

# Non-Linear Statistical Modelling of GaAs FET Integrated Circuits Using Principal Component Analysis

Marco Balsi, Francesco Centurelli, Alessandra Forte, Giuseppe Scotti,  
Pasquale Tommasino, Alessandro Trifiletti

Dept. of Electronic Engineering, University of Rome "La Sapienza",  
Via Eudossiana 18, 00184, Rome, Italy. Phone: +3944585679

**Abstract** — A statistical non-linear model of GaAs FET MMIC's which makes use of principal components to describe distance-dependent variations of technological parameters is presented. The proposed model is composed of a nominal non-linear model for the GaAs FET and a correlation matrix which accounts for correlation among parameters of the active devices on the MMIC. The correlation matrix is orthogonalised and reduced by using the Principal Component Analysis (PCA) technique. Capability to reproduce statistical distribution has been checked by comparing measured and modelled S-parameters in the 1-50 GHz frequency range by means of hypothesis tests of equivalence.

## I. INTRODUCTION

The availability of statistical model libraries is a key feature to design MMIC's with short-length III-V technologies within commercial CAD tools. Several empirical statistical linear models of MESFET and HEMT devices [1]-[3] have been developed in last years to evaluate circuit performance by means of yield-oriented design techniques. In recent years, a procedure to accurately reproduce mean values, standard deviations, correlations, and non-linear relationships among S-parameters in a given bias point, has been proposed [4] to develop a fully non-linear model (i.e. a model able to cover bias-dependency). The final step of the extraction procedure is the orthogonalisation and reduction of the empirical parameters multivariate distribution by means of the Principal Components Analysis (PCA), a technique used in [2] and in [3] to develop statistical empirical linear models of MESFET and HEMT devices. In [5] we proposed a non-linear statistical model of a given MMIC composed of FET devices in which statistical parameters were considered as a Gaussian multivariate random variable and correlation between on-chip process parameters was modelled as a decreasing function of the distance between the devices.

In this paper, we propose a procedure to represent the statistical non-linear model by means of the principal components. At the best of our knowledge it is the first time that PCA technique is used to describe the statistical behaviour of the active portion of an MMIC in the non-linear operating regime. A considerable reduction of the

variable numbers is achieved with the PCA technique, so leading to a less complex and more portable CAD model.

## II. THE UNCORRELATED NON-LINEAR STATISTICAL MODEL

The statistical model of the MMIC comprises a nominal non-linear model for the single device and a covariance matrix, which accounts for statistical correlation among the active devices. The nominal model is composed of a set of bias independent parameters and a set of empirical non-linear functions of the instantaneous voltages  $V_{gsi}$  and  $V_{dsi}$ . Some empirical parameters of such non-linear functions have been chosen to represent statistical variability of the technological process by means of a sensitivity analysis:  $M = 11$  parameters have been chosen as random variables with multivariate Gaussian distribution. Moreover, as the mismatch between devices on the same chip is an increasing function of their distance, the covariance matrix of the  $M$  statistical parameters has been considered as a function of the distance. A procedure has been proposed to determine both the device nominal model and the distance-dependent covariance matrix from standard measurements performed on a test-chip. In this work, the correlation matrix of the MMIC non-linear model has been expressed in terms of the principal components. The eigenvalues of the correlation matrix have been computed and ordered according to the amount of variation in the original data set that they describe.

The statistical model of the MMIC comprising  $N$  active devices is composed of a nominal non-linear model and  $N \times M$  uncorrelated statistical variables  $V_i$  expressed in terms of their mean values  $\langle V_i \rangle$ , standard deviations  $\sigma_{Vi}$ , and principal factors  $P_i$ :

$$V_i = \langle V_i \rangle + \sigma_{Vi} \cdot (P_1 F_1 + P_2 F_2 + \dots + P_{N \times M} F_{N \times M}) \quad (1)$$

where  $F_i$  are uncorrelated random variables with standardised (i.e. with zero mean value and unitary standard deviation) normal distribution. The uncorrelated random variables in Eq. 1 are suitable for Monte Carlo analysis even within CAD simulators which do not provide multivariate distributions. Moreover, a reduction of the parameters space can be achieved by considering

