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Abstract

A popular way of obtaining essential inputs requires the establishment of an input production joint venture (IPJV) in the upstream (U) section of the vertical chain of production by firms competing and selling final goods in the downstream (D) section of the vertical chain. In spite of the apparently simple arrangement there are many possible governances for the management of the IPJV according to the ownership structure and to the degree of delegation granted to the IPJV by parent firms. We explore the best sustainable governance arrangement for the IPJV. We address this question in a duopoly framwork and we find a large area of impossible vertical arrangements associated with technological asymmtery. The most likely governance of the vertical arrangment associated to the IPJV is total independence.

JEL codes: L24, L42

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1 Introduction

A popular way of obtaining intermediate goods requires the establishment of an input production joint venture (IPJV) in the upstream (U) section of the vertical chain of production by firms competing and selling goods in the downstream (D) section of the vertical chain. Many examples may be found in almost all industries (Hewitt, 2008; Rossini and Vergari, 2010; Chen and Ross, 2003). Firms jointly build and own a firm which is specialized in the production of an input sold to the owner firms.

In spite of the apparently simple arrangement there are many possible governance structures which may be adopted for the management of the IPJV.The main differences among them depend on the ownership structure of the IPJV and on the degree of delegation and/or freedom granted to the IPJV by parent firms.

A challenging question concerns the best sustainable governance arrangement for the IPJV from a private and a social perspective. However, a crucial and, may be, hotter question regards the feasibility of most IPJV governance settings, in particular in the cases in which there are asymmetries between the parents firms as to production costs in D and ownership stakes in the IPJV. To this purpose we shall focus also on the cases in which D firms with different degrees of efficiency adopt a specific vertical arrangement, since asymmetric firms turn out to be quite often unable to give rise to vertical coordinated settings, making many governances of the IPJV simply not feasible. Indeed, this is main aim of this paper: to go through the large area of impossible vertical arrangements associated with technological asymmtery, which casts many doubts on the ability of firms to efficiently jointly manage IPJV.

More precisaly, we develop a simple duopoly framework with linear pricing.[1] We investigate both a symmetric and asymmetric framework. We first consider the case in which the IPJV is left totally independent of the owners and pursues its objective, namely it maximizes its own profit obtained in the U section. In a second case the D firms delegate U to pursue an objective that takes into account the joint vertical profits, namely its objective consists of both the profit raised in U and the profit raised by both D firms. Comparing these two cases and several intermediate settings we wonder whether it is better from the D firms' standpoint to let or not to let the U firm behave independently. We shall see that the second incentive scheme (i.e., the maximization of the joint profit) leads to a cartel outcome which is clearly the best solution from the industry point of view. Nevertheless the governance arrangement underlying this incentive scheme requires coordination among the D owner firms as they have to agree on the market strategy of the U firm. If firms are not equal in all respects, i.e., they show different degree of efficiency, the two firms may not be able to coordinate on the input price charged by the U firm they own. Then, the cartel-clone solution is not possible. A way out of this impossibility could be a bargaining. Unfortunately, there are large areas of the parameter sets in

[1] The adoption of linear pricing seems the most realistic approach to vertical pricing as recent literature points out (Sappington (2005), Arya *et al.* (2008)).

which the bargaining solution does not exist. This occurs mainly when the two firms are different but own the U in equal shares. If the two firms have different effciency levels and different stakes in the IPJV, for instance proportional to their relative efficiency, a the bargaining solution may exist.

A general available way out of impossibility dead ends is delegation. However, when deciding about delegating U, the D firms face a trade-off. The more the U firm is independent of the D firms, the higher the negative externality that it imposes on the D firms and the lower the joint profits. The lower the degree of delegation granted to the U firm, the higher the joint profit obtained in U (that in the limit corresponds to the monopoly profit) but the lower the probability of reaching an agreement about the U strategy. As we shall see, this trade-off points to the existence of an optimal degree of delegation. We shall then see that the most likely governance of the vertical arrangment associated to the IPJV is total independence, leaving aside most of remaing alternative governance schemes.

The paper is organized as follows. In Section (2) we describe the model. We first study a symmetric framework, where firms are equal in all respects and may have the incentive to choose an intermediate degree of delegation to grant to the U producer they own. We then investigate an asymmetric framework where firms differ in their efficiency levels. This is the case where impossibility results are more likely to arise. We conclude in Section (3).

