A remark on the experimental evidence from tacit coordination games

Lidia Bagnoli[*], Giorgio Negroni[†]

February 6th, 2008

Abstract

This paper proposes an interpretation of the experimental evidence on tacit coordination games involving randomly matched players provided by Van Huyck, Battalio and Beil (1990), based on the notion of stochastic stability. When the model is calibrated with the parameters chosen in the experiment, it predicts that every strict Nash equilibrium is stochastically stable; therefore, in the long run we should not observe the emergence of any particular pattern of behavior, as suggested by the experimental evidence. The model is also compatible with the experimental evidence provided by Goeree and Holt (2005).

keywords: coordination games, stochastic stability.

1 Introduction

In this note we are concerned with the interpretations of part of the experimental evidence from tacit coordination games put forward by Van Huyck, Battalio and Beil (1990) (hereafter VHBB).

Consider a simple symmetric game in which two agents have to decide the effort they want to put into a joint project. Because of the tecnological complementarities, the output is determined by the minimum effort provided. Denoting by e_i, e_j the actions taken by two players, the payoff function of the generic agent i is

$$V_i(e_i, e_j) = a \min(e_i, e_j) - be_i \tag{1}$$

where, for any i, $e_i \in [1, 2, .., \overline{e}]$ and where $a > b$. This last condition ensures that the game has a multiplicity of symmetric strict, Pareto rankable, Nash equilibria. The Nash equilibrium $(\overline{e}, \overline{e})$ is Pareto dominant. Call G this game. In VHBB the parameter values are $e_i \in [1, 2, .., 7]$, $a = 0.2$ and $b = 0.1$.

[*]Regione Emilia Romagna, Bologna.
[†]Bologna University and DARRT. E-mail: giorgio.negroni@unibo.it

We are interested in disussing VHBB's treatment C where the stage game G is played for a sequence of periods by two randomly selected players (experiments 6 and 7). No pre-play negotiation is allowed and, after each repetition of the stage game, the minimum action was publicly announced. The only common historical data available to the players is then the minimum effort. The outcome of these experiments are reported in the Table below (Table 5 in VHBB).

The main conclusions which can be derived from this Table are the following: (a) there is no evidence that successive repetitions of the basic stage game (where each repetition involves two newly randomly matched players) converge to any equilibrium; (b) the first-best outcome (which is the payoff dominant equilibrium in which both agents choose to supply the maximum effort, i.e. 7) is quite an unlikely outcome.

	Period				
	21	22	23	24	25
Experiment 6					
No. of 7's	5	5	4	10	8
No. of 6's	0	1	3	0	0
No. of 5's	2	5	3	3	4
No. of 4's	3	1	1	1	1
No. of 3's	1	1	1	0	0
No. of 2's	1	1	2	2	2
No. of 1's	4	2	2	0	1
Experiment 7					
No. of 7's	-	-	6	5	5
No. of 6's	-	-	1	0	1
No. of 5's	-	-	0	3	0
No. of 4's	-	-	2	1	4
No. of 3's	-	-	2	0	0
No. of 2's	-	-	0	0	1
No. of 1's	-	-	3	5	3

Several students have found an explanation of these results; see in particular Crawford (1990, 1995), Anderson and alii (2001) as well as Goere and Holt (2005).

Crawford proposes an interpretation based on agents' adaptive behavior. The idea is that in each stage, each player adjusts his strategy by choosing a best reply action to his beliefs, where these beliefs are influenced by what he can learn from past strategy choices. These adjustments then determine a new choice, which will be part of the experience available to the players in the subsequent stage, and so on. By assuming that players' beliefs are subject to idiosyncratic random shocks – reflecting the differences in which they interpret their experience – Crawford shows that, although the influence of such shocks on players' adaptive adjustment can be expected to die out over time, they can nevertheless alter the course of players' adjustments, and thereby the final outcome. A full model along these lines is developed in Crawford (1995).

Anderson and alii (2001) makes the observation that the human decision processes, instead of being so clearly defined as in our game G, exhibit some randomness. This induces the authors to model the decision process as a logit probabilistic choice framework which allows them to derive a logit equilibrium which determines a unique probability distribution of decisions in an analogue of game G. They show that in a minimum-effort coordination game with a continuum of Pareto-ranked Nash equilibria, the introduction of even a small amount of noise results in a unique equilibrium distribution over effort choices.

In a similar model Goere and Holt (2005) discuss the effect of changes of the effort-cost parameter. They show that when the effort-costs is low the more frequently observed effort is the maximum one while the opposite occurs when the effort-cost is high.

