Competition, Reputation and Cheating*

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November 13, 2009

Abstract

Under repeated market interaction, reputation and competition may drive out of the market those firms that do not comply with their quality promises. One may thus presume that competitive pressure improves average market quality. This paper shows that the opposite may be true in an endogenous entry, repeated interaction, linear demand oligopoly model, in which introductory prices may be used as quality signals. Cheating firms may enter the market, fool even rational consumers, and exit the market when discovered, implying a failure of the basic reputation mechanism and an increasing time path of prices. Markets for closer substitutes tend to have a lower initial average quality and less trusting consumers, whereas the number of competitors has no clear relationship with average quality.

JEL-Classification: L13, L14, L15

Key-words: Oligopoly, Quality, Price Signals, Consumers' Trust

*I would like to thank Heski Bar-Isaac, Antonio Cabrales, Matteo Cervellati, Sjaak Hurkens, Doh-Shin Jeon, José Luis Moraga-González, Joel Shapiro, Yaron Yehezkel and seminar participants at Pompeu Fabra University and University of Padova for useful discussions. I also benefited from comments from conference participants at EDP Jamboree in Louvain, EEA-ESEM in Budapest, EARIE in Valencia and ASSET in Padova. A previous version of this article appeared as a chapter in my Ph.D. dissertation at UPF.

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1 Introduction

A buyer purchasing an experience good needs to trust the seller. Confidence is needed that the effective quality, which is discovered only after purchase, corresponds to the expected (and paid for) one. Lack of trust may cause loss of efficient trade opportunities. In general terms, one may say that compliance with promises by the seller depends on punishment threats and on efficiency payments (on stick and carrot). Such incentives may be provided by an external authority, say, the state with its judicial system. Yet, in some cases compliance is hard to verify and, even when it is verifiable, state enforcement is generally costly. Alternatively, market interaction might autonomously shape the adequate incentives. One obvious way is through repeated interaction and reputation mechanisms, as suggested by Klein and Leffler (1981) and investigated by the subsequent literature discussed below. When the net return to the investment in reputation is high such mechanisms work well and guarantee compliance with promises. Yet when it is low they may fail. When this happens, rational buyers become less trusting and some sellers cheat on quality.

This paper investigates the dynamic interaction between market structure and sellers' trustworthiness when, as it is often observed in markets for experience goods, reputation fails to guarantee promise compliance and no other enforcement mechanisms are available. To this end, it displays a dynamic oligopoly model, in which promise compliance is conceived as supply of high quality goods. More specifically, it considers a game with four stages: entry, quality selection (of an experience good) and twice repeated market interaction.[1]

The number of sellers entering the market may be fundamental to determine incentives for high quality provision, because it affects the net returns to the investment in reputation. Yet the direction of this effect is not obvious, as competition may both lower monopoly rents, and thus weaken the carrot, and offer buyers more alternatives, and thus strengthen the stick. Moreover, competition itself depends on entry and exit, which depend on expected profits and therefore on net returns to reputation.[2]

[1] In order to focus on market structure and quality choice, I consider price signals of quality alone and disregard the role of warranties, advertising, brand name investment, quality disclosure policies and any kind of external enforcement.

[2] The literature on reputation, discussed below, either takes the number of firms as given, or does not explicitly model market interaction.

Upon entry, sellers make a long lasting quality choice, which affects their variable costs. This corresponds to what happens in many service markets, in which quality depends on hiring skilled employees, such as financial promoters, auditors, lawyers, doctors, professors, journalists, chefs, mechanics, and so on, depending on the considered industry.[3]

In the introductory phase of market interaction, since prices are observed before purchase and quality is not, prices may be used as signals of quality.[4] To focus on the case in which price signals do not allow high quality providers to separate from low quality ones, I assume that low quality, if recognized as such, would not be bought at any profitable price. This makes separation through high prices impossible, because for low quality sellers it is always profitable to mimic such prices.[5] To eliminate the possibility of separation through low prices, I assume that profits from repeated purchase are insufficient to compensate the initial losses implied by a credible investment in reputation.

Given the impossibility of separation, some sellers in equilibrium choose low quality and cheat rational consumers. When cheating sellers are discovered, they are forced out of the market, whereas high quality ones compete for repeat purchase at the mature stage of the market. Market dynamics is therefore characterized by an entry phase followed by a shake-out, a phenomenon widely documented in the product life cycle literature, for instance by Gort and Klepper (1982) and Klepper (1996).[6] The time path of prices is rising, since rational consumers correctly anticipate the lower initial average quality, and therefore demand rises over time. Interestingly, a higher competitive pressure, due to the fact that different varieties are closer substitutes, reduces initial average market quality and consumers' trust, because it reduces relatively more high quality than low quality firms' profits, due to

[3]The fact that quality often lasts longer than it takes consumers to experience and find it out has been neglected by the literature on reputation, which has rather focused on the short term moral hazard problem of quality choice every period.

[4]Although I let quality expectations depend on prices alone, without any direct signaling role for the number of entrants, such number has an indirect role, because it determines equilibrium prices.

[5]This, in turn, makes consumers skeptical when they observe different market prices, unless such prices are so low that they cannot be profitably imitated by low quality firms.

[6]This phenomenon is usually attributed to supply rather than demand factors, namely sellers' experimentation with new technologies and varieties, only some of which will turn out to be successful. By contrast, the explanation proposed here relies on information diffusion among buyers and on strategic cheating by sellers.

the fact that high quality firms compete repeatedly. By contrast, no clear relationship between the number of entrants and average quality and trust emerges, because entry by a firm of a given quality favors the relative profitability of the other quality.

The remainder of the paper is organized as follows. Section 2 discusses related literature. Section 3 presents the model. Section 4 analyzes its equilibrium and Section 5 concludes. Technical lemmas are presented in Appendix.

2 Related literature

The basic mechanism proposed by Klein and Leffler (1981) is as follows. If buyers pay a price premium for high quality, if they are informed on past compliance and never buy from a seller who cheated on quality in the past, the present value of the stream of future profits granted by compliance may be higher than the one period deviation gain that can be obtained by cheating consumers, so that the seller is indeed induced to be trustworthy. Shapiro (1983) formally investigates this mechanism and shows that low and rising prices guarantee high quality in a competitive market, because premiums for high quality ensure that no firm has an incentive to cut on quality and cheat the market, but competition for such premiums induces firms to set initially low, loss-making prices, which correspond to an investment in reputation, to which later profits are the normal market return. Yet, this result relies on consumers' expectations on new products' quality, which are on average wrong: it is the fact that consumers (irrationally) mistrust new products' quality that forces firms to signal high quality through low prices. Subsequent developments have overcome this limitation.

An important strand of the literature has clarified how a monopolist might signal private information on quality through either low and rising prices or through high and declining prices, depending on the role played by taste uncertainty and information diffusion.[7] The analysis has been extended in two

[7] Milgrom and Roberts (1986) show that a monopolist may signal the exogenous quality of a newly introduced experience good to rational consumers through low and rising prices, possibly used together with dissipative advertising. In their model, prices are initially low, because consumers' initial uncertainty about their own preferences lowers initial demand, and in turn initial demand is positively related to the subsequent demand from repeated purchase. With no uncertainty about preferences, Bagwell and Riordan (1991) argue that high and declining prices may guarantee high quality, because in principle a high quality monopolist might separate itself from a low quality one through either low or high initial

main (partially overlapping) directions. One strand has focused on different means to communicate private information on (exogenously determined) quality, such as warranties, brand name investments, advertising and disclosure policies, under the maintained assumption of monopoly.[8] The other one has discussed how results change under duopoly and oligopoly.[9] In all this literature, market structure and quality are assumed as exogenously given. Overgaard (1994) and Bester (1998) show that relaxing such assumptions yields interesting and non standard results.[10]

As for competitive models, Allen (1984) extends Klein and Leffler (1981) and reconciles quality guaranteeing prices with a competitive market, by

prices, but only high introductory prices are intuitive, in the sense of Cho and Kreps (1987), and should therefore be expected; moreover, prices decline over time as information about quality spreads among consumers, reducing the signaling distortion.

[8]See Lutz (1989) on the role of warranties in presence of consumers' moral hazard. Linnemer (2002) extends Bagwell and Riordan (1991) by allowing the monopolist to use both price and advertising signals, and shows that only (high) prices are used for new products, both signals are used at intermediate levels of information diffusion and no signaling is needed for mature products. Daughety and Reinganum (2008a) discuss disclosure and signalling, and Levin et al. (2009) disclosure and competition.

[9]Hertzendorf and Overgaard (2001) consider a vertically differentiated duopoly, in which production costs are constant across qualities (and normalized to zero), and show that the high quality producer uses (high) price signals alone when quality differentials are high, and both price and advertising when quality differentials are low. Fluet and Garella (2002) consider the four possible combinations of the two firms' quality, assume that high quality is more costly to produce than low quality, and characterize the parameter space for which separating equilibria exist and the kind of signal mix that is used. Yehezkel (2008) considers both price and advertising signals of quality in a vertically differentiated duopoly. Daughety and Reinganum (2007) discuss the case in which each duopolist's quality is its own private information, whereas Daughety and Reinganum (2008b) consider the case of an n-firm oligopoly.

