), independently of the level of cost savings measured by c. The former result is a direct consequence of the way we model cost savings, although it highlights that prices for independent components may even decrease, as reported in Corollary 1. When $c = 1/2$, for example, firms always gain by bundling, although prices for stand-alone products increase. The latter result is already well accepted in the literature (Matutes and Regibeau, 1992; Maruyama and Minamikawa, 2009; *inter alli*) and it is explained by the fact that (mixed) bundling is usually adopted to soften competition when the degree of substitutability across systems is particularly high. This is valid also for very low values of cost savings, as one can see by considering the case in which $c = 1/100$.

Our analysis reveals a third case in which mixed bundling is beneficial for firms. In particular, this occurs when *(iii)* at least a certain degree of cost savings are present ($c = 1/10$, for example; more on this value later), and the systems are sufficiently differentiated ($\gamma < 0.308$). When this happens, firms avoid the prisoner's dilemma in $c \in (c^N, \bar{c})$; however, the interplay between c and γ becomes so puzzling that c^N is *non-monotone* in γ. This is a crucial result of our paper, and it deserves particular attention, as different forces are simultaneously at work. In order to have an economic interpretation, we need to understand what happens to total profits, prices, and demands when firms adopt mixed bundling *vis à vis* the benchmark case represented by separate pricing. We can decompose the profits into two main components to demonstrate that:

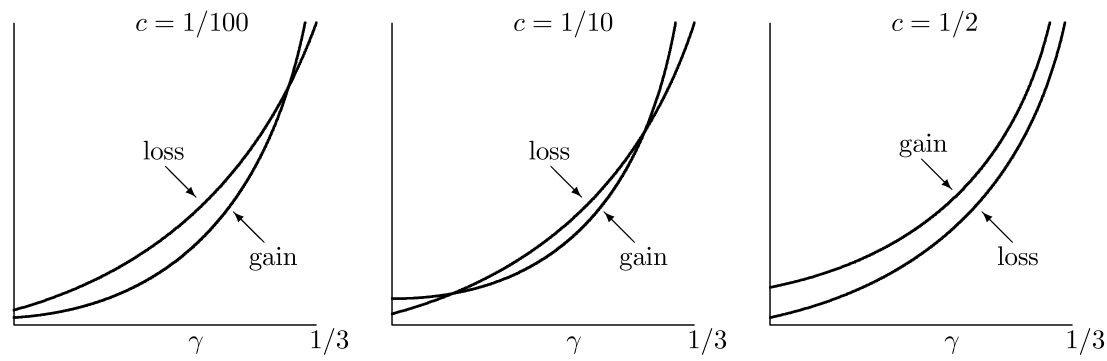
Lemma 1 *As compared to separate pricing, when both firms adopt mixed bundling, they enjoy a profit gain from the sale of the bundled system, while they suffer a profit loss from the sale of the hybrid systems.*

Proof. See Appendix. ■

From Corollary 1 we know that prices for stand-alone components may rise or fall. When they increase, it can be easily demonstrated that the price surge does not compensate for the demand reduction. In general, the demand for hybrid systems always shrinks when firms adopt bundling. Hence, also in the parametric region where hybrid systems are cheaper ($c \in [c^P, \bar{c})$), their demand turns out to be lower than under separate selling, given that bundle systems are indeed more convenient. This explains why the profit accruing from hybrid systems diminishes under bundling. As for the bundled system, it guarantees a profit gain as compared to separate pricing. Its price is always lower than the sum of the prices of stand-alone components, however the increase in the demand (at the expenses of hybrid systems) more than compensates for the discounted price.

It is also shown in the Appendix that an increase in γ expands the gain in the bundled system while simultaneously aggravates the losses in the hybrid systems. On the contrary, while such gain typically increases in c, the losses decrease with the intensity of cost savings. Hence, it is again the interplay between c and γ which becomes crucial. When $\gamma > 0.308$, the gain is always higher than the loss, thus solving the prisoner's dilemma. When $\gamma \in [0, 0.308]$, the profitability of bundling at the firms' level is determined by the intensity of marginal cost savings, as Proposition 1 suggests. Figure 2 represents gains and losses for the three different values of c previously represented in Figure 1 as well:

Figure 2 : Bundled system's gain and hybrid systems' losses



For low values of c, examplified by $c = 1/100$, the gain in the bundled system is obviously scarce, while the losses from the hybrid systems, although not very significant, are usually higher than the gain, unless systems are very substitutable. In fact, only for high values of γ, does the bundling strategy pay off. For sufficiently higher values of c the opposite holds, and the bundling strategy is always Pareto efficient, as one can when $c = 1/2$. Finally, notice that for intermediate values of c ($c = 1/10$), the two curves intersect twice, thus contributing to explain the non-monotone shape of c^N appearing in Figure 1. In order to obtain more insight, however, let us consider the bundled discount, defined as

