CONDITIONAL LEPTOKURTOSIS IN ENERGY PRICES: MULTIVARIATE EVIDENCE FROM FUTURES MARKETS

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ABSTRACT

We study the joint movements of the returns on futures for crude oil, heating oil and natural gas. We model the leptokurtic behavior through the multivariate GARCH with dynamic conditional correlations and elliptical distributions introduced by Pelagatti and Rondena (2004). Energy futures markets co-vary strongly. The correlation between the futures prices of natural gas and crude oil has been rising over the last 5 years. However, this correlation has been low on average over two thirds of the sample, indicating that futures markets have no established tradition of pricing natural gas as a function of developments on oil markets.

JEL classification: C22, G19.
Keywords: multivariate GARCH, kurtosis, energy prices, futures markets.

"For decades, natural gas prices (as well as those of gasoline, heating oil, propane, et cet) have hinged off crude oil. But as more investors pile into an energy market that no longer holds the crude benchmark sacred, it looks like gas is poised to cut the cord."

'Crude Oil-Natural Gas Price Connection Unraveling?' in *Trader Daily*, May 2007

1. Introduction

Energy commodities are widely priced in financial markets through futures on crude oil, natural gas and heating oil. Although a large amount of research has been devoted to studying the comovements between energy spot prices, little effort has been dedicated to the study of the joint movements among the prices of energy futures. Like most financial assets, energy futures exhibit changes in volatility through time. This feature makes the use of the GARCH framework of Engle (1982) especially valuable. Assets are also typically characterized by extreme observations of prices that make the empirical distribution non-normal. For instance, Bollerslev (1987) shows that the t distribution performs better in order to capture the higher observed kurtosis of some selected exchange rates.

In this paper, we model the conditional correlation between prices of energy futures traded in the New York Mercantile Exchange. We use the Dynamic Conditional Correlation model — DCC — proposed by Engle (2002). However, since leptokurtosis is a key feature of the data, we resort to the extension of the DCC proposed by Pelagatti and Rondena (2004), and focus on elliptically-contoured distribution for the returns.

The results indicate that the correlation over the last 5 years between the the futures prices of natural gas and crude oil has been rising. However, the correlation has been weak on average over two thirds of the sample. This suggests that futures markets have no established tradition of pricing natural gas as a function of the developments on oil markets.

This paper is organized as follows. Section 2 provides a general overview of multivariate GARCH models, with a focus on the DCC model. Section 3 discusses the properties of the dataset and the results. Section 4 proposes some concluding remarks.

2. An overview of multivariate garch models

We model a vector r_t of returns on energy futures as

$$r_t = \mu_t + \epsilon_t, \tag{1}$$

where μ_t indicates the conditional mean, and ϵ_t denotes the residuals. Conditioning on information up to time $t-1$, the multivariate model for the second moments takes the form:

$$\epsilon_t = H_t^{1/2} z_t \tag{2}$$

where H_t has dimension $n \times n$. The conditional mean is usually a function of its lagged values. For the vector z_t, $\mathsf{E}[z_t] = 0$ and $\mathsf{Var}[z_t] = I$, where I is a suitable identity matrix.

Parametrizing a multivariate GARCH needs to ensure positive-semidefiniteness of H_t. Bollerslev (1990) proposes to model the conditional covariances as proportional to the conditional standard deviations, the so-called constant correlation model. Assuming known constant correlations, the model gives

$$H_t = V_t F V_t \tag{3}$$

$$V_t = \text{diag}(h_{11,t}^{1/2}, \ldots h_{nn,t}^{1/2}) \tag{4}$$

and F contains the (time-invariant) conditional correlations. Bollerslev (1990) rewrites each of the conditional variances as:

$$h_{ii,t} = \omega_i \sigma_{it}^2 \tag{5}$$

where ω_i is a scalar. It should be noted that positive-definitess of H_t follows from both F being positive definite, and the conditional variances being all positive. The estimates of $h_{ii,t}^{1/2}$ can be obtained from univariate GARCH-type models. This implies that there can be heterogeneity in the source of the conditional variances. Given the conditional variance matrix, one then estimates the constant correlations from the multivariate model.