[10]Overgaard (1994) shows that potential entry by an uninformed high quality firm may inefficiently distort an incumbent monopolist's price upwards, because it strengthens a low quality incumbent's incentive to convince both consumers and the potential entrant that its quality is high, in order to deter entry, and therefore forces a high quality incumbent to distort its price further up to effectively separate. Bester (1998) shows that uncertainty on (endogenously chosen) quality may drive duopolists to locate near one another on a line, because quality guaranteeing prices relax competition. My work is complementary, since I endogenize the number of firms on the market, rather than the degree of horizontal differentiation. Huck et al. (2006) find in an experiment that competition fosters trust. The present paper emphasizes the importance of distinguishing between competitive pressure, due to higher product substitutability, and competition in the sense of the number of competitors.

observing that, if firms produce in the increasing returns to scale range, prices may be at the same time equal to average cost and higher than marginal cost, so that no initial investment in reputation is needed to have zero profits. Cooper and Ross (1984) consider a static competitive framework, with beliefs similar to those of the present paper: besides finding an analogous result as Allen (1984) under U-shaped average cost (with the fraction of uninformed agents serving to equalize cheating firm's average cost with the unique market price), they also show that a competitive equilibrium does not exist under constant returns to scale, thus further motivating my focus on oligopoly.

More recently, Hörner (2002) presents a dynamic version of Klein and Leffler (1981), in which firms enter the market, choose quality every period and use prices to signal quality. Each firm's (quality guaranteeing) price rises over time (as its reputation increases), until bad luck drives it out of the market. Consumers' knowledge of a firm's customer base implies that it cannot raise its price to mimic higher reputation firms. While this is a plausible assumption in some contexts, it is often the case that buyers ignore at the same time sellers' quality and their customer base, with the implication that upwards price mimicry is feasible. Besides for this aspect, the present work also differs from Hörner (2002) because it explicitly considers strategic interaction, rather than featuring a constant continuous mass of firms on the market.

A limited number of recent papers deal with dynamic oligopolistic interaction with quality choice. Kranton (2003) displays a homogenous good model in which competition for (uncertain) market shares may prevent prices from assuring high quality. Bar-Isaac (2005) provides an example in which, with quantity being chosen but not used as a quality signal, a high quality equilibrium exists for either low or high degrees of substitutability, but not at intermediate levels. Dana and Fong (2008) show that, with firms rather than consumers punishing deviators, reputation for high quality is easier to maintain under oligopoly than under monopoly or competitive market. All three of these papers maintain the number of firms as exogenously given. This limitation is removed in Toth (2008), who presents a dynamic oligopoly model with stochastic entry and with investment in quality every period, and shows that market concentration may alleviate moral hazard. Yet his work is focused on firms' survival contest and does not present an explicit model of market interaction (in particular, prices are not used as signals of quality).

3 Model

3.1 Structure

I consider a game with the following four-stage structure. At stage one $N \in \mathbb{N}$ firms simultaneously decide whether to enter the market or not. Each entering firm pays a fixed entry cost $\zeta > 0$, which is sunk after entry, and chooses a different variety of an experience good. Varieties are imperfect substitutes. The number of firms who enter the market is denoted by n. At stage two the n firms on the market simultaneously choose whether to produce high or low quality. The result of these choices is a vector $\mathbf{z} \in \{0,1\}^n$, with $z_j = 1$ meaning that firm j has chosen high quality. Denote $h = \sum_{j=1}^n z_j$ the number of high quality firms. Once decided, the quality level remains the same in the two following market stages. To simplify and concentrate only on asymmetric information on consumers' side, I assume that, once chosen, a firm's quality becomes known to all firms on the market, but not to consumers. Consumers may learn a firm's quality either through direct experience with its products or by information extraction from equilibrium price signals. At stage three firms and consumers interact on the market for the first time. They move sequentially: first, firms simultaneously choose prices, determining a price vector $\mathbf{p^1} \in \mathbb{R}^n_+$. Next, having observed $\mathbf{p^1}$, consumers (indeed, a representative consumer) decide how much to demand to each firm, determining the demand vector $\mathbf{q^1} \in \mathbb{R}^n_+$. Stage four is analogous to stage three, but consumers now have additional information: if they have consumed a positive quantity of a firm's product, they are fully informed about its quality. Again, first firms simultaneously choose prices and determine the new price vector $\mathbf{p^2} \in \mathbb{R}^n_+$ and then consumers choose the new demand vector $\mathbf{q^2} \in \mathbb{R}^n_+$.

3.2 Preferences and technology

Preferences are assumed in such a way as to generate linear demands. This is done by extending a model first presented by Shubik and Levitan (1980) and more recently used by Motta (2004). I assume the following expected utility function:

$$U(\mathbf{q}, \mathbf{e}) = \sum_{j=1}^n \alpha(e_j) q_j - \frac{n}{2(1+\mu)} \left[\sum_{j=1}^n q_j^2 + \frac{\mu}{n} \left(\sum_{j=1}^n q_j \right)^2 \right] + y, \qquad (1)$$

where $\mathbf{e} \in [0,1]^n$ is a vector of beliefs, i.e., its elements are the probability attributed by the representative consumer to the fact that each good is of high quality, conditional on information about previous play of the game (which I omit to write for notational simplicity): $e_j = \Pr\{z_j = 1\}$; $\alpha(e_j)$ reflects the utility value attributed to good j's expected quality, defined as $\alpha(e_j) = \beta + e_j\gamma$, where $\beta \geq 0$ and $\gamma \geq 0$ are parameters: β captures the value attributed to a unit of a low quality good and γ the additional value of high over low quality; $\mu \in [0, \infty)$ is a parameter capturing the degree of substitutability between different varieties; y is a perfectly competitive outside good, introduced only to make partial equilibrium analysis justified. This representation extends Shubik and Levitan (1980) and Motta (2004) by allowing for imperfect observability and product-specific quality, two features which are absent in the baseline model (which may be seen as corresponding to the special case in which $\forall j,\ \alpha(e_j) = \alpha > 0$, a known parameter).

One feature of this model is that (at interior consumers' choices) market size does not depend upon either the degree of substitutability or the number of products, but only upon average expected quality and average price. To see this, notice that maximization with respect to $\mathbf{q}$ of (1) under an exogenous income constraint (and given prices and beliefs) implies the system of n FOC's $\frac{\partial U}{\partial q_j} = \alpha(e_j) - \frac{1}{1+\mu}\left(nq_j + \mu \sum_{i=1}^{n} q_i\right) = p_j$, by inverting which one obtains each product's demand

$$q_j(\mathbf{p}, \mathbf{e}, n) = \frac{1}{n}\left\{\left[\frac{n + \mu(n-1)}{n}\right][\alpha(e_j) - p_j] - \frac{\mu}{n}\sum_{i \neq j}[\alpha(e_k) - p_k]\right\}, \quad (2)$$

or, in matrix notation, $\mathbf{q}(\mathbf{p}, \mathbf{e}, n) = E(n) \cdot [\alpha(\mathbf{e}) - \mathbf{p}]$, where $E(n)$ is an $n \times n$ matrix with elements $E_{ii}(n) = \frac{n+\mu(n-1)}{n^2}$ and $E_{ik}(n) = -\frac{\mu}{n^2}$, $\alpha(\mathbf{e})$ is the vector of $\alpha(e_j)$'s and $\mathbf{p}$ is the price vector[11]. The size of the market is then $Q(\mathbf{p}, \mathbf{e}, n) \equiv \sum_{j=1}^{n} q_j(\mathbf{p}, \mathbf{e}, n) = \frac{1}{n}\sum_{j=1}^{n}[\alpha(e_j) - p_j] = \bar{\alpha} - \bar{p}$. In the special case in which all products are expected to be of the same quality α and have the same price p, individual demands are simply $q_j = \frac{\alpha - p}{n}$.

Although I later let consumers' beliefs depend upon the previous history of play, it is useful to see how firms with constant returns to scale react to this demand under exogenous quality expectations. In this case, the general

[11] The procedure to invert the system of FOC's is analogous to that described in Motta (2004, pp. 578-579), and is not reported here. Clearly, equation (2) characterizes demand only when all the FOC's hold with equality.

expression of Nash equilibrium prices can be calculated[12]. If all firms have the same marginal cost c and all products are expected to be of the same quality $\alpha(e) > c$, this expression simplifies to

$$p^E(n, e, c) = \frac{n\alpha(e) + [n + \mu(n-1)]c}{2n + \mu(n-1)}, \tag{3}$$

which is increasing in e and c, decreasing in n and μ, converges to c as $\mu \to \infty$ and further simplifies to the usual monopoly price $\frac{\alpha(e)+c}{2}$ if $n = 1$.

To later consider deviations from equilibrium, notice that if firm j manages to convince consumers that it is the only one offering high quality, i.e., if $e_j = 1$ and $\forall i \neq j,\ e_i = 0$, then $\forall n > 0$ and $\forall \mathbf{p}$ such that $p_j < \alpha(1)$ and $p_i \geq \alpha(0)\ \forall i \neq j$, it holds that $q_j(\mathbf{p}, \mathbf{e}, n) = \frac{1+\mu}{n+\mu}[\alpha(1) - p_j]$ and $\forall i \neq j,\ q_i(\mathbf{p}, \mathbf{e}, n) = 0$[13].

All goods are produced with a constant returns to scale technology, with higher quality being more expensive to produce. Marginal costs of low and high quality are $c_L \geq 0$ and $c_H > c_L$, respectively. Firms can always exit the market if it is in their interest to do so.

3.3 Equilibrium concept and parameter restrictions

I look for a pure strategy weak perfect Bayesian equilibrium (WPBE) of the entire game and restrict attention to equilibria that are symmetric, in the sense that all firms choosing the same quality also set the same price. Since several equilibria are possible, depending on how consumers form quality expectations based on observed prices, and on how firms use prices to signal (or hide) their quality, I restrict attention to a simple class of belief functions (specified below), characterized by the fact that consumers distrust price

[12]Let $\mathbf{c}$ be the vector of marginal costs. Nash equilibrium prices are $p_j^E(n, \mathbf{e}, \mathbf{c}) = \frac{n+\mu(n-1)}{[2n+\mu(2n-1)][2n+\mu(n-1)]}\left\{\left[\frac{n^2(1+\mu)}{n+\mu(n-1)} + n + \mu(n-1)\right]\alpha(e_j) + n(2+\mu)c_j + \mu\sum_{i\neq j}[c_i - \alpha(e_i)]\right\}$.

