Bidimensional quality competition and scope economies

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Abstract

In markets where product quality is important, more than one characteristic is usually necessary to define product quality and the amelioration of the goods characteristics is usually costly. Then, properties of production technologies, in particular if they exhibit economies or diseconomies of scope in producing quality along several dimensions, should matter for the industry configuration at equilibrium. To date, however, the literature has neglected the issue. We analyze a duopoly model where two characteristics can be used to vertically differentiate the products. Our results are that the existence of economies of scope (resp. diseconomies of scope) leads to the emergence of a quality leader in all dimensions (resp. to a different quality leader in each characteristic).

Keywords: Oligopoly equilibrium, vertical differentiation, quality, scope economies.

JEL: L10, L13, L15

1 Introduction

Oligopolistic industries are often characterized by fierce competition on quality. In theory, quality of a good relates strictly speaking to attributes which can be measured and which, if increased, increase consumers' satisfaction. Often firms claim about the superior quality of their products, however, there can be several characteristics which affect quality. In statements to the public each firm emphasizes the strong points of its product. However, most theory models only allow for one quality dimension. If one considers *quality* as represented by one parameter only, then the theory predicts that firms will differentiate in that attribute. This is perhaps the best known result of the literature on vertical differentiation[1] originated by Gabszewicz and Thisse (1979) and Shaked and Sutton (1982, 1983). If two vertical attributes exist, only one of them will be used to differentiate and the other will be equal across firms (see Neven and Thisse, 1990; Vandenbosch and Weinberg, 1995; Dos Santos Ferreira and Thisse, 1996). One should then expect that *quality leaders* emerge among firms, while followers sell goods of inferior quality. This is consistent with what we observe in many but not all markets and hence this theory description does not seem to exhaust the industry configurations which could be encountered in an empirical analysis. Surely there are cases where some firm succeeds to be a leader in all attributes of the product it makes, but often the products on the market are characterized by different quality levels in different dimensions. A brand of Hi-Fi system with excellent sound may have poor electronic facilities incorporated, as opposed to a rival brand with the opposite pattern of characteristics. We shall analyze the particular case where products have two vertical quality attributes: one which is desirable at the same degree by every consumer, and the other that corresponds to the definition of a hedonic characteristic: consumers desire more of it but with different intensity[2].

A first guess is that technology should matter for what is the equilibrium configuration. For instance one could assume that quality leadership in all di-

[1]For the case of *horizontal* differentiation, multicharacteristics models exist. Irmen and Thisse (1998) find maximum differentiation along one dimension, while minimum differentiation is observed along all other dimensions. Vendorp and Majeed (1995) and Tabuchi (1994) obtains a similar result in a bidimensional setting. For mixed models with a horizontal and a vertical dimension see Häckner (2000) and Garella and Petrakis (2008).

[2]The non hedonic variable has the same utility as money in our example. In the real world one may think of fuel efficiency in cars as an illustrative example.

mensions is easier to achieve under a complementarity assumption: when the improvement in one dimension reduces the costs of making improvements in the others, a case that we shall define of "scope economies". *Vice versa*, if the cost effect are reversed, one would presume that absolute quality leadership should not prevail. The aim of the present paper is to relate the equilibrium outcome to "economies of scope" in producing quality along several dimensions. The result is in accordance with intuition, namely that economies of scope are associated with the emergence of a unique quality leader in all dimensions. By contrast, diseconomies of scope lead to what one may term *cross leadership*, that is, each firm is quality leader in a different dimension. In our model too, however, if quality improvements were obtained at zero-cost or with separable costs, strategic interaction would lead to: quality leadership by one firm along the hedonic characteristic and to zero differentiation in the other characteristic, in accordance with the existing theory. Therefore, the analysis below shows that, as soon as there are interlinked costs of improvements in different attributes, the outcome of strategic interaction is drastically modified. This opens up for further refinements in the theory of vertical differentiation.