Engle (2002) modifies the constant-correlation model by allowing for time-dependent conditional correlations in the DCC model, where $H_t = V_t F_t V_t$. The matrix V_t follows from the recursion

$$V_t^2 = \text{diag}(\omega_i) + \text{diag}(\nu_i) \otimes \epsilon_{t-1}\epsilon_{t-1}' + \text{diag}(\gamma_i) \otimes V_{t-1}^2 \tag{6}$$

where $\otimes$ denotes element-by-element multiplication. The expression for the conditional correlation matrix becomes

$$F_t = \text{diag}(q_{11,t}^{-1/2}, \ldots q_{nn,t}^{-1/2}) Q_t \text{diag}(q_{11,t}^{-1/2}, \ldots q_{nn,t}^{-1/2}). \tag{7}$$

The matrix Q_t is symmetric positive definite

$$Q_t = S \otimes (11' - A - B) + A \otimes u_{t-1}u_{t-1}' + B \otimes Q_{t-1}, \tag{8}$$

and the standardized error u_t is

$$u_t = V_t^{-1}\epsilon_t. \tag{9}$$

It can be proved that the matrix S is equal to the unconditional correlation matrix, and that it can be estimated from u_t. The matrices A and B are often reduced to scalars in order to limit the number of parameters to be estimated.

When the number of stocks is large, Pelagatti and Rondena (2004) suggest that a three-step procedure can be employed to estimate the DCC model. First, the set of parameters $\{\omega_i, \nu_i, \gamma_i\}$ can be obtained from the maximum-likelihood maximization of the univariate GARCH models. Estimates of V_t are computed from the recursion 6, along with the standardized residuals u_t. Then, the correlation matrix F_t is estimated from equation 7. Finally, given F_t and V_t, the likelihood function

$$\mathcal{L} = \sum_{t=1}^{T}\left[\log(c) - \frac{1}{2}\log|F_t| - \log|V_t| + \log\left(g(u_t F_t^{-1} u_t')\right)\right] \tag{10}$$

is maximized to obtain the estimates of A and B. The function $g(\cdot)$ is the density generator, and refers the distributional assumption on the residuals.

Pelagatti and Rondena (2004) extend the DCC model through fat-tailed elliptically contoured — or leptokurtic — distributions. They focus on the multivariate Student's t distribution

$$f(\epsilon_t) = \frac{\Gamma\left[(\theta+\vartheta)/2\right]}{\left[\pi(\theta-2)\right]^{\vartheta/2}\Gamma(\theta/2)|H_t|^{1/2}}\left|1 + \frac{\epsilon_t' H_t^{-1}\epsilon_t}{\theta-2}\right|^{-(\theta+\vartheta)/2}, \tag{11}$$

and on the multivariate Laplace

$$f(\epsilon_t) = \frac{2}{(2\pi)^{\vartheta/2}|H_t|^{1/2}}\left(\frac{\epsilon_t' H_t^{-1}\epsilon_t}{2}\right)^{\theta/2}\kappa_v\left(\sqrt{2\epsilon_t' H_t^{-1}}\right) \tag{12}$$

with a modified Besel function κ_v.

2.1. *Testing for constant conditional correlation*

The main advantage of the constant correlation model consists in parsimony, i.e. it involves a lower number of parameters to be estimated than the dynamic correlation model. However, this aspect should be weighed against the costs of eventual mis-specification. Engle and Sheppard (2001) propose a test for the constancy of the conditional correlations. They apply the idea underlying the DCC model. Their null and alternative hypotheses are

$$\mathcal{H}_0: \ F_t = F, \tag{13}$$

$$\mathcal{H}_1: \ \text{vec}[F_t] = \text{vec}[F] + \varpi_1 \text{vec}[F_{t-1}] + \ldots \varpi_m \text{vec}[F_{t-m}]. \tag{14}$$

This procedure requires defining the auxiliary vector-autoregression

$$\hat{w}_t = \omega_0 + \omega_1 \hat{w}_{t-1} + \ldots \omega_m \hat{w}_{t-m} \tag{15}$$

$$\hat{w}_t = \text{vec}(\hat{l}_t \hat{l}_t^T - I_n), \tag{16}$$

where $\hat{l}_t$ is a vector of standardized residuals:

$$\hat{l}_t = \hat{F}^{-1/2} \hat{V}_t^{-1} \hat{\epsilon}_t. \tag{17}$$

The null implies that the coefficients ω_ι of equation 15 equal zero. The test statistics is asymptotically distributed as a $\chi^2(m-1)$.