only the principal factors which allow the description of a given amount of variation in the original data set. Parameter space reduction is performed according to two alternative criteria: it is possible to select only the principal components related to eigenvalues greater than a given amount, or to take all the eigenvectors that explain a given percentage of the total cumulative variation. A routine has been added to the extraction procedure of the statistical model presented in [5], to perform orthogonalisation and reduction of the MMIC correlation matrix. The flow diagram of the complete extraction procedure is reported in Fig. 1.

### III. MODEL VALIDATION

The PCA-based MMIC model has been validated by means of hypothesis testing procedure, within AGILENT-ADS CAD tool. The measurements database of  $Q=10 \times 10$  devices (gate length 0.2  $\mu\text{m}$ , gate width 4x30  $\mu\text{m}$ ) created in [5] from the measurement performed on a single device, has been used together with the procedure in Fig. 1 to extract the PCA-based statistical non-linear model of a MMIC composed of  $N=3$  active devices, and so containing  $N \times M=33$  correlated statistical parameters. The hypothesis to find a percentage error between the 8 measured and modelled S-parameters (the real and imaginary parts of each S-parameter are considered) lower than a given amount  $d$  has been checked for the 8 mean values, the 8 standard deviations, the 28 auto-correlation and the 36 cross-correlation coefficients with a cumulative significance level  $\alpha$ . The complete model and several reduced models  $x \cdot \sigma^2$  ( $x = 0.7, 0.8, 0.9, 0.95$ ), which account for the  $x$  % of the total cumulative variation, have been extracted and compared. We have found that 19 principal factors are required to describe a cumulative variation of 95 %, 14 factors for a 90 % variation, 10 and 7 factors for 80 % and 70 % variations, respectively. A population of modelled devices has been built for the complete and the reduced models: for each of the models a Monte Carlo analysis has been performed at the bias point  $BP_1$  ( $V_{ds}=3.0V$ ,  $V_{gs}=-0.2V$ ) for the small-signal operation region. The percentage errors of the standard deviation for the reduced models with respect to the complete model have been evaluated to check how much the reduced models are able to reproduce the original population variance. Statistical hypothesis tests have been performed to compare the distributions of the real and the imaginary parts of the S-parameters for the complete and the reduced models in the 1-50 GHz frequency range, with a cumulative level of significance  $\alpha = 0.1$ . Tab. I reports on the left the percentage error of standard deviations of the reduced models with respect to the PCA complete model, and on the right the percentage error which allows the statistical test to be passed in 75 % of the tested frequencies (i.e. 1-50 GHz band). Hypothesis tests have been performed for the auto-correlation and for the cross-correlation coefficients at the distance  $d_1 = d_{min}$  (the reticular distance of the test-chip), by checking a maximum difference of 0.35. The percentage of

homologue coefficients showing the same sign has been also evaluated. From the analysis of results in Tab. I, the best trade-off between model accuracy and complexity seems to be the model which allows to describe the non-linear behaviour of a 3-device MMIC by using only 14 principal factors. A further analysis has been carried out to compare both the PCA reduced  $0.9 \cdot \sigma^2$  model and the non-orthogonalised model to the measured database. In Tab. II results of the hypothesis tests on auto correlation-coefficients and on cross-correlation coefficients at the distances  $d_1 = d_{min}$  and  $d_2 = 2 \cdot d_{min}$  are reported: the percentage of passed tests on S-parameter correlation coefficients with a 0.35 maximum error in 1-50 GHz band are reported. In Fig. 2 standard deviations and in Fig. 3 some cross-correlation coefficients of the measurements and the PCA-based  $0.9 \cdot \sigma^2$  reduced model are reported for the 1-50 GHz band.