2 Model

We consider a Cournot duopoly model with 2 firms producing a differentiated output, q_i sold at the unit price p_i and variable production costs equal to $c_i q_i$. The demand system is given by linear inverse schedules $p_i = a - q_i - bq_j$ in the region of quantities where prices are positive. The parameter $a > 0$ represents market size; $b \in [0, 1]$ measures the degree of substitutability between the final products (if $b = 1$, products are perfect substitutes; if $b = 0$, products are specialized, i.e., perfectly differentiated).

Manufacturing a final good requires an essential input produced by an upstream (U) enterprise owned either in equal or in different stakes by the downstream (D) firms (Input Production Joint Venture - IPJV). More precisely, the D firms may set up an Equity Joint Venture (Hewitt, 2008) whose profits accrue ultimately to the D firms themselves, making for their consolidated profits.

As it is customary in the literature on vertical relationships we assume that one unit of input is embodied in each unit of output (perfect vertical complementarity). Input production requires a fixed commitment equal to $f \geq 0$.

A two stage game is developed where the two firms first (possibly) agree on the price of the essential input and then compete in the D market. The D firms' consolidated profits read as follows:

$$\pi_{iD} = (p_i - c_i - w)\, q_i + s_i \left[w(q_i + q_j) - f \right], i = 1, 2 \text{ and } i \neq j, \qquad (1)$$

where w is the input price set in the first stage if an agreement among the D firms is reached; $s_i \in (0,1)$ is the share of the U IPJV owned by firm i, $s_i = 1/2$ means that U firm is owned in equal stakes by Ds. The consolidated profits are then made up of two parts. The first $(p_i - c_i - w)\, q_i$ is the own profit gained in the D section by each D firm. The second $s_i\,[w(q_i + q_j)]$ is the share of profit gained in the U section. Proceeding backwards in the second stage, profit maximization by the D firms gives rise to the equilibrium quantities and prices which depend on w; in the first stage the input price w is chosen. The input price is a crucial variable which "filters" the externality affecting many vertical relationships. In our model w is set by the IPJV and the setting rule depends on the degree of delegation that the D firms decide to grant.

2.1 Symmetric framework

We first analyse the case of symmetric technology and for the sake of simplicity we assume $c_i = c_j = 0$. This also let us reasonably set $s_i = s_j = 1/2$.

2.1.1 Minimal and maximal delegation

Consider first the case in which the U firm completely complies with the D guidelines (hence, the degree of delegation granted to the U firm is zero, *minimal delegation*). Each firm first chooses its preferred input price w_i and then the output q_i maximising π_{iD} defined in (1). This scenario has been analysed by Chen and Ross (2003) that confine the analysis to the case of perfect symmetry. The equilibrium variables are:

$$
\begin{aligned}
w &= \frac{ab}{1+b}, \\
p_M &= \frac{a}{2}, && (2) \\
q_M &= \frac{a}{2\,(1+b)}, && (3) \\
\pi_{iD} &= \frac{a^2}{4\,(1+b)},
\end{aligned}
$$

and industry profits are:

$$\Pi_M = \frac{a^2}{2\,(1+b)} - f. \qquad (4)$$

This equilibrium is equivalent to the horizontal merger between the two D firms, which decide to produce in-house (vertical integration) the essential input and set the monopoly price downstream. Unfortunately this governance arrangement suffers a drawback which raises a feasibility issue. An agreement between the two D firms as to the input price is reached if and only if $w_i = w_j = w$. It is easily shown that the two firms would like to charge different prices for the input when, in general, they are not equal, i.e., for example $c_i \neq c_j$. In other words the degree of disagreement depends directly upon the difference in their cost structure, i.e., the degree of asymmetry.