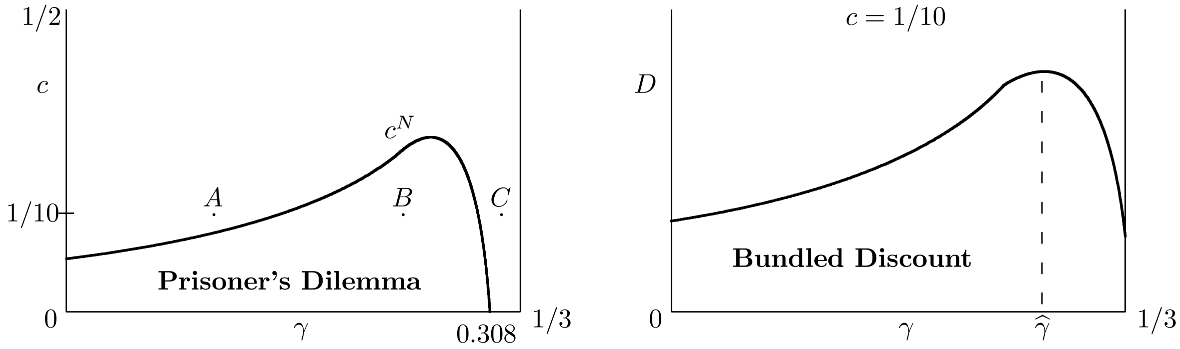
$$D = \left| p_{B_i}^{BB} - (p_{\alpha_i}^{II} + p_{\beta_i}^{II}) \right| . \tag{29}$$

While all the other price and quantity differences (in absolute values) are always increasing in γ, as it can be easily ascertained, when it comes to the discount we find that $\partial D/\partial\gamma \gtrless 0$ in $c \in (0, 1)$. In particular,

$$\frac{\partial D}{\partial \gamma} \geq 0 \text{ when } \gamma \leq \frac{85 + 201c - 2\sqrt{(5+c)(17+8c)}}{255 + 511c} \equiv \widehat{\gamma}. \tag{30}$$

In the right panel of Figure 3, we represent D under the assumption that $c = 1/10$, while in the left panel of the same figure we report the representation already appearing in Figure 1, limiting our analysis to $c \in (0, 1/2)$. In this interval, firms increase the price of stand-alone components when adopting mixed bundling, and such a price surge is increasing in γ, as it is already known.

Figure 3 : Explaining the prisoner's dilemma



Consider the left panel of Figure 3 at $c = 1/10$ and start at point A. Notice from the right panel that, for low γ, the discount proposed by both producers on their respective bundled system is moderate. Nonetheless, for such an intermediate value of cost savings, this discount is such that the increase in the demand of the bundled system more than compensates for the decrease in the demand of hybrid systems. Thus, firms are gaining from the bundled system, thanks to cost savings, and not losing too much from the hybrid systems, whose prices are subject to a contained increase (low γ), and whose demand is not crowded out by the bundle discount. Consumers indeed still perceive the systems as sufficiently differentiated. The balance of gains and losses is therefore in favour of mixed bundling, which ensures a consistent gain in the bundle system in front of relatively small losses in the hybrid systems.

On the contrary, in point B the game is characterized by a prisoner's dilemma. Systems are more substitutable than in point A, which implies a simultaneous increase in the price of stand-alone components (and a higher loss in such systems), and a consistent discount for the bundled systems, as it can be noticed from the right panel of Figure 3. For such intermediate levels of product substitutability, firms still care about both their respective bundled system and the two hybrid systems. However, in the attempt to capture more and more consumers, each firm has to offer a large discount, thereby eroding the profit gain that derives from the bundled system.

Finally, in point C, the four systems are extremely substitutable and consumers tend to neglect the hybrid systems given that similar alternative are offered at a discount. However, precisely for the fact that hybrid systems are virtually not an option, then competition between bundled systems smooths out, thus allowing firms to reduce the discount. This can be observed in the right panel of Figure 3 for relatively high values of γ. Hence, each producer is able to recoup some gains when selling the bundle system with respect to the previous case, thus contributing to explaining the private profitability of mixed bundling in such an interval region.

