Alma Mater Studiorum - Università di Bologna
DEPARTMENT OF ECONOMICS

Imperfect Substitutes for Perfect Complements: Solving the Anticommons Problem

Matteo Alvisi
Emanuela Carbonara

Quaderni - Working Papers DSE N° 708

Imperfect Substitutes for Perfect Complements: Solving the Anticommons Problem*

Matteo Alvisi† and Emanuela Carbonara‡

June 16, 2010

Abstract

An integrated monopoly, where all complements forming a composite good are offered by a single firm, is typically welfare superior to a complementary monopoly. This is "the tragedy of the anticommons". We consider the possibility of competition in the market for each complement. We present a model with two perfect complements and introduce n imperfect substitutes for one and then for both complements. We prove that, if one complementary good is produced by a monopolist, and if competition for the other complement does not vary the average quality in the market, then an integrated monopoly is still superior. In such case, favoring competition in some sectors, leaving monopolies in others would be detrimental for consumers and producers alike. Competition may be preferred if and only if the substitutes of the complementary good differ in their quality, so that as their number increases, average quality and/or quality variance increases. Results change when competition is introduced in each sector. In this case, if goods are close substitutes, we find that competition may be welfare superior for a relatively small number of competing firms in each sector, even with no quality differentiation.

JEL Codes: D43, D62, K11, L13.

Keywords: Price competition, Anticommons, Complements and Substitutes, Welfare Effects.

*We thank Paola Bortot, Giuseppe Dari-Mattiacci, Vincenzo Denicolò, Francesco Parisi and participants to the Bologna Workshop in Law and Economics, February 2010 for fruitful suggestions.

†Department of Economics, University of Bologna, 2 Piazza Scaravilli, 40126 Bologna Italy. E-mail:m.alvisi@unibo.it.

‡Department of Economics, University of Bologna, 2 Piazza Scaravilli, 40126 Bologna Italy. E-mail:emanuela.carbonara@unibo.it.

1 Introduction

A complementary monopoly is characterized by the presence of multiple sellers, each producing a complementary good. It has been known for quite some time in the literature that a complementary monopoly may be worse than an integrated monopoly, in which all such complementary goods are offered by a single firm (Cournot, 1838 and, more recently, Economides and Salop, 1992, Matutes and Regibeau, 1988). An individual firm producing a single complement takes into account only the impact of a price raise on its own profits, without considering the negative externality caused to the sellers of other complementary goods. The quantity demanded would be reduced for everyone, but each seller benefits fully of an increase in its own price. As a consequence, separated producers of complementary goods will set a higher price than an integrated monopolist, thus resulting in a lower consumer surplus.[1]

The complementary monopoly problem is also known as "the tragedy of the anticommons", in analogy with its mirror case, the more famous "tragedy of the commons" and has been applied in the legal literature to issues related to the fragmentation of physical and intellectual property rights.[2] Recently, the issue of complementarity has been brought to the attention of the economics profession by some important antitrust cases, both in the United States and in Europe, in particular the Microsoft case (discussed before both American and European Courts) and the General Electric-Honeywell Case, decided by the European Commission.[3] In the Microsoft case, the American decision is especially interesting. Judge Jackson ordered the firm to divest branches of its business other than operating systems, creating a new company dedicated to application development. The break-up (later abandoned) would have created two firms producing complementary goods, with the likely result of increasing prices in the market. However, far from being unaware of the potential tragedy of the anticommons, Judge Jackson motivated his decision with the need to reduce the possibility for Microsoft to engage in limit pricing, thus deterring entry. Separation would have facilitated entry, possibly driving prices below pre-separation levels.[4] A similar economic argument motivated the European Commission's Decision in the General Electric-Honeywell Case. In such case, the EC indicated that the post-merger prices would be so low as to injure new entrants, so that a merger would reduce the number of potential and actual competitors in both markets.[5]

[1]Complementary monopoly is similar to the problem of double marginalization in bilateral monopoly, with the important difference that here each monopolist competes "side by side", possibly without direct contacts with each other. In bilateral monopoly, the "upstream" monopolist produces an input that will be used by the "downstream" one, who is then a monopsonist for that specific input (see Machlup and Taber, 1960).

[2]For an application to property rights, see Heller (1998), Buchanan and Yoon (2000) and Parisi (2002). Heller and Eisenberg (1998) argue that patents may produce an anticommons problem in that holders of a specific patent may hold up potential innovators in complementary sectors. Particularly, they focus on the case of biomedical research, showing how a patent holder on a segment of a gene can block the development of derivative innovations based on the entire gene. Emblematic, in this respect, the case of Myriad Genetics Inc., which held patents on specific applications of the BRCA1 and BRCA2 genes, and blocked the development of cheaper breast-cancer tests (see Paradise, 2004).

[3]See European Commission Decision of 03/07/2001, declaring a concentration to be incompatible with the common market and the EEA Agreement Case, No. COMP/M.2220 - General Electric/Honeywell.

[4]United States v. Microsoft Corp., 97 F. Supp. 2d. 59 (D.D.C. 2000). See Gilbert and Katz (2001) for a thorough analysis of the Microsoft case.

[5]On the possibility that an integrated monopolist engages in limit pricing to deter entry, see Fudenberg and

Both these decisions indicate that separation may not be an issue (and may even be welfare improving) if the post-separation market configuration is not a complementary monopoly in the Cournot's sense, i.e., the market for each complement is characterized by competition. The higher prices due to the tragedy of the anticommons may in fact encourage entry in the market. If competition increases sufficiently, the resulting market structure may yield a higher welfare than the initial integrated monopoly. The question then is how much competition is needed in the supply of each complement in order to obtain at least the same welfare as in the original monopoly?

Investigating the impact of competition on welfare when complementary goods are involved, Dari-Mattiacci and Parisi (2007) note that, when n perfect complements are bought together by consumers and firms compete *à la* Bertrand, two perfect substitutes for $n-1$ complements are sufficient to guarantee the same social welfare experienced when an integrated monopolist sells all n complements. In fact, all competitors in the $n-1$ markets price at marginal cost, thus allowing the monopolist in the n-th market to extract the whole surplus, fixing its price equal to the one set by an integrated monopolist for the composite good. The tragedy of the anticommons is therefore solved by competition.

Our analysis of competition among several composite goods maintains this framework when it considers perfect complements. However, we extend it in several directions. First, differently from previous literature, the competing goods are imperfect substitutes[6] and are vertically differentiated. Second, we consider the presence of substitutes in all markets for the n components.

Particularly, we consider two perfect complements, proving that, if one complementary good is still produced in a monopolistic setting and if competition for the other complement does not vary the average quality in the market, then an integrated monopoly remains welfare superior to more competitive market settings. In fact, with imperfect substitutability the competing firms retain enough market power as to set relatively high prices. As a result, the equilibrium prices of the composite goods under competition remain always higher than in an integrated monopoly. Hence, favoring competition in some sectors only while leaving monopolies in others may actually be detrimental for consumers. Competition may be preferred if and only if the substitutes of the complementary good produced competitively differ in their quality, so that average quality and/or quality variance increase as their number increases.

Results change when competition is introduced for both components. In this case we find that the tragedy may be solved for a relatively small number of competing firms in each sector whenever goods are close substitutes. Not surprisingly, the higher the degree of substitutability and the number of competitors in one sector, the more concentrated the remaining sector can be and still produce a better performance than an integrated monopoly in terms of consumer surplus.

The welfare loss attached to a complementary monopoly has been analyzed, among others, by Economides and Salop (1992) who show, in a duopoly model with complements, that a merger

Tirole (2000).

[6]Imperfect substitutability in this case means that the cross-price elasticity is lower than own-price elasticity.

reduces prices because it allows the coalition firm to absorb positive externalities. Gaudet and Salant (1992) study price competition in an industry producing perfect complements and prove that welfare-improving mergers may fail to occur endogenously. Tan and Yuan (2003) are concerned with the opposite issue, i.e., they consider a market in which two firms sell imperfectly substitutable composite goods consisting of several complementors. They show that firms have the incentive to divest along complementary lines, because the price raise due to competition among producers of complements counters the downward pressure on prices due to Bertrand competition in the market for imperfect substitutes. McHardy (2006) demonstrates that, in general, ignoring demand complementarities when breaking up firms that produce complementary goods may lead to substantial welfare losses. However, if the break-up stops limit-pricing practices by the previously merged firm, even a relatively modest degree of post-separation entry may lead to higher welfare than an integrated monopoly. He assumes a setting in which firms producing the same component compete *a la* Cournot among them, whereas competition is *a la* Bertrand among complements (i.e., among sectors). Differently from McHardy (2006), we analyze the impact of complementarities and entry in a more consistent model, where all firms choose prices when competiting both intra and inter layer and in such framework we also study the impact of product differentiation and imperfect substitutability.

Previous literature on the relationship between complementary goods and market structure is scanty and deals mostly with bundling practices (Matutes and Regibeau, 1988, Anderson and Leruth, 1993, Denicolò, 2000, Nalebuff, 2004, Alvisi et el., 2009).[7]

The paper is organized as follows. Section 2 introduces the model when one sector is a monopoly; Section 3 presents the reference cases of complementary and integrated monopoly. Section 4 analyzes the impact of competition on welfare when one complement is produced by a monopolist; Section 5 extends the model considering competition in the markets for all complements. Section 6 concludes. Appendix A contains the proofs of the Lemmas and Propositions.

2 The Model

Consider a composite good (a system) consisting of two components, A and B. The two components are perfect complements and are purchased in a fixed proportion (one to one for simplicity). Initially, we assume that complement A is produced by a monopolist, whereas complement B is produced by n oligopolistic firms.[8] The number of competitors in each sector is exogenous. Marginal costs are the same for all firms and are normalized to zero.[9] Firms compete by setting

[7] Matutes and Regibeau (1988) study compatibility and bundling in markets in which complementary goods have to be assembled into a system. Anderson and Leruth (1993) study bundling choices under different market structures. Denicolò (2000) analyzes compatibility and bundling choices when an integrated firm selling all complements in a system competes with non-integrate firms, each producing a single, different complement. Nalebuff (2004) analyzes the incentives to bundle by oligopolistic firms, showing that bundling is a particularly effective entry-deterrent strategy. On the opposite, Alvisi et el. (2009) show that, when firms sell complementary goods, integration along complementary lines may actually be pro-competitive, favoring entry.

[8] We will remove this assumption later and consider a market configuration in which n_1 firms produce complement A, whereas n_2 firms produce complement B.

[9] This assumption is with no loss of generality, because results would not change for positive, constant marginal costs (see Economides and Salop, 1992).

prices. We also assume full compatibility among components, meaning that the complement produced by the monopolist in sector A can be purchased by consumers in combination with any of the n versions of complement B. This assumption is made because we are interested in the effect of competition on the pricing strategies of the firms operating in the various complementary markets. If we let firms decide to restrict compatibility, competition may be limited endogenously (for instance, the monopolist could allow combination with a subset of producers in sector B only) and the purpose of our analysis would be thwarted.[10] Finally, we assume that the systems have different qualities and that consumers perceive them as imperfect substitutes.[11]

More specifically, the representative consumer has preferences represented by the following utility function, quadratic in the consumption of the n available systems and linear in the consumption of all the other goods (as in Dixit, 1979, Beggs, 1994):

$$U(q,I)=\sum_{j=1}^{n}\alpha_{1j}q_{1j}-\frac{1}{2}\left[\beta\sum_{j=1}^{n}q_{1j}^{2}+\gamma\sum_{j=1}^{n}q_{1j}\left(\sum_{s\neq j}q_{1s}\right)\right]+I \tag{1}$$

where I is the total expenditure on other goods different from the n systems, $q=[q_{11},q_{12},..,q_{1n}]$ is the vector of the quantities consumed of each system and q_{1j} represents the quantity of system $1j$, $(j=1,....,n)$, obtained by combining q_{1j} units of component A purchased from the monopolist, indexed by the number 1 (component $A1$), and $q_{Bj}=q_{1j}$ units of component B purchased from the jth firm in sector B (component Bj).[12]Also, $\alpha=(\alpha_{11},\alpha_{12},..,\alpha_{1n})$ is the vector of the qualities of each system (with α_{1j} representing the quality of system $1j$, $(j=1,....,n)$, γ measures the degree of substitutability between any couple of systems, $\gamma\in[0,1]$, and β is a positive parameter. The representative consumer maximizes the utility function (1) subject to a linear budget constraint of the form $\sum_{j=1}^{n}p_{1j}q_{1j}+I\leq M$, where

$$p_{1j}=p_{A1}+p_{Bj},\quad j=1,....,n \tag{2}$$

is the price of system $1j$ (expressed as the sum of the prices of the single components set by firm 1 in sector A and firm j in sector B, respectively) and M is income.

[10]The assumption of perfect compatibility is common to many contributions in the literature on complementary markets, see Economides and Salop (1992), McHardy (2006), Dari-Mattiacci and Parisi (2007).