[13]The reason why firm j's demand depends on n is that, although j is the only one selling a positive quantity, it is not the only one initially on the market. Consumers are 'tempted' by the other goods, although they do not buy them: the presence of other firms posting prices and offering their products reduces the marginal utility derived from j's good, so that j is able to sell at p_j a lower quantity than it would, at the same price, if it were alone on the market (i.e., if $n = 1$). Technically, only j's FOC holds with equality, whereas all the other ones hold with strict inequality. Notice that, given $n > 1$ and $\mathbf{p}$, j's demand increases in μ, since a higher degree of substitutability reduces consumers' temptation from different goods.

signals whenever they are easy to imitate, and, if imitation occurs, to intuitive prices, which are the most profitable ones for high quality firms, given that low quality firms set the same price. Three assumptions will be used in the analysis and are introduced and discussed here. Let $\bar{\gamma} \equiv \frac{N+2\left(1+\sqrt{N+1}\right)}{N}(c_H - c_L)$.[14]

Assumption 1. $\alpha(0) = c_L$

Assumption 2. $\alpha(1) > c_H$

Assumption 3. $\gamma < \bar{\gamma}$

Under perfect information, Assumption 1, which equalizes the intrinsic utility of low quality goods and their production cost, makes demand for low quality goods insufficient even for the profitable entry of a single low quality monopolist, since its demand would be positive only at prices strictly below marginal cost. This implies that, under imperfect information, firms can profitably produce goods only as long as they manage to convince consumers of their high quality (or count to recoup initial losses in the future). It also implies that separation (of high from low quality firms) through upward distorted prices is impossible, because, if any price above c_L were a credible signal of high quality, it would be imitated by low quality firms, thus losing its credibility.

Assumption 2 is needed to ensure that high quality firms receive positive demand in equilibrium. Given Assumption 1, it is equivalent to $\gamma > c_H - c_L$, so that the utility difference between high and low quality is higher than their cost difference.[15]

Assumption 3 rules out the possibility that high quality firms separate from low quality ones by pricing at c_L, because in that case initial losses would not be compensated by future profits.

Assumption 1 implies that low quality firms, if recognized as such, leave the market.[16] By Assumption 2, high quality firms, if recognized as such, stay

[14]The origin of $\bar{\gamma}$ is made clear in the proof of Proposition 3 below. Notice that $\bar{\gamma}$ is unboundedly increasing in $(c_H - c_L)$ and decreasing in N, with $\lim_{N\to\infty} \bar{\gamma} = (c_H - c_L)$, but with $\bar{\gamma} > (c_H - c_L)$ for any finite N.

[15]This implies that an expected increase in firm j's competitors' quality makes them more competitive and therefore forces j to react by lowering its own price.

[16]I assume that firms leave the market if they expect zero profits, thus resolving the indifference case. An arbitrary small cost of staying on the market would fully rationalize it.

on the market and price according to (3). This is reflected in the full information equilibrium at stage 4, captured by Proposition 1. By the same logic, it is impossible that at stage 3 both high and low quality firms stay on the market and set two different prices (Proposition 2). This justifies our focus on beliefs that support equilibria with pooling prices. If high quality firms are able to separate themselves from low quality ones through low prices, then the market dries up for low quality firms, and these are forced out of the market, implying that nobody at stage 2 would choose low quality.[17] Yet, under Assumption 3, high quality firms are unable to set a quality-assuring introductory price (Proposition 3), so that the price they set is imitated by low quality firms (Proposition 4).[18] Then whether firms choose low or high quality at stage 2 depends on the balance between higher initial profits from cheating and repeated purchase profits from compliance (Proposition 5). Finally, entry costs determine the number of entrants, thus closing the model and allowing to make comparative statics exercises (Proposition 6).

In what follows I formalize and deepen the implications of these assumptions. I order firms on the market by assigning lower indices to high quality ones. I start solving the model by backward induction, establishing sequential rationality of strategies and deferring to the end the consistency requirement between beliefs and strategies along the equilibrium path of play.

4 Analysis

4.1 Stage 4: second market interaction

When consumers choose demand in the last move before the game ends, they are fully informed about the quality of goods on the market, because they have already experienced them. In other words, beliefs are $e_j^2(\mathbf{p^1}, \mathbf{q^1}, \mathbf{p^2}) = z_j$ if $q_j^1 > 0$[19]. I assume $e_j^2 = 0$ if $q_j^1 = 0$, to rule out the possibility that a firm finds it optimal to produce only at stage 4. Thus consumers never buy either goods indexed $j > h$, because they know these goods are of low quality,

[17] In a companion paper (Vanin, 2009) I show that if, in violation of Assumption 3, γ is sufficiently high, then the reputation mechanism (through low and rising prices) indeed grants high quality.

[18] Given this, I focus on the price that is most profitable for high quality firms (and that in this sense is intuitive).

[19] The superscript 2 is due to the fact that beliefs are relevant only in the two stages of market interaction and stage 4 is the second one.

or goods they have not experienced in the first stage of market interaction, because they expect them to be of low quality. Anticipating this behavior, all low quality firms exit the market, together with all firms that did not previously sell a positive quantity. High quality firms stay on the market and, taking as given other firms' prices, set their own profit maximizing price. This leads to the following proposition.

Proposition 1. ***(unique stage 4 equilibrium)***
Let Assumptions 1 and 2 hold. For any $n \geq 0$, $h \in \{0, \dots, n\}, \mathbf{p^1}$ and $\mathbf{q^1}$, there exists a unique pure strategy Nash equilibrium in the corresponding stage 4 subgame, at which all firms remaining on the market receive strictly positive demand: all firms that at stage 3 did not sell anything (if any) exit the market; among firms that at stage 3 sold positive quantities, low quality ones (if any) exit the market and high quality ones (if any) set prices, receive demand and make profits as follows: $\forall j \in \{1, ..., h\} : \ q_j^1 > 0, \quad p_j^2 = p^E(h, 1, c_H) = p^2(h)$, $q_j^2 = q_j(\mathbf{p^2}, \mathbf{e^2}, h) = q^2(h)$ and $\pi_j^2 = \pi^2(h)$, where

$$p^2(h) = \frac{h\alpha(1) + [h + \mu(h-1)]c_H}{2h + \mu(h-1)}, \tag{4}$$

$$q^2(h) = \frac{h + \mu(h-1)}{h[2h + \mu(h-1)]}[\alpha(1) - c_H], \tag{5}$$

$$\pi^2(h) = \frac{h + \mu(h-1)}{[2h + \mu(h-1)]^2}[\alpha(1) - c_H]^2. \tag{6}$$

Proof. Suppose first all firms produce a positive quantity at stage 3, so their quality is known at stage 4. By Assumption 1, once its quality is known, any low quality firm receives zero demand if it stays on the market, at any price (weakly) higher than its marginal cost, independently of what other firms do. Thus, whatever $h \in \{0, \dots, n\}$, low quality firms indeed choose to exit and high quality firm's best response (if $h \geq 1$) is indeed to stay on the market and set their profit maximizing price. See Motta (2004), formulae (8.65) and (8.66) on p. 569, for the precise expression of high quality firms' choices.

Now, by the assumption that $e_j^2 = 0$ if $q_j^1 = 0$, any firm, which does not produce at stage 3, is forced out of the market at stage 4 as well, since it is regarded as a low quality firm.[20] □

[20] To account for subgames in which some high quality firms do not produce at stage 3,

Observe that full information prices, quantities and profits in (4), (5) and (6) are all decreasing in the number of high quality firms.

4.2 Stage 3: first market interaction

At stage 3 (first market interaction) firms set prices $\mathbf{p^1}$, consumers observe them, formulate beliefs on each firm's quality and then choose demand. While at stage 4 any collection of previous histories of play identifies a proper subgame, this is not the case at stage 3, because, for any n, any price vector $\mathbf{p^1} \in \mathbb{R}^n_+$ identifies one information set for the representative consumer, independently of $\mathbf{z} \in \{0,1\}^n$. To analyze the game, we need to specify each firm's introductory price after any possible $n > 0$ and $\mathbf{z} \in \{0,1\}^n$, since this identifies any possible information set at which firms may be called to set prices.[21]

I first show that no pure strategy equilibria exist, in which both low and high quality firms are present and choose different introductory prices. Hence, I subsequently focus on equilibria with pooling introductory prices. I specify beliefs which are best suited to support such equilibria, introduce an intuitive introductory price function and show its sequential rationality given these beliefs under Assumption 3.

Proposition 2. ***(no equilibria with separation)***
Let Assumptions 1 and 2 hold. There exist no pure strategy weak perfect

in expressions (4), (5) and (6) h should be replaced by the cardinality of the set $\{j \in \mathbb{N} : j \leq h,\ q^1_j > 0\}$. I write just h to reduce the notational burden, given that such subgames are out of equilibrium.