2 Costless Quality Improvements

In a market for an indivisible good produced by two firms, the quality of a good depends upon two attributes, x and y. Consider a population of buyers with taste parameter θ uniformly distributed over the $[0, 1]$ interval. Their utility is given by

$$u = x + y\theta - p$$

if they buy one unit of the good, and zero otherwise. Characteristic x can be called the 'non-hedonic' or 'homogeneous taste characteristic' and y the 'hedonic' or 'heterogeneous taste characteristic'.

The two firms are indexed as firm 1 and firm 2. The competition process is described by a two stage game. At stage 1 firms choose the product quality attributes x_i and y_i, for $i = 1, 2$. At stage two they choose prices, p_1 and p_2. We limit here the action space at the first stage assuming that both x and y must belong to technically feasible intervals, $[x', x'']$, and $[y', y'']$, with $x' > 0$ and $y' > 0$. Let $d_x = x_2 - x_1$ and $d_y = y_2 - y_1$. Assume, without loss of generality, $d_y > 0$. Also. assume for the time being that $d_x > 0$ as well (the relation between d_y and d_x shall become apparent in the analysis

in Section 3). The consumer indifferent between product 1 and 2 is defined by the consumer with taste parameter $\tilde{\theta}$:.

$$\tilde{\theta} = (p_2 - p_1 + d_x)/d_y.$$

The demand to firm 2, is then equal to

$$D_2(p_1, p_2) = 1 - (p_2 - p_1 + d_x)/d_y$$

if $p_2 \in [(p_1 + d_x), (d_y - d_x + p_1)]$, it is equal to zero if $p_2 > d_y - d_x + p_1$ and it is equal to 1 if $0 < p_2 < p_1 + d_x$.

The demand to firm 1, is then equal to

$$D_1(p_1, p_2) = (p_2 - p_1 + d_x)/d_y$$

if $p_1 \in [(p_2 + d_x - d_y), (p_2 + d_x)]$, it is equal to zero if $p_1 > p_2 + d_x$, and it is equal to one if $p_1 < p_2 + d_x - d_y$.

First, we analyze the case where firms incur no production costs. This will allow us to describe the pure incentives towards quality choices. Simple calculations show that the second stage equilibrium prices are

$$p_1 = \frac{d_y - d_x}{3}, \qquad p_2 = \frac{2d_y + d_x}{3}$$

and the corresponding profits are

$$\Pi_1(d_y, d_x) = \frac{(d_y - d_x)^2}{9d_y}, \qquad \Pi_2(d_y, d_x) = \frac{(2d_y + d_x)^2}{9d_y}.$$

The equilibrium quantities are $D_1 = \frac{d_y - d_x}{3d_y}$ and $D_2 = \frac{2d_y + d_x}{3d_y}$, therefore at equilibrium firm 1 survives only if $d_y - d_x > 0$ (this condition would be met more easily if d_x were negative, so our assumption that $d_x > 0$ is more restrictive than necessary to make competition by firm 1 viable). It is easy to see then that the choices at the first stage are determined uniquely by the sign of the derivatives of the equilibrium profits with respect to d_y and d_x.

In particular, since

$$\frac{\partial \Pi_1}{\partial d_x} = -\frac{2(d_y - d_x)}{9d_y}$$

this implies that

$$\frac{\partial \Pi_1}{\partial x_1} > 0.$$

This means that unambiguously, firm 1 (i.e., the firm with the lower quality) chooses $x_1 = x''$ as a best reply to any possible choice by firm 2. Similarly,

$$\frac{\partial \Pi_2}{\partial d_x} = \frac{2(2d_y + d_x)}{9d_y},$$

which is always positive. Therefore

$$\frac{\partial \Pi_2}{\partial x_2} > 0.$$

The sign of the derivatives of the profit functions with respect to the non hedonic variable, then imply that there is a quality race in this characteristic. This means that first stage choices under the pure incentives will lead to $x_1 = x_2 = x''$.

Then, consider the choices of y_1 and y_2, the hedonic characteristic. One has

$$\frac{\partial \Pi_1}{\partial d_y} = \frac{d_y^2 + d_y d_x - d_x^2}{9d_y^2} > 0,$$

and

$$\frac{\partial \Pi_2}{\partial d_y} = \frac{4d_y^2 - d_x^2}{9d_y^2} > 0.$$

Therefore firm 1 will choose $y_1 = y'$ as a best reply against any possible pair (x', y_2), and firm 2 will choose $y_2 = y''$ against any possible pair (x', y_1) by firm 1. This implies that the pure incentives point toward maximal differentiation in y. The results so far can be summarized in the following Proposition.