4 Welfare analysis

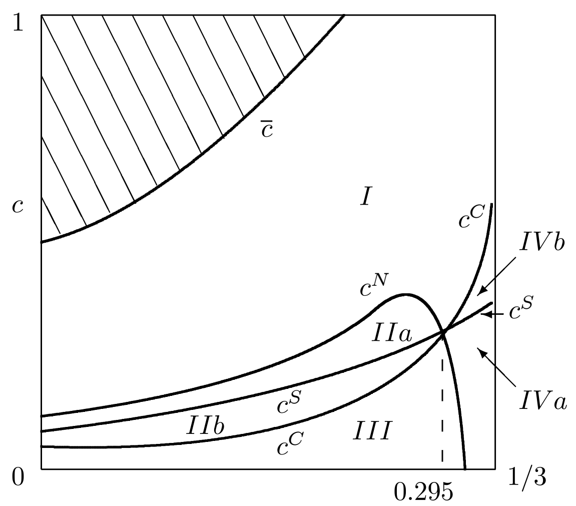
In this section we evaluate the impact of the bundling decision carried out by firms on consumer surplus and total welfare. Recall from Proposition 1 that mixed bundling is a dominant strategy for both firms. The following result holds:

Lemma 2 *Mixed bundling benefits consumers when $c \geq c^C$, while it maximises total welfare when $c \geq c^S$. Both c^C and c^S are increasing in γ. Moreover, $c^S \geq c^C$ when $\gamma \in (0, 0.295]$. When $c < c^S$, total welfare is maximised under separate pricing.*

Proof. See Appendix, where we also report the precise expression of c^S and c^C. ■

The threshold values c^S and c^C are represented in Figure 4, together with c^N. When the cost savings effect is substantial (relatively high values of c), both consumers and producers unambiguously gain from bundling. Consider lower levels of c. When systems progressively become less differentiated, the prices for stand-alone components tend to increase while the discount for the bundled systems initially augments and then shrinks. As we noticed in the previous section, this has contrasting effects on firms' profitability, thus explaining the shape of c^N. As for consumers, this means that, the higher the value of γ, the higher the cost savings effect required for mixed bundling to increase their surplus with respect to separate pricing. One can notice that c^S represents a sort of "mediation" between c^N and c^C. It is also evident that c^C increases more rapidly when c^N starts decreasing in γ, precisely for the fact that the bundle discount diminishes, as we know from (30).

Figure 4 : Consumer surplus, profit and social welfare



The following proposition summarizes the most relevant results in terms of comparing the private and social incentive to bundle:

Proposition 3 *The decision of the two multiproduct firms to adopt mixed bundling:*

(I) *is Pareto efficient and it also maximises consumer surplus when* $c \in \left(\max\left\{c^N, c^C\right\}, \overline{c}\right)$*, and mixed bundling is therefore socially efficient;*

(II) *maximises consumer surplus but results in a prisoner's dilemma when* $\gamma \in (0,\ 0.295]$ *and* $c \in \left(c^C, c^N\right)$*, and mixed bundling is socially efficient only when* $c \in \left(c^S, c^N\right)$*;*

(III) *does not maximise consumer surplus and it generates a prisoner's dilemma when* $c \in \left(0, \min\left\{c^N, c^C\right\}\right)$*, hence separate pricing would be socially efficient;*

(IV) *is Pareto efficient but does not maximise consumer surplus when* $\gamma \in (0.295, 1/3]$ *and* $c \in \left(\max\left\{c^N, 0\right\}, c^C\right)$*; mixed bundling is socially efficient only when* $c \in \left(c^S, c^C\right)$*.*

Proof. The results of Proposition 3 can be easily obtained by combining the results of Proposition 1 and those of Lemma 2. ■

In Figure 4 we represent the four regions described in Proposition 3. In Region *I* the bundling activity generates a sufficiently high cost savings effect to benefit both producers and consumers. Such an effect has to be comparatively higher for intermediate values of γ, otherwise firms would lose from bundling given the significant price reduction required for bundled systems to be competitive. In Region *II* product substitutability is not excessive and cost savings are intermediate. Consumers always gain but producers are trapped in a prisoner's dilemma. In terms of total welfare, bundling is socially efficient only in Subregion *IIa*, where the cost savings is sufficiently high to guarantee that the gain enjoyed by consumers overcomes the profit loss suffered by producers. In subregion *IIb* the opposite holds, and bundling should be prohibited. This would benefit producers, but damage consumers, who would still prefer mixed bundling. In Region *III*, the cost savings effect is too low to benefit consumers. Moreover, the intensity of product substitutability is not enough for producers to avoid the prisoner's dilemma. Here separate pricing would increase both consumers surplus and firms' profits, and the social planner should therefore prohibit bundling. Finally, in Region *IV* another conflict of interests between producers and consumers takes place. Contrary to Region *II*, in Region *IV* product substitutability is very intense. It follows that producers gain, whereas consumers suffer, as shown in the previous analysis. The loss for consumers is higher than the gain for producers in Subregion *IVa*, where the cost savings effect is limited. The opposite holds in Subregion *IVb,* where bundling should be allowed, although still damaging consumers with respect to independent selling.