[11]This implies that the consumption possibility set consists of n imperfectly substitutable systems. Later on, when we consider n_1 components in sector A, consumers will have the opportunity to combine each of these components with any of the n_2 complements produced in Sector B. We would then have $n_1\times n_2$ imperfectly substitutable systems in the market.

[12]Note that when referring to a particular system, we use a couple of numbers indicating the two firms in sector A and B, respectively, selling each component of such system. When referring instead to separate components, we use a couple of one letter and one number, the first indicating the sector (the component) and the second the particular firm selling it. This might appear redundant for $A1$ when component A is sold by a monopolist, but it will become useful when we introduce competition in sector A.

2.1 Equilibrium Prices and Demand

The first order condition determining the optimal consumption of a particular system $1k$ is

$$\frac{\partial U}{\partial q_{1k}} = \alpha_{1k} - \beta q_{1k} - \gamma \sum_{j \neq k} q_{1j} - p_{1k} = 0 \tag{3}$$

Summing (3) over all firms in the B sector, we obtain the demand for system $1k$

$$q_{1k} = \frac{(\beta + \gamma(n-2))(\alpha_{1k} - p_{A1} - p_{Bk}) - \gamma \left(\sum_{j \neq k} \alpha_{1j} - (n-1)p_{A1} - \sum_{j \neq k} p_{Bj} \right)}{(\beta - \gamma)\left(\beta + \gamma(n-1)\right)} \tag{4}$$

Summing the demands of all firms in sector B we obtain the total market size

$$Q = \sum_{j=1}^{n} q_{1j} = \frac{\sum_{j=1}^{n} (\alpha_{1j} - p_{Bj}) - n p_{A1}}{\beta + \gamma\,(n-1)} \tag{5}$$

Following Shubik and Levitan (1980), to prevent changes in γ and n to affect Q, we set

$$\beta = n - \gamma(n-1) > 0. \tag{6}$$

so that, substituting such expression into (5), the normalized market size becomes

$$Q = \bar{\alpha} - \bar{p}_B - p_{A1} \tag{7}$$

where $\bar{\alpha} = \frac{\sum_{j=1}^{n} \alpha_{1j}}{n}$ is the average quality of the n available systems and $\bar{p}_B = \frac{\sum_{j=1}^{n} p_{Bj}}{n}$ is the average price in the market for the second component.

Note that expression (7) also represents the demand function for the monopolist in sector A - given that component $A1$ is part of all the n systems - so that the monopolist's profit function can be written as $\Pi_{A1} = p_{A1} Q = (\bar{\alpha} - \bar{p}_B)\, p_{A1} - p_{A1}^2$, whereas profit for a single producer of component B is $\Pi_{Bk} = p_{Bk} \cdot q_{Bk}$, where $q_{Bk} = q_{1k}$ is given in (4). Each firm chooses its price to maximize its own profits, taking the prices of others as given.[13] Equilibrium prices for the monopolist $A1$ and for the k-th oligopolist are, respectively

$$p_{A1}^M = \frac{\bar{\alpha}(n - \gamma)}{n(3-\gamma) - 2\gamma} \tag{8}$$

$$p_{Bk}^M = \frac{\bar{\alpha} n (1-\gamma)}{n(3-\gamma) - 2\gamma} + \frac{n(\alpha_{1k} - \bar{\alpha})}{2n - \gamma} \tag{9}$$

where the superscript M stands for "monopoly in sector A". It is immediate to verify that p_{A1}^M

[13] The second order conditions for maximization of $U(q, I)$ requires $\gamma \leq \frac{\beta}{2}$, i.e., $\gamma < \frac{n}{n+1}$.

is increasing in average quality $\bar{\alpha}$. In fact, being $A1$ part of all systems, an increase in their average quality allows an higher profit-maximizing price for the monopolist. p_{A1}^{M} also depends positively on the number of systems sold, n, and on the degree of substitutability between any couple of systems, γ. As we will show below, the increase in competition in the market of the second component (either because of a greater number of firms or of an higher degree of substitutability among systems) reduces all oligopolistic prices, allowing the monopolist in sector A to extract part of the surplus created by such price decrease.[14] Not surprisingly, from (9), producers of below-average quality charge lower than average prices (since $(\alpha_{1k} - \bar{\alpha}) < 0$), whereas the opposite is true for producers of above-average quality. However, quality "premiums and discounts" cancel out on average. In fact, the average price in the market for the second component is

$$\bar{p}_B = \frac{\sum_{k=1}^{n} p_{Bk}^{M}}{n} = \frac{\bar{\alpha}n(1-\gamma)}{n(3-\gamma)-2\gamma} \tag{10}$$

Combining (8) and (9), the equilibrium price of system $1k$ is

$$p_{1k}^{M} = p_{A1}^{M} + p_{Bk}^{M} = \frac{(n(2-\gamma)-\gamma)\bar{\alpha}}{n(3-\gamma)-2\gamma} + \frac{n\,(\alpha_{1k}-\bar{\alpha})}{2n-\gamma} \tag{11}$$

so that, the average system price becomes

$$\bar{p}_{1k}^{M} = p_{A1}^{M} + \bar{p}_B = \frac{(n(2-\gamma)-\gamma)\bar{\alpha}}{n(3-\gamma)-2\gamma} \tag{12}$$

Finally, using (4), (8) and (9), we derive the equilibrium quantities

$$q_{1k}^{M} = \frac{\bar{\alpha}(n-\gamma)}{n(n(3-\gamma)-2\gamma))} + \frac{(\alpha_{1k}-\bar{\alpha})(n-\gamma)}{n(2n-\gamma)(1-\gamma)} \tag{13}$$

We are now ready to compute profits and consumer welfare.

2.2 Consumer and producer surplus when sector A is a monopoly

Given (8) and (4), the monopolist's profits in sector A are equal to

$$\Pi_{A1}^{M} = p_{A1}^{M}\sum_{j=1}^{n} q_{1j}^{M} = \frac{\bar{\alpha}^2(n-\gamma)^2}{(n(\gamma-3)+2\gamma)^2} \tag{14}$$

As for the $k-$th oligopolist's profit, note first that

$$p_{Bk}^{M} = t \cdot q_{1k}^{M} \tag{15}$$

[14] It should be noted that the impact of an increase in n on p_{A1}^{M} is analyzed assuming a constant $\bar{\alpha}$, which implies that we are concentrating on mean-preserving distributions of quality across firms.

where $t = \frac{n^2(1-\gamma)}{(n-\gamma)}$. Hence

$$\Pi_{Bk}^M = t\left(q_{1k}^M\right)^2 = \frac{n^2(1-\gamma)}{(n-\gamma)}\left(\frac{\bar{\alpha}(n-\gamma)}{n(n(3-\gamma)-2\gamma)} + \frac{(\alpha_{1k}-\bar{\alpha})(n-\gamma)}{n(2n-\gamma)(1-\gamma)}\right)^2, \tag{16}$$

so that aggregate profits in sector B are equal to

$$\Pi_B^M = \sum_{j=1}^n \Pi_{Bj}^M = t\sum_{j=1}^n \left(q_{1j}^M\right)^2 = n(1-\gamma)(n-\gamma)\left(\frac{\bar{\alpha}^2}{n(n(3-\gamma)-2\gamma)^2} + \frac{\sigma_\alpha^2}{n(2n-\gamma)^2(1-\gamma)^2}\right) \tag{17}$$

where $\sigma_\alpha^2 = \frac{\sum_{j=1}^n(\alpha_{1j}-\bar{\alpha})^2}{n}$ represents the variance of the qualities of the n available systems.

We now turn to consumer surplus. Given the utility function in (1), consumer surplus is defined as

$$CS = U(q,I) - \left(\sum_{j=1}^n p_{1j}q_{1j} + I\right) \tag{18}$$

Following Hsu and Wang (2005), we can rewrite the expression above as

$$CS = \frac{n(1-\gamma)}{2}\sum_{j=1}^n q_{1j}^2 + \frac{\gamma}{2}\left(\sum_{j=1}^n q_{1j}\right)^2 = \frac{n(1-\gamma)}{2}\sum_{j=1}^n (q_{1j}-\bar{q})^2 + \frac{n^2}{2}(\bar{q})^2 \tag{19}$$

where $\bar{q} = \frac{\sum_{j=1}^n q_{1j}}{n} = \frac{Q}{n}$ is average quantity. Using (13), we can write

$$\bar{q} = \widetilde{A}\bar{\alpha} \tag{20}$$

and

$$q_{1k} - \bar{q} = \widetilde{B}\left(\alpha_{1k} - \bar{\alpha}\right) \tag{21}$$

where $\widetilde{A} = \frac{(n-\gamma)}{n(n(3-\gamma)-2\gamma)}$ and $\widetilde{B} = \frac{(n-\gamma)}{n(1-\gamma)(2n-\gamma)}$. Also, using (21),

$$\sum_{j=1}^n (q_{1j}-\bar{q})^2 = \widetilde{B}^2 n\sigma_\alpha^2 \tag{22}$$

Finally, substituting (20) and (22) into (19), we obtain

$$CS^M = \frac{n^2(1-\gamma)}{2}\tilde{B}^2\sigma_\alpha^2 + \frac{n^2}{2}\tilde{A}^2\bar{\alpha}^2 \tag{23}$$

Given that our goal is to compare equilibrium outcomes under competition and under both an integrated and a complementary monopoly à la Cournot, in the remainder of this Section we compare equilibrium prices and quantities under competition with those obtained under integrated and complementary monopolies. Particularly, prices and quantities in an integrated monopoly are $p_{IM} = \frac{\alpha_{IM}}{2}$ and $Q_{IM} = \frac{\alpha_{IM}}{2}$ respectively, so that profits and consumer surplus are

$$\Pi_{IM} = \frac{\alpha_{IM}^2}{4}; \qquad CS_{IM} = \frac{\alpha_{IM}^2}{8}. \tag{24}$$

In should be immediately noticed that when $\sigma_\alpha^2 = 0$ and the common quality level among all systems coincides with that of an integrated monopoly ($\alpha_{1k} = \bar{\alpha} = \alpha_{IM} = \alpha^*$, $k = 1, ..., n$), component prices in sector B are lower than the price set by the integrated monopolist, p_{IM}, while system prices are higher. In fact,

$$p_{Bk}^M - p_{IM} = -\frac{\alpha^*(n-\gamma)}{(3n-\gamma(2+n))} < 0 \tag{25}$$

$$p_{1k}^M - p_{IM} = \frac{\alpha^*(n-\gamma(4-n))}{2(3n-\gamma(2+n))} > 0 \tag{26}$$

for all $\gamma \in \left[0, \frac{n}{n+1}\right]$. Thus, competition lowers prices in the oligopolistic sector, but the monopolist in sector A optimally reacts to this by extracting more surplus and setting higher prices, so that overall $p_{1k}^M > p_{IM}$. This has a negative impact on the number of systems sold in the market and, as a matter of fact, it is immediate to check that $Q^M = nq_{1k}^M < Q_{IM}$.

In a complementary monopoly, two separate firms $A1$ and $B1$ produce one component each of the composite good (i.e., $n = 1$), and, in the equilibrium, set prices equal to $p_{CM}^i = \frac{\alpha_{CM}}{3}$, $i = A, B$ (where CM stands for "complementary monopoly"). Hence the equilibrium price and quantity of the composite good are $p_{CM} = \frac{2\alpha_{CM}}{3}$ and $Q_{CM} = \frac{\alpha_{CM}}{3}$. Profits and consumer surplus then are:

$$\Pi_{CM}^i = \frac{\alpha_{CM}^2}{9}, \quad i = A, B; \quad CS_{CM} = \frac{\alpha_{CM}^2}{18}, \tag{27}$$

where $CS_{CM} < CS_{IM}$, obviously. It is easy to check that, when σ_α^2=0 and the common quality level among all systems coincides with that of a complementary monopoly ($\alpha_{1k} = \bar{\alpha} = \alpha_{CM}$, $k = 1, ..., n$), component and system prices are lower with competition than with a complementary monopoly (i.e. $p_{Bk}^M < p_{CM}^B$and $p_{1k}^M < p_{CM}$, respectively). This implies that $Q^M > Q_{CM}$, even if each oligopolist sells less than a complementary monopolist ($q_{1k}^M < Q_{CM}$).

3 Competition and Welfare When Sector A is a Monopoly

In this section we verify the impact of changes in the number of firms in Sector B, n, in the degree of substitutability among systems,γ, and in the distribution of the quality parameters (the α_{1k}'s) on equilibrium prices and welfare.

Along the way, we will verify how the assumption of imperfect substitutability among systems changes the impact of n on the extent of the tragedy of the anticommons with respect to the case studied by Dari-Mattiacci and Parisi (2007).[15]

The following Lemma illustrates first the relationship between p_{Bk}^M, $\bar{\alpha}$, γ, and n.

Lemma 1. *Oligopolistic prices decrease with n and γ.*

Proof. See Appendix A. □

[15]One should recall that, in their simple model, two firms competing in the market for the second component would be enough to guarantee a surplus equal to that attained in the presence of a single, integrated firm.