[21]Recall that firms' quality is not randomly determined by Nature, but is rather chosen by each firm at stage 2. If such quality choice is in pure strategies, and if consumers can identify each firm and correctly anticipate its equilibrium strategy, then, even after observing a pooling price at stage 3 (at least as long as this price is along the equilibrium path of play), consumers would be able to precisely anticipate each firm's quality. Yet, given firms' initial symmetry to consumers' eyes, it is more interesting and reasonable to assume that, upon observing a pooling price (at least on the equilibrium path of play), consumers may extract information about average quality, but cannot precisely identify which firm has chosen which quality. To make this idea formally consistent with equilibrium, one may either let firms choose their quality in mixed strategies, or allow Nature to initially randomly choose, for any possible n, a permutation of the n firms (drawn from a uniform distribution), and assume that consumers cannot observe this move by Nature. This second route is chosen here and it means that any firms setting the same price are initially indistinguishable for consumers.

Bayesian equilibria, in which, along the equilibrium path of play, at stage 3 both high and low quality firms are present on the market and set two different prices (one for each quality level).

Proof. Suppose such an equilibrium exists. By observing two prices on the market, consumers infer each firm's quality. Then the result follows from the proof of Proposition 1. □

Beliefs

It is convenient to specify consumers' beliefs in terms of a prior e^0, which is updated upon observation of the introductory price vector, because such prior may be interpreted as the degree of consumers' initial trust in firms' product quality, which is assumed to be common knowledge.

When n firms post their price at stage 3, and consumers have a prior $e^0 \in [0,1]$, the space of possible posterior beliefs is the set of all functions $\mathbf{e^1} : \mathbb{R}^n_+ \times [0,1] \to [0,1]^n : (\mathbf{p^1}, e^0) \mapsto \mathbf{e^1}(\mathbf{p^1}, e^0)$, mapping prices and prior to quality expectations. In order to study equilibria with pooling introductory prices, I restrict attention to a specific simple class of beliefs, which is especially likely to support such equilibria: $\forall n > 0$, $\forall j \in \{1, ..., n\}$, $\forall \mathbf{p^1} \in \mathbb{R}^n_+$, $\forall e^0 \in [0,1]$,

$$e^1_j(\mathbf{p^1}, e^0) = \begin{cases} e^0 & , \text{ if } \exists p^1 \in (c_L, \alpha(1)) : \ \forall i, \ p^1_i = p^1 \\ 1 & , \text{ if } p_j \le c_L \\ 0 & , \text{ otherwise} \end{cases} \tag{7}$$

This means that, upon observing a pooling introductory price (in the range of profitable prices for low quality firms), consumers receive no information from price signals and do not revise their prior.[22].

If a firm's price is (weakly) lower than c_L, then consumers conclude it must be a high quality firm. Indeed, at such introductory price no firm makes positive initial profits, but only a high quality firm may expect to recoup initial losses at stage 4, whereas a low quality firm, expecting to exit at stage 4, would never stay on the market unless at stage 3 it can price above its marginal cost.

[22]Differently from what happens in Bayesian games, the prior is not implied by an exogenous type distribution. If in equilibrium n^* firms enter the market, h^* of them choose high quality, and all of them set the same introductory price $p^1 \in (c_L, \alpha(1))$, then beliefs (7) imply $e^1_j(\mathbf{p^1}, e^0) = e^0 \ \forall j$, so Bayes' rule implies $e^0 = \frac{h^*}{n^*}$. If in equilibrium p^1 does not belong to $(c_L, \alpha(1))$, then Bayes' rule imposes no restrictions on the prior, since e^0 does not affect posterior beliefs along the equilibrium path of play.

Further, beliefs (7) entail a high degree of distrust in price signals (not to be confused with trust in product quality, captured by the prior), in the sense that, if different prices are observed on the market, consumers interpret any price, at which low quality firms could make profits, as a trial to cheat them, and hence expect low quality. Indeed, if low quality firms anticipated that consumers are going to interpret a certain price deviation as a signal of high quality, they would deviate themselves to such price, as long as it is above their marginal cost. That is why no deviation above c_L is a credible signal.

In summary, I am considering an environment in which consumers do not trust price signals, because they are too easy to imitate, unless they convey the information that a firm is indeed willing to incur losses to build a good reputation, losses that low quality firms could never recoup.

Introductory prices, quantities and profits

I consider each firm's strategy as specifying an introductory price function of the form $p^1(n, h, e^0)$.[23] When all potential entrants adopt the same $p^1(n, h, e^0)$, I call it a pooling introductory price function.[24]

Consider a pooling introductory price function $p^1(n, h, e^0)$. Given $e^0 \in [0, 1]$, at any stage 3 information set such that $n > 0$ and $h \in \{0, \dots, n\}$, denoting $p^1 = p^1(n, h, e^0)$ and $e^1 = e^1(p^1, e^0)$, if $p^1 < \alpha(e^1)$, then each firm j on the market expects to receive demand according to (2) as $q_j(\mathbf{p^1}, \mathbf{e^1}, n) = q^1(p^1, e^1, n)$ (where $\mathbf{p^1}$ and $\mathbf{e^1}$ denote the $n \times 1$ vectors whose elements are all p^1 and e^1, respectively), so that initial profits for low and high quality firms are $\pi^1_L(p^1, e^1, n)$ and $\pi^1_H(p^1, e^1, n)$, respectively, where

$$q^1(p^1, e^1, n) = \frac{\alpha(e^1) - p^1}{n}, \tag{8}$$

$$\pi^1_L(p^1, e^1, n) = (p^1 - c_L) \cdot q^1(p^1, e^1, n), \tag{9}$$

$$\pi^1_H(p^1, e^1, n) = (p^1 - c_H) \cdot q^1(p^1, e^1, n), \tag{10}$$

[23] Since e^0 is common knowledge, introductory prices may depend on it. To study equilibria with pooling introductory prices, it is enough to summarize $\mathbf{z}$ through h. This will imply some slight abuse of language when talking about firms' stage 3 information sets. Such imprecisions are immaterial to the analysis and simplify the exposition.

[24] Observe that in this case consumers' quality expectations at stage 3 are the same for all firms and, using (7), can be written as $e^1(p^1, e^0)$, where $p^1 = p^1(n, h, e^0)$.

Overall profits

From equations (6), (7), (9), (10), letting again $p^1 = p^1(n, h, e^0)$ and $e^1 = e^1(p^1, e^0)$, define low and high quality firms' overall expected profits at a pooling price $p^1 < \alpha(e^1)$ as $\pi_L(n, h, p^1, e^1) \equiv \pi_L^1(p^1, e^1, n)$ and $\pi_H(n, h, p^1, e^1) \equiv \pi_H^1(p^1, e^1, n) + \pi^2(h)$, respectively. At any price $p^1 \geq \alpha(e^1)$, any firm receives zero demand and expects zero overall profits: in this case, let $\pi_L(n, h, p^1, e^1) = 0$ and $\pi_H(n, h, p^1, e^1) = 0$. If $n > 1$, denote a firm's overall deviation profits, if it sets $p \neq p^1$ when all other firms set p^1, as $\pi'_L(n, h, p, p^1, e')$ and $\pi'_H(n, h, p, p^1, e')$, for a low and a high quality firm respectively, where e' is still derived from (7), but taking into account that the deviating firm prices at p and all the other ones price at p^1. In particular, if $n \geq h > 0$, $e^0 > 0$ and $p^1 > c_L$, then by deviating to $p = c_L$, a high quality firm earns overall deviation profits $\pi'_H(n, h, c_L, p^1, 1) = -(c_H - c_L)\left(\frac{1+\mu}{n+\mu}\right)\gamma + \pi^2(1)$.

Sequential rationality

To be part of a WPBE, a pooling introductory price function must be sequentially rational.[25] In particular, it must specify a sequentially rational price for a monopolist. Lemma 1 in Appendix shows that an implication of Assumptions 1 and 2 is that a high quality monopolist has no sequentially rational introductory prices if $\alpha(e^0) \in (c_L, c_H]$.[26] Noting that $\alpha(e^0) \in (c_L, c_H]$ is equivalent to $e^0 \in \left(0, \frac{c_H - c_L}{\gamma}\right]$, restricts the initial levels of trust, for which we may find sequentially rational pooling introductory price functions, to $e^0 \in \{0\} \cup \left(\frac{c_H - c_L}{\gamma}, 1\right]$.

Proposition 3 shows the key implication of Assumption 3, together with beliefs (7), namely that a necessary condition for sequential rationality of a

[25]Suppose everybody expects that at stage 4 the unique pure strategy Nash equilibrium is played. A pooling introductory price function $p^1(n, h, e^0)$ is sequentially rational given beliefs (7), with $e^0 \in [0, 1]$, if at any stage 3 information set at which firms choose (hence, $\forall n > 0$, $\forall h \in \{0, \ldots, n\}$), it specifies a price p^1, which is sequentially rational (given such beliefs and at the corresponding information set) for each firm on the market, provided all other firms set the same price (i.e., $\forall p \neq p^1$ it holds that, if $h < n$, then $\pi_L(n, h, p^1, e^1) \geq \pi'_L(n, h, p, p^1, e')$, and if $h > 0$, then $\pi_H(n, h, p^1, e^1) \geq \pi'_H(n, h, p, p^1, e')$, where e' is derived from (7), taking into account that the deviating firm prices at p and all the other ones price at p^1).

[26]In this case, it must incur initial losses if it wants to sell a positive quantity. But then it has no profit maximizing initial price, since, from any price at which demand is strictly positive, a slight price increase would reduce initial losses.

pooling introductory price function is that it does not involve investment in reputation. Indeed, when high quality, relative to low quality, is not much more important to consumers than more costly to firms, repeated purchase is not enough remunerative to justify the initial losses incurred to invest in reputation.[27]

Proposition 3. ***(no investment in reputation)***
Let Assumptions 1, 2 and 3 hold. If a pooling introductory price function $p^1(n, h, e^0)$ *is sequentially rational given beliefs (7), then* $\forall(n, h) : N \geq n \geq h \geq 1,\ \forall e^0 \in \{0\} \cup \left(\frac{c_H - c_L}{\gamma}, 1\right],\quad p^1(n, h, e^0) > c_L$.