3. Results

We use daily data on futures prices on light crude oil, natural gas and heating oil traded in the New York Mercantile Exchange between November 1 1990 and November 22 2005. The futures are front month. The sample includes 3929 observations. Log-returns are computed as $r_t = 100 * \log(y_t/y_{t-1})$, where y_t indicates the price.

Figures 1-3 depict the series of returns. The QQ plots suggest that there are considerable deviations from the normality assumption for all the returns. Table 1 shows that all the returns are skewed left, which implies that the empirical density is fat-tailed. The estimated kurtosis is largely in excess of 3, which indicates a peaked distribution. The Jarque-Bera test statistics rejects the null of normality very strongly. Table 1 resports also the normality test of Anderson and Darling (1952). This is a modification of the Kolmogorov-Smirnov test, and gives more weight to the tails than the Kolmogorov-Smirnov test itself. Also in this case, there is a rejection of the null of normality. As an additional step of preliminary investigation, we implement a test for general nonlinear dependence, namely the BDS test of Brock et al. (1996). The $p-$values of the BDS test statistics provide strong evidence against the indpendence and identical distribution for each of the returns. We also test for the presence of ARCH effects in each series using the Lagrange Multiplier test of Engle (1982). Table 2 reports large rejections of the null of no ARCH. Finally, the test of Engle and Sheppard (2001) shows that the correlation structure should be time-varying. Summing up, the evidence suggests that the returns can be well characterized by heteroskedasticity, and approximated through a distribution with fat tails. The correlation structure between the returns is not constant over time.

The multivariate model of volatility is estimated on the residuals from a vector autoregression. The Bayesian and Schwartz Information Criteria point to the adequacy of a VAR of order 2. The estimated coefficients of the VAR are reported in table 3. The subsequent issue concerns the selection of the distribution of errors for the GARCH model. We computed the values of the likelihood corresponding to different distribution functions. These are reported in table 4. The results confirm the relevance of the t distribution, which largely outperforms the normal. It should be noted that, for less than four degrees of freedom, estimation is unfeasible and runs into numerical problems. The low number of degrees of freedom suggests that heavy tails is a key feature of the model.

Table 5 reports the estimated parameters of alternative specifications for the DCC GARCH. The constants A and B are restricted to scalars. The estimates change largely across distributions. Figure 4 plots the conditional variances. The returns on crude oil and on heating oil are characterized by two peaks in volatility. These correspond to the spike in prices due to the first Gulf War. There are two other peaks in the volatility of futures prices on heating oil. The first one occurs in 1997, and anticipates cold Winter temperatures both in Europe and in the U.S. This is also marked by a U.S. attack into southern Iraq following an Iraqi-supported invasion of Kurdish safe areas The second one affects also crude oil futures, and relates to an increase in spot prices of three times between January 1999 and September 2000 due to strong world oil demand and OPEC oil production cutbacks. Although global events are key determinants of oil prices, prices of natural gas are mostly determined by domestic determinants, such as developments in the distribution network and weather conditions. For instance, the plot shows that a third peak is located around December 2000, and is due to an especially cold Winter and fall in inventories

Figure 5 reports the conditional covariances. The second panel on the upper-right side indicates that futures prices on crude and heating oil have moved very closely on average. The largest drop in covariance corresponds to the U.S. strike on Iraq of 1997, to which heating oil prices responded strongly despite the lack of reaction of crude oil prices. The second stylized facts emerging from figure 5 has to do with the correlation among the returns on natural gas and crude oil. A Reuters headline of May 4 2007 reports that "*(b)uyers and sellers alike are wrestling with whether to break a decades-old practice of pricing natural gas on the basis of the dominant commodity oil*".[1] The upper-right panel of figure 5 shows that the correlation between the futures prices of

[1] The article is titled '*Buyers, sellers puzzle over gas-oil price divorce*', and is written by Barbara Lewis.

natural gas and crude oil has risen over the last 5 years. However, the correlation has been weak on average over two thirds of the sample. This suggests that futures markets have no established tradition of pricing natural gas as a function of the developments in oil markets.