The negative relationship between p_{Bk}^M, γ and n is intuitive. The higher the number of firms in sector B and the degree of substitutability among systems, the fiercer the competition for the second component and the lower the Bertrand equilibrium prices in sector B. Similarly, it is immediate to verify from (10) that the impact of a change in n and γ on $\bar{p}_B$ is the usual and negative one.[16]

When checking the relationship between system prices p_{1k}^M, the number of firms n and the degree of substitutability γ, we notice from (11) that it is influenced by opposite forces. On the one hand, p_{A1}^M increases as either n or γ increase, whereas p_{Bk}^M decreases. However, the following Proposition indicates that the first effect is always smaller in magnitude than the second, so that, overall, p_{1k}^M decreases with n and γ.

Proposition 1. *The equilibrium system prices decrease with n and γ. Then, consumer surplus increases with n and γ.*

Proof. See Appendix A. □

When p_{Bk}^M decreases because of the increased competition in the market for component B, the monopolist's optimal response is to increase its price, given that goods $A1$ and Bk are complements. However, since the monopolist sets the same price p_{A1} for all the n systems, such an increase negatively affects the demand of all systems. The monopolist then internalizes such negative externality, thus limiting the increase in p_{A1}. The same applies to the degree of substitutability γ.

Finally, notice that the result in (26) indicates that, with a common quality value, no matter the extent of competition in sector B (i.e., no matter n), "unbundling" the two components of a system, having them sold by different firms, always leads to higher prices compared to an integrated monopoly. This seems to indicate that, when goods are not perfect substitutes, the tragedy of the anticommons is never solved by introducing competition in sector B only contrarily to what happens with perfect substitutes (Dari-Mattiacci and Parisi, 2007).[17] In order to confirm such prediction, we now compare equilibrium consumer surplus with the integrated monopoly case, establishing the following result

Proposition 2. *When sector A is a monopoly and n firms compete in sector B,*

1) if $\alpha_{1k} = \alpha_{IM} = \alpha_{CM}$ $(k = 1, ..., n)$, consumer surplus with competition in sector B is always lower than with an integrated monopoly but higher than with a complementary monopoly ($CS_{CM} < CS^M < CS_{IM}$).

[16] p_{Bk}^M is also negatively related to $\bar{\alpha}$. In fact, it is defined for a given α_{1k}, so that if $\bar{\alpha}$ increases it is because the quality of some systems other than $1k$ has increased. In such circumstance, the ratio $\frac{\alpha_{1k}}{\bar{\alpha}}$ actually decreases, reducing the price that firm k can charge. However, $\bar{p}_B$ is positively affected by $\bar{\alpha}$: as the average quality of the available systems increases, their average price also increases.

[17] In such case, two perfect substitutes in sector B would be enough to solve the tragedy. Our conclusion seem to contradict also the results obtained by McHardy (2006). In his paper, a very low number of competitors selling imperfect substitutes is sufficient to attain the level of social welfare of a complementary monopoly, even if the other sector remains monopolistic.

2) if systems differ in quality, then consumer surplus is higher with competition in sector B than with an integrated monopoly if and only if

$$\sigma_\alpha^2 > \sigma_{CS}^2 = \frac{1}{(1-\gamma)\widetilde{B}^2}\left[\frac{\alpha_{IM}^2}{4n^2} - \tilde{A}^2\bar{\alpha}^2\right] \tag{28}$$

where σ_{CS}^2 is decreasing in γ and n. If quality variance is sufficiently high, competition may be preferred even if $\bar{\alpha} < \alpha_{IM}$.

Proof. See Appendix A. □

When goods are imperfect substitutes and quality is the same across systems and market structures, competition in one sector can certainly improve consumer welfare with respect to a complementary monopoly, but it is never enough to solve the anticommons problem ($CS^M < CS_{IM}$). Competition can effectively increase consumer surplus above CS_{IM} only if both average quality and variance play a role. Particularly, while it is not surprising that competition increases consumer welfare when it also increases average quality, from (23) it can be verified that also quality variance has a positive effect. In other words, our representative consumer benefits from variety (*varietas delectat*). Finally, we observe that both parameters n and γ have a negative effect on σ_{CS}^2. This is because an increase in n and γ decreases equilibrium prices under competition, thus raising consumer surplus, *ceteris paribus*.[18]

The results in Proposition 2 are shown graphically in Figure 1, presenting simulations for different parameter values. Panel a) illustrates a case in which $n = 2$, $\bar{\alpha} = \alpha_{IM} = \alpha_{CM} = 1$ and $\sigma_\alpha^2 = 0$. It shows that consumer surplus under integrated monopoly (CS_{IM}) is always greater than consumer surplus under competition. Panel b) represents the same case, this time letting the number of firms n vary and setting $\gamma = \frac{1}{3}$. Panel c) considers instead a case in which $\sigma_\alpha^2 = 0.25$. and again $\bar{\alpha} = \alpha_{IM} = \alpha_{CM} = 1$. It is possible to verify that now $CS^M > CS_{IM}$ for a sufficiently high value of γ. Finally, panel d) depicts the case in which average quality under competition is slightly lower than the quality of an integrated monopoly ($\bar{\alpha} = 0.95$ and $\alpha_{IM} = 1$). Here, $\gamma = \frac{1}{3}$ and variance is set sufficiently high ($\sigma_\alpha^2 = 0.37$), so that, for $n > 4$, the representative consumer prefers an oligopoly in sector B to an integrated monopoly. [19]

In order to analyze equilibrium profits and the corresponding producer surplus in the various market configurations, we establish first the following results regarding equilibrium quantities.

Lemma 2. *(a) Equilibrium quantities q_{1k}^M are decreasing in n; (b) There exists $\hat{\alpha}_{1k} < \bar{\alpha}$, such that q_{ik}^M is increasing in γ for $\alpha_{1k} > \hat{\alpha}_{1k}$ and is decreasing in γ for $\alpha_{1k} < \hat{\alpha}_{1k}$; (c) Total quantity sold in the market is increasing in n and γ.*

Proof. See Appendix A. □

[18] Obviously, when $\alpha_{IM} > \bar{\alpha}$, the greater the gap between α_{IM} and $\bar{\alpha}$, the greater σ_{CS}^2 to compensate for lower quality.

[19] In the simulations presented here, consumer surplus in complementary monopoly, (CS_{CM}), is always lower than CS^M whenever $\alpha_{IM} = \alpha_{CM}$. This is due to the assumption that $\bar{\alpha}$ is only slightly smaller than or equal to α_{IM}. If $\bar{\alpha}$ were smaller enough, we might have $CS^M < CS_{CM}$, at least for low values of γ and n.

When n increases, both oligopolistic prices and total system prices in (9) and (11) decrease due to enhanced competition. Moreover, as assumed, such increase in the number of competiting firms takes place leaving average quality $\bar{\alpha}$ unchanged, so that the difference $\alpha_{1k} - \bar{\alpha}$ is not affected by the entry of new available system. Thus, overall, demands for all systems raises proportionately. The case in which γ changes is more complex. As γ increases, systems become closer substitutes and their prices decrease (see Lemma 1). However, this does not necessarily translate into a greater demand for each of them. In fact, as implied by the utility function (1), consumers have a taste for quality so that, ceteris paribus, they prefer systems characterized by a higher α_{1k}. Then, as systems become closer substitutes, consumers will demand more high-quality systems at the detriment of low-quality ones. Hence, the demand for some low-quality systems (those with $\alpha_{1k} < \hat{\alpha}_{1k}$) decreases as γ increases. This has immediate repercussions on profits, as we will see below. Finally, and not surprisingly, given that an increase in γ or n decreases the price of all systems, their total demand will increase as well.

The following Corollary and Proposition use Lemmas 1 and 2 to discuss and compare equilibrium profits.

Corollary 1. Π_{A1}^{M} *is increasing in n and γ. Both Π_{Bk}^{M} and Π_{B}^{M} are decreasing in n.*

Corollary 1 states that the monopolist in sector A always benefits from an increase in competition in sector B (produced by an increase in either n or γ). This is because both the monopolist's equilibrium price p_{A1}^{M} and total demand (from Lemma 2) increase in n and γ. The Corollary also establishes a clear relationship between individual profits and the number of firms in sector B: as n increases, competition gets fiercer and each firm sets a lower price, sells a lower quantity and obtains lower profits. This implies that also aggregate profits in sector B decrease with n, "counterbalancing" the growth in the monopolist's profits in sector A. Regarding the relationship between γ and Π_{Bk}^{M}, we know from Lemmas 1 and 2 that both p_{Bk}^{M} and q_{1k}^{M} decrease with γ for low-quality systems, but also that q_{1k}^{M} increases with γ when the quality of system $1k$ is sufficiently high, i.e. $\alpha_{1k} > \hat{\alpha}_{1k} > \bar{\alpha}$. Then for high-quality systems such positive impact of γ on quantities might prevail and Π_{Bk}^{M} can be increasing with γ. Such possibility also influences the relationship between γ and Π_{B}^{M}, as the following Proposition shows. Particularly, this is more likely to happen when quality variance is high and then the chance of having firms in sector B with $\alpha_{1k} > \hat{\alpha}_{1k}$ is greater.

Proposition 3. *If $\bar{\alpha} = \alpha_{IM} = \alpha_{CM}$,*

(a)$\Pi_{IM} > \Pi_{A1}^{M} > \Pi_{CM}^{A}$ for any $n \geq 2$ and $\gamma \in \left[0, \frac{n}{n+1}\right]$. When systems are perfect substitutes ($\gamma = 1$), $\Pi_{A1}^{M} = \Pi_{IM} > \Pi_{CM}^{A}$;

(b)Π_{B}^{M} is increasing in γ if and only if σ_{α}^{2} is sufficiently high;

(c) If $\sigma_{\alpha}^{2} = 0$ then Π_{B}^{M} is lower than Π_{CM}^{B} and Π_{IM}. Also, Producer Surplus ($PS \equiv \Pi_{B}^{M} + \Pi_{A1}^{M}$) is such that $\Pi_{IM} > PS > \Pi_{CM}^{A} + \Pi_{CM}^{B}$.

(d) If $\sigma_{\alpha}^{2} > 0$, $n = 2$, then $\Pi_{B}^{M} < \Pi_{IM}$. If $\sigma_{\alpha}^{2} > 0$, $n \geq 3$, then$\Pi_{B}^{M} \geq \Pi_{IM}$ for sufficiently high σ_{α}^{2}. Also, for $n \geq 2$, $PS \geq \Pi_{IM}$ if and only σ_{α}^{2} is sufficiently high..

Proof. See Appendix A. □

The positive relationship between Π^M_{A1} and n illustrated in Corollary 1 also explains why, as indicated in part (a) of the Proposition, the monopolist's profits are higher when sector B is an oligopoly than when it is a complementary monopoly. Note however that the monopolist's profits are always lower than those obtained by an integrated monopolist. Only in the limit case in which $\gamma = 1$, the monopolist in sector A is able to extract the whole surplus from sector B, thus behaving like an integrated monopolist.[20] Interestingly, whenever $\gamma < 1$, even an infinite number of competitors would not allow the monopolist to obtain the same profits of an integrated monopolist. This is because systems are not perfect substitutes, so that prices in the oligopolistic sector remain, on average, above marginal cost.[21] When quality variance in sufficiently high, part (b) of the Proposition indicates that the increase in profits of high-quality producers more than compensates the decrease in the profits of low-quality ones, confirming the intuition given above.

In the remaining two parts, the Proposition compares industry profits in sector B and total producer surplus with the respective values obtained under a complementary and an integrated monopoly. In the simple case of a common quality level (part (c)), industry profits in sector B (and then *a fortiori* individual profits) are smaller than both the profits of a complementary and of an integrated monopolist producing the same quality level. The relationship between Π^M_B and Π_{IM} is not surprising and is a direct implication of the results in section 2.2, according to which $p^M_{1k} > p_{IM}$ and $Q^M < Q_{IM}$, no matter the number of competing firms. Once more, when quality variance is zero, increasing the number of competitors in one sector only is not enough to eliminate the tragedy of the anticommons. Note that in the same section we also established that both q^M_{Bk} and p^M_{Bk} are lower than q_{CM} and p^B_{CM}, respectively, so that $\Pi^M_{Bk} < \Pi^B_{CM}$. Part (c) states that this result holds in aggregate, as well, and that $\Pi^M_B < \Pi^B_{CM}$: introducing competition in sector B unambiguously lowers industry profits, no matter the degree of substitutability. As for producer surplus, results are ambivalent. On one side, the idea that post-separation entry of new firms in sector B is never able to overcome the tragedy is supported also in terms of the sum of all firms' profits in the economy (so that $\Pi^M_B + \Pi^M_{A1} < \Pi_{IM}$). On the other, we verify that competition in sector B increases the profits of the monopolist in sector A in a way that more than compensates the losses in industry profits in sector B, so that overall producer surplus under competition is greater than under a complementary monopoly ($\Pi^M_B + \Pi^M_{A1} > \Pi^A_{CM} + \Pi^B_{CM}$).