Proof. Given $\gamma < \bar{\gamma}$, high quality firms would deviate from a pooling introductory price equal to c_L, since at that price they make negative profits. Indeed, $\forall(n, h) : N \geq n \geq h \geq 1, \quad \pi_H(n, h, c_L, 1) \leq \pi_H(N, 1, c_L, 1)$; and $\pi_H(N, 1, c_L, 1) < 0 \iff \gamma < \bar{\gamma}$. □

Recall that along the equilibrium path of play posterior beliefs at stage 3 must correctly reflect average market quality, and that if they specify $e^1 = 0$, then markets are closed. According to (7) $e^1 = e^0$ if equilibrium introductory prices pool on some $p^1 \in (c_L, \alpha(1))$. In light of Proposition 3, equilibrium pooling prices must be above c_L. If they were (weakly) above $\alpha(1)$, then markets would be closed. It follows that no equilibria exist with active markets, pooling introductory prices and $e^0 = 0$. Together with the above mentioned result, that sequentially rational pooling introductory price functions require $e^0 \in \{0\} \cup \left(\frac{c_H - c_L}{\gamma}, 1\right]$, this implies the following remark.

Remark 1. ***(high trust)***
Let Assumptions 1, 2 and 3 hold. A necessary condition for existence of a WPBE with beliefs (7), active markets and pooling introductory prices is $e^0 > \frac{c_H - c_L}{\gamma}$*, that is,* $\alpha(e^0) > c_H$.

Given $e^0 > \frac{c_H - c_L}{\gamma}$ and Assumptions 1, 2 and 3, necessary and sufficient conditions for a pooling price function to be sequentially rational are stated in Lemma 2 in Appendix. The main message can be summarized as follows.

[27] Formally, Assumption 3 adds to Assumptions 1 and 2 the following implications: first, $\forall n \in \{1, \ldots, N\}$, $\forall h \in \{1, \ldots, n\}$, it holds that $\pi_H(n, h, c_L, 1) < 0$; second, if $p^1 > c_L$, then $\pi'_H(n, h, c_L, p^1, 1) < 0$. In a different setting, a short time horizon or a high discount rate would have the analogous effect of reducing the repeated purchase incentive to invest in reputation.

If consumers were to believe any price signal that can be profitably imitated by low quality firms, these latter firms would obviously send the same signal. Thus, price signals are not credible, unless they cannot be profitably imitated. This would be the case of prices (weakly) below c_L, but under low γ such prices are not profitable. When consumers distrust price signals, it becomes profitable for firms to coordinate on a single price, which yields positive profits to all firms (or initial losses for high quality firms, later recouped through future gains).[28] Provided firms coordinate, there may be several profitable pooling prices.[29] Yet pooling prices are uninformative signals, so that consumers' demand depends on their initial trust e^0. If they distrust product quality so much that they are never induced to try it, the natural consequence is that no firm finds it profitable to start selling, so all markets remain closed. If, by contrast, $e^0 > 0$, then I focus on the most intuitive pooling pricing function, which has firms setting the most profitable price for high quality firms, given that low quality firms, whenever this is rational, set the same price $p^E(n, e^0, c_H)$.[30]

Definition 1. ***(intuitive pooling introductory price function)***
Let the pooling introductory price function $p^1_E(n, h, e^0)$ be defined as follows. If $e^0 = 0$, then all markets remain closed. $\forall e^0 \in (0, 1]$, firms set $p^1_E(1, 0, e^0) = p^E(1, e^0, c_L)$ and $p^1_E(n, h, e^0) = p^E(n, e^0, c_H)$ in any other case.

Proposition 4. ***(seq. rationality of intuitive introductory prices)***
Let Assumptions 1, 2 and 3 hold. The pooling introductory price function $p^1_E(n, h, e^0)$ is sequentially rational if and only if $e^0 \in \{0\} \cup \left(\frac{c_H - c_L}{\gamma}, 1\right]$.

Proof. If $e^0 = 0$, expected demand and profits are zero at any price, so no profitable deviation is possible. By Lemma 1 in Appendix, $p^1_E(1, 1, e^0)$ is not

[28] Obviously, no coordination issues arise for monopolists, whose unique sequentially rational price, given $e^0 > \frac{c_H - c_L}{\gamma}$, is determined by equation (3).

[29] Equilibrium refinements developed for signaling games cannot be straightforwardly applied to the present game and the application of more general refinements turns out to be prohibitively complex. Yet, it can be shown that a reasonable application of forward induction arguments, in the spirit of the intuitive criterion introduced by Cho and Kreps (1987), does not eliminate this multiplicity.

[30] See Vanin (2007) for an analysis of sequential rationality of the simplest class of pooling price functions: those in which firms set the same price both on and off the equilibrium path of play, so that the price schedule is flat in n and in h. That analysis is omitted here, because the gain in simplicity is not compensated by the loss in plausibility.

sequentially rational if $e^0 \in \left(0, \frac{c_H - c_L}{\gamma}\right]$, so the same is true for the entire function $p_E^1(n, h, e^0)$. Given $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$, it is easy to check that, under Assumptions 1, 2 and 3, all the conditions of Lemma 2 in Appendix hold for $p_E^1(n, h, e^0)$. □

So the intuitive introductory price schedule, whereby low quality oligopolists act as if they had high quality goods, is sequentially rational, given beliefs (7), when trust is either zero (in which case markets are closed) or high (in which case they are open).

4.3 Stage 2: quality choice

At stage 2 the n firms on the market simultaneously choose whether to specialize their technology to produce high or low quality goods. This choice determines whether their marginal cost in the two subsequent periods will be c_H or c_L, respectively. Indeed, firms' strategies must specify such quality choice at any possible stage 2 information set[31]. This yields a vector of quality choice profiles $\mathbf{z}(n, e^0)$. To investigate its sequential rationality, given the intuitive pooling introductory price function $p_E^1(n, h, e^0)$, let $\pi_i^E(n, h, e^0) \equiv \pi_i\left(n, h, p_E^1(n, h, e^0), e^1(p_E^1(n, h, e^0), e^0)\right)$, for $i \in \{L, H\}$.[32]

For $e^0 = 0$ firms are indifferent between choosing either quality and exiting the market. Thus in particular market exit if $e^0 = 0$ is sequentially rational at stage 2.

[31] Order the N potential entrants so that if n of them enter the market, it is the first ones who do. Any potential entrant i's strategy should specify a quality choice for any $n \geq i$ (i.e., whenever it enters the market) and for any possible permutation of n. To simplify, I assume symmetry in position-contingent quality choice sequences, so that for any potential entrant i, its strategy specifies, $\forall n \geq i$, $\forall j \in \{1, \ldots, n\}$, $z_j^i(n)$, i.e., its quality choice whenever it enters the market, together with $(n-1)$ other firms, and is assigned position j in a permutation. Symmetry also implies that $\forall n > 0$, $\forall i, j, k \in \{1, \ldots, n\}$, $z_j^i(n) = z_j^k(n) = z_j(n)$. Thus, $\forall n > 0$, consumers face the same vector $\mathbf{z}(n)$, independently of the particular permutation of n.

[32] Suppose everybody expects that that at stage 3 all firms pool on the intuitive introductory price function $p_E^1(n, h, e^0)$, and that at stage 4 the unique pure strategy Nash equilibrium is played. A sequence of quality choice profiles $\mathbf{z}(n, e^0)$ is sequentially rational if $\forall n > 0$, the number of high quality firms $h(n, e^0) = \sum_{j=1}^{n} z_j(n, e^0)$ satisfies the following properties: if $h(n) > 0$, then $\pi_H^E\left(n, h(n, e^0), e^0\right) \geq \pi_L^E\left(n, h(n, e^0) - 1, e^0\right)$; and if $h(n, e^0) < n$, then $\pi_L^E\left(n, h(n, e^0), e^0\right) \geq \pi_H^E\left(n, h(n, e^0) + 1, e^0\right)$.

Lemma 3 in Appendix investigates sequential rationality of quality choice profiles when $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$, under Assumptions 1, 2 and 3. It first shows that a monopolist chooses low quality, because the additional demand granted by re-purchase of high quality goods is not sufficient to compensate for their additional cost. Next, for $n > 1$, it implies that sequentially rational quality choices depend on the initial level of consumers' trust and on the number of firms on the market in the following way.

Proposition 5. ***(trust, competition and market quality)***
Let Assumptions 1, 2 and 3 hold. Assume that prices are set according to the pooling introductory price function $p_E^1(n, h, e^0)$ at stage 3, and that the unique Nash equilibrium is played at stage 4. $\forall n \in \{2, \ldots, N\}$, $\exists e_B(n), e_T(n)$: $\frac{c_H - c_L}{\gamma} < e_B(n) < e_T(n)$, such that sequentially rational quality choices depend on consumers' trust in the following way.