4. Conclusion

This paper proposes studies the conditional correlation between prices of energy futures traded in the New York Mercantile Exchange. We find that leptokurtosis is a key feature of the data. Hence, we resort to the extension of the DCC proposed by Pelagatti and Rondena (2004), and focus on elliptically-contoured distribution for the returns.

Our results suggest that the correlation between the futures prices of natural gas and crude oil has risen between 2001 and 2006. However, this correlation has been weak on average before 2001. This finding indicates that the widely-held belief that prices of natural gas are set according to oil prices finds no ground within futures markets.

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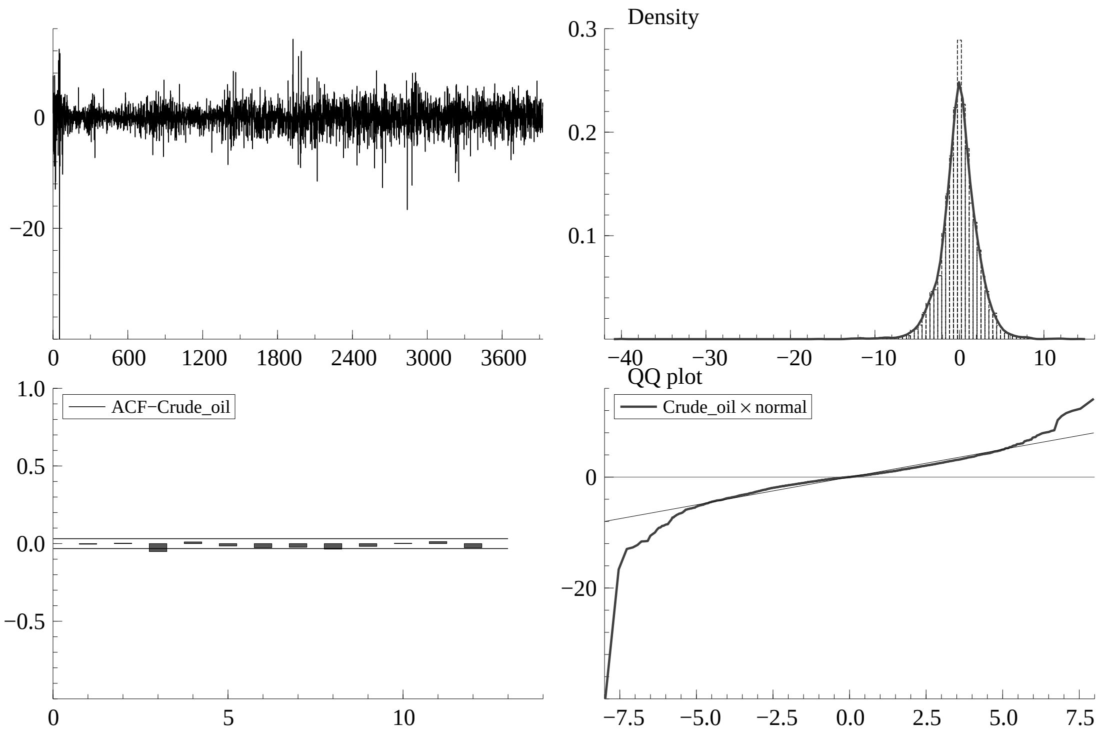