Proposition 1 *Pure profit incentives lead to maximal quality level (zero differentiation) by both firms in the non hedonic characteristic and to maximal differentiation in the hedonic characteristic.*

The above Proposition relates pure incentives to a unique equilibrium configuration. The question is, then, whether this configuration can be altered when the costs of ameliorating the good are considered.

3 Properties of Technology and Quality Leadership

We shall define the following possible cases. *Double quality leadership* by one firm: this happens when at equilibrium the firms' choices are such that $y_i > y_j$ and $x_i > x_j$, for $i, j = 1, 2$, and $i \neq j$. *Cross quality leadership*: this happens when at equilibrium the firms' choices are such that either the two inequalities $y_i > y_j$ and $x_i < x_j$, both hold, or $y_i < y_j$ and $x_i > x_j$ both hold, for $i, j = 1, 2$, and $i \neq j$. Finally, *single quality leadership* obtains when either $y_i \neq y_j$ and $x_i = x_j$ both hold, or $y_i = y_j$ and $x_i \neq x_j$ both hold, for $i, j = 1, 2$, and $i \neq j$.

Cost functions summarize the available technology. The cost function of producing quantity z is assumed to be given by

$$C_i(z) = cz + F(x_i, y_i),$$

where c is a constant marginal cost and $F(x, y)$ is a fixed cost which depends upon the quality choices. Since the variable cost is assumed not to depend upon the quality variables, we shall assume henceforth that $c = 0$. The function $F(x_i, y_i)$ is assumed to be continuous and with continuous first and second derivatives in both arguments, over the admissible ranges. Its first derivatives are assumed to be both positive for all values strictly larger than x' and y' respectively: $F_x(x, y) > 0$ for $x > x'$ and $F_y(x, y) > 0$ for $y > y'$; also, $F_x(x, y) = 0$ for $x = x'$ and $F_y(x, y) = 0$ for $y = y'$. Second derivatives are denoted by $F_{xx}(x, y)$, $F_{yy}(x, y)$ and $F_{xy}(x, y)$. The second cross derivative $F_{xy}(x, y)$ plays a special role in the remainder.

The cost function is said to exhibit diseconomies of scope in characteristics if $F_{xy}(x, y) > 0$. And it is said to exhibit economies of scope in characteristics if $F_{xy}(x, y) < 0$. The cost function is separable if $F_{xy}(x, y) = 0$.

The analysis of the first stage choices can make use of the results for the case of pure incentives analyzed in Section 2 above. Define the profits net of fixed costs as $\Pi_i = \pi_i - F(x_i, y_i)$

To start, consider the choice of firm 1, where for a convention and without loss of generality it is assumed that $y_2 \geq y_1$. Then clearly,

$$\frac{\partial \pi_1}{\partial y_1} = -\frac{\partial \Pi_1}{\partial d_y} - \frac{\partial F}{\partial y_1}$$

since from the results in Section 2 and from the assumptions on the derivatives of $F(x, y)$ this derivative is always negative, then one has that $y_1 = y'$,

i.e., that the inclusion of cost does not change the choice of the firm with low quality in the heterogeneous dimension. It is useful to save this result as a lemma.

Lemma 2 *For all technologies, there is always a low quality firm in the hedonic characteristic y. This firm is denoted as firm 1, and it chooses the minimum quality $y_1 = y^{'}$, irrespective of the choice made by the rival firm.*

As to firm 2, its choice of quality variable y, when it is interior[3], will derive from the first order condition

$$\frac{\partial \pi_2}{\partial y_2} = \frac{\partial \Pi_2}{\partial d_y} - \frac{\partial F}{\partial y_2} = 0.$$

Since the term $\dfrac{\partial \Pi_2}{\partial d_y}$ is positive, and $F_y(x, y) = 0$ for $y = y'$, this condition leads to $y_2 > y'$. This is true obviously also when $\dfrac{\partial \pi_2}{\partial d_y} - \dfrac{\partial F}{\partial y} > 0$ for all values of y_2, i.e., when the result is $y = y''$. This can also be summarized by a lemma.