1. *Low trust implies full high quality, i.e., if $e^0 \in \left(\frac{c_H - c_L}{\gamma}, e_B(n)\right)$, then $h(n, e^0) = n$ characterizes sequentially rational quality choice profiles.*

2. *High trust and low competition may imply full low quality, i.e., if $e^0 \in (e_T(n), 1]$ (which is an empty interval for high n), then $h(n, e^0) = 0$ characterizes sequentially rational quality choice profiles.*

3. *If either trust is at intermediate levels or both trust and competition are sufficiently high, i.e., if $e^0 \in [e_B(n), e_T(n)]$ (for high n, $e_T(n) > 1$), then sequentially rational quality choice profiles are characterized by initial coexistence on the market of both high and low quality firms. Average market quality falls into an interval of length $1/n$, whose boundaries decrease in consumers' trust.*

Proof. It follows from Lemma 3 in Appendix and the fact that $e_B(n)$ and $e_T(n)$ there defined are respectively decreasing and increasing. □

Low trust implies low initial demand, so that high quality is always more profitable than low quality, independently of the number of competitors, since it yields profits from repeated purchase. High trust makes the initial profit advantage of low quality firms higher, the more so, the lower the number of competitors, so that in particular a combination of high trust and low competition may induce all firms to cheat the market. For intermediate trust levels, given n, average quality decreases in trust; yet, given e^0, average market quality needs not be a monotonic function of n. The reason is that entry of additional low quality firms makes high quality relatively more profitable, and entry of additional high quality firms tends to make low quality relatively more profitable. Thus, although a proportional increase in n and h raises the relative profitability of low quality, so that, if we take a sufficiently long sequence of n, a decreasing trend in average quality, as determined by sequentially rational choices, should be detectable, this may not be the case when N is low, since then discrete jumps in n and h may determine non monotonic jumps in average quality, with no observable trend.

4.4 Stage 1: entry and consistency

At stage 1 firms decide whether to enter the market or not. Given Assumptions 1, 2 and 3, beliefs (7) and e^0, under the above considered sequentially rational continuation of the game, the following holds. If $e^0 = 0$, then overall expected profits are zero and the entry cost is not worth paying, so that in equilibrium $n = 0$ and all markets are closed.[33] If $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$, then overall expected profits, for both high and low quality firms, are decreasing in n, so that the sequentially rational number of entrants is uniquely determined as the highest integer such that each firm's overall expected profits are higher than the fixed entry cost ζ. This yields n as an increasing function of $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$.

At a WPBE beliefs must be consistent with strategies along the equilibrium path of play. Whenever equilibrium introductory pooling prices imply that beliefs are determined by initial trust, consistency imposes equality in equilibrium between trust and average expected quality.

A technical problem is due to the fact that nothing grants the joint existence of a sequentially rational pure strategy quality choice profile and of an initial level of consumers' trust, which, under beliefs (7), makes quality expectations consistent with equilibrium strategies. Yet, this is only due to an integer problem, which is easily solved by allowing just one firm to choose quality in mixed strategies along the equilibrium path of play. Lemma 4 makes this precise and thus grants equilibrium existence. The analytical expressions of the equilibrium number of entrants and of expected average

[33]With closed markets, e^0 matters only out of equilibrium, so no restriction on it is imposed.

quality can be obtained, but this requires solving a third degree polynomial equation, whose solution is essentially unreadable. Yet, comparative statics on n is easy to draw, based on our previous results.

Proposition 6. ***(equilibrium and comparative statics)***
Let Assumptions 1, 2 and 3 hold and suppose that ζ is sufficiently low as to allow equilibrium entry by a number of firms higher than $\frac{\gamma}{c_H - c_L}$. Suppose further that beliefs are given by (7) and that firms play the unique Nash equilibrium at stage 4, pool on the intuitive introductory price function at stage 3, make sequentially rational quality choices at stage 2, with the possibility that one firm randomizes along the equilibrium path of play, and keep entering until expected profits exceed entry costs. Then there exists only one level of initial consumers' trust, which makes beliefs consistent with strategies along the equilibrium path of play. So under these assumptions a WPBE exists, is unique and has the following properties.

1. *The equilibrium number of entrants is increasing in N, decreasing in ζ, μ and c_H, and trend increasing in γ.*

2. *The equilibrium values of average expected quality and consumers' trust are trend increasing in c_H, trend decreasing in μ, unrelated to N and their relationship with ζ and γ has no stable sign. As the market matures, there is a shake-out, after which prices rise.*

Proof. Existence and uniqueness of a WPBE follow from Lemma 4 in Appendix.

1. The number of potential entrants matters only when entry costs are so low that all N firms enter the market. For higher costs, N does not affect the equilibrium number of entrants. An increase in entry costs obviously reduces the number of entrants. An increase in the elasticity of substitution makes competition tougher both at stage 3 and 4. This reduces firms' profits and therefore the number of entrants. A rise in the production cost of high quality obviously discourages high quality production, and thus decreases consumers' confidence and reduces the size of the market, allowing entry by a lower number of firms[34]. A rise in γ expands consumers' demand and allows more firms to enter

[34] I do not make comparative statics exercises on c_L, since, under Assumption 1, their interpretation would be unclear

the market. Yet, as shown below, a rise in γ may be associated to a reduction in trust, which reduces consumers' demand and thus works in the opposite direction. Numerical analysis shows that the former effect tends to prevail, with the number of entrants increasing stepwise in γ, but that this trend is compatible with occasional stepwise falls of n after an increase in γ.

2. A rise in c_H has two effects (besides that of reducing the number of entrants). First, it tends to make low quality relatively more profitable, since this becomes comparatively cheaper to produce. Second, it raises firms' prices. Since low quality firms' introductory prices are already distorted upwards with respect to their full information optimum, and since moreover they set a higher markup than high quality firms initially do, the profit loss may be higher for low quality than for high quality firms, so that equilibrium average expected quality may go up after a rise in c_H. Numerical analysis indicates that indeed this tends to be the case. The elasticity of substitution, the number of potential entrants, entry costs and consumers' preference for high quality, all have the same effects on equilibrium average expected quality, and for the same reasons (in the case of γ, this is due to the fact that in Lemma 3 in Appendix $\frac{\partial g}{\partial \gamma} < 0 \iff e^0 > \frac{2(c_H - c_L)}{\gamma + 2(c_H - c_L)}$, i.e., given n, and provided that equilibrium trust is sufficiently high, a rise in γ makes high quality relatively more profitable, thus fostering an increase in average quality and in trust; but this, in turn, raises the size of the market and makes more firms enter, with counterbalancing effects, since a proportional increase in n and h makes low quality relatively more profitable). Finally, whenever average expected quality is initially lower than 1 (as is the case in all the numerical exercises performed), either with certainty or with positive probability there is a shake-out, and since prices are decreasing in the number of firms on the market, their time path is increasing.

□

While all parameters have the expected impact on the equilibrium number of entrants, more interesting is to discuss their effect on equilibrium average expected quality, trust and prices.

Bagwell and Riordan (1991) show that a monopolist uses high and declining prices to signal high quality in markets with information diffusion. The intuition is that price increases are less costly for high quality firms, since their mark-up is lower, and that in equilibrium low quality firms prefer to be recognized as such, rather than setting high quality firms' (upward distorted) prices. This intuition does not work in the present context, because, by Assumption 1, low quality firms prefer any price above their marginal cost to being recognized as such. They therefore have a strong incentive to pool on the same price set by high quality firms, provided it covers their marginal cost. Thus high prices are not credible signals of high quality. In turn, the possibility to guarantee high quality through low prices is ruled out by Assumption 3, which states that the utility difference is not much higher than the cost difference between high and low quality. These two assumptions depict an environment in which some firms rationally choose low quality, mimic the pricing strategy of high quality firms, are able to initially cheat the market, but, upon discovery, are forced out of the market, and prices rise over time both because the number of competitors is lower and because average quality, consumers' confidence and hence demand are higher.[35]

Average quality and quality expectations tend to be lower in markets in which products are closer substitutes, because this erodes the price premium that motivates firms to produce high quality goods. More specifically, at equilibrium levels of n, h and e^0, a rise in μ lowers high quality firms' profits both at stage 3 and 4, whereas it raises low quality firms' profits at stage 3 (because it reduces their upward price distortion and allows them to sell more). It thus makes low quality relatively more profitable, and this effect tends to remain predominant even after considering the equilibrium decrease in n and in e^0.[36] More broadly, a higher competitive pressure due to higher product substitutability imposes a trade-off to consumers between lower prices and lower average quality.

The number of potential entrants, entry costs and consumers' preference for high quality (within the range allowed by the fact that we are considering the case in which γ is low, relative to $c_H - c_L$), have no clear effect on average quality and on quality expectations. While this is intuitive for N

[35] In Vanin (2007) I show that, if firms pool on a sufficiently high flat introductory price function (in which firms set the same introductory price on and off the equilibrium path of play), then high prices may still guarantee high quality, since the initial profits accruing to both kind of firms are virtually driven to zero, and only high quality allows repeated purchase. Yet, although this is the simplest pooling pricing rule, it is not a plausible one.

[36] To have a concrete example of this, think for instance of many souvenir markets for tourists.

and ζ, since these parameters directly affect the number of entrants, but not the relative profitability of high vs. low quality, it is surprising for γ. Indeed, the first intuition would be that a higher consumers' preference for high quality should raise average market quality. This effect is at work, but there are also counterbalancing effects: given n and e^0, a higher number of high quality firms increases the relative profitability of low quality, since it decreases high quality firms' stage 4 profits; moreover, if consumers expect a higher average quality, the size of the market increases and this may raise the number of entrants; additional entrants may then be tempted to choose low quality, precisely because it has become relatively more profitable; yet, if this happens and is anticipated, average quality and trust may decrease. Thus, it is not at all straightforward that a higher consumers' preference for high quality translates into higher average market quality.

Finally, it is interesting to notice that a rise in c_H, besides reducing the number of entrants, also has the independent and additional effect of raising prices. Higher prices favor high over low quality at stage 3 and low over high quality at stage 4, with the former effect that tends to prevail on the latter.

In summary, when price signals of quality do not work, and provided no other credible signals are available, the reputation mechanism fails to grant high quality production by all firms, and a number of interesting phenomena emerge, like rational consumers who are fooled by some firms, a demand-driven shake-out, an increasing time path of market prices and a negative impact of the competitive pressure due to product substitutability on average market quality and consumers' trust.