Finally, in part (d) we establish that industry profits in sector B can actually be larger than those of an integrated monopolist (and then *a fortiori*, of a complementary monopolist) when variance is positive. As indicated by equation (17), the higher the quality variance, the larger the value of aggregate profits in sector B, so that it may happen indeed that $\Pi^M_B \geq \Pi_{IM}$. Then, provided a sufficiently large value for σ^2_α, producer surplus under competition might also be greater than with an integrated monopoly.[22] In conclusion, quality variance is an indicator of

[20]One should notice the analogy between this case and the results in Dari-Mattiacci and Parisi (2007).

[21]From (10), $lim_{n\to\infty}\bar{p}_B = \frac{\bar{\alpha}(1-\gamma)}{3-\gamma} > 0$ if $\gamma < 1$.

[22]Note that, for a given average quality, variance is obviously weakly increasing in the number of firms in sector B. In other terms, the higher n, the higher the maximum value that quality variance can take while still satisfying

product differentiation and *varietas delectat* not only for consumers, but for sector B as a whole as well. Then, joining the results in Propositions 2 and 3, the following Corollary holds

Corollary 2. *(a) Total Surplus increases with quality variance. (b) When $\sigma_\alpha^2 = 0$ and $\alpha_{1k} = \alpha_{IM} = \alpha_{CM}$ $(k = 1, ..., n)$, total surplus with competition in sector B is greater than with a complementary monopoly but lower than with an integrated monopoly. (c) When $\sigma_\alpha^2 > 0$, $\bar{\alpha} = \alpha_{IM}$, there exists a value for σ_α^2 such that total surplus with competition in sector B is greater than with an integrated monopoly.*

Summing up, consumers are always worse off in a complementary monopoly. They prefer competition to an integrated monopoly if quality variance is very high, so that they can enjoy the benefits of some very high-quality goods and some lower-quality goods with little price. As for producer surplus, total profits can be higher under competition if variance is large enough. Again, in such case some very high-quality firms are able to earn sufficiently high profits to compensate for the low profits of their low-quality competitors and for the loss in market power due to competition vis a vis both complementary and integrated monopolies. When quality variance is high, such possibility is actually *favoured* by an high degree of subtitutability, given that in such instance Π_B^M increases with γ.

Total surplus follows a similar trend. As long as quality is uniform across systems, the tragedy prevails also in welfare terms and competition in sector B is not able to raise social welfare above the integrated monopoly case. However, separating an integrated firm into independent units producing one component each can be welfare improving if this generates post-separation entry and competition for at least one component and if the competing systems in the market exhibit enough quality differentiation. Note that in Proposition 3 we assumed that $\bar{\alpha} = \alpha_{IM} = \alpha_{CM}$, but our result would be qualitatively the same for $\bar{\alpha} \neq \alpha_{IM}$. Particularly, competition in one sector can still be welfare enhancing even if post-separation entry in such sector reduces average quality, provided a sufficiently high value for quality variance.[23]

In the next Section, we extend the model to consider competition in Sector A, too.

4 Oligopolies in the markets for both complements

In this Section we change the setting analyzed so far and we assume that both complements A and B are produced in oligopolistic markets. Particularly, component A is produced by n_1 different firms, whereas component B is produced by n_2 firms. Again, firms compete by setting prices.

Since consumers can "mix and match" components at their own convenience, there are $n_1 \times n_2$

the constraints of the model (that is non-negative prices). This is the reason why this result holds only if $n \geq 3$. Two firms only in sector B are not enough to generate a sufficiently high quality variance (or equivalently a sufficiently high value of the parameterα_{1k}^{max} introduced in the proof of Lemma 1) such that $\Pi_B^M \geq \Pi_{IM}$.

[23]In this respect, our paper integrates the main conclusion in Economides (1999), according to which separation of the monopolized production of complementary goods may damage quality.

systems in the market and the utility function in (1) becomes

$$U(q,I) = \sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\alpha_{ij}q_{ij} - \frac{1}{2}\left[\beta\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}q_{ij}^2 + \gamma\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\left(q_{ij}\sum_{z=1}^{n_1}\sum_{s=1}^{n_2}q_{zs} - q_{ij}^2\right)\right] + I \tag{29}$$

where q_{ij} represents the quantity of system ij, $(i = 1, \ldots.., n_1;\ j = 1, \ldots., n_2)$, obtained by combining q_{ij} units of component A purchased from the ith firm in sector A (component Ai), and q_{ij} units of component B purchased from the jth firm in sector B (component Bj). Also in this case, $\alpha_{ij} > 0$ $(i = 1, ..., n_1;\ j = 1, ..., n_2)$, $\gamma \in [0, 1]$. The budget constraint now takes the form $\sum_{i=1}^{n_1}\sum_{j=1}^{n_2} p_{ij}q_{ij} + I \leq M$, where $p_{ij} = p_{Ai} + p_{Bj}$ $(i = 1, ..., n_1;\ j = 1, ..., n_2)$ is the price of system ij.

The first order condition determining the optimal consumption of system tk is

$$\frac{\partial U}{\partial q_{tk}} = \alpha_{tk} - (\beta - \gamma)\, q_{tk} - \gamma\sum_{i=1}^{n_1}\sum_{j=1}^{n_2} q_{ij} - p_{tk} = 0 \tag{30}$$

After some tedious algebra, we obtain the demand function for system tk

$$q_{tk} = \frac{b\,(\alpha_{tk} - p_{At} - p_{Bk}) - \gamma\left[\sum_{j\neq k}(\alpha_{tj} - p_{Bj}) - p_{At}\,(n_2 - 1)\right] - \gamma\sum_{i\neq t}\sum_{j=1}^{n_2}(\alpha_{ij} - p_{ij})}{(\beta - \gamma)\,[\beta + \gamma\,(n_1 n_2 - 1)]} \tag{31}$$

where $b = \beta + \gamma\,(n_1 n_2 - 2)$.

As before, to prevent total market size to change with γ, n_1 and n_2 we normalize β as follows[24]

$$\beta = n_1 n_2 - \gamma(n_1 n_2 - 1) \tag{32}$$

Given that component At is possibly bought in combination with all n_2 components produced in sector B, total demand for firm t in sector A is obtained summing q_{tk} in (31) over all possible values of k, i.e., $D_{At} = \sum_{j=1}^{n_2} q_{tj}$. Similarly, total demand for firm k in sector B is $D_{Bk} = \sum_{i=1}^{n_1} q_{ik}$. Then, maximizing profits $\Pi_{At} = p_{At}D_{At}$ with respect to p_{At} and $\Pi_{Bk} = p_{Bk}D_{Bk}$ with respect to p_{Bk}, the equilibrium prices p_{At}^O and p_{Bk}^O (the superscript "O" stands for "oligopoly in both sectors") are, respectively

$$p_{At}^O = A\bar{\alpha} + B\,(\bar{\alpha}_t - \bar{\alpha}) \tag{33}$$

$$p_{Bk}^O = C\bar{\alpha} + D\,(\bar{\alpha}_k - \bar{\alpha}) \tag{34}$$

where $\bar{\alpha} = \frac{\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\alpha_{ij}}{n_1 n_2}$ is the average quality of all systems available in the market, $\bar{\alpha}_t = \frac{\sum_{j=1}^{n_2}\alpha_{tj}}{n_2}$ is the average quality of the systems containing component t, and $\bar{\alpha}_k = \frac{\sum_{i=1}^{n_1}\alpha_{ik}}{n_1}$ is the average quality of systems containing component k. Parameters A, B, C and D are defined as follows: $A = \frac{n_1(1-\gamma)(n_2-\gamma)}{n_1 n_2(3-2\gamma)+\gamma^2(1+n_1+n_2)-2\gamma(n_1+n_2)}$, $B = \frac{n_1}{2n_1-\gamma}$, $C = \frac{n_2(1-\gamma)(n_1-\gamma)}{n_1 n_2(3-2\gamma)+\gamma^2(1+n_1+n_2)-2\gamma(n_1+n_2)}$ and

[24] The second-order condition then becomes $\gamma \leq \frac{n_1 n_2}{n_1 n_2 + 1}$.

$D = \frac{n_2}{2n_2-\gamma}$. The equilibrium price of system tk, $p^O_{tk} = p^O_{At} + p^O_{Bk}$, is therefore

$$p^O_{tk} = (A+C)\,\bar{\alpha} + B\,(\bar{\alpha}_t - \bar{\alpha}) + D\,(\bar{\alpha}_k - \bar{\alpha}) \tag{35}$$

Equilibrium quantities are

$$q^O_{tk} = z\bar{\alpha} + \frac{\alpha_{tk} - \bar{\alpha}}{n_1 n_2 (1-\gamma)} + \frac{\bar{\alpha}_t - \bar{\alpha}}{n_2(2n_1-\gamma)(1-\gamma)} + \frac{\bar{\alpha}_k - \bar{\alpha}}{n_1(2n_2-\gamma)(1-\gamma)} \tag{36}$$

where

$$z = \frac{(n_1-\gamma)(n_2-\gamma)}{n_1 n_2 (n_1 n_2 (3-2\gamma) + \gamma^2(1+n_1+n_2) - 2\gamma(n_1+n_2))} \tag{37}$$

In order to compute profits, we need to calculate the total quantity of each component sold in equilibrium. Given that component At ($t = 1, ..., n_1$) is sold in combination with all its n_2 complements, its total quantity will be

$$q^O_{At} = \sum_{k=1}^{n_2} q^O_{tk} = \frac{(n_1-\gamma)(\gamma-n_2)\overline{\alpha}}{n_1(\gamma(n_2(2-\gamma)-\gamma) + n_1((2-\gamma)\gamma + n_2(2\gamma-3)))} + \frac{(n_1-\gamma)(\bar{\alpha}_t - \bar{\alpha})}{n_1(2n_1-\gamma)(1-\gamma)} \tag{38}$$

Similarly, the total quantity of complement Bk ($k = 1, ..., n_2$) sold in equilibrium is

$$q^O_{Bk} = \sum_{t=1}^{n_1} q^O_{tk} = \frac{(n_2-\gamma)(\gamma-n_1)\overline{\alpha}}{n_2(\gamma(n_1(2-\gamma)-\gamma) + n_2((2-\gamma)\gamma + n_1(2\gamma-3)))} + \frac{(n_2-\gamma)(\bar{\alpha}_k - \bar{\alpha})}{n_2(2n_2-\gamma)(1-\gamma)} \tag{39}$$

Hence profits for each firm are

$$\Pi^O_{At} = p^O_{tk} \cdot q^O_{At} \tag{40}$$

$$\Pi^O_{Bk} = p^O_{tk} \cdot q^O_{Bk} \tag{41}$$

In our $n_1 \times n_2$ model, the expression for the consumer surplus can be rewritten as

$$CS = \frac{n_1 n_2 (1-\gamma)}{2} \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} (q_{ij} - \bar{q})^2 + \frac{n_1^2 n_2^2}{2}\bar{q}^2 \tag{42}$$

Using (36), we find that

$$\bar{q} = \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} q^O_{ij} = z\bar{\alpha} \tag{43}$$

so that we can define

$$Var(q) = \frac{\sum_{i=1}^{n_1}\sum_{j=1}^{n_2} \left(q^O_{ij} - \bar{q}\right)^2}{n_1 n_2} \tag{44}$$

i.e. the variance of the quantities of the systems sold in equilibrium in the whole market. Finally, substituting expressions (43) and (44) into the definition of consumer surplus in (42), we obtain

$$CS^O = \frac{n_1^2 n_2^2}{2}\left(z^2\bar{\alpha}^2 + (1-\gamma)Var(q)\right) \tag{45}$$

In the remainder of this section we want to investigate the impact that the introduction of competition in sector A has on consumer surplus and on profits, compared to less competitive options, like complementary or integrated monopoly. The comparison is rather straightforward in case all systems produced in oligopoly have the same quality of the unique system produced under monopolistic settings (so that, by symmetry, $Var(q) = 0$). For more general cases, however, the complexity of the expressions for prices, quantities and profits renders the algebraic analysis rather difficult. We will therefore perform numerical simulations.

First, we assume that $Var(q) = 0$, with $\alpha_{tk} = \alpha_{IM} = \alpha_{CM} = \alpha^*$, $(t = 1, ..., n_1;\ k = 1, ..., n_2)$ and we establish the following results.

Proposition 4. *When both sectors are oligopolies,* $Var(q) = 0$, $\alpha_{tk} = \alpha_{IM} = \alpha_{CM} = \alpha^*$ $(t = 1, ..., n_1;\ k = 1, ..., n_2)$,

(a) $CS^O > CS_{CM}$;

(b) $CS^O > CS_{IM}$ *if and only if*

$$n_1 > n_1^* = \frac{(n_2 - 1)\,\gamma^2}{n_2(2\gamma - 1) - \gamma^2} \tag{46}$$

where n_1^* *decreases both with* n_2 *and* γ.

(c) Oligopolistic profits Π_{At} *and* Π_{Bk} *are always smaller than* Π^i_{CM}, *hence than* Π_{IM}.