The above analysis reveals that social and private incentives for mixed bundling are often not aligned. Moreover, the interval regions where firms and society disagree may change, depending on the measure of social welfare taken into account. In fact, if one considers total welfare, then bundling should be prohibited when $c < c^S$. Such a decision would however hurt consumers in Subregion *IIb*, where they would have actually benefited from consistent discounts on the bundled systems. On the contrary, focusing again on total welfare, bundling should be allowed in Subregion *IVb.* Here consumers would lose from bundling; compared to separate pricing, not only stand-alone components become extremely expensive, but the discounts on bundled systems are also not that substantial.

5 Robustness of the Baseline Model

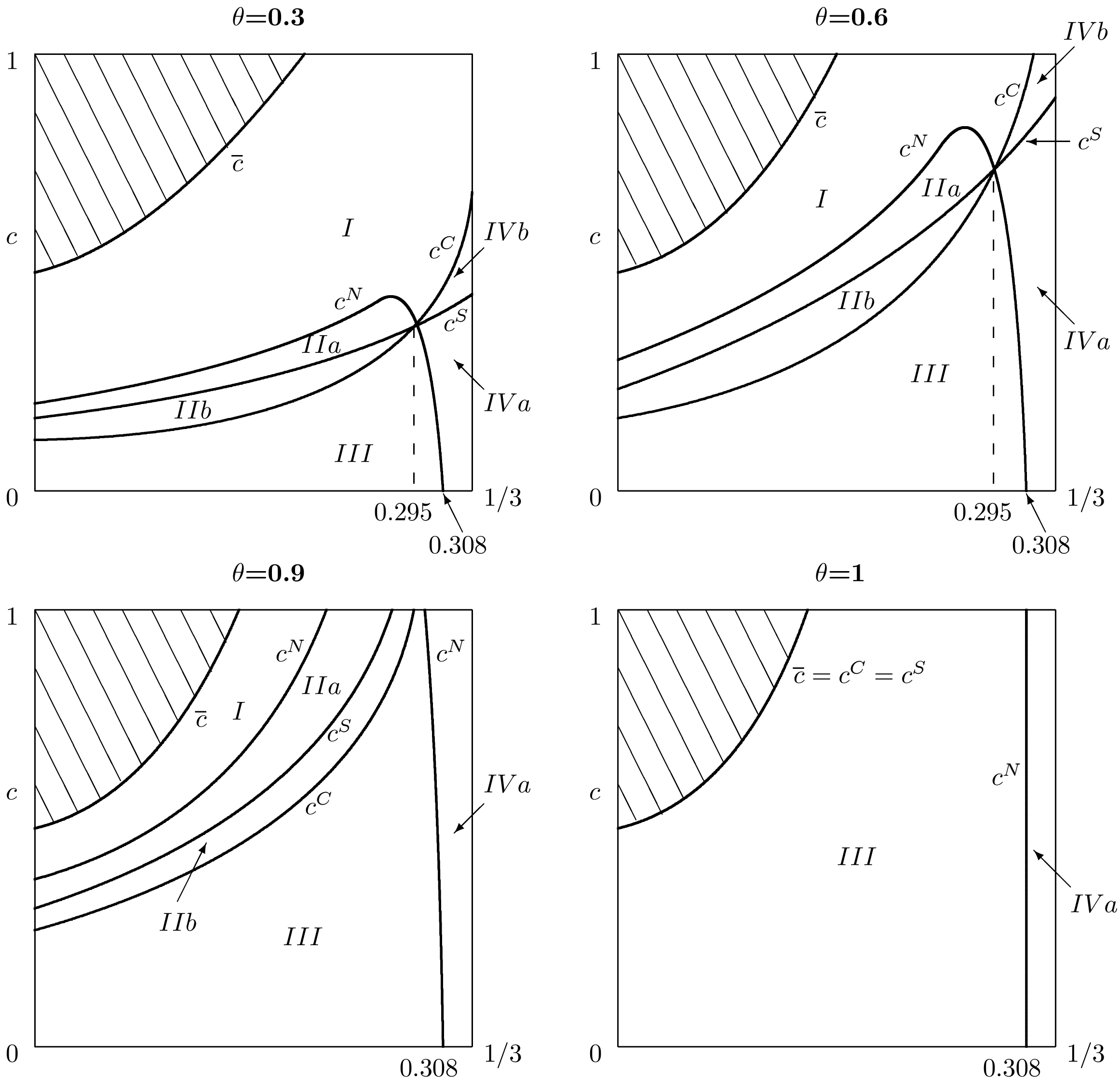
One of the most important assumption of our model regards the extreme synergetic effect induced by bundling. Following Assumption 1, we carried out the calculations for the three possible scenarios (independent pricing, one firm adopting mixed bundling, both firms adopting mixed bundling) and the relative comparisons assuming that $\theta = 0$. However, one may wonder what happens when $\theta \in (0, 1)$. In such a case, θ becomes a precise inverse measure of the cost savings effect. As we noticed in Section 2, the higher its value (or, the lower the value of $(1 + \theta)$), the lower the cost synergy brought forth by selling the two complementary products in a bundle.