In what follows we suggest an interpretation of VHBB 's results based on the notion of stochastic stability introduced by Young (1993). As we shall show, the evidence reported in Table 1 above is very well compatible with the absence of a unique stochastically stable equilibrium that occurs when the considered model is calibrated with the parameters chosen by VHBB. We shall also show that the predictions of our model are compatible with the evidence provided by Goere and Holt (2005). With respect to the previously quoted literature, we believe that our interpretation has the great merit of simplicity.

2 Stochastic stability

In this Section we briefly discuss the notion of stochastically stable state introduced by Young (1993). Let $t = 1, 2, ...$ denote successive time periods and consider a fixed (but large) population of N players. Let N_1 and N_2 be the sub-populations of agents 1 and 2 respectively. In each period the finite basic game G is played once by two agents randomly selected from these sub-populations. When selected, each agent has to form an expectation on the behavior of his opponent. Let $h(t) = (e(1), ..., e(t))$ be the history of plays at the end of period t; it consists of the past plays of the game where each play denotes the profile of strategies played in that period, i.e.$e(t) = (e_1(t), e_2(t))$. Since gathering information is costly, each agent bases his current action not on the whole past plays available but rather on a sample of k plays taken from the most recent m (with $1 \leq k \leq m$) past plays.

Thus at the beginning of period $t + 1$ (with $t \geq m$) each agent consults k plays of the game and derives the empirical frequency with which each (pure) strategy was played in the past by other agents; then with probability $1 - \epsilon$ he chooses an action which is a best reply to the derived empirical frequencies while with probability ϵ he chooses an action which is not the best reply to the derived empirical frequencies (i.e. he makes a mistake). Current actions are recorded[1] and the economy moves from the current state h to the successor state h'. Period $t+1$ then closes. In the new period $t+2$, the game is played again by two other randomly selected agents. As before, each consults k plays of the game from the most recent m periods and with probability $1-\epsilon$ chooses an action which is the best reply to the newly derived empirical frequencies while with probability ϵ he makes a mistake. This moves the economy from the current state h' to the successor state, h'' and so on.

The transition from one state to its successor is governed by the perturbed Markov process P^{ϵ} with transition function $P^{\epsilon}_{hh'}$. If we assume that all mistakes are possible and that the probability to make a mistake is time-independent, then the transition matrix associated with $P^{\epsilon}_{hh'}$ is strictly positive and the Markov process is irreducible and aperiodic (ergodic). The process has thus a unique stationary distribution μ^{ϵ}. When the probability of mistakes is small, the stationary distribution of the perturbed process P^{ϵ} coincides with one of the stationary distributions of the unperturbed process. Hence we say that a state h is *stochastically stable* relative to the process P^{ϵ} if $\lim_{\epsilon\to 0}\mu^{\epsilon}(h) > 0$. When there is a unique stochastically stable state, this stationary distribution is concentrated around just one equilibrium; this is the state that, in the long run, will be observed with probability close to one.

In order to apply Young's result we have to derive the stochastic potential of the game. We start by observing that a Markov chain can be represented by a tree having a node in each state. A tree rooted at h consists of directed edges such that from every node $h' \neq h$ there exists one and only one direct path from h to h'. Let the *resistance* $r(h,h')$ be the total number of mistakes needed to move from state h to the successor[2] state h'. Let the *total resistance* of a rooted tree to be the sum of resistances associated with its edges and let the stochastic potential of a state h be the least total resistance among all $h-$trees. If the basic game is acyclic and the sample is sufficiently limited[3], then Corollary of Theorem 2 in Young (1993) establishes that a stochastically stable state is a convention with minimum stochastic potential.

Each stochastically stable state is thus a convention $h = (\widehat{e}, ..., \widehat{e})$ in which the same Nash equilibrium profile $\widehat{e} = (\widehat{e}_1, \widehat{e}_2)$ is played m times in succession. If k and m are sufficiently large, the stochastically stable state is unique. In order to detect the stochastically stable state it is sufficient to consider, for each strict Nash equilibrium $\widehat{e}$, all the trees rooted at this equilibrium $\widehat{e}$ (rather than at h), where the resistance on each directed edge now tells us the minimum number of mistakes needed to move the economy from one equilibrium to another. Hence, even if the basic game has many strict Nash equilibria, the adaptive process

[1] Since the memory size is finite, the inclusion of the current period implies that agents disregards the most distant play.

[2] If state h' is not a successor of state h, then $r(h,h') = \infty$.

[3] The precise condition is if $k \leq m(L_{\Gamma}+2)^{-1}$ where $L(e)$ denote the length of the shortest best reply path originated in the profile e and $L_{\Gamma} = \max L(e)$.

with mistakes can converge to one of these. This, in turn, gives a theory of equilibrium selection.

As put forward by Binmore, Samuelson and Young (2003), the computation of the minimum stochastic potential is made easier when the game satisfies the bandwagon property. This because the minimum number of mistakes needed to switch from the strict Nash equilibrium $\widehat{s} = (\widehat{s}_1, \widehat{s}_2)$ to the strict Nash equilibrium $\overline{s} = (\overline{s}_1, \overline{s}_2)$ is found when agents by mistake play an action belonging to the profile $\overline{s}$.