Proof. See Appendix A. □

Thus, when there is competition in both sectors, a competitive industry may be preferred to an integrated monopoly even for a low number of firms in both sectors. Particularly, two firms in both sectors may be enough to solve the tragedy if γ is sufficiently high, as shown in Figure 2.

Figure 2 is obtained assuming $Var(q) = 0$, $\alpha^* = 1$, $n_1 = 2$ and $\gamma = 0.62$. As it can be readily verified, consumer surplus is always higher under competition than in a complementary monopoly. It can be further noticed that consumer surplus under competition is increasing in n_1 and lies below CS_{IM} for low n_1 but becomes larger than CS_{IM} for $n_1 > 4$ ($n_1^* = 4.021$). Part (b) of the proposition also suggests that the degree of competition required in one sector (say, sector A) to increase consumer surplus above CS_{IM} decreases as either the number of firms in the other sector or the degree of substitutability increase (as n_1^* is decreasing in both n_2 and γ). This happens because an increase in n_2 and/or in γ not only reduces the prices of each single component sold in sector B but also the prices of all systems, thus increasing consumer welfare.[25] Finally, part (c) confirms the relationships among profits found in the $n \times 1$ case, with oligopolists always earning the lowest profits and an integrated monopolist the highest.

If $Var(q)$ were positive, the value n_1^* at which CS^O and CS_{IM} cross would be lower. If firms produce different qualities and $Var(q) > 0$, the number of competing firms required to make consumer surplus under competition preferred to that obtained in an integrated monopoly

[25] As we will also see in the simulations below, oligopolists in sector A react to a decrease in the prices in the complementary sector B by increasing their own price. Such increase is however limited, and total system prices overall decrease.

decreases. In fact, a positive $Var(q)$ increases CS^O in (42), thus increasing the range of the parameters for which $CS^O > CS_{IM}$.[26] The exact changes in prices, quantities, profits and welfare as the number of firms and the degree of substitutability between systems vary are analyzed in the following simulations.

In the first simulation we assume a common quality level for all systems ($\alpha_{tk} = \alpha_{IM} = \alpha_{CM} = 1, t = 1, ..., n_1; k = 1, ..., n_2$), We then consider an increase in competition in sector B, keeping the number of firms in sector A fixed at $n_1 = 2$ thoroughout the simulation.[27] Table 1.1 summarizes the welfare obtained by consumers and producers under integrated and complementary monopoly, whereas Table 1.2 presents the prices for firm $A1$ across the various market configurations and the quantity sold of system q_{11}.[28] The equilibrium quantity of each system falls steadily as competition in sector B increases. As expected, the increase in n_2 produces a decrease in the prices in sector B, whereas it raises prices in sector A. This is the effect of two distinct forces. On the one hand there is the traditional cross-price effect characterising complementary goods (for which the cross-price elasticity is negative). On the other hand, the increase in competition in sector B allows firms in sector A to take advantage of an increasing relative market power for the provision of the essential component A. Such behavior of quantities and prices can be observed both when the degree of substitutability among systems is relatively low ($\gamma = 0.2$) and when it is higher ($\gamma = 0.62$). The effect on welfare is different according to the level of γ. At $\gamma = 0.2$, consumer surplus in oligopoly is always lower than in an integrated monopoly but higher than in a complementary monopoly. When $\gamma = 0.62$, consumer surplus in oligopoly is larger than in the $\gamma = 0.2$ case (higher substitutability implies fiercer competition among systems), increases with competition and in particular, for $n_2 > 4$, is also larger than in an integrated monopoly. This is the main difference with the $n \times 1$ case: an high degree of substitutability allows consumers to buy systems at very low prices when competition prevails in both sectors, and this happens even when quality is uniformly distributed, so that we might indeed observe $CS^O > CS_{IM}$. We then consider profits. When $\gamma = 0.2$, total profits in sector A increase with n_2, whereas they decrease in sector B. In sector A, the increase in prices and the larger range of complements each firm can combine with its product more than compensate for the decrease in the quantity demanded of existing systems. On the contrary, the decrease in prices in sector B drives this sector's profits down. The overall effect is however positive, with total aggregate profits across sectors increasing with competition.[29] Anyway, as in the $n \times 1$ case, an integrated monopoly yields the highest profits, a complementary monopoly the lowest, with the competitive market structure somewhere in the middle. When $\gamma = 0.62$, profits in sector A are higher compared to the $\gamma = 0.2$ case (and still increasing in n_2), whereas profits in sector B are higher than in the $\gamma = 0.2$ case only for $n_2 = 2$ and $n_2 = 3$. Then they get lower (and they steadily decrease in

[26] Clearly, a fortiori, $CS^O > CS_{CM}$ always when quality variance is positive.

[27] This is with no loss of generality, given the symmetry of the setting. Note also that we have already provided analytical results for this case, but the simulation will serve as a benchmark.

[28] We have chosen firm $A1$ and system q_{11} because they are respectively always present and sold in positive amount as n_2 increases (the same applies for firms $A2$, $B1$ and $B2$ and quantities q_{12}, q_{21}, q_{22}). Given symmetry, the behavior of all firms and systems is the same in any case.

[29] Total profits Π^O are obtained summing over all firms in each sector and then summing across sectors.

n_2). Consequently, total aggregate profits are U-shaped, increasing up to $n_2 = 4$ (and reaching the profits of an integrated monopoly) and then slowly decreasing. They are however always higher than in the $\gamma = 0.2$ case. Summing up consumer and producer surplus, we note that competition in both sectors can be welfare enhancing compared to an integrated monopoly, even with a common quality level across systems. Specifically, total surplus can get larger than TS_{IM} with a large degree of substitutability, and this is because the significant increase in consumer surplus above CS_{IM} more than counterbalances the slight decrease in total profits when $n_2 > 4$.

In the second simulation we assume that firms are heterogeneous, so that the two sectors A and B can be characterized by different quality distributions which get reflected on systems' qualities. Specifically, we assume that the entry of new firms in one sector allows the composition of ever better systems, so that competition increases average quality in the market. To obtain the effect of quality decreasing with competition, we set α_{tk} ($t = 1, ..., n_1$; $k = 1, ..., n_2$) as follows

$\alpha_{11} = 8$	$\alpha_{12} = 8.5$	$\alpha_{13} = 9$	$\alpha_{14} = 9.5$	$\alpha_{15} = 10$
$\alpha_{21} = 7.5$	$\alpha_{22} = 8$	$\alpha_{23} = 8.5$	$\alpha_{24} = 9$	$\alpha_{25} = 9.5$

Due to our chosen values, the set of systems $\{1k\}$ ($k = 1, ..., 5$) has high average quality than the set $\{2k\}$ and systems denoted by higher k are better in quality. Table 2 reports equilibrium prices, quantities and welfare when competition increases in sector B. It can be verified that quantity q_{11} still decreases with n_2, although being larger than quantities q_{11} obtained in simulation 1 for given γ. As argued above, quality variance increases demand.

As in Simulation 1, prices in sector A increase with n_2, whereas prices in sector B decrease. System prices however decrease in n_2. Unsurprisingly, prices are higher with $\gamma = 0.2$ than with $\gamma = 0.62$, since competition is fiercer in the second case. When $\gamma = 0.2$, consumer, producer and total surplus are always higher under integrated monopoly. Things change when $\gamma = 0.62$; now fiercer competition among closer substitutes leads to substantially lower system prices, thus benefitting consumers (for $n_2 \geq 3$). This more than compensates for the lower producer surplus in oligopoly, so that total surplus in the latter configuration is the highest. Complementary monopoly yields the lowest surplus, both for consumers and producers. As in the previous simulation, individual profits decrease in sector B, experiencing the increase in competition, whereas sector A takes advantage of this by increasing its own profits.[30]

In the last simulation, we assume that competition worsens average quality in the market, so that, the larger the number of active firms, the lower $\bar{\alpha}$, $\bar{\alpha}_t$ and $\bar{\alpha}_k$. Again, with no loss of generality, we assume that competition increases in sector B, whereas $n_1 = 2$ throughout the simulation. To obtain the effect of quality decreasing with competition, we set α_{tk} ($t = 1, ..., n_1$; $k = 1, ..., n_2$) as follows[31]

[30] It should be noticed that both consumer surplus and profits under monopolistic configurations increase in n_2. This happens because each oligopoly structure (for each n_2) is compared with both types of monopoly at the same average quality and here, by assumption, $\bar{\alpha}$ increases with n_2.

[31] It should be noticed that the coefficients α_{tk} are the same as in Simulation 2, but in reversed order.

$\alpha_{11} = 10$	$\alpha_{12} = 9.5$	$\alpha_{13} = 9$	$\alpha_{14} = 8.5$	$\alpha_{15} = 8$
$\alpha_{21} = 9.5$	$\alpha_{22} = 9$	$\alpha_{23} = 8.5$	$\alpha_{24} = 8$	$\alpha_{25} = 7.5$

When $\gamma = 0.2$, Table 3 shows that individual firms' and system prices decrease with competition. Interestingly, prices are declining and lower in sector A. This reverts the trend obseved in the previous simulations, in which the sector not affected by competition was able to limit the impact or even to take advantage of the increased competition in the complementary sector. Moreover, demand of a given system (say q_{11}) decreases with competition. In fact, equation (36) implies that equilibrium quantities are positively affected by both the system's quality α_{tk} and by average qualities. For a given α_{tk}, a decrease in average quality has a negative impact on demanded quantity.[32] However, firms in sector A enjoy higher (though declining) profits; they are still able to extract a higher surplus than their complementors operating in the more competitive sector. Overall profits are lower than their integrated monopoly counterpart but higher than in a complementary monopoly. Consumer surplus decreases with competition: lower prices and increased variance are not enough to compensate for the decline in quality. Again, consumer surplus is highest in integrated monopoly and lowest in complementary monopoly.[33] When $\gamma = 0.62$, a fifth firm in sector 2 obtains no demand because of a too low quality level. This is why the most competitive feasible market structure is at $n_2 = 4$. System prices and quantities decrease as n_2 increases (and prices are lower than in the $\gamma = 0.2$ case, whereas quantities are higher). Interestingly, comparing consumer surplus across market configurations, it can be noticed that $CS^O < CS_{IM}$ for $n_2 = 2$ but $CS^O > CS_{IM}$ for $n_2 \geq 3$. This happens because the comparison is performed for the same quality level (α_{IM} is set equal to $\bar{\alpha}$ for each value of n_2), but quality variance is increasing. Similarly to the $n \times 1$ case, then, as variance increases, consumer welfare might be greater in competition than with an integrated monopoly. Finally, although p_{B1} has the usual pattern (as competition increases in sector B, p_{B1} decreases), p_{A1} has a non-monotonic behavior. First, it increases from $n_2 = 2$ to $n_2 = 3$. This is the same behavior displayed in previous simulations; as competition increases in sector B and the price of the complements decrease, firms in sector A react by raising their prices. However, we have just checked that when $\gamma = 0.2$ and competition decreases quality, p_{A1} instead decreases in n_2. And in fact, when $n_2 = 4$, p_{A1} is lower than in both the $n_2 = 2$ and the $n_2 = 3$ cases. What happens is that average quality is getting so low that firms in sector A are forced to lower their prices. The initial increase when competition is still relatively low (and average quality high) is made possible by the high degree of substitutability γ, that renders competition especially fierce in sector B. This is not possible anymore when further competition takes quality to very low levels. Profits follow the same pattern: they increase in sector A when n_2 goes from 2 to 3 but then decrease. The fiercer competition due to high substitutability does not allow firms in sector A to counteract the decline in demand due to lower average quality by reducing price as it was

[32] At $n_2 = 6$ the quantity of the lowest quality systems becomes negative, implying that increased competition is not sustainable in such market configuration. That's why simulation 3 considers n_2 only up to 5.

[33] Here consumer surplus and profits under monopolistic configurations decrease in n_2 since $\bar{\alpha}$ decreases with higher n_2.

able to do when $\gamma = 0.2$. Finally, profits in sector B always decrease and so do total profits. However, $\Pi^O > \Pi_{IM} > \Pi_{CM}$ because of the high quality variance exogenously produced in the simulation, and this result, combined with the trend observed for consumer surplus, produces an increasing trend for sociale welfare. In fact, as n_2 increases, total surplus increases as well, surpassing the corresponding integrated monopoly value for $n_2 \geq 3$.

5 Conclusions

Complementary monopoly may be worse than an integrated monopoly, in which all such complementary goods are offered by a single firm. This is "the tragedy of the anticommons". We have considered the possibility of competition in the market for each complement, presenting a model in which n imperfect substitutes for each perfect complement are produced. We have proven that, if at least one complementary good is produced in a monopoly, an integrated monopoly is always superior to a more competitive market setting. Consequently, favoring competition in some sectors, leaving monopolies in others may be detrimental for consumers. Competition may be justified if and only if the goods produced by competitors differ in quality, so that also average quality and variance become important factors to consider.