Consider then $\theta \in (0, 1)$. The benchmark case (separate pricing) does not change, while the equilibrium values for prices, profits and social welfare in both the asymmetric case and in the symmetric case in which both firms bundle now depend on θ. Such expressions, although not reported here, are still tractable. However, when turning to comparing profits and social welfare expressions, the threshold values of c become extremely long. For this reason, we decided to omit them from the current analysis.

It is however possible to demonstrate that all the results stated in Propositions 1 and 2 are still valid, at least when some sort of cost savings effect is present.[16] The only difference is represented by the fact that the interval regions reported in Figure 4 are affected by θ. More precisely, all the relevant threshold values of c increase in θ, as one can also easily guess. Indeed, the lower the cost savings effect, the lower the parametric region in which bundling is both privately and socially efficient (or, the higher the threshold value of c above which bundling is both privately and socially beneficial). The graphical representation provided in Figure 5 allows one to easily interpret the consequences of considering progressively higher values of θ on the main results that we found in the baseline model.

[16] A formal demonstration can be provided upon request, together with the precise expressions of the threshold values of c that depend on θ.

Figure 5 : Private and social incentives when θ=0.3, θ=0.6, θ=0.9, θ=1



In particular, what we can therefore learn from considering different values of θ is that only the relative dimensions of our main regions change, but not the results. The only relevant exception is represented by $\theta = 1$. In this extreme case, cost savings are ruled out, and only regions *III* and *IVa* persist. This replicates the results obtained by Maruyama and Minamikawa (2009) if one considers only multiproduct firms. Firms adopt mixed bundling, and they are caught in a prisoner's dilemma when $\gamma < 0.3084$. Moreover, the bundling decision is never socially optimal, independently on the measure of social welfare that one considers. It follows that such a market practise should be *tout court* prohibited. Apart from social welfare, this dramatically reduces the richness of results and the underlying strategic interaction between firms that appears when a certain degree of cost savings are generated. For this reason, we believe that the inclusion of cost savings plays a crucial role when evaluating the social desirability of (mixed) bundling activities in markets for complemetary goods.

6 Conclusions

In this paper we considered two multiproduct firms that each sell two complementary products. They have to decide whether to fix a price for each product separately or to adopt mixed bundling. We assumed that selling the two products together in a package generates a (marginal) cost reduction. The telecommunications sector provides a useful scenario to justify our theoretical framework. In fact, the increasing digitalization of network services has enabled operators to provide a variety of retail services over a single platform, thus benefiting from significant cost savings. A report conducted by the OECD in 2011 revealed that broadband services in the OECD were overwhelmingly sold as mixed bundles.[17] Moreover, it reported examples of the costs of set-top boxes to support the cost savings argument for the recent development of the telecommunications sector. In fact, triple-play providers include now video, voice and data functionality into a single set-top box, thus allowing for relevant cost savings.

Based on such stylized case, we analyzed a two-stage game in which firms had to decide whether to adopt mixed bundling or separate pricing in the first stage, and then had to compete in prices in the second stage. We initially confirmed that mixed bundling is a dominant strategy for each player, and pointed out the interval regions in which firms were trapped in a prisoner's dilemma. Interestingly, we highlighted how the profitability of mixed bundling crucially depends on the interplay between the cost savings effect and the degree of substitutability between systems. In particular, in comparison with the benchmark case represented by separate pricing, when firms adopt mixed bundling they enjoy a profit gain in the bundled system, whereas they suffer from a profit loss in the hybrid systems. Moreover, we discovered that component prices and bundled prices behave differently when product differentiation decreases. The former tend to increase, while the bundle discount first rises and then declines. When bundling generates sufficiently high cost savings, firms unambiguously gain. Furthermore, when cost synergies are extremely profound, component prices are lower than in the case of separate selling. On the contrary, for low

[17] The data collection consisted of over 2, 000 offers of stand-alone and bundled services from 90 firms across 30 OECD countries.

levels of cost savings, bundling is profitable for firms only when systems are highly substitutable, as the bundle discount is low, whereas individual components become extremely expensive.

We also evaluated the social welfare consequences of adopting mixed bundling as compared to the benchmark case represented by the case in which firms opt for separate pricing. We demonstrated that mixed bundling maximizes social welfare when the cost savings effect is sufficiently high. The social welfare analysis carried out in the paper also allowed us to evaluate consumer surplus. Again, we showed that the parametric regions where the interests of producers and those of consumers were not aligned depended on the degree of system substitutability. A case which deserves particular attention occurs when systems are perceived as extremely substitutable. As we highlighted above, component prices increase substantially, while the discount on the bundled system is modest. In such scenario, consumers could be extremely penalized by the bundling strategy adopted by the firms, unless a large cost savings effect is generated.