Independently of firms' asymmetries, any coordination problem as to the input price can be fixed in the case of complete delegation when the D firms let the IPJV decide on the input price in a totally autonomous way (the degree of delegation granted to the U firm is one, *maximal delegation*). Formally, first, the input price is chosen by the IPJV maximising its own objective function:

$$\pi_U = w\left(q_i + q_j\right) - f. \tag{5}$$

In the second stage we face two alternative routes: the D firms compete in the downstream market maximising either their consolidated profit or their operative profit. We first consider the former alternative where the D firms maximize π_{iD} defined in (1). The equilibrium variables are:

$$\begin{aligned} w &= a \qquad (6)\\ p_i &= \frac{a\left(3+b\right)}{2\left(2+b\right)}, \qquad (7)\\ q_i &= \frac{a}{2\left(b+2\right)},\\ \pi_{iD} &= \frac{a^2\left(b+3\right)}{4\left(b+2\right)^2} \qquad (8)\\ \Pi &= \frac{a^2\left(3+b\right)}{2\left(b+2\right)^2} \qquad (9) \end{aligned}$$

It is easily shown that the operative profit in D, i.e., $\left(p_i - w\right) q_i$ is negative as the input price set by U is too large. This is a first impossibility result that we shall generalize in the asymmetric framework (Section 2.2). If we consider the latter alternative for the second stage, i.e., maximization of Ds' operative profits rather than the consolidated ones, the same equilibrium price, quantity and industry profit arise; however, the equilibrium input price is at the monopoly level, i.e., $w_M = a/2$ and the distribution of profits along the vertical chain is such that both the operative profit in D and U are nonnegative.[2]

After having compared the two extreme cases, we realize that under complete delegation there is no coordination problem as to the input price to be charged by U. On one hand complete delegation is good since it is feasible. On the other hand, the market equilibrium turns out to be worse than the cartel case (delegation = 0) from both a social and a private point of view (indeed comparing equilibrium price and industry profit we find that p_M defined in (2) is lower than p_i defined in (7), and Π_M defined in (4) is lower than Π defined in (9)).

2.1.2 Optimal degree of delegation

Given the above results in the two extreme cases of total and no delegation we may wonder whether there exists a level of delegation which is feasible and

[2]For details see Rossini and Vergari (2010) where we investigate IPJV in the symmetric case.

sustainable as an equilibrium, not coincident with neither of the two extremes, the cartel case and total delegation.

To this purpose we introduce uncertainty about the possibility to reach an agreement on the U strategy and assume that the D owner firms strategically decide the degree of delegation to grant to U, $d \in [0,1]$. More precisely, we consider a situation where the U objective function is:

$$\pi_u = dw\left[q_1 + q_2\right] + (1-d)\left[w\left(q_1+q_2\right) + \left(p_1 - w\right)q_1 + \left(p_2 - w\right)q_2\right], \qquad (10)$$

where $d \in (0,1)$ represents delegation parameter, i.e., the incentive structure that the D owner firms determine for the managers governing the IPJV.[3] Namely, the objective function of the U producer depends on the degree of delegation granted, $d \in [0,1]$: if $d = 1$, the IPJV is completely autonomous; if $d = 0$, the IPJV complies with the D guidelines; if $d \in (0,1)$ its objective is somewhere in between the two extreme cases.

With this aim, we develop the following three stage game. First, the D firms choose d; second, the IPJV in U sets w; finally, the D firms compete in the D market.

The objective function of each D firm is the operative profit raised in D plus the share of operative U profit if they reach an agreement on the U strategy; it is the sheer operative D profit otherwise. The objective function of the D firms is thus stochastic as it depends on the exogenous probability, $\delta \in [0,1]$, to reach an agreement on the U strategy. Formally, the expected operative profit is:

$$E(\pi_{iD}) = \delta\left[\left(p_i - w\right)q_i + (1/2)\,w\left(q_i + q_j\right)\right] + (1-\delta)\left(p_i - a/2\right)q_i.$$

Note that if an agreement on w is not reached the IPJV ends up behaving in an independent fashion.[4] In this case, we would have a monopoly in U with input price equal to $w_M = a/2$ and a duopoly in D.

Proceeding backwards, the third stage maximization problem leads to the following quantity and price which depend on the variable w and on the parameters, δ and b:

$$q_i\left[w(d);\delta,b\right] = \frac{a + \delta\left(a - w\right)}{2\left(b+2\right)}$$

$$p_i\left[w(d);\delta,b\right] = \frac{3a + ab + \delta\left(w - a\right)\left(b+1\right)}{2\left(b+2\right)}$$

If $\delta = 0$, the game is over, the remaining two stages are not feasible as the IPJV will not be set up. In this case, we have a U monopoly and a D duopoly, with equilibrium quantities $q_i\left(\delta = 0\right) = a/(2(2+b)$. Therefore, the following results hold for $\delta \in (0,1]$.