### VI. CONCLUSION

A non-linear statistical model of GaAs FET MMIC's has been developed which accounts for correlation between device parameters on the same chip and correlation variation as a function of the distance between devices. The PCA technique allows statistical description by means of uncorrelated statistical variables and significant reduction of the number of variables without significant loss of accuracy, so leading to straightforward implementation and simulation in most commercial circuitual CAD tools. The model has been successfully extracted and implemented in AGILENT-ADS CAD tool. Statistical hypothesis tests have highlighted statistical equivalence between measured and modelled populations with 0.1 cumulative level of significance even for reduced models, and encourage use of the model to evaluate and optimise circuit yield in MMIC design within commercial CAD tools.

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	% error				Hypothesis test: 75 % of success			
	(0.7 $\sigma^2$ )	(0.8 $\sigma^2$ )	(0.9 $\sigma^2$ )	(0.95 $\sigma^2$ )	(0.7 $\sigma^2$ )	(0.8 $\sigma^2$ )	(0.9 $\sigma^2$ )	(0.95 $\sigma^2$ )
<b>Re[S<sub>11</sub>]</b>	8.46	6.41	6.44	2.22	26	24	24	18
<b>Im[S<sub>11</sub>]</b>	11.78	9.47	7.79	3.83	25	23	22	17
<b>Re[S<sub>12</sub>]</b>	56.84	39.1	8.2	5.81	92	73	28	25
<b>Im[S<sub>12</sub>]</b>	29.69	22.15	6.17	3.49	46	40	24	21
<b>Re[S<sub>21</sub>]</b>	8.66	7.01	2.65	1.03	30	28	20	18
<b>Im[S<sub>21</sub>]</b>	14.46	11.95	3.63	1.29	36	32	21	19
<b>Re[S<sub>22</sub>]</b>	12.59	10.88	5.47	4.63	33	31	22	21
<b>Im[S<sub>22</sub>]</b>	7.24	5.06	1.17	1.16	28	24	21	19

TABLE I

COMPARISON BETWEEN THE S-PARAMETER STANDARD DEVIATIONS OF THE PCA-BASED COMPLETE MODEL AND SOME REDUCED MODELS IN 1-50 GHz BAND.

	$d = d_{min}$		$d = 2 \cdot d_{min}$	
	(0.9 $\sigma^2$ )	Multivariate model	(0.9 $\sigma^2$ )	Multivariate model
<b>Auto-correlation</b>	64	67	59	70
<b>Auto-correlation sign</b>	87	87	87	88
<b>Cross- correlation</b>	76	84	62	53
<b>Cross- correlation sign</b>	96	87	99	94

TABLE II

PERCENTAGE OF TEST SUCCESS IN THE COMPARISON BETWEEN S-PARAMETER CORRELATION COEFFICIENTS FOR  $D_1 = D_{MIN}$  AND  $D_2 = 2 \cdot D_{MIN}$  OF BOTH PCA-BASED 0.9  $\cdot \sigma^2$  REDUCED MODEL AND MULTIVARIATE MODEL