3 Stochastic stability in VHBB's model

Consider the game G. The game is acyclic and satisfies the bandwagon property.[4] Acyclicity means that the best reply graph contains no directed cycles, a property satisfied by all coordination games. A sufficient condition for the (marginal) bandwagon property to hold for generic (i.e. not necessarily acyclic) symmetric games has been proved by Kandori and Rob (1998). A reformulation which holds for acyclic but not necessarily symmetric two players games is given by Binmore, Samuelson and Young (2003). This essentially says that, for both agents, deviations from the equilibrium strategy are more costly when the opponent plays his part of the equilibrium.

Consider two Nash equilibria $e^* = (e^*, e^*)$ and $\overline{e} = (\overline{e}, \overline{e})$ and let e^* be the fixed initial state. The bandwagon property implies that the path of least resistance from e to $\overline{e}$ is the direct path. This implies that, in order to derive the resistance of this path it is sufficient to analyze the restrict game where the only strategies available are those corresponding to the two equilibria considered. Two cases are possible: either $\overline{e} > e^*$ or $\overline{e} < e^*$. The former corresponds to a situation in which we exit from the state e^* to the right (i.e. such that $\overline{e} > e^*$) while the latter corresponds to a situation in which we exit from the state e^* to the left (i.e. such that $\overline{e} < e^*$).

Let $\overline{e} = e^+ > e^*$ and consider the following payoff matrix

	e^*	e^+
e^*	$e^*(a-b); e^*(a-b)$	$e^*(a-b); e^*a - e^+b$
e^+	$e^*a - e^+b; e^*(a-b)$	$e^+(a-b); e^+(a-b)$

Let (σ_1, σ_2) be a mixed strategy profile where σ_1 (resp. σ_2) is the probability that agent 1 (resp. 2) plays e^*. The best reply correspondence is

$$V_i(e^*, \sigma_j) \geq V_i(e^+, \sigma_j) \iff \sigma_j \geq 1 - \frac{b}{a}. \tag{2}$$

Let m be the memory size and k be the sample size used by both agents. Suppose that in the past plays agents 2 choose e^+ by mistake from period

[4]See Bagnoli and Negroni (2008) for further details.

$t = m + 1$ to $t = m + k'$ inclusive, where $k' \leq k$. If agent 1 draws a sample that includes these k' choices of e^+, as well as $k - k'$ choices of e^*, then agent 1 deduce that $\sigma_2 = 1 - \frac{k'}{k}$ and $1 - \sigma_2 = \frac{k'}{k}$. It then follows from (2) that the minimum numbers of mistakes past agents 2 must make in order to induce actual agent 1 to choose $\overline{e}_1$ as best reply is

$$k' \geq \frac{b}{a} k. \tag{3}$$

In other words, k' mistakes by agent 2 are sufficient to move the economy from e^* to e^+. Due to the symmetry of the game, analogous considerations holds when the mistakes are made by agent 1. Hence the resistance of the path from e^* to e^+ is $r(e^*, e^+) = \frac{b}{a}$.

Let $\overline{e} = e^- < e^*$ and consider the following payoff matrix

	e^*	e^-
e^*	$e^*(a-b); e^*(a-b)$	$e^- a - e^* b; e^-(a-b)$
e^-	$e^-(a-b); e^- a - e^* b$	$e^-(a-b); e^-(a-b)$

Let (σ_1, σ_2) be a mixed strategy profile where σ_1 (resp. σ_2) is the probability that agent 1 (resp. 2) plays e^*. The best reply correspondence is

$$V_i(e^*, \sigma_j) \geq V_i(e^+, \sigma_j) \iff \sigma_j \geq \frac{b}{a}. \tag{4}$$

Proceeding as before, the minimum numbers of mistakes past agents 2 must make in order to induce actual agent 1 to choose e^- as best reply is

$$k' \geq \left(1 - \frac{b}{a}\right) k \tag{5}$$

Due to the symmetry of the game, analogous considerations holds when the mistakes are made by agent 1. Hence the resistance of the path from e^* to e^- is $r(e^*, e^-) = 1 - \frac{b}{a}$. A direct application of Binmore, Samuelson and Young's (2003) naive minimization test leads to the following Proposition.[5]

Proposition 1 *The exit resistances are constant over all possible states; this means that they do not depend neither on the state from which the path originates nor on the state into which the path flows. The following cases are possible.*

(a) If $r(e^*, e^+) < r(e^*, e^-)$ the stochastically stable state corresponds to the Nash equilibrium in which agents supply their maximum effort. This occurs when $b < \frac{a}{2}$.