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[3] As studied by Fershtam and Judd (1987), in oligopoly profit maximizing owners may give their managers incentives different from profit maximization.

[4] The market equilibrium is the same as the case in which the non-agreement situation occurs with maximum delegation and IPJV.

The second stage maximization problem results in the following input price:

$$w(d;\delta,b)=\frac{a\left[\delta\left(\delta+b\delta-1\right)+d\left(b+3\delta+b\delta-\delta^{2}-b\delta^{2}+2\right)\right]}{\delta\left[\delta\left(b+1\right)+d\left(2b-\delta-b\delta+4\right)\right]}.$$

The D firm expected profit thus becomes

$$E(\pi_{iD})=\frac{\left[d\left(3-\delta^{2}+3\delta-b\delta^{2}+b\delta+b\right)+\delta^{2}\left(b+1\right)\right]\left(d+\delta\right)a^{2}}{4\left(d\delta-2bd-\delta-b\delta-4d+bd\delta\right)^{2}}$$

and the maximization with respect to d results in the following optimal degree of delegation:

$$d^{*}\left(\delta,b\right)=\frac{\delta\left(b+1\right)\left(1-\delta\right)}{\left(-\delta^{2}+b\delta+3\delta-b\delta^{2}+2\right)}\in[0,1).$$

As expected, when $\delta=1$, that is the D firms reach an agreement with certainty, the optimal degree of delegation is $d^{*}=0$, i.e., the D firms obtain the cartel outcome. Since $\delta>0$, there is always a positive probability to reach an agreement. In other words, $d^{*}=1$ is never an equilibrium. Comparative statics with respect to δ and b allow us to write the following.

Proposition 1 *There exists an optimal degree of delegation, $d^{*}(\delta,b)$, which is a non linear function of the probability to reach an agreement between the D firms and is growing in the degree of product substitutability.*

Proof. See Appendix. ■

Discussion. An optimal degree of delegation exists; it grows for low levels of the probability to reach an agreement and it decreases for high levels of this probability. This means that when the likelihood of the agreement is growing but low, firms tend to increase the level of delegation and prefer to make the IPJV more accountable (the incentive to delegate is larger than the incentive to provide guidelines to the U firm to get cartel profits). When the agreement becomes almost sure, the firms tend to impose to the IPJV their policies (the incentive to delegate becomes lower since the higher probability of reaching an agreement makes the cartel solution a quite safe arrangement). A more interesting feature is the one concerning the degree of competition in D embodied in the level of b: the tougher the competition in D, the higher the delegation that firms are willing to grant. As product market competition goes up the profit-reservoir role of the IPJV is back and becomes the main engine behind the degree of delegation.

2.2 Asymmetric framework

As pointed out in the introduction, the generalization of the analysis of vertical arrangements associated to the IPJV requires the investigation of cases in which the D parent firms differ. As we shall see it is in these cases that most of the impossibility outcomes appear. Then, we turn to these cases trying to provide some way out of impossibility results.

We assume that firms' marginal costs are $c_1 - c_2 > 0$. For the sake of simplicity we set $c_2 = 0$. Thus, $c_1 \in [0,1]$ measures the cost difference among the two firms. Note that when the D owner firms show specific efficiency levels, they may differ also in the ownership shares of the IPJV. Assume, for instance, that ownership shares depend upon cost asymmetry in the following way: $s_1 = (1 - c_1)/2$ and $s_2 = (1 + c_1)/2$. The consolidated profit are then:

$$\pi_{1D} = (p_1 - c_1 - w)\, q_1 + \frac{1 - c_1}{2}\left[w(q_1 + q_2) - f\right], \tag{11}$$

$$\pi_{2D} = (p_2 - w)\, q_2 + \frac{1 + c_1}{2}\left[w(q_1 + q_2) - f\right]. \tag{12}$$

In words, the most efficient firm, which is presumably the largest one, gets a higher share of U profit.