We have also proven that, when competition is introduced in each sector, the tragedy may be solved for relatively small numbers of competing firms in each sector if systems are close substitutes, and this even in the limit case of a common quality level across systems. Unsurprisingly, the higher the degree of substitutability and the level of competition in other sector, the more concentrated a sector can be, while still performing better than an integrated monopoly in terms of consumer surplus.

References

[1] Alvisi, M., Carbonara E., Dari-Mattiacci, G., and Parisi, F. (2009). Complementing Substitutes: Bundling, Compatibility and Entry, Amsterdam Center for Law & Economics Working Paper No. 2009-10.

[2] Anderson, S.P. and Leruth, L. (1993). Why Firms May Prefer not to Price Discriminate via Mixed Bundling, International Journal of Industrial Organization, 11: 49-61.

[3] Beggs, A. (1994). Mergers and Malls, Journal of Industrial Economics 42: 419–428.

[4] Buchanan, J. and Y.J. Yoon (2000). Symmetric Tragedies: Commons and Anticommons, Journal of Law and Economics, 43: 1–13.

[5] Cournot, A.A. (1838). Researches into the Mathematical Principles of the Theory of Wealth (translation 1897). New York: Macmillan.

[6] Dari-Mattiacci, G.and F. Parisi (2007). Substituting Complements, Journal of Competition Law and Economics, 2(3): 333-347.

[7] Denicolò, V. (2000). Compatibility and Bundling with Generalist and Specialist Firms, Journal of Industrial Economics, 48: 177–188.

[8] Dixit, A. (1979). A Model of Duopoly Suggesting a Theory of Entry Barriers, Bell Journal of Economics, 10: 20–32.

[9] Eonomides, N. (1999), Quality, Choice and Vertical Integration, International Journal of Industrial Organization, 17:903-914.

[10] Economides, N. and Salop, S.C. (1992). Competition and Integration Among Complements, and Network Market Structure, Journal of Industrial Economics, 40: 105 –123.

[11] Fudenberg, D. and Tirole, J. (2000). Pricing a Network Good to Deter Entry, Journal of Industrial Economics, 48: 373–390.

[12] Gaudet, G. and Salant, S. W. (1992). Mergers of Producers of Perfect Complements Competing in Price, Economics Letters, 39: 359–364.

[13] Gilbert, R.J. and Katz, L.M. (2001). An Economist's Guide to U.S. v. Microsoft, Journal of Economic Perspectives, 15: 25–44.

[14] Häckner, J. (2000). A Note on Price and Quantity Competition in Differentiated Oligopolies, Journal of Economic Theory, 93: 233–239.

[15] Heller, M.A. (1998). The Tragedy of the Anticommons: Property in the Transition from Marx to Markets, Harvard Law Review, 111: 621.

[16] Heller, M.A. and Eisemberg, R.S. (1998). Can Patents Deter Innovation? The Anticommons in Biomedical Research, Science, 280: 698–701.

[17] Hsu, J. and X. H. Wang (2005). On Welfare under Cournot and Bertrand Competition in Differentiated Oligopolies, Review of Industrial Organization, 27: 185–191.

[18] Machlup, F. and Taber, M. (1960). Bilateral Monopoly, Successive Monopoly, and Vertical Integration, Economica, 27: 101-119.

[19] Matutes, C. and P. Regibeau (1988). "Mix and Match": Product Compatibility without Network Externalities, RAND Journal of Economics, 19: 221-234.

[20] McHardy, J. (2006). Complementary Monopoly and Welfare: Is Splitting Up So Bad? The Manchester School, 74: 334-349.

[21] Nalebuff, B. (2004). Bundling as an Entry Barrier, Quarterly Journal of Economics, 1:159-187.

[22] Paradise, J. (2004). European Opposition to Exclusive Control Over Predictive Breast Cancer Testing and the Inherent Implications for United States Patent Law and Public Policy: A Case Study of the Myriad Genetics' BRCA Patent Controversy, Food & Drug Law Journal, Vol. 59: 133-154.

[23] Parisi, F. (2002). Entropy in Property, American Journal of Comparative Law, 50: 595–632.

[24] Plackett, R. (1947). Limits of the Ratio of Mean Range to Standard Deviation. Biometrika, 34: 120-122.

[25] Shubik, M. and Levitan, R. (1980). Market Structure and Behavior, Cambridge, MA: Harvard University Press.

[26] Singh, N. and Vives, X. (1984). Price and Quantity Competition in a Differentiated Monopoly, Rand Journal of Economics, 15: 546– 554.

[27] Sonnenschein, H. (1968). The Dual of Duopoly Is Complementary Monopoly: or, Two of Cournot's Theories Are One, Journal of Political Economy, 76: 316–318.

[28] Tan, G. and Yuan, L. (2003). Strategic Incentives of Divestitures of Competing Conglomerates, International Journal of Industrial Organization, 21: 673–697.

A Proofs

A.1 Proof of Lemma 1

In order to prove that $\frac{\partial p_{Bk}^M}{\partial \gamma} < 0$ we note first that this is always true if $\alpha_{1k} < \bar{\alpha}$. In fact, $\frac{\partial}{\partial \gamma} \frac{n(1-\gamma)}{n(\gamma-3)-2\gamma} = -\frac{2(n-1)n}{(n(3-\gamma)-2\gamma)^2} < 0$ and $\frac{\partial}{\partial \gamma} \frac{n}{2n-\gamma} = \frac{n}{(2n-\gamma)^2} > 0$. If $\alpha_{1k} > \bar{\alpha}$, it may be that $\frac{\partial p_{Bk}^M}{\partial \gamma} > 0$ for a sufficiently high value of α_{1k}, and in particular for $\alpha_{1k} > \tilde{\alpha}_{1k}$, where $\tilde{\alpha_{1k}}$ is obtained solving $\frac{\partial p_{Bk}^M}{\partial \gamma} = 0$ with respect to α_{1k}. We then check whether $\tilde{\alpha}_{1k}$ is a feasible value for an above-average quality. To do that, we compute first the highest α_{1k} compatible with a given average $\bar{\alpha}$, α_{1k}^{max}, which is obtained when the remaining $n-1$ firms produce such low-quality systems $\alpha_{1s}^{min} < \bar{\alpha}$, $s \neq k$ as to optimally set their price equal to marginal cost (so that they remain active in sector B), that is $p_{Bs}^M = 0$. From (9), we obtain:

$$\alpha_{1s}^{min} = \frac{\bar{\alpha}(n-\gamma)(1+\gamma)}{n(3-\gamma)-2\gamma} \tag{47}$$

Setting $\alpha_{1s} = \alpha_{1s}^{min}$ for all firms $s \neq k$, we obtain α_{1k}^{max} solving

$$\frac{(n-1)\,\alpha_{1s}^{min} + \alpha_{1k}^{max}}{n} = \bar{\alpha} \tag{48}$$

i.e., $\alpha_{1k}^{max} = n\bar{\alpha} - (n-1)\,\alpha_{1s}^{min}$. Substituting such value into $\frac{\partial p_{Bk}^M}{\partial \gamma}$, we have

$$\left.\frac{\partial p_{Bk}^M}{\partial \gamma}\right|_{\alpha_{1k}=\alpha_{1k}^{max}} = \frac{(n-1)n\bar{\alpha}\left[2\gamma^2 - n\left(1+4\gamma-\gamma^2\right)\right]}{(2n-\gamma)\left[n(\gamma-3)+2\right]^2} < 0.$$

Hence, $\alpha_{1k}^{max} < \tilde{\alpha}_{1k}$ always and $\frac{\partial p_{Bk}^M}{\partial \gamma} < 0$ for all $\gamma \in \left[0, \frac{n}{n+1}\right]$.

Similarly, in order to prove that $\frac{\partial p_{Bk}^M}{\partial n} < 0$ for all $n \geq 2$, we note from (9) that $\frac{\partial p_{Bk}^M}{\partial n} < 0$ always if $\alpha_{1k} > \bar{\alpha}$, since $\frac{\partial}{\partial n} \frac{n(1-\gamma)}{n(3-\gamma)-2\gamma} = -\frac{2(1-\gamma)}{(n(3-\gamma)-2\gamma)^2} < 0$ and $\frac{\partial}{\partial n} \frac{n}{2n-\gamma} = -\frac{\gamma}{(2n-\gamma)^2} < 0$. If $\alpha_{1k} < \bar{\alpha}$, it may be that $\frac{\partial p_{Bk}^M}{\partial n} > 0$ for a sufficiently low value of α_{1k}. We then prove that $\frac{\partial p_{Bk}^M}{\partial n} < 0$ at the minimum possible value of α_{1k}, α_{1k}^{min}. Substituting α_{1k}^{min} from (47) we obtain $\left.\frac{\partial p_{Bk}^M}{\partial n}\right|_{\alpha_{1k}=\alpha_{1k}^{min}} = \frac{n\bar{\alpha}\gamma(\gamma^2-1)}{(2n-\gamma)(n(3-\gamma)-2\gamma)^2}$, which is negative for all relevant values of γ and n.

A.2 Proof of Proposition 1

Differentiating $\frac{\partial p_{1k}^M}{\partial \bar{\alpha}} = -\frac{(2\gamma-1)[1+(n-2)\gamma]}{[3+\gamma(2n-5)][2+\gamma(2n-3)]} > 0$ for all $\gamma < \frac{n}{n+1}$.

We now prove that $\bar{p}_{1k}^M$ decreases with n. From (8) it can be readily verified that $\frac{\partial p_{A1}^M}{\partial n} > 0$, whereas Lemma 1 demonstrates that $\frac{\partial p_{Bk}^M}{\partial n} < 0$. It is then sufficient to prove that $\frac{\partial p_{A1}^M}{\partial n} < \left|\frac{\partial p_{Bk}^M}{\partial n}\right|$ when $\left|\frac{\partial p_{Bk}^M}{\partial n}\right|$ takes its minimum value (i.e., when it is closest to zero). Note first that $\frac{\partial p_{Bk}^M}{\partial n} = -\frac{\gamma(\alpha_{1k}-\bar{\alpha})}{[2+(2n-3)\gamma]^2} - \frac{2(1-\gamma)\bar{\alpha}}{[3+(2n-5)\gamma]^2}$, which reaches its minimum value when $\alpha_{1k} = \alpha_{1k}^{min}$ (where α_{1k}^{min} is defined in the proof of Lemma 1), since $-\frac{\gamma(\alpha_{1k}-\bar{\alpha})}{[2+(2n-3)\gamma]^2}$ is positive and maximum at α_{1k}^{min}. It is then easy to verify that $\frac{\partial p_{A1}^M}{\partial n} - \left|\frac{\partial p_{Bk}^M}{\partial n}\right|_{\alpha_{ik}=\alpha_{ik}^{min}} = -\frac{\alpha(1-\gamma)(2+4\gamma(n-\bar{2})+(8-7n+2n^2)\gamma^2)}{(1+\gamma(n-1))(3+\gamma(2n-5))^2(2+\gamma(2n-3))} < 0$ for all γ and n.

A similar proof works for $\frac{\partial p_{1k}^M}{\partial \gamma}$.

The effect on CS^M is a direct consequence of the influence of γ and n on system prices.

A.3 Proof of Proposition 2

Part 1). In this case $\alpha_{1k} = \bar{\alpha}$, $(k = 1, ..., n)$ and $\sigma_\alpha^2 = 0$. From (23), consumer surplus under competition is $CS^M = \frac{n^2}{2}\tilde{A}^2\bar{\alpha}^2$. Comparing it with consumer surplus under integrated and complementary monopoly, given by (24) and (27), respectively, we note immediately that the difference $CS^M - CS_{IM} = \frac{\bar{\alpha}^2 n(1-\gamma)(n(\gamma-5)+4\gamma)}{8(n(3-\gamma)-2\gamma)^2}$ is negative, while the difference $CS^M - CS_{CM} = \frac{\bar{\alpha}^2\left(n(6\gamma(n-1)+\gamma)+5\gamma^2\right)}{18(n(3-\gamma)-2\gamma)^2}$ is positive, for all $n \geq 2$ and $\gamma \in [0, 1]$.

Part 2). When $\sigma_\alpha^2 > 0$, subtracting CS_{IM} from CS^M and solving for σ_α^2, it is immediate to obtain σ_{CS}^2 in expression (28). When $\sigma_{CS}^2 < 0$, competition is always preferred. The relevant case is thus $\sigma_{CS}^2 > 0$, which holds when $\bar{\alpha} < \frac{\alpha_{IM}}{2\tilde{A}n}$. It can be verified that $\alpha_{IM} < \frac{\alpha_{IM}}{2\tilde{A}n}$, so that it is possible to have a case in which $\bar{\alpha} < \alpha_{IM}$ and $CS^M > CS_{IM}$.

Given that CS^M is increasing in n and γ, the minimum value of σ_α^2 required to have $CS^M \geq CS_{IM}$, σ_{CS}^2, must be decreasing in n and γ.