Appendix

Proof of Proposition 1

The following Table represents the first stage of the game, where firms have to decide whether to resort to mixed bundling or not. The precise expressions of the equilibrium profits are reported in (8), (17), (18), and (24), respectively.

Table A: Payoff matrix

		Firm 2			
		I		B	
Firm 1	I	π_1^{II}	π_2^{II}	π_1^{IB}	π_2^{IB}
	B	π_1^{BI}	π_2^{BI}	π_1^{BB}	π_2^{BB}

First, it can be easily obtained that (mixed) bundling is a dominant strategy for both firms, given that we can prove that: *(i)* $\pi_1^{BI} > \pi_1^{II}$ and $\pi_1^{BB} > \pi_1^{IB}$; *(ii)* $\pi_2^{IB} > \pi_2^{II}$ and $\pi_2^{BB} > \pi_2^{BI}$. Hence, the unique equilibrium is represented by both firms opting for such a strategy.

Secondly, turning to Pareto efficiency, we need to evaluate the following difference:

$$\pi_i^{BB} - \pi_i^{II} = \frac{1}{9\left(7-17\gamma\right)^2\left(2-5\gamma\right)^2}\cdot\Theta,$$

$$\begin{aligned}\Theta &= \left(7-17\gamma\right)^2\{32\gamma - c\left(1-2\gamma\right)\left[146\gamma + 41c - \gamma c\left(228-319\gamma\right) - 50\right] - 17\} \\ &\quad -9\left(2-5\gamma\right)^2 8\left(3-5\gamma\right)\left[1 + c\left(-2+6\gamma\right)\right]^2.\end{aligned}$$

For any given $\gamma \in (0, 1/3)$, this expression is a second degree polynomial in c. When $c = 0$, this polynomial is negative for $\gamma \in (0, 0.308)$ and positive for $\gamma \in (0.308, 1/3)$, as in Maruyama and Minamikawa (2009). However, for each $c > 0$, the polynomial has two real roots,

$$\begin{aligned}c_{\pm}^N &= \frac{\left[4727\gamma - 16107\gamma^2 + \gamma^3(23305 - 11806\gamma) - 503\right] \pm 3\left(7-17\gamma\right)\left(2-5\gamma\right)\sqrt{2\Psi}}{18828\gamma - 96140\gamma^2 + \gamma^3(240458 - 293781\gamma + 139618\gamma^2) - 1447}, \\ \Psi &= (1-3\gamma)(59-364\gamma) + \gamma(699-446\gamma) > 0 \quad \text{for each } \gamma \in (0, 1/3).\end{aligned}$$

By denoting with c_+^N the bigger root and by c_-^N the lower one, it is possible to demonstrate that $c_+^N > \bar{c} > c_-^N$, and that c_-^N is non-negative when $\gamma < 0.308$. We can therefore discard c_+^N and keep $c_-^N \equiv c^N$. To sum up, $\pi_i^{BB} \geq \pi_i^{II}$ always when $\gamma \in (0.308\,43, 1/3)$, and when $c \geq c^N$ in $\gamma \leq 0.308$, whereas $\pi_i^{BB} < \pi_i^{II}$ when $c < c^N$, which can occur only for $\gamma \leq 0.308$.

Proof of Lemma 1

On the one hand, when both firms adopt mixed bundling, the profit of firm 1 profit can be decomposed as follows:

$$\begin{aligned}\pi_1^{BB} &= \pi_{11}^{BB} + \pi_{12}^{BB} + \pi_{21}^{BB} \\ &= (p_{B_1}^{BB} - c)q_{11}^{BB} + (p_{\alpha_1}^{BB} - c)q_{12}^{BB} + (p_{\beta_1}^{BB} - c)q_{21}^{BB},\end{aligned}$$

with:

$$\pi_{11}^{BB} = \frac{[1-c(1-3\gamma)]\{3-4\gamma+c[\gamma(17-22\gamma)-3]\}}{3(2-5\gamma)^2},$$
$$\pi_{12}^{BB}+\pi_{21}^{BB} = \frac{2[1-c(2-4\gamma)][2-c(4-11\gamma)]}{9(2-5\gamma)}.$$

On the other hand, in the benchmark case in which none of them bundled:

$$\begin{aligned}\pi_1^{II} &= \pi_{11}^{II}+\pi_{12}^{II}+\pi_{21}^{II} \\ &= (p_{\alpha_1}^{II}+p_{\beta_1}^{II}-2c)q_{11}^{II}+(p_{\alpha_1}^{II}-c)q_{12}^{II}+(p_{\beta_1}^{II}-c)q_{21}^{II},\end{aligned}$$

with:

$$\pi_{11}^{II} = \pi_{12}^{II}+\pi_{21}^{II} = \frac{4(3-5\gamma)[1-c(2-6\gamma)]^2}{(7-17\gamma)^2}.$$