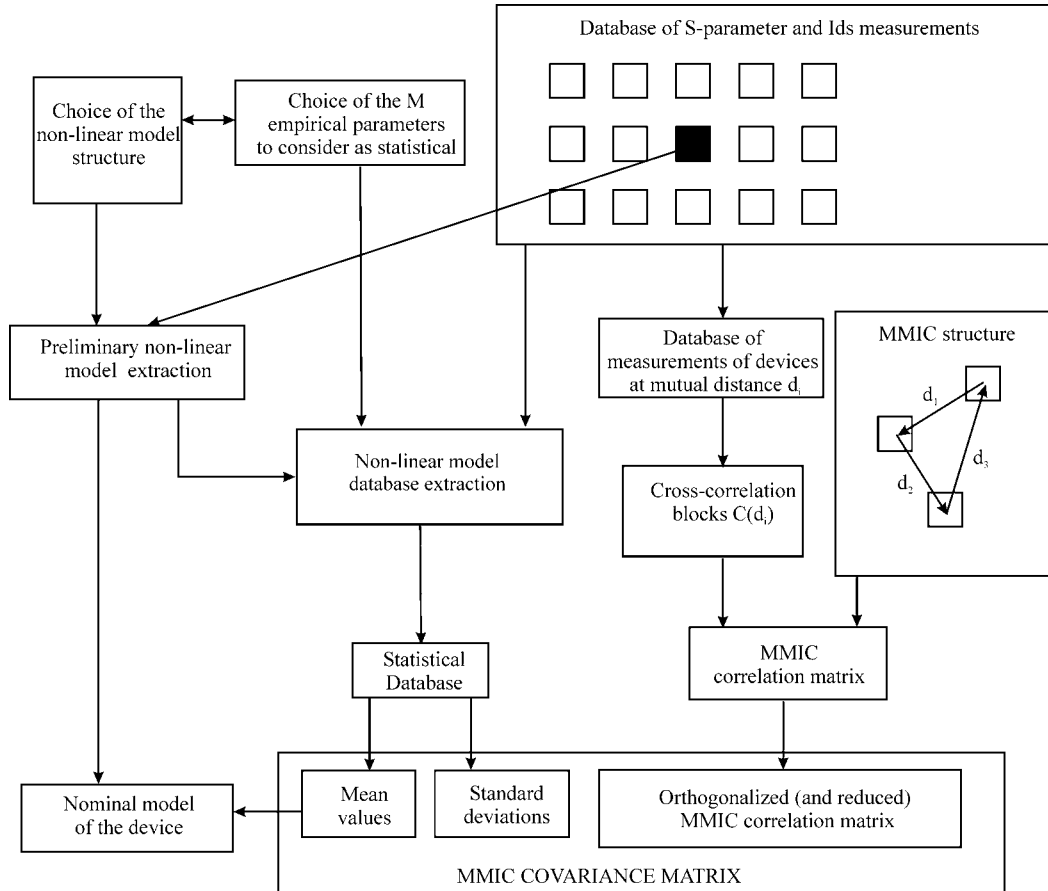


Fig. 1. Extraction procedure algorithm of the MMIC PCA-based statistical model.