2.2.1 Bargaining on the input price

Due to the results pointed out in the symmetric framework, we wonder whether we may think of a bargaining process on the input price w as a way out of the kind of “impossibility" arising in the cartel case when the two D firms are not equal, so they have different preferred input prices. We think of D firms bargaining over the input price to be set by the IPJV. The bargaining will substitute a stage of the game played by the two D firms.

In the first stage firms choose the input price through a bargaining process.[5] The predicted input price is given by:

$$w = \arg\max[\pi_{1D} - \pi_1^{duopoly}]^{\gamma}[\pi_{2D} - \pi_2^{duopoly}]^{1-\gamma},$$

where π_{1D} and π_{2D} are defined either by (1) with $s_i = 1/2$, or by (11) and (12); $\pi_i^{duopoly}$ represents firm i's outside option (with $i = 1, 2$) which is the equilibrium outcome of an asymmetric duopoly downstream and an independent monopoly upstream; $\gamma = \frac{1-c_1}{2}$ is firm 1's baldness consistent with a larger bargaining power by the most efficient firm (firm 2). In the second market stage firms compete in quantities. Solving backwards this two-stage game, we can prove the following.

Proposition 2 *When firms differ in their efficiency levels, i) they are not able to reach an agreement upon the input price if they own the U producer in equal shares; ii) an agreement can be reached instead if the D firms own U in asymmetric shares.*

[5] We model the outcomes of the bargaining via the formula of an asymmetric Nash bargaining solution which is interpreted as the limit of the subgame perfect equilibrium of the Rubinstein (1982) bargaining model when the lag between offers converges to zero (see Binmore *et al.*, 1986).

Proof. See Appendix. ■

Discussion This result may be tought to reflect commonly observed organization and practical wisdom. If two firms give rise to any kind of joint venture it is quite unlikely that they may accept equal stakes if they enjoy different degrees of efficiency. The lowest cost firm, which is quite often the largest company, will certainly require a stronger voice in the executive board of the IPJV. In the absence of an asymmetric governance the bargaining will be deemed to failure. In the presence of an asymmetric governance the bargaining process may succed.

2.2.2 Degree of delegation

Let us consider again delegation now in the asymmetric case. Is there an optimal level of delegation? To answer this question we introduce a more interesting setting where the probability of reaching an agreement is no longer exogeneous but it is a function of the cost difference, i.e., $\delta(c_1, c_2) = 1 - (c_1 - c_2)$ with $c_2 = 0$. Thus, the probability to reach an agreement decreases with c_1 and it is equal to 1 in the symmetric case, $c_1 = 0$.

The game structure remains as in Subsection (1). Then, we have a three-stage game where, in the third stage we get the D quantities as a function of w, in the second stage we obtain w as a function of the degree of delegation d, and in the first stage (choice of d), we solve the coordination problem over the governance of the IPJV. As one may guess, in this new asymmetric framework, firms prefer different degrees of delegation, in particular $d_1 > d_2$. The most efficient firm would choose a lower degree of delegation which in turn results in a lower input price and a more efficient market outcome. An agreement on d is not easily attainable. For instance, a bargaining on d leads to results close to those drawn for the bargaining on w. This further result let us write the following.

Proposition 3 *Only complete delegation ($d = 1$) allows to solve any coordination problem in the asymmetric framework.*

Proof. The proof is a kind of clone of the Proposition 2 and we do not report it.[6] ■

In this case the IPJV objective function is simply $\pi_U = w(q_1 + q_2)$. The equilibrium results are proposed in following subsections in the two cases of symmetric and asymmetric shares in the IPJV.

Symmetric shares In this case, the D owner firms' objective function is defined by (1) with $s_i = s_j = 1/2$. The second stage quantity competition leads to the following quantities:

$$q_1(w) = \frac{(2a - w)(2 - b) - 4c_1}{2(b+2)(2-b)}$$

$$q_2(w) = \frac{(2a - w)(2 - b) + 2bc_1}{2(b+2)(2-b)}.$$

[6] We make it available only for interested readers.

The first stage input price set by the IPJV is

$$w = a - \frac{c_1}{2}.$$

It is easy to show that with this input price the operative profits of the least efficient firm are always negative since the input price is too large and U extracts too much profit from the D parent firms. Then, we end up with a new impossibility.