Lemma 3 *For all technologies, there exists a high quality firm in the hedonic characteristic y. This firm is denoted as firm 2, and its choice y_2 is strictly larger than y'.*

The choice of the quality level in the non hedonic space can be analyzed in turn. Assume first that there are economies of scope. Then, the choice of the hedonic level $y_1 = y'$ against $y_2 > y'$ is reflected in the property that the first derivative for firm 1, $F_x(x, y_1)$, for all levels of x is higher than the corresponding derivative for firm 2. In other words, firm 2 has an endogenous cost advantage in the production of high quality in the x characteristic if it has a leadership in the y characteristic. Indeed, recalling that the pure incentives $\dfrac{\partial \Pi_2}{\partial d_x}$ and $\dfrac{\partial \Pi_1}{\partial d_x}$ are of opposite sign, and that $\dfrac{\partial \Pi_2}{\partial d_x} > -\dfrac{\partial \Pi_1}{\partial d_x} > 0$, then for $x_1 = x_2$

$$\frac{\partial \pi_2}{\partial x_2} = \frac{\partial \Pi_2}{\partial d_x} - \frac{\partial F(x, y_2)}{\partial x_2},$$

[3]Given the assumption that $\partial F(x_2, y_2)/\partial y_2$ is continuous over all admissible region of (x, y) values, and that it is equal to zero when $y_2 = y'$, if the solution is not interior it must be $y_2 = y''$.

will be larger than

$$\frac{\partial \pi_1}{\partial x_1} = -\frac{\partial \Pi_1}{\partial d_x} - \frac{\partial F(x, y_2)}{\partial x_1}.$$

It follows that the incentive to increase quality in the x dimension is larger for firm 2 than for firm 1. This leads to the following proposition.

Proposition 4 *If the cost function exhibits economies of scope in characteristics, then the unique equilibrium outcome entails double leadership in qualities by one of the two firms.*

Proposition 4 implies that if there is complementarity in quality improvements, through the cost function, vertical differentiation spills over from the hedonic to the non hedonic characteristic and a better quality brand is better in all attributes.

Assume, by contrast, that there are diseconomies of scope, i.e., that $F_{xy}(x, y) > 0$. Then, the nature of the argument leading to lemma 1 and lemma 2 is unaltered. This implies that the firm which has an advantage in the hedonic quality y will have an endogenous cost disadvantage in the non hedonic quality, x. Then it is immediate to verify that firm 1, i.e., the firm with the lowest value of y, will be lead to choose a higher level of x than its rival firm 2.

Therefore, the existence of diseconomies of scope is unambiguously associated to an outcome of cross leadership.

Proposition 5 *If the cost function exhibits diseconomies of scope in characteristics, the unique equilibrium outcome entails cross leadership. One of the two firms chooses a higher level of x than the rival, but a lower level of y.*

Finally,

Proposition 6 *If the cost function is separable (no scope effects) at equilibrium the products are identical in the non hedonic attribute and are maximally differentiated in the hedonic one.*

The case with separable costs is obviously one where the existence of costs in the production of quality does not interfere with pure incentives. The optimal choices then will then lead to single quality leadership. In particular, the hedonic characteristic will be differentiated, while the non hedonic will be the same across firms. The opposite, however, is not possible.

4 Conclusion

The analysis of vertical differentiation traditionally leads to the prediction that a single attribute shall be used to strategically differentiate products so as to relax price competition. This obviously implies the emergence of a single firm as the quality leader. The present paper shows that this need not be the case. If there are scope economies in increasing the good's attributes, then a quality leader emerges in all attributes. If there are diseconomies of scope, then a situation of cross leadership arises, with one firm producing a better product in terms of the hedonic attribute and the other supplying a better good in terms of the non hedonic attribute.

These results are conveyed by a specific model, as usual in the literature on vertical differentiation. Generalizations and extensions are left for future research.

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