A.4 Proof of Lemma 2

Differentiating q_{ik}^M in (13) with respect to γ we get

$$\frac{\partial q_{ik}^M}{\partial \gamma} = \frac{(n-1)\bar{\alpha}}{(n(3-\gamma)-2\gamma)^2} + \frac{(2n^2+\gamma^2-n(1+2\gamma))(\alpha_{1k}-\bar{\alpha})}{n(1-\gamma)^2(2n-\gamma)^2} \tag{49}$$

When $n \geq 2$ and $\gamma \in \left[0, \frac{n}{n+1}\right]$, the first term on the right-hand side of (49) is positive. The second term is positive if $\alpha_{1k} > \bar{\alpha}$ and negative otherwise. Thus, $\frac{\partial q_{ik}^M}{\partial \gamma} > 0$ always if $\alpha_{1k} > \bar{\alpha}$. If $\alpha_{1k} < \bar{\alpha}$, the maximum negative value of the second term in (49) is taken when α_{1k} reaches its minimum feasible value, α_{1s}^{min} (see equation (47) in the proof of Lemma 1). Evaluating $\frac{\partial q_{ik}^M}{\partial \gamma}$ at $\alpha_{1k} = \alpha_{1k}^{min}$ we obtain $\left.\frac{\partial q_{ik}^M}{\partial \gamma}\right|_{\alpha_{1k}=\alpha_{1s}^{min}} = -\frac{(n(4n-6\gamma-1)+\gamma^2(2+n))(n-\gamma)\bar{\alpha}}{n(2n-\gamma)(1-\gamma)(n(3-\gamma)+2\gamma^2} < 0$. Thus, given that $\frac{\partial q_{ik}^M}{\partial \gamma}$ is continuous in α_{1k}, there exists $\hat{\alpha}_{1k} < \bar{\alpha}$ such that $\frac{\partial q_{ik}^M}{\partial \gamma} \geq 0$ for $\alpha_{1k} \geq \hat{\alpha}_{1k}$ and negative otherwise.

Differentiating q_{ik}^M in (13) with respect to n we get

$$\frac{\partial q_{ik}^M}{\partial n} = \frac{((3-\gamma)n(2-n)-2\gamma^2)\bar{\alpha}}{n^2(n(3-\gamma)-2\gamma)^2} + \frac{(2n(n-2\gamma)+\gamma^2)(\alpha_{1k}-\bar{\alpha})}{n^2(\gamma-1)(2n-\gamma)^2} \tag{50}$$

When $n \geq 2$ and $\gamma \in \left[0, \frac{n}{n+1}\right]$, the first term on the right-hand side of (50) is negative. The second term is negative if $\alpha_{1k} > \bar{\alpha}$ and positive otherwise. Thus, $\frac{\partial q_{ik}^M}{\partial n} < 0$ always if $\alpha_{1k} > \bar{\alpha}$. If $\alpha_{1k} < \bar{\alpha}$, the maximum positive value for the second term of (50) occurs when $\alpha_{1k} = \alpha_{1k}^{min}$. Evaluating $\frac{\partial q_{ik}^M}{\partial n}$ at $\alpha_{1k} = \alpha_{1s}^{min}$ we obtain $\left.\frac{\partial q_{ik}^M}{\partial n}\right|_{\alpha_{1k}=\alpha_{1s}^{min}} = -\frac{(n-\gamma)\gamma(1-\gamma)\bar{\alpha}}{n(2n-\gamma)(n(3-\gamma)-2\gamma)^2} < 0$. Thus, $\frac{\partial q_{ik}^M}{\partial n} < 0$.

Define total quantity as

$$Q^M \equiv \sum_{k=1}^{n} q_{ik}^M = \frac{\bar{\alpha}(n-\gamma)}{n(3-\gamma)-2\gamma} \tag{51}$$

Differentiating (51) with respect to γ and n we obtain $\frac{\partial Q^M}{\partial \gamma} = \frac{n(n-1)\bar{\alpha}}{(n(3-\gamma)-2\gamma)^2} > 0$ and $\frac{\partial Q^M}{\partial n} = \frac{\gamma(1-\gamma)\bar{\alpha}}{(n(3-\gamma)-2\gamma)^2} > 0$ in the admissible range of the parameters.

A.5 Proof of Proposition 3

Part (a). Comparing Π_{A1}^M in (14) and Π_{CM}^A in (27), we obtain $\Pi_{A1}^M - \Pi_{CM}^A = \frac{(n-1)\bar{\alpha}^2\gamma(n(6-\gamma)-5\gamma)}{9(n(3-\gamma)-2\gamma)^2} > 0$ in the relevant parameters' range. We then compare Π_{A1}^M with the profit of an integrated monopoly and $\Pi_{A1}^M - \Pi_{IM} = -\frac{n\bar{\alpha}(1-\gamma)(n(5-\gamma)-4\gamma)}{4(n(3-\gamma)-2\gamma)^2} < 0$. Note that $lim_{n\to\infty}\Pi_{A1}^M = \frac{\bar{\alpha}^2}{(3-\gamma)^2}$, which is in any case smaller than Π_{IM} when $\gamma \in \left[0, \frac{n}{n+1}\right]$. Only at $\gamma = 1$ we would have $\Pi_{A1}^M = \Pi_{IM}$.

Part (b). From Lemmas 1 and 2, both p_{Bk}^M and q_{1k}^M decrease with n. Then both Π_{Bk}^M and Π_B^Malso decrease with n.

To prove the impact of γ on Π_B^M, let us differentiate expression (17) with respect to γ. We find:

$$\frac{\partial \Pi_B^M}{\partial \gamma} = \frac{n(n(2n-3\gamma)+\gamma(2-\gamma))}{(2n-\gamma)^3(1-\gamma)^2}\sigma_\alpha^2 - \frac{n(n-1)(n+\gamma(n-2))}{(n(3-\gamma)-2\gamma)^3}\bar{\alpha} \tag{52}$$

It might then happen that $\frac{\partial \Pi_B^M}{\partial \gamma} > 0$ if σ_α^2 is high enough for given $\bar{\alpha}$. It is a well-known result in statistics that the maximum variance $\sigma_{\alpha max}^2$ in a discrete distribution is attained when $\frac{n}{2}$ firms have quality equal to the minimum value in the range and $\frac{n}{2}$ firms have quality equal to the maximum value in the range (see Plackett, 1947). In our specific case, the minimum value in the range is given by α_{1k}^{min}, whereas the maximum value has to be computed given the average $\bar{\alpha}$ and the fact than $\frac{n}{2}$ firms produce α_{1k}^{min}. Define such maximum $\breve{\alpha} = \bar{\alpha}\left(n - \frac{(n-\gamma)(1+\gamma)}{n(3-\gamma)-2\gamma}\right)$. Then maximun variance would be $\sigma_{max}^2 = \frac{1}{2}\bar{\alpha}^2\left(\frac{(1-\gamma)^2(2n-\gamma)^2}{(n(3-\gamma)-2\gamma)^2}\right) + \left(n-1+\frac{(n-\gamma)(1+\gamma)}{n(3-\gamma)-2\gamma}\right)^2$. By differentiating Π_B^M with respect to γ and solving the derivative with respect to σ_α^2, it is possible to verify that $\frac{\partial \Pi_B^M}{\partial \gamma} \geq 0$ iff $\sigma_\alpha^2 \geq \sigma_0^2 = \frac{(n-1)\bar{\alpha}^2(2n-\gamma)^3(1-\gamma)^2(n-2\gamma+n\gamma)}{(n(3-\gamma)-2\gamma)^3(2n^2-3n\gamma-\gamma(1-2\gamma))}$. To compare σ_0^2 with σ_{max}^2, we evaluate the expression $\sigma_{max}^2 - \sigma_0^2$ numerically for all admissible values of γ and we find that $\sigma_{max}^2 > \sigma_0^2$ for all $n \geq 2$, implying that $\frac{\partial \Pi_B^M}{\partial \gamma} > 0$ for σ_α^2 sufficiently high.

Part (c). When $\sigma_\alpha^2 = 0$, then all systems have the same quality level α_{1k}, $k = 1, ..., n$. If this level is such that $\alpha_{1k} = \alpha_{IM} = \alpha_{CM}$, then the difference $\Pi_B^M - \Pi_{CM}^B = -\frac{(n-1)\bar{\alpha}^2\gamma(3n+\gamma(n-4))}{9(n(3-\gamma)-2\gamma)^2}$ is always negative in the admissible parameters' range. We know that $\Pi_{CM}^B < \Pi_{IM}$, hence, *a fortiori*, $\Pi_B^M - \Pi_{IM} < 0$. As for Producer Surplus, $PS \equiv \Pi_{A1}^M + \Pi_B^M = \frac{\bar{\alpha}^2\left[n^2(2-\gamma)-n(3-\gamma)\gamma+\gamma^2\right]}{[n(3-\gamma)-2\gamma]^2}$. It is easy to check that $PS - \Pi_{IM} = -\frac{n\bar{\alpha}^2(1-\gamma)^2}{4(n(3-\gamma)-2\gamma)^2} < 0$. Also, $\Pi_{A1}^M + \Pi_B^M - \Pi_{CM}^A - \Pi_{CM}^B = PS - 2\Pi_{CM}^i = \frac{(n-1)\bar{\alpha}^2\gamma(n(3-2\gamma)-\gamma)}{9(n(3-\gamma)-2\gamma)^2}$ which is always positive in the relevant parameters' range.

Part (d). The final result is immediate and is obtained solving $\Pi_{Bk}^M = \Pi_{IM}$ with respect to σ_α^2. Then $\Pi_{Bk}^M \geq \Pi_{IM}$ iff $\sigma_\alpha^2 \geq \sigma_{\Pi_B}^2 = \frac{(n-1)\bar{\alpha}^2(1-\gamma)\gamma(2n-\gamma)^2(n(3+\gamma)-4\gamma)}{9n(n-\gamma)(n(3-\gamma)-2\gamma)^2}$, where $\sigma_{\Pi_B}^2 < \sigma_{max}^2$ for all $n \geq 3$ (numerical evaluation for all admissible values of γ). For $n = 2$, $\sigma_{\Pi_B}^2 > \sigma_{max}^2$, implying that $\Pi_{Bk}^M < \Pi_{IM}$. As for Producer Surplus, the result is obtained solving $\Pi_{A1}^M + \Pi_B^M = \Pi_{IM}$ with respect to σ_α^2. Then $\Pi_{A1}^M + \Pi_B^M \geq \Pi_{IM}$ iff $\sigma_\alpha^2 \geq \sigma_{PS}^2 = \frac{n\bar{\alpha}^2(1-\gamma)^2(2n-\gamma)^2}{4(n-\gamma)(n(3-\gamma)-2\gamma)^2}$. Also, it is possible to establish (through numerical evaluation) that $\sigma_{PS}^2 < \sigma_{max}^2$ for all $n \geq 2$.

A.6 Proof of Proposition 4

Part (a). The proof is immediate, setting $Var(q) = 0$ in (42) and comparing the resulting expression with CS_{CM}.

Part (b). Solving $CS^O - CS_{IM} = 0$ with respect to n_1, i.e. $\frac{n_1^2 n_2^2}{2}z^2q^2 - \frac{\alpha^{*2}}{8} = 0$, yields two solutions, $n_{11} = \frac{(n_2-1)\gamma^2}{n_2(2\gamma-1)-\gamma^2}$ and $n_{12} = \frac{\gamma(n_2(4-\gamma)-3\gamma)}{n_2(5-2\gamma)-(4-\gamma)\gamma}$, so that $CS^O > CS_{IM}$ iff either $n_1 < n_{12}$ or $n_1 > n_{11}$. It is possible to verify, however, that $n_{12} < 1$ for all γ and n_2 in the admissible range of the parameters. Therefore, $CS^O \geq CS_{IM}$ iff $n_1 \geq n_{11}$ and $n_{11} = n_1^*$ in (46).

Differentiating (46) with respect to γ, $\frac{\partial n_1^*}{\partial \gamma} = -\frac{2(n_2-1)n_2(1-\gamma)\gamma}{(n_2(1-2\gamma)+\gamma^2)^2} < 0$, whereas differentiating it with respect to n_2 yields $\frac{\partial n_1^*}{\partial n_2} = -\frac{(1-\gamma)^2\gamma^2}{(n_2(1-2\gamma)+\gamma^2)^2} < 0$.