Consider now the same two-stage game. Yet D firms maximize their operative profits rather than the consolidated ones. Formally:

$$q_i^*(w) = \arg\max_{q_i} (p_i - c_i - w)\, q_i.$$

Second stage equilibrium quantities are:

$$\begin{aligned} q_1(w) &= \frac{(a-w)(2-b) - 2c_1}{(b+2)(2-b)}, \\ q_2(w) &= \frac{(a-w)(2-b) + bc_1}{(b+2)(2-b)}. \end{aligned}$$

The first stage equilibrium input price is

$$w = \frac{2a - c_1}{4}.$$

Equilibrium quantities, prices and profits are:

$$\begin{aligned} q_1^* &= \frac{(2ab - 4a + 6c_1 + bc_1)}{4(b+2)(b-2)} \\ q_2^* &= \frac{(4a - 2ab + 2c_1 + 3bc_1)}{4(b+2)(2-b)} \\ \pi_U^* &= \frac{(-2a + c_1)^2}{8(2+b)} \\ \pi_{1D}^* &= \frac{(2ab - 4a + 6c_1 + bc_1)^2}{16(b+2)^2(b-2)^2} \\ \pi_{2D}^* &= \frac{(4a - 2ab + 2c_1 + 3bc_1)^2}{16(b+2)^2(b-2)^2}. \end{aligned}$$

Asymmetric shares Here we assume asymmetric stakes, with the larger stake for the most efficient firm. Namely, consider the consolidated profits defined by (11) and (12). Second stage quantity competition leads to the following quantities:

$$\begin{aligned} q_1(w) &= \frac{(2a-w)(b-2) + c_1(2w + bw + 4)}{2(b+2)(b-2)} \\ q_2(w) &= \frac{(2a-w)(2-b) + c_1(2b + 2w + bw)}{2(b+2)(2-b)}. \end{aligned}$$

The first stage input price set by the IPJV under complete delegation is

$$w = a - \frac{c_1}{2}$$

We get the same result as in the symmetric shares case and we end up with another impossibility. Turning again to a quantity competition such that the D firms maximize their operative profits rather than their consolidated profits the equilibrium outcome is the same as in the symmetric shares case.

These results are summarized in the ensuing Proposition and in the subsequent discussion.

Proposition 4 *When the D firms show different (or equal) efficiency levels and own the IPJV in proportions which are directly proportional to their relative efficiency levels, the maximization of consolidated profits gives rise to an impossibility result since at least the inefficient firm always faces negative operative profits.*

Discussion As it appears the only possible solution is one in which the D firms maximize their operative profits, this holds for both the symmetric and the asymmetric framework. In this case the rules adopted for the sharing of the IPJV profits may change the preference of the least efficient firm vis à vis the rival, but it is not going to make the equilibrium variables undergo any variation.

3 Conclusions

In our investigation of the governance of an IPJV we came across several impossibility results. Most of them arise in the more general cases of asymmetric cost structure, which may hint different firm sizes. The bulk of impossibility outcomes makes an independent IPJV the most likely setting. Indeed the IPJV with maximum delegation seems to be the most viable and likely governance solution which turns out to be adopted even in asymmetric circumstances. Firms may decide to bargain over the input price. In this case they reduce drastically the extent of delegation. Here, a solution is possible if the IPJV has an ownership structure which reflects the different degrees of efficiency of the two parent firms in D.

4 Appendix

4.1 Proof of Proposition 1

Partial derivatives of the optimal degree of delegation are such that:

$$\begin{aligned}
\frac{\partial}{\partial \delta} d^* (\delta, b) &> 0 \iff \delta \in (0, 0.414), \\
\frac{\partial}{\partial b} d^* (\delta, b) &> 0, \text{ always.}
\end{aligned}$$

4.2 Proof of Proposition 2

i) Maximising the consolidated profits defined in (1) with $s_i = 1/2$, we get the following second stage equilibrium quantities and profits:

$$
\begin{aligned}
q_1 &= \frac{(2a-w)(2-b)-4c_1}{2(b+2)(2-b)} > 0 \iff c_1 < \frac{(2a-w)(2-b)}{4} \\
q_2 &= \frac{(2a-w)(2-b)+2bc_1}{2(b+2)(2-b)} \\
\pi_{1d} &= \frac{16c_1^2+2c_1(2-b)\left(4w-8a+2bw+b^2w\right)+(b-2)^2(2a-w)(2a+w+bw)}{4(b+2)^2(b-2)^2} \\
\pi_{2d} &= \frac{4b^2c_1^2+8c_1(2-b)(ab-w-bw)+(b-2)^2(2a-w)(2a+w+bw)}{4(b+2)^2(b-2)^2}
\end{aligned}
$$