5 Conclusion

I have displayed a linear demand oligopoly model, in which firms endogenously decide whether to enter the market and whether to specialize on high or low quality products, and then repeatedly interact to sell experience goods to consumers, who are able to precisely discover a firm's product quality only after the first purchase, but who are sufficiently rational to form correct expectations about average market quality. Although introductory prices may be used as signals of quality, consumers do not trust them if such signals are too easy to imitate. This creates a strong incentive for firms to pool on the same introductory price, independently of their quality. This coordination incentive, in turn, creates the scope for multiple pooling price equilibria.

Results are presented for the most intuitive pricing rule, in which low quality firms mimic high quality firms' introductory price and, given this, high quality firms' introductory price is the most profitable they can set. This pricing rule is labeled intuitive because not only it survives an adapted version of the Intuitive Criterion, but it also considers that, since high quality firms would like to be recognized and low quality firms would not like it, the former have a sort of advantage, so that it is intuitive to expect that they set the introductory price that is best for them, given that they are going to be imitated. The assumption is maintained that the utility difference between high and low quality is not much higher than the cost difference, thus ruling out the possibility to guarantee high quality through low prices. As a result, the market initially hosts both high and low quality firms, which pool on the same introductory price, which is therefore uninformative.

A number of interesting results emerge: first, rational consumers know that they are going to be fooled by some firms at the early stage of market interaction, and therefore do not trust firms' initial quality claims very much; second, cheating firms exit the market when discovered and this tends to raise both average quality and prices at the mature stage of market interaction; third, higher product substitutability raises competitive pressure, but tends to lower, besides the number of entrants, both prices and average market quality.

These main results have been derived under the assumption that low quality products cannot be profitably sold under perfect information. This allows to focus on equilibria with pooling introductory prices, which are the only ones in which actual cheating occurs and consumers are not able to distinguish a firms' product quality based on its price. Relaxing this assumption makes the analysis of equilibria with pooling introductory prices significantly and unnecessarily more complicated, although it would be necessary to investigate markets in which different qualities are sold at different prices, with no cheating. Preliminary investigation indicates that this extension promises to yield new interesting results, but that it also poses new subtleties and therefore requires a separate work.

An analogous argument applies to the assumption that the utility difference between high and low quality is not much higher than the cost difference, so that low introductory prices, as an investment in reputation, are not profitable, independently of the degree of market competition. In a companion paper (Vanin, 2009) I show that the symmetric case, in which high quality is so much more valuable to consumers than more costly to firms, naturally yields full high quality provision, which is signalled through low introductory prices. While this is intuitive, working out the details of the four stage game also requires a separate work. The analysis of the intermediate case, in which investing in reputation may be profitable when competition is low but not when it is high, would make derivation and presentation of results unnecessarily cumbersome, without adding much to intuition. It is to be expected that, if entry costs are high and the equilibrium number of entrants is low, then all firms entering the market would choose high quality; in turn, if entry costs are low and the equilibrium number of entrants is high, results would resemble those obtained here.

Appendix

Sequentially rational pooling introductory price functions

Lemma 1. ***(either high or zero trust)***
Let Assumptions 1 and 2 hold. A high quality monopolist has no sequentially rational introductory price, given beliefs (7), if $e^0 \in \left(0, \frac{c_H - c_L}{\gamma}\right]$.

Proof. Under Assumption 1, $\alpha(e^0) \in (c_L, c_H] \iff e^0 \in \left(0, \frac{c_H - c_L}{\gamma}\right]$. In this case, if a high quality monopolist sets $p^1 \geq \alpha(e^0)$, it receives zero demand and makes zero overall profits. This is not sequentially rational, because there exists an introductory price $p \in (c_L, \alpha(e^0))$, sufficiently close to $\alpha(e^0)$, which grants strictly positive overall profits (thanks to Assumption 2). In turn, $\forall p^1 < \alpha(e^0)$, $\exists\, p \in (p^1, \alpha(e^0))$, which grants strictly lower initial losses and the same future profits, and hence strictly higher overall profits. □

Lemma 2. ***(seq. rational pooling introductory price functions)***
Let Assumptions 1, 2 and 3 hold. A pooling introductory price function $p^1(n, h, e^0)$ *is sequentially rational given beliefs (7), with* $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$*, if and only if it satisfies the following conditions.*[37]

1. $\forall n > 0, \forall h \in \{0, \ldots, n\},\ p^1(n, h, e^0) \geq c_L$.

[37] In the cases not explicitly considered no additional constraints are imposed. See Vanin (2007) for a generalization of this lemma to Assumptions 1 and 2 alone, with $e^0 \in \{0\} \cup \left(\frac{c_H - c_L}{\gamma}, 1\right]$.

2. $p^1(1, 0, e^0) = p^E(1, e^0, c_L)$.

3. $p^1(1, 1, e^0) = p^E(1, e^0, c_H)$.

4. *If* $n > 1$ *and* $h \in \{1, \ldots, n\}$, *then*

   (a) $p^1(n, h, e^0) > c_L$;

   (b) *if* $p^1(n, 1, e^0) \in (c_L, \alpha(e^0))$, *then* $\pi_H(n, h, p^1(n, 1, e^0), e^0) \geq 0$.

Proof. 1. If, for some $n > 0$ and $h \in \{0, \ldots, n\}$, $p^1(n, h, e^0) < c_L$, then at the corresponding information set any firm would strictly gain by deviating to $p = c_L$.

2. Given Assumption 1, $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$ implies $\alpha(e^0) > c_H$ and therefore $\alpha(e^0) > c_L$. In this case, under beliefs (7) a low quality monopolist faces exogenous quality expectations e^0, whatever price it may choose in the interval $(c_L, \alpha(e^0))$. Only if it chooses its optimal monopoly price in this interval, no profitable deviations are possible.

3. For a high quality monopolist, an analogous argument applies.

4. When several firms initially enter the market and a pooling introductory price $p^1(n, h, e^0) \geq c_L$ is expected, low quality ones have no profitable deviations. Any high quality firm ($h > 0$) may guarantee itself zero overall expected profits through a deviation to $p > c_L$; if it deviates to $p \leq c_L$, it monopolizes the market at both stages 3 and 4, but it makes initial losses (so that the best such deviation is to $p = c_L$).

   (a) Condition $p^1(n, h, e^0) > c_L$ follows from Proposition 3.

   (b) A pooling price $p^1(n, h, e^0) \in (c_L, \alpha(e^0))$ is sequentially rational if and only if two inequalities hold: $\pi_H(n, h, p^1(n, h, e^0), e^0) \geq 0$ and $\pi_H(n, h, p^1(n, h, e^0), e^0) \geq \pi'_H(n, h, c_L, p^1(n, h, e^0), 1)$. Assumption 3 implies that $\pi'_H(n, h, c_L, p^1(n, h, e^0), 1) < 0$, so the only relevant condition is $\pi_H(n, h, p^1(n, h, e^0), e^0) \geq 0$.[38]

[38] Observe that any pooling price $p^1(n, h, e^0) \geq \alpha(e^0)$ is sequentially rational. At such price, all firms expect zero demand and zero overall profits. By deviating to $p = c_L$, a high quality firm earns overall deviation profits $\pi'_H(n, h, c_L, p^1(n, h, e^0), 1) = -(c_H - c_L)\left(\frac{1+\mu}{n+\mu}\right)\gamma + \pi^2(1)$, which are negative under Assumption 3, because $\pi'_H(n, h, c_L, p^1(n, h, e^0), 1) \leq \pi_H(N, 1, c_L, 1) < 0$.

□

Sequentially rational quality choices

Let $f(x,n) \equiv \frac{[2n+\mu(n-1)][(1+\mu)x-\frac{\mu}{n}]}{[n+\mu(n-1)][(2+\mu)x-\frac{\mu}{n}]^2}$ and $g(e^0) \equiv \frac{(c_H-c_L)[\alpha(e^0)-c_H]}{[\alpha(1)-c_H]^2}$. The functions f and g have the following properties: $f(\frac{1}{n},n) = \frac{n[2n+\mu(n-1)]}{4[n+\mu(n-1)]}$; $f(1,n) = \frac{n}{2n+\mu(n-1)}$; given $n \geq 1$, $f(x,n)$ is decreasing in $x \in \left[\frac{1}{n},1\right]$ and $f\left(x+\frac{1}{n},n\right)$ defines a new function of x, which is just a leftwards shift of $f(x,n)$ by $1/n$; given $x \in \left[\frac{1}{n},1\right]$, $f(x,n)$ is decreasing in n; given $x \in [0,1]$, $f\left(x+\frac{1}{n},n\right)$ defines a new function of n, which tends to be increasing (in the sense that its trend is increasing); given $h \in \{1,\ldots,n\}$, $f(\frac{h}{n},n)$ defines a new function of n, which is increasing; $g(e^0)$ is increasing; $\forall n \in \{1,\ldots,N\}$ it holds that $f\left(\frac{1}{n},n\right) > g\left(\frac{c_H-c_L}{\gamma}\right)$ and that, if $N \geq 3$, then $f(1,n) < g(1)$.
Let $e_B(n) \equiv \frac{c_H-c_L}{\gamma} + \frac{n[\alpha(1)-c_H]^2}{[2n+\mu(n-1)](c_H-c_L)\gamma}$ and $e_T(n) \equiv \frac{c_H-c_L}{\gamma} + \frac{n[2n+\mu(n-1)][\alpha(1)-c_H]^2}{4[n+\mu(n-1)](c_H-c_L)\gamma}$.
Let $x_1(n,e^0)$ and $x_2(n,e^0)$ be the solutions of $g(e^0) = f\left(x+\frac{1}{n},n\right)$ and $g(e^0) = f(x,n)$, by $x>0$, given $e^0 \in [e_B(n), e_T(n)]$, respectively.
Let $e_M(n)$ be the solution by $x>0$ of $g(x) = f\left(x+\frac{1}{n},n\right)$; and $e_N(n)$ be the solution by $x > \frac{1}{n}$ of $g(x) = f(x,n)$.