Part (c). For this part, it suffices to prove that either Π_{At} or Π_{Bk} is always smaller than Π_{CM}^i. The other is implied by the clear symmetry. Moreover, being $\Pi_{CM}^i < \Pi_{IM}$, this implies also that Π_{At} and Π_{Bk} are smaller than Π_{IM}. By comparing Π_{At} with Π_{CM}^A, we find that

$$\Pi_{At} - \Pi_{CM}^A = \frac{1}{9}\alpha^{*2}\left(\frac{9(n_1-\gamma)(n_2-\gamma)^2(1-\gamma)}{n_2(\gamma(n_2(\gamma-2)+\gamma)+n_1(n_2(3-2\gamma)+(\gamma-2)\gamma))^2} - 1\right) \tag{53}$$

Numerically solving (53) with respect to n_1 for given values of n_2 and considering all the admissible values $\gamma \in \left[0, \frac{n_1 n_2}{n_1 n_2 + 1}\right]$, it is possible to check that (53) admits two solutions $\tilde{n}_a$ and $\tilde{n}_b$ and that both are always lower than 1 when not imaginary. Simulations show that $\Pi_{At} - \Pi_{CM}^A \geq 0$ for $\tilde{n}_a \leq n_1 \leq \tilde{n}_b$ (when $\tilde{n}_a$ and $\tilde{n}_b$ are real) and $\Pi_{At} - \Pi_{CM}^A < 0$ when $\tilde{n}_a$ and $\tilde{n}_b$ are imaginary. This implies that $\Pi_{At} - \Pi_{CM}^A < 0$ in the relevant range of the parameters. The same proof can be applied to Π_{Bk}.

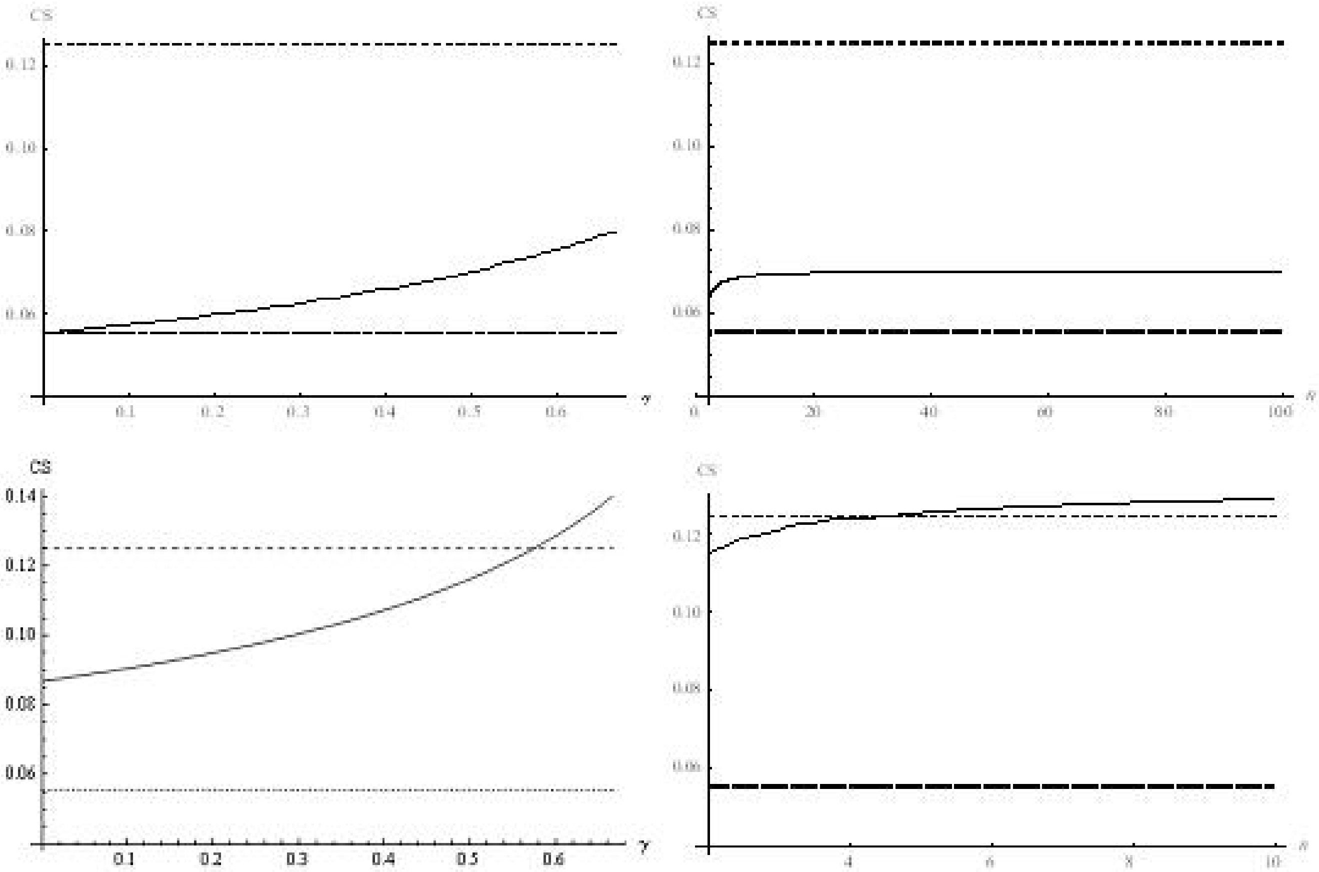

Figure 1: Comparing consumer surplus under three regimes – competition, integrated and complementary monopoly (___ CS^M, --- CS_{IM}, ….. CS_{CM}).

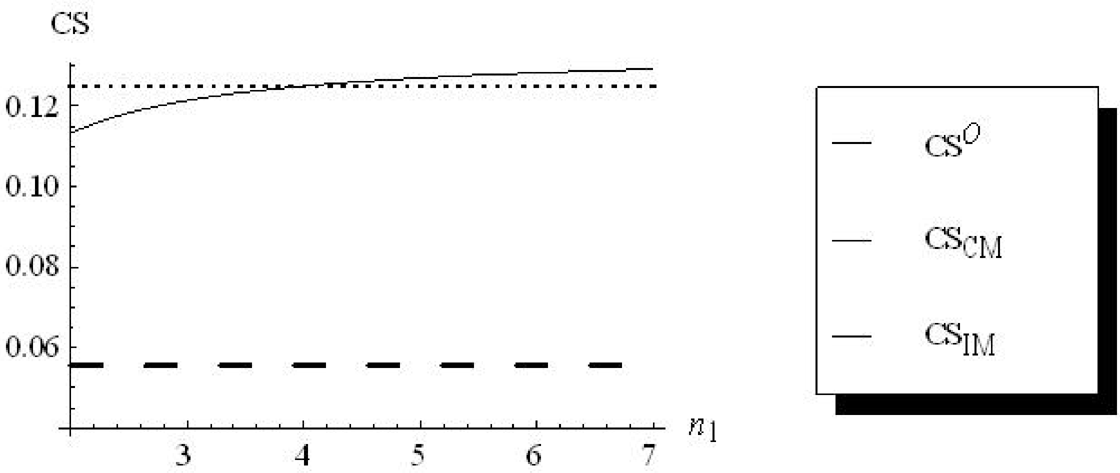

Figure 2: Consumer surplus under three different regimes when competition is present in both sectors.

CS_{IM}	CS_{CM}	Π_{IM}	Π_{CM}	TS_{IM}	TS_{CM}
0.125	0.055	0.25	0.22	0.375	0.275

Table 1.1: Welfare under integrated and complementary monopoly in simulation 1.

		$n_2 = 2$	$n_2 = 3$	$n_2 = 4$	$n_2 = 5$	$n_2 = 6$	$n_2 = 7$	$n_2 = 8$	$n_2 = 9$	$n_2 = 10$
$\gamma = 0.2$	p_{A1}	0.32	0.324	0.325	0.326	0.327	0.3277	0.328	0.3285	0.3286
	p_{B1}	0.32	0.31	0.308	0.306	0.305	0.304	0.303	0.302	0.301
	p_{11}	0.64	0.634	0.633	0.632	0.632	0.6317	0.631	0.6305	0.6296
	q_{11}	0.09	0.06	0.045	0.036	0.03	0.026	0.023	0.02	0.018
	CS^O	0.0648	0.0663	0.067	0.0675	0.0677	0.0679	0.0681	0.0682	0.0683
	Π_{A1}	0.0576	0.0589	0.0595	0.0599	0.0602	0.0604	0.0605	0.0606	0.0607
	Π_{B1}	0.0576	0.0378	0.0282	0.0224	0.0186	0.0156	0.0139	0.0124	0.0111
	Π^O	0.2304	0.2315	0.2321	0.2324	0.2326	0.2327	0.2328	0.2329	0.2330
	TS^O	0.295	0.298	0.2991	0.2999	0.3	0.3007	0.3009	0.3012	0.3013
$\gamma = 0.62$	p_{A1}	0.2621	0.2713	0.2753	0.2775	0.2789	0.2798	0.2806	0.2811	0.2816
	p_{B1}	0.2621	0.2359	0.2248	0.2186	0.2146	0.2119	0.2098	0.2083	0.2781
	p_{11}	0.5242	0.5073	0.5001	0.4961	0.4935	0.4918	0.4905	0.4895	0.4887
	q_{11}	0.1189	0.0821	0.0624	0.0504	0.0422	0.0363	0.0318	0.0283	0.0255
	CS^O	0.1132	0.1213	0.1249	0.1269	0.1282	0.1291	0.1298	0.1303	0.1307
	Π_{A1}	0.0623	0.0668	0.0688	0.0699	0.0706	0.0711	0.0715	0.0717	0.0719
	Π_{B1}	0.0623	0.0387	0.0281	0.0220	0.0181	0.0154	0.0134	0.0118	0.0106
	Π^O	0.2494	0.24995	0.25	0.24998	0.24995	0.24993	0.2499	0.24989	0.24987
	TS^O	0.3626	0.3713	0.3749	0.3769	0.3782	0.3791	0.3797	0.3802	0.3806

Table 1.2: Impact of competition when firms are homogeneous.

	$\gamma = 0.2$				$\gamma = 0.62$			
	$n_2 = 2$	$n_2 = 3$	$n_2 = 4$	$n_2 = 5$	$n_2 = 2$	$n_2 = 3$	$n_2 = 4$	$n_2 = 5$
p_{A1}	2.69	2.80	2.90	2.99	2.24	2.39	2.49	2.58
p_{B1}	2.69	2.32	2.24	2.17	2.24	1.67	1.50	1.38
p_{11}	5.38	5.12	5.14	5.16	4.49	4.06	3.99	3.96
q_{11}	0.72	0.42	0.27	0.18	0.95	0.51	0.28	0.14
CS^O	4.33	4.84	5.38	5.97	7.66	9	10.21	11.47
CS_{IM}	8	8.51	9.03	9.57	8	8.51	9.03	9.57
CS_{CM}	3.55	3.78	4.01	4.25	3.55	3.79	4.01	4.25
Π_{A1}	4.07	4.42	4.72	5.02	4.57	5.17	5.62	6.02
Π_{B1}	4.07	2.09	1.48	1.13	4.57	1.94	1.26	0.88
Π^O	14.78	15.83	16.88	17.96	16.04	17.16	18.31	19.51
Π_{IM}	16	17.01	18.06	19.14	16	17	18.06	19.14
Π_{CM}	14.22	15.125	16.05	17.01	14.22	15.12	16.05	17.01
TS^0	19.11	20.67	22.27	23.94	23.70	26.16	28.52	30.98
TS_{IM}	24	25.52	27.09	28.72	24	25.52	27.09	28.71
TS_{CM}	17.78	18.91	20.07	21.27	17.78	18.91	20.07	21.27

Table 2: Impact of competition when firms are heterogeneous and competition increases quality.

	$\gamma = 0.2$				$\gamma = 0.62$			
	$n_2 = 2$	$n_2 = 3$	$n_2 = 4$	$n_2 = 5$	$n_2 = 2$	$n_2 = 3$	$n_2 = 4$	$n_2 = 5$
p_{A1}	3.17	3.12	3.06	2.99	2.64	2.66	2.62	-
p_{B1}	3.17	3.14	3.16	3.19	2.64	2.46	2.43	-
p_{11}	6.34	6.26	6.22	6.18	5.28	5.12	5.05	-
q_{11}	1.1	0.8	0.65	0.56	1.65	1.27	1.07	-
CS^O	6.03	6	5.97	5.96	10.63	11.12	11.3	-
CS_{IM}	11.3	10.7	10.12	9.57	11.28	10.7	10.12	-
CS_{CM}	5	4.75	4.5	4.25	5	4.75	4.5	-
Π_{A1}	5.65	5.5	5.27	5.02	6.32	6.41	6.25	-
Π_{B1}	5.65	3.84	2.96	2.44	6.32	4.22	3.28	-
Π^O	20.83	19.88	18.91	17.97	22.6	21.5	20.5	-
Π_{IM}	22.56	21.39	20.25	19.14	22.6	21.4	20.25	-
Π_{CM}	20.05	19.01	18	17	20	19	18	-
TS^0	26.86	27.13	24.88	23.94	33.22	32.66	31.8	-
TS_{IM}	33.86	32.08	30.37	28.71	33.84	32.08	30.04	-
TS_{CM}	25.07	23.77	22.5	21.27	27.07	23.77	22.5	-

Table 3: Impact of competition when firms are heterogeneous and competition decreases quality.

Alma Mater Studiorum - Università di Bologna
DEPARTMENT OF ECONOMICS

Strada Maggiore 45
40125 Bologna - Italy
Tel. +39 051 2092604
Fax +39 051 2092664
http://www.dse.unibo.it