The outside option is an asymmetric D duopoly and an U monopoly:

$$
\begin{aligned}
w_M &= \frac{(2a-c_1)}{4} \\
q_1^{duopoly} &= \frac{(2ab-4a+6c_1+bc_1)}{4(b+2)(b-2)} > 0 \iff c_1 < \frac{4a-2ab}{(6+b)} \\
\pi_1^{duopoly} &= \frac{(2a(-2+b)+(6+b)c_1)^2}{16(-4+b^2)^2} \\
\pi_2^{duopoly} &= \frac{\left(-2a(-2+b)+(2+3b)c_1\right)^2}{16(-4+b^2)^2}
\end{aligned}
$$

Computing the difference $\pi_{iD} - \pi_i^{duopoly}$ we check whether (and for which values of w), both firms prefer the IPJV with respect to the outside option. We consider the case of homogeneous goods, i.e., $b = 1$ so that the constraint for the non-negativity of $q_1^{duopoly}$ becomes $c_1 < 2a/7$ and the constraint for the non-negativity of q_1 becomes $c_1 < \frac{(2a-w)}{4}$. We restrict our attention to this case to simplify computations, moreover in this case we eliminate a source of heterogeneity among firms, therefore if an agreement is not reached under $b = 1$, we conclude that a fortiori it is not reached for $b \in [0, 1)$.

$$\pi_{1d} - \pi_1^{duopoly} = \frac{3(c_1-2a)(5c_1-2a)-8w^2+w(8a+56c_10)}{144} > 0 \iff w \in (w_1, w_2),$$

where $w_2 = \frac{1}{2}\left(a + 7c_1 + \frac{1}{2}\sqrt{2}\sqrt{14a^2 - 8ac_1 + 113c_1^2}\right) > 0$, and $w_1 = \frac{1}{2}(a + 7c_1) - \frac{1}{4}\sqrt{2}\sqrt{14a^2 - 8ac_1 + 113c_1^2} < 0$ given the constraint $c_1 < \frac{2}{7}a$ for the non-negativity of $q_1^{duopoly}$.

$$\pi_{2d} - \pi_2^{duopoly} = -\frac{8w^2+w(64c_1-8a)+3(c_1-2a)(2a+3c_1)}{144} > 0 \iff w \in (w_3, w_4),$$

where $w_3 = \frac{1}{2}a - 4c_1 - \frac{1}{4}\sqrt{2}\sqrt{14a^2 - 20ac_1 + 119c_1^2}$ and $w_4 = \frac{1}{2}a - 4c_1 + \frac{1}{4}\sqrt{2}\sqrt{14a^2 - 20ac_1 + 119c_1^2}$. $w_3 > 0 \iff \frac{1}{2}a - 4c_1 > \frac{1}{4}\sqrt{2}\sqrt{14a^2 - 20ac_1 + 119c_1^2}$. If $\frac{1}{2}a - 4c_1 > 0 \iff \frac{a}{8} > c_1$, then $w_3 > 0 \iff (c_1 - 2a) > 0$, never.

If $\frac{1}{2}a - 4c_1 < 0$, then $w_3 < 0$. We conclude that $w_3 < 0$. $w_4 > 0 \iff \left(\frac{1}{2}a - 4c_1\right) + \frac{1}{4}\sqrt{2}\sqrt{14a^2 - 20ac_1 + 119c_1^2} > 0$, always if $\frac{a}{8} > c_1$, else, $w_4 > 0 \iff \left(-\frac{3}{8}\right)(c_1 - 2a)(2a + 3c_1) > 0$, always. We conclude that $w_4 > 0$. $w_2 - w_4 > 0 \iff 11c_1 + \frac{1}{2}\sqrt{2}\sqrt{14a^2 - 8ac_1 + 113c_1^2} > \frac{1}{4}\sqrt{2}\sqrt{14a^2 - 20ac_1 + 119c_1^2}$ always. The range of w such that both $\pi_{1d} - \pi_1^{duopoly} > 0$ and $\pi_{2d} - \pi_2^{duopoly} > 0$ is $w \in (0, w_4)$ which is a non-empty interval as shown above. We next compute

$$w^* = \arg\max[\pi_{1D} - \pi_1^{duopoly}]^{\frac{1-c_1}{2}}[\pi_{2D} - \pi_2^{duopoly}]^{\frac{1+c_1}{2}}$$

and find that $w^* > w_4$ and $w^* \to w_4$ as $c_1 \to 0$, that is the symmetric case.