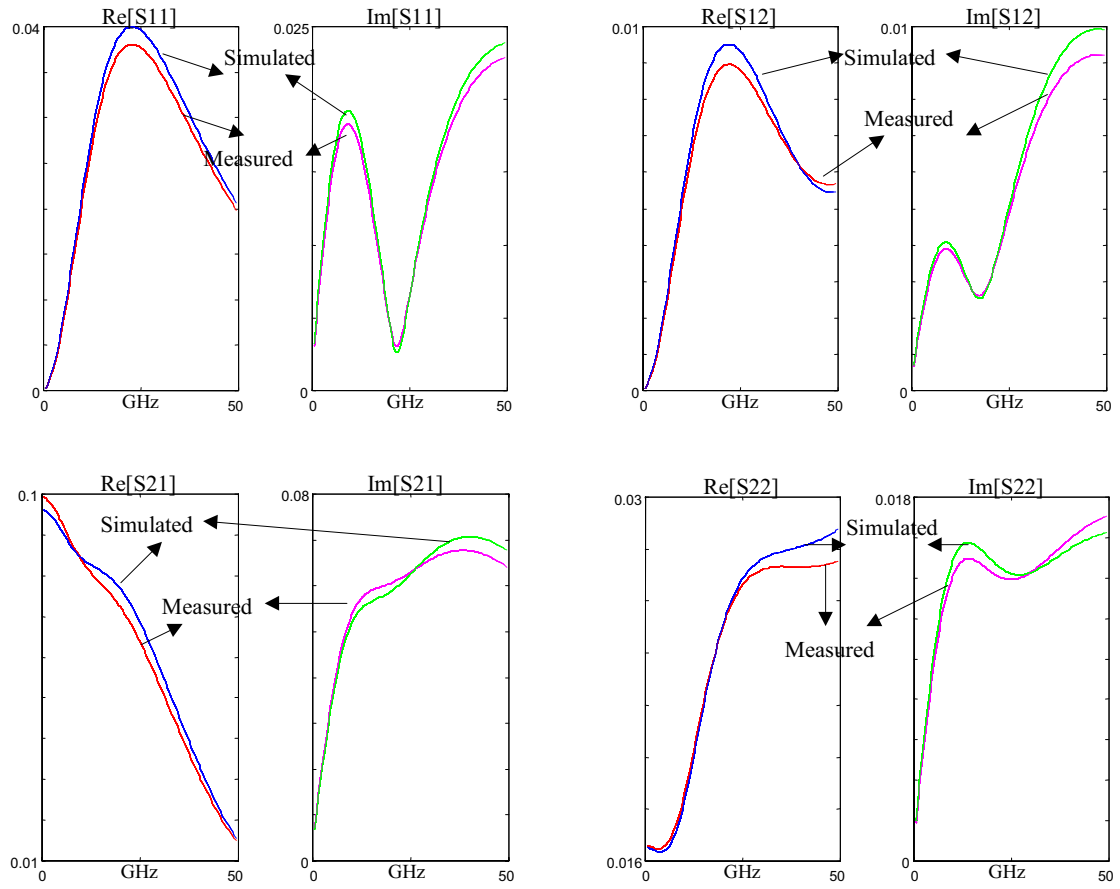


Fig. 2. Standard deviations of S-parameters for the measured data and the PCA-based  $0.9 \cdot \sigma^2$  reduced model.

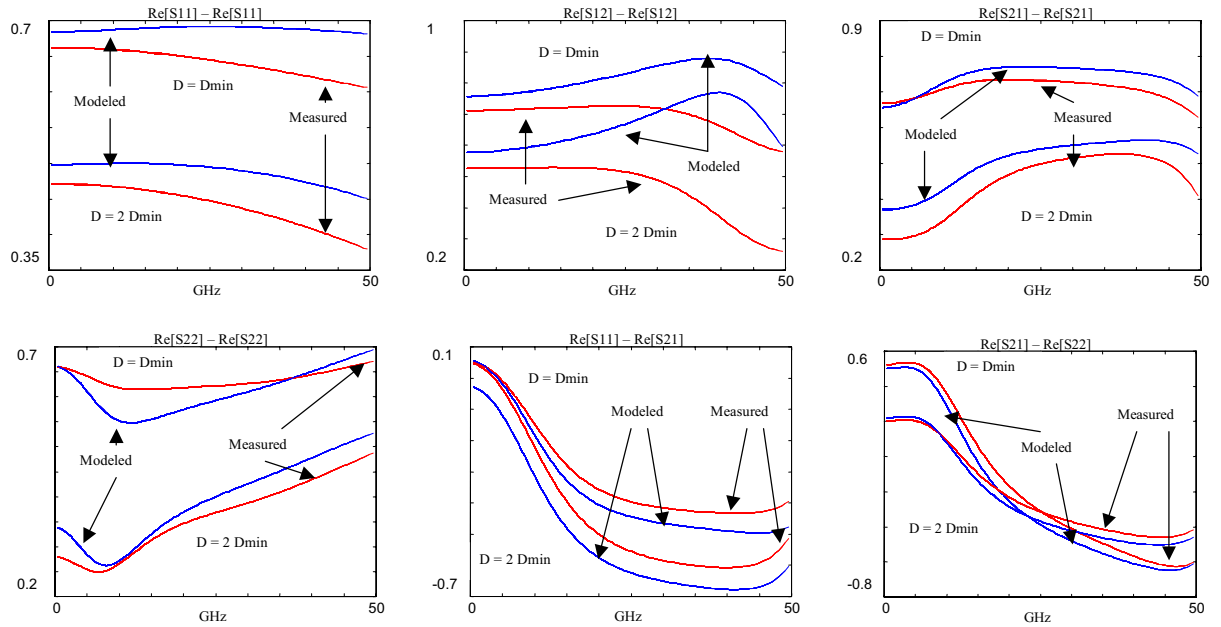


Fig. 3. Some cross-correlation coefficients of S-parameters for the measured data and the PCA-based  $0.9 \cdot \sigma^2$  reduced model.