(b) If $r(e^*, e^-) < r(e^*, e^+)$ the stochastically stable state corresponds to the Nash equilibrium in which agents supply their minimum effort. This occurs when $b > \frac{a}{2}$.

[5] Notice that the Proposition holds true even when the action set is continuous and bounded, i.e. when $e_i \in [\underline{e}, \overline{e}]$ for any i

(c) If $r(e^*, e^-) = r(e^*, e^+)$ any equilibrium profile has the same stochastic potential. Therefore any equilibrium profile is stochastically stable This occurs when $b = \frac{a}{2}$.

We can use this Proposition to evaluate treatment C in VHBB. Recall that in this treatment, agents are randomly matched to play the stage game; we are thus in the same framework studied by Young (1993). In that case the parameters are $b = 0.1$ and $a = 0.2$. Hence, since $b = \frac{a}{2}$, we are exactly in the case in which Point (c) of the above Proposition applies. Since a unique stochastically stable equilibrium does not exist, the theory predicts that in the long run we should not observe the emergence of a particular pattern of behavior. This is exactly what emerges in the VHBB's experiments involving randomly matched players.

Our interpretation is also compatible with the experimental evidence provided by Goeree and Holt (2005) and illustrated in their Figure 2 (bottom), plotting the effort choice frequencies. If we let $c = b/a$, our stage game G coincide with the stage game considered by these authors where $e_i \in [110, 170]$. They report evidence from two treatments, one in which $c = 1/4$ and another in which $c = 3/4$. They show that when $c = 1/4$ the more frequently observed effort is the maximum one while the opposite occurs when $c = 3/4$. This evidence is in accord with the predictions of our model. In fact, from Proposition 1 we deduce that the Nash equilibrium in which agents supply their maximum effort is the stochastically stable state when $c = \frac{b}{a} = \frac{1}{4} < \frac{1}{2}$ while the Nash equilibrium in which agents supply their minimum effort is the stochastically stable state when $c = \frac{b}{a} = \frac{3}{4} > \frac{1}{2}$. Hence, in the long run, we expect to observe more frequently the equilibrium with minimum (resp. maximum) effort when the effort-cost is low (resp. high).

4 Conclusions

This paper has given an interpretations of the experimental evidence on tacit coordination games involving randomly matched players provided by Van Huyck, Battalio and Beil (1990), treatment C. The experimental results found no evidence that successive repetitions of the basic stage game converge to any equilibrium. Our interpretation was based on the notion of stochastic stability introduced by Young (1993). We have shown that, when the model is calibrated with the parameters chosen by VHBB, then a unique stochastically stable equilibrium does not exist. The theory thus predicts that in the long run we should not observe the emergence of any particular pattern of behavior, which is compatible with the experimental evidence. With respect to the previously literature our interpretation has the great merit of simplicity. Our model is also compatible with the experimental evidence provided by Goeree and Holt (2005).

References

[1] Anderson S. P., J. K. Goeree and C. A. Holt, (2001), Minimum-effort coordination games: stochastic potential and logic equilibrium, Games and Economic Behavior, 34, 177-199.

[2] Bagnoli L. and G. Negroni (2008), The emergence of norms of cooperation in stag hunt games with production, University of Bologna, mimeo.

[3] Binmore K., L. Samuelson and P. Young, (2003), Equilibrium selection in bargaining games, Games and Economic Behavior, 45, 296-328.

[4] Crawford V., (1990), An "evolutionary" interpretation of Van Huyck, Battalio and Beil's experimental results on coordination, Games and Economic Behavior, 3, 25-59.

[5] Crawford V., (1995), Adaptive dynamics in coordination games, Econometrica, 63, 103-143.

[6] Ellison G., (2000), Basin of attraction, long-run stochastic stability, and the speed of step-by-step evolution, Review of Economic Studies, 67, 17-45.

[7] Goere J. and C. Holt, (2005), An experimental study of costly coordination, Games and Economic Behavior, 51, 349-364.

[8] Anderson S. P., J. Goere and C. Holt, (2001), Minimun-effort coordination games: stochastic potential and logit equilibrium, Games and Economic Behavior, 34, 177-199.

[9] Kandori M. and R. Rob (1998), Bandwagon effects and long run technology choice, Games and Economic Behavior, 22, 30-60.

[10] Van Huyck J., R. Battalio and R. O. Beil (1990), Tacit coordination games, strategic uncertainty and coordination failures, American Economic Review, 80, 234-248.

[11] Van Huyck J., J. Cook and R. Battalio (1997), Adaptive behavior and coordination failures, Journal of Economic Behavior and Organization, 32, 483-503.

[12] Young P., (1993), The evolution of conventions, Econometrica, 61, 57-84.

[13] Young P., (1998), Individual Strategies and Social Structure, Princeton University Press.