Lemma 3. ***(sequentially rational quality choice)***
Let Assumptions 1, 2 and 3 hold. Assume that $e^0 \in \left(\frac{c_H-c_L}{\gamma},1\right]$, that prices are set according to the pooling introductory price function $p_E^1(n,h,e^0)$ at stage 3, and that the unique Nash equilibrium is played at stage 4. Assume as well $N \geq 8$. Then $\forall e^0 \in \left(\frac{c_H-c_L}{\gamma},1\right]$, the unique sequentially rational quality choice for a monopolist is low quality. $\forall n \in \{2,\ldots,N\}$,

1. *if $e^0 \in \left(\frac{c_H-c_L}{\gamma}, e_B(n)\right)$, then the unique sequentially rational quality choice profile has $h(n,e^0) = n$;*

2. *if $e^0 \in [e_B(n), e_T(n)]$, then a quality choice profile is sequentially rational if and only if $\frac{h(n,e^0)}{n} \in [x_1(n,e^0), x_2(n,e^0)]$; in particular, $x_1(n,e^0)$ and $x_2(n,e^0)$ are both decreasing in e^0; $x_1(n,e^0)$ and $e_M(n)$ tend to be increasing in n; $x_2(n,e^0)$ and $e_N(n)$ are decreasing in n; the two intervals $[x_1(n,e^0), x_2(n,e^0)]$ and $[e_M(n), e_N(n)]$ are both contained in $[0,1]$, may or not overlap, satisfy $x_2(n,e^0) - x_1(n,e^0) = \frac{1}{n}$ and $e_N(n) - e_M(n) < \frac{1}{n}$, and*

(a) $e^0 > e_N(n) \Rightarrow x_2(n, e^0) < e^0$;

(b) $e^0 \in [e_M(n), e_N(n)] \Rightarrow e^0 \in [x_1(n, e^0), x_2(n, e^0)]$;

(c) $e^0 < e_M(n) \Rightarrow x_1(n, e^0) > e^0$.

3. *if* $e^0 \in (e_T(n), 1]$, *then the unique sequentially rational quality choice profile has* $h(n, e^0) = 0$.

Proof. Under Assumptions 1, 2 and 3, and $N \geq 8$, a monopolist pricing according to $p_E^1(n, h, e^0)$ strictly prefers low to high quality $\forall e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$[39]. Given $n \in \{2, \ldots, N\}$ and $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$, sequential rationality of the quality choice profile $\mathbf{z}(n, e^0)$ may be conveniently rewritten in terms of the following condition:

$$f\left(\frac{h}{n}, n\right) \geq g(e^0) \geq f\left(\frac{h+1}{n}, n\right), \tag{11}$$

where only the first inequality matters if $h = n$ and only the second one if $h = 0$. $\forall n \in \{2, \ldots, N\}$, $e_B(n)$ and $e_T(n)$ are the solutions by $e^0 > 0$ of $f(1, n) = g(e^0)$ and $f\left(\frac{1}{n}, n\right) = g(e^0)$, respectively.[40]

1. If $e^0 \in \left(\frac{c_H - c_L}{\gamma}, e_B(n)\right)$, then the second inequality of (11) is always violated for $h < n$, whereas the first one always holds for $h = n$.

2. If $e^0 \in [e_B(n), e_T(n)]$, then condition (11) holds if and only if $\frac{h(n,e^0)}{n} \in [x_1(n, e^0), x_2(n, e^0)]$. The monotonicity properties of the boundaries of this interval, as well as those of $[e_M(n), e_N(n)]$, follow from those of f and g, and the same is true for whether $e^0 \in [x_1(n, e^0), x_2(n, e^0)]$ or

[39]The assumption of $N \geq 8$ plays no other role than to simplify the exposition of a monopolist's choice.

[40]Given $n > 1$ and $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$, the following holds: if $h > 0$, then $\pi_H^E(n, h, e^0) \geq \pi_L^E(n, h-1, e^0) \iff g(e^0) \leq f\left(\frac{h}{n}, n\right)$; if $h < n$, then $\pi_L^E(n, h, e^0) \geq \pi_H^E(n, h+1, e^0) \iff g(e^0) \geq f\left(\frac{h+1}{n}, n\right)$; $\pi_H^E(n, n, e^0) \geq \pi_L^E(n, n-1, e^0) \iff e^0 \leq e_B(n)$; $\pi_L^E(n, 0, e^0) \geq \pi_H^E(n, 1, e^0) \iff e^0 \geq e_T(n)$.

not[41]. Since the length of the $[x_1(n,e^0), x_2(n,e^0)]$ interval is $1/n$, an integer $h(n,e^0) \in \{0,\dots,n\}$ such that (11) holds exists[42].

3. If $e^0 \in (e_T(n), 1]$ (but observe that this interval is empty for high n), then the first inequality of (11) is always violated for $h > 0$, whereas the second one always holds for $h = 0$.

□

Trust and consistent expectations

Let $\bar{h}(n)$ be the unique integer such that $\frac{\bar{h}(n)}{n} \in \left(e_N(n) - \frac{1}{n}, e_N(n)\right]$.

Lemma 4. ***(consistency)***
Under the assumptions of Lemma 3, suppose that in equilibrium $n \in \{2,\dots,N\}$ firms enter the market and allow one firm to randomize between high and low quality. Then both sequential rationality at stages 2, 3 and 4 and consistency of beliefs (7) along the equilibrium path of play simultaneously hold if and only if $\bar{h}(n)$ firms choose high quality in pure strategies and

1. *if $\frac{\bar{h}(n)}{n} \in [e_M(n), e_N(n)]$, then $(n - \bar{h}(n))$ firms choose low quality in pure strategies and $e^0 = \frac{\bar{h}(n)}{n}$;*

2. *if $\frac{\bar{h}(n)}{n} \in \left(e_N(n) - \frac{1}{n}, e_M(n)\right)$, then $(n - \bar{h}(n) - 1)$ firms choose low quality in pure strategies,*

$$e^0 = \frac{c_H - c_L}{\gamma} + \frac{n[2n + \mu(n-1)][\bar{h}(n) + 1 + \mu\bar{h}(n)][\alpha(1) - c_H]^2}{[n + \mu(n-1)][2\bar{h}(n) + 2 + \mu\bar{h}(n)]^2(c_H - c_L)\gamma} \quad (12)$$

and the mixing firm chooses high quality with probability $w = ne^0 - \bar{h}(n)$.

Proof. Under the above assumptions, consistency of beliefs requires $e^0 = \frac{\bar{h}(n)}{n}$ if all firms play in pure strategies, with $\bar{h}(n)$ of them choosing high quality; it requires $e^0 = \frac{E(h)}{n} = \frac{\bar{h}(n)+w}{n}$ if one firm chooses high quality with

[41] Observe that $e_M(n)$ and $e_N(n)$ are decreasing in μ, increasing in $(c_H - c_L)$, and increasing in γ when they are high, but decreasing in γ when they are low

[42] If $n \cdot x_1(n,e^0) \in \{0,\dots,n-1\}$, then both $h(n,e^0) = n \cdot x_1(n,e^0)$ and $h(n,e^0) = n \cdot x_1(n,e^0) + 1$ satisfy condition (11). In any other case, the integer $h(n,e^0)$ such that (11) holds is unique.

probability $w \in (0,1)$, $\bar{h}(n)$ firms choose high quality in pure strategies and $(n-\bar{h}(n)-1)$ firms choose low quality in pure strategies. By Lemma 3, given this, sequential rationality of quality choices implies that either $\bar{e}_B(n) > 1$, or $e^0 \in [e_M(n), e_N(n)]$. In the former case, consistency and sequential rationality imply $e^0 = 1$ and $h(n,1) = n$. In the latter case, $p_E^1(n,h,e^0)$ is sequentially rational. In this case, if $\frac{\bar{h}(n)}{n} \in [e_M(n), e_N(n)]$, then we have consistency and sequential rationality in pure strategies, with $e^0 = \frac{\bar{h}(n)}{n}$. If this does not hold and one firm chooses high quality with probability $w \in (0,1)$, this firm must be indifferent between the two qualities. Given the other firms' pure strategy quality choices (i.e., given that $\bar{h}(n)$ other firms choose high quality and $(n-\bar{h}(n)-1)$ choose low quality), this condition may be expressed as $g(e^0) = f\left(\frac{\bar{h}(n)+1}{n}, n\right)$, which, solved for e^0, yields (12). Firms choosing low quality in pure strategies then expect exactly the same profits as the mixing firm. In turn, firms choosing high quality in pure strategies expect slightly higher profits, because with probability w there will be $\bar{h}(n)+1$ competitors at stage 4, but with probability $(1-w)$ there will be only $\bar{h}(n)$. Yet, no firm has a profitable deviation, since if $\bar{h}(n)+1$ firms were to choose high quality in pure strategies, then expected profits would be lower than in equilibrium. Then w is determined by the consistency equation $e^0 = \frac{\bar{h}(n)+w}{n}$, and this indeed grants that $e^0 \in [e_M(n), e_N(n)]$). Finally, Lemma 3 implies that for any other number of firms choosing high quality in pure strategies and any other specification of w, sequential rationality of quality choices and consistency of beliefs cannot hold at the same time.

□

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