ii) Consider now the case in which each firm owns a share of the U producer which is proportional to its efficiency:

$$\begin{aligned} \pi_{1D} &= (p_1 - c_1 - w)\, q_1 + \frac{1 - c_1}{2} w\,(q_1 + q_2) \\ \pi_{2D} &= (p_2 - w)\, q_2 + \frac{1 + c_1}{2} w\,(q_1 + q_2) \end{aligned}$$

Second stage equilibrium quantities and profits (a stands for asymmetric shares) are then

$$\begin{aligned} q_2^a &= \frac{(2a - w)(2 - b) + c_1(2b + 2w + bw)}{2(b + 2)(2 - b)} \\ q_1^a &= \frac{(2a - w)(2 - b) - c_1(2w + bw + 4)}{2(b + 2)(2 - b)} > 0 \iff c_1 < \frac{(2a - w)(2 - b)}{(2w + bw + 4)} \\ \pi_{1D}^a &= \tfrac{c_1^2 A + 2c_1(2-b)B + C}{4(b+2)^2(b-2)^2} \text{ and } \pi_{2D}^a = \tfrac{c_1^2 D + 2c_1(b-2)E + F}{4(b+2)^2(b-2)^2}, \end{aligned}$$

where $A = b^3 w(w + 2) + 3b^2 w^2 - 4\left(w^2 - 4w - 4\right)$, $B = b^2 w(1 + a) + bw(3w - 2a + 2) + 2\left(2w - 4a - 4aw + 3w^2\right)$, $C = (b - 2)^2(2a - w)(2a + w + bw)$, $D = b^3 w^2 + b^2(w + 2)(3w + 2) + 4w(2b - w - 4)$, $E = ab^2 w + b\left(4w - 4a - 2aw + 3w^2\right) - 2w(4a - 3w - 2)$ and $F = (b - 2)^2(2a - w)(2a + w + bw)$. Again we consider a particular case to simplify computations: suppose $b = 1$. Then, $q_1^a > 0 \iff c_1 < \frac{(2a-w)}{(3w+4)}$ and $q_1^{duopoly} > 0 \iff c_1 < \frac{2}{7}a$ with $\frac{(2a-w)}{(3w+4)} - \frac{2}{7}a = -\frac{(7w-6a+6aw)}{7(3w+4)} > 0 \iff w < \frac{6a}{(6a+7)}$. For firm 1 to stay in the market in both cases we need $c_1 < \min\left\{\frac{(2a-w)}{(3w+4)}, \frac{2}{7}a\right\}$. Suppose also $c_1 = \frac{a}{10}$. $\pi_{1D}^a - \pi_1^{duopoly} = \frac{w^2(720a-800) + w\left(1360a - 648a^2\right) + 855a^2}{14\,400} > 0$ for $a > 1.24$ because the root $w = \frac{\left(4a(81a-170) - 4\sqrt{a^2(6561a^2 - 66\,015a + 71\,650)}\right)}{720a-800}$ is not real. $\pi_{2D}^a - \pi_2^{duopoly} = -\frac{w^2(720a+800) - wa(160+720a) - 1311a^2}{14\,400} > 0 \iff w^2(720a + 800) + w\left(-160a - 720a^2\right) - 1311a^2 < 0$. The numerator is zero in $w = \frac{\left(80a + 360a^2 - 4\sqrt{5}\sqrt{a^2(12\,519a + 1620a^2 + 13\,190)}\right)}{720a+800} < 0$. We conclude that $\pi_{2D}^a - \pi_2^{duopoly} > 0$ always. Thus, there are ranges of the parameters where the IPJV is preferred to the outside option for any w and so also for w^*.

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