It is straightforward to demonstrate that, under the basic assumptions of our model:

$$\pi_{11}^{BB}-\pi_{11}^{II}>0,$$
$$\left(\pi_{12}^{BB}+\pi_{21}^{BB}\right)-\left(\pi_{12}^{II}+\pi_{21}^{II}\right)<0.$$

It follows that multiproduct firm 1 gains from selling the bundled system, while it loses in the hybrid systems in which it participates by selling one component. Moreover,

$$\frac{\partial\left|\pi_{11}^{BB}-\pi_{11}^{II}\right|}{\partial\gamma}>0,\ \frac{\partial\left|\left(\pi_{12}^{BB}+\pi_{21}^{BB}\right)-\left(\pi_{12}^{II}+\pi_{21}^{II}\right)\right|}{\partial\gamma}>0$$

Hence, both the gain and the losses are increasing in the degree of system substitutability. Similar results obviously hold for firm 2.

Finally,

$$\frac{\partial\left|\pi_{11}^{BB}-\pi_{11}^{II}\right|}{\partial c}\geq 0 \text{ when } c\in(0,\widetilde{c}],$$
$$\widetilde{c}=\frac{\gamma^2[(7219-4087\gamma)\gamma-4690]+1335\gamma-141}{(1-3\gamma)\{4021\gamma-\gamma^2[13929+\gamma(11642\gamma-21051)]-429\}}<1\Leftrightarrow\gamma<0.225;$$
$$\frac{\partial\left|\left(\pi_{12}^{BB}+\pi_{21}^{BB}\right)-\left(\pi_{12}^{II}+\pi_{21}^{II}\right)\right|}{\partial c}<0.$$

While the losses are always decreasing in the degree of cost savings, the gain in the bundled system increases in c only when cost savings are not excessive, provided $\gamma<0.225$. When $\gamma\geq 0.225$, the gain is always increasing in the intensity of cost savings, captured by c. In Figure 2 in the text we considered the relevant case in which $\partial\left|\pi_{11}^{BB}-\pi_{11}^{II}\right|/\partial c\geq 0$. Similar results obviously hold for firm 2.

Proof of Lemma 2

In order to derive the consumer surplus, we assume that the utility function U of the representative consumer is separable and linear in the numeraire good m:

$$\begin{aligned}
& U(q_{11}, q_{12}, q_{21}, q_{22}) \\
= \; & m + (\lambda + \mu)\,\sigma(q_{11} + q_{12} + q_{21} + q_{22}) - \frac{\lambda - 2\mu}{2}(q_{11}^2 + q_{12}^2 + q_{21}^2 + q_{22}^2) \\
& -\mu(q_{11}q_{12} + q_{11}q_{21} + q_{11}q_{22} + q_{12}q_{21} + q_{12}q_{22} + q_{21}q_{22}),
\end{aligned}$$

where $\lambda = \frac{\eta}{(\eta-3\gamma)(\eta+\gamma)} = \frac{1}{(1-3\gamma)(1+\gamma)}$ and $\mu = \frac{\gamma}{(\eta-3\gamma)(\eta+\gamma)} = \frac{\gamma}{(1-3\gamma)(1+\gamma)}$, as we normalized to 1 both σ and η. We can then compute consumer surplus and social welfare, for each of the three cases of interest.

In the benchmark case where both firms sell the stand alone products, consumer surplus amounts to:

$$CS^{II} = \frac{2(3-5\gamma)^2[1+2c(3\gamma-1)]^2}{(7-17\gamma)^2\,(1-3\gamma)},$$

and social welfare is equal to:

$$SW^{II} = \frac{2(5\gamma-3)\,(29\gamma-11)\,[1+2c(3\gamma-1)]^2}{(7-17\gamma)^2\,(1-3\gamma)}.$$

In the second case, when only one firm adopts mixed bundling:

$$\begin{aligned}
CS^{BI} = \; & \frac{349 + \gamma\,\{\gamma\,[3463 + 3\gamma\,(264\gamma - 925)] - 1837\}}{8\,(1-3\gamma)\,[11+\gamma(24\gamma-37)]^2} + \\
& + \frac{c\,(1-2\gamma)\,\{\gamma\,[1950 + \gamma\,(744\gamma - 2105)] - 577\}}{4[11+\gamma(24\gamma-37)]^2} + \\
& \frac{c^2\,(1-2\gamma)\left\{1033 - 6552\gamma + \gamma^2\,[14061 + 2\gamma\,(1864\gamma - 6129)]\right\}}{8[11+\gamma(24\gamma-37)]^2}, \\
SW^{BI} = \; & \frac{1279 + \gamma\left[17101\gamma + 3\gamma^2\,(1560\gamma - 5023) - 7951\right]}{8\,(1-3\gamma)\,[11+\gamma(24\gamma-37)]^2} + \\
& + \frac{c\left[13744\gamma + \gamma^2(26558\gamma - 8304\gamma^2 - 2195)\right]}{4[11+\gamma(24\gamma-37)]^2} + \\
& \frac{c^2\left\{4027 + 125911\gamma + 4\gamma^3\,[\gamma\,(38055 - 10712\gamma) - 50717] - 36606\right\}}{8[11+\gamma(24\gamma-37)]^2}.
\end{aligned}$$

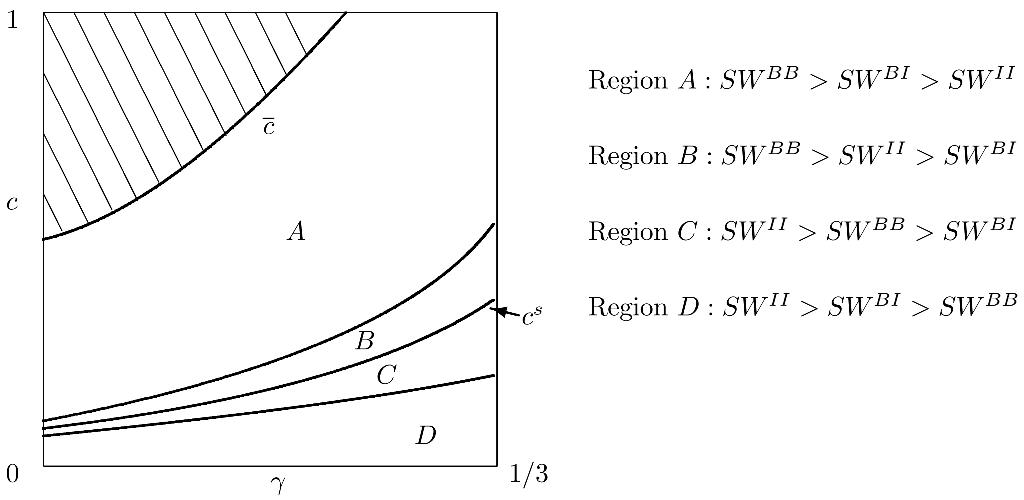
In the last case, where both firms adopt mixed bundling:

$$CS^{BB} = \frac{c\,(1-2\gamma)\,[2\,(32\gamma-17) + c\,(1-2\gamma)\,(25-73\gamma)]}{9\,(2-5\gamma)^2} + \frac{13 + \gamma\,(39\gamma - 46)}{9\,(1-3\gamma)\,(2-5\gamma)^2} \quad (31)$$

$$SW^{BB} = \frac{c\,(1-2\gamma)\,(2\,(178\gamma-67) + c\,(107 + \gamma\,(784\gamma - 579)))}{9\,(2-5\gamma)^2} + \frac{47 + \gamma\,(231\gamma - 212)}{9\,(1-3\gamma)\,(2-5\gamma)^2} \quad (32)$$

Regarding social welfare, after tedious but rather straightforward computations, the following ranking applies:

Figure A : Social welfare ranking



It is interesting to notice that the asymmetric case is never optimal. The threshold values of c that delimit the above interval regions are omitted for brevity, with the exception of c^S, given that it discriminates between the two relevant cases SW^{BB} and SW^{II}:

$$\begin{aligned} c^S &= \frac{12974\gamma + \gamma^2[8\gamma(7163 - 3452\gamma)] - 1469 + 3(7 - 17\gamma)(2 - 5\gamma)\sqrt{\Xi}}{\gamma^2[\gamma(613115 - 719778\gamma + 329848\gamma^2) - 256073] + 52605\gamma - 4261}, \\ \Xi &= 1047 - 8360\gamma + \gamma^2[24259 + 2\gamma(7140\gamma - 15227)]. \end{aligned}$$

Regarding consumer surplus, a similar representation as Figure A can be obtained, although with different threshold values of c. We decided to omit it in order to reduce the length of the paper. However, it is relatively easy to demonstrate that: *(i)* when $c \geq c^C$, consumer surplus is at its highest when both firm bundle; *(ii)* when $c < c^C$, consumer surplus is maximized under independent selling. The threshold value c^C is as follows:

$$\begin{aligned} c^C &= \frac{\gamma[3520 - 9543\gamma + 2\gamma^2(5347 - 2002\gamma)] - 463 + 3(7 - 17\gamma)(2 - 5\gamma)\sqrt{\Upsilon}}{\gamma\{14949\gamma - \gamma^2[63793 - 132199\gamma + 4\gamma^2(33054 - 12653\gamma)]\} - 1367}, \\ \Upsilon &= 113 - 626\gamma + \gamma^2[1205 + 4\gamma(63\gamma - 235)]. \end{aligned}$$

Finally, $c^S \geq c^C$ when $\gamma \leq 0.295$, and $c^S < c^C$ when $\gamma \in (0.295, 1/3]$.

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