FIGURE 1—: Returns on crude oil futures

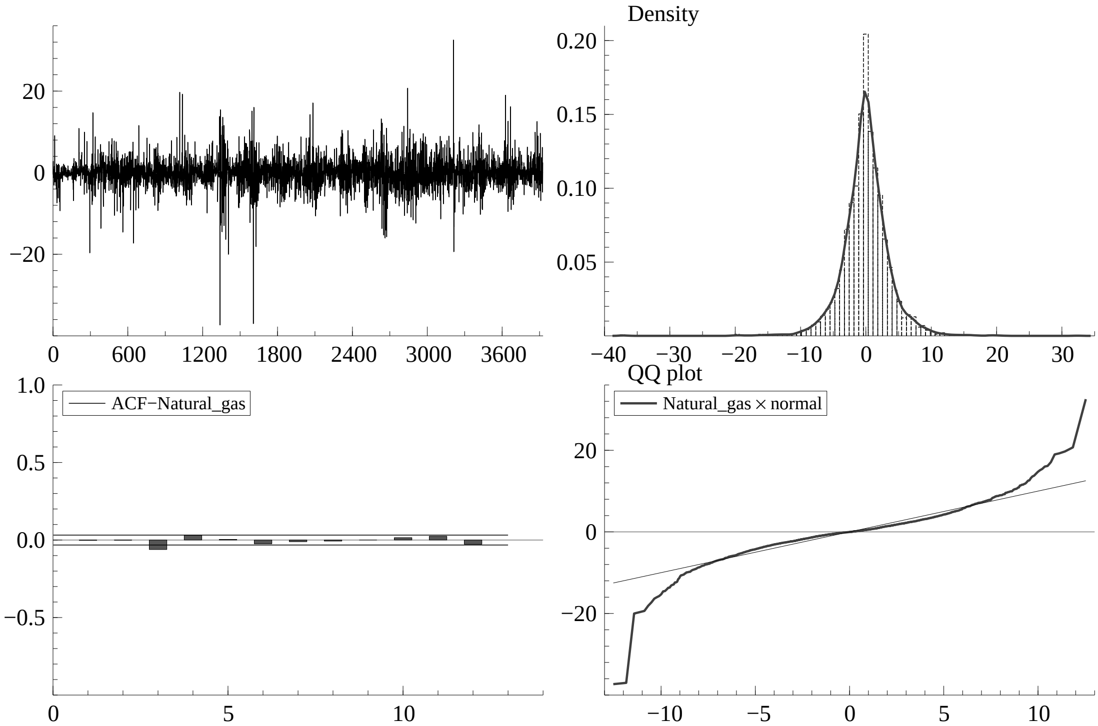

FIGURE 2—: Returns on natural gas futures

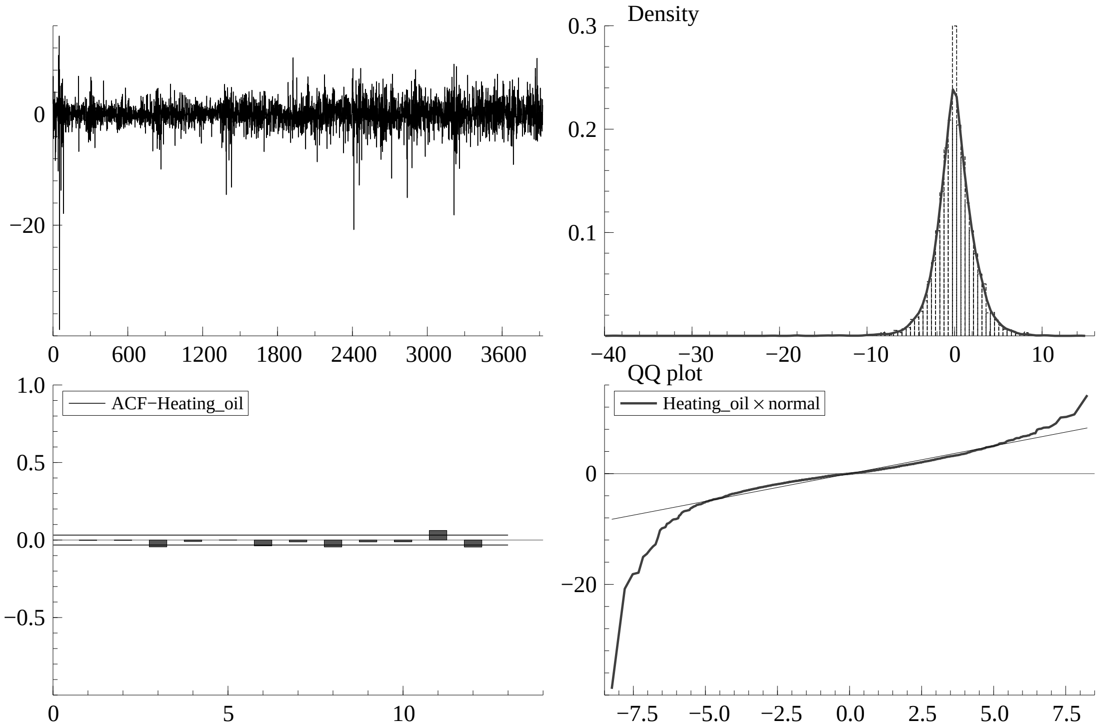

FIGURE 3—: Returns on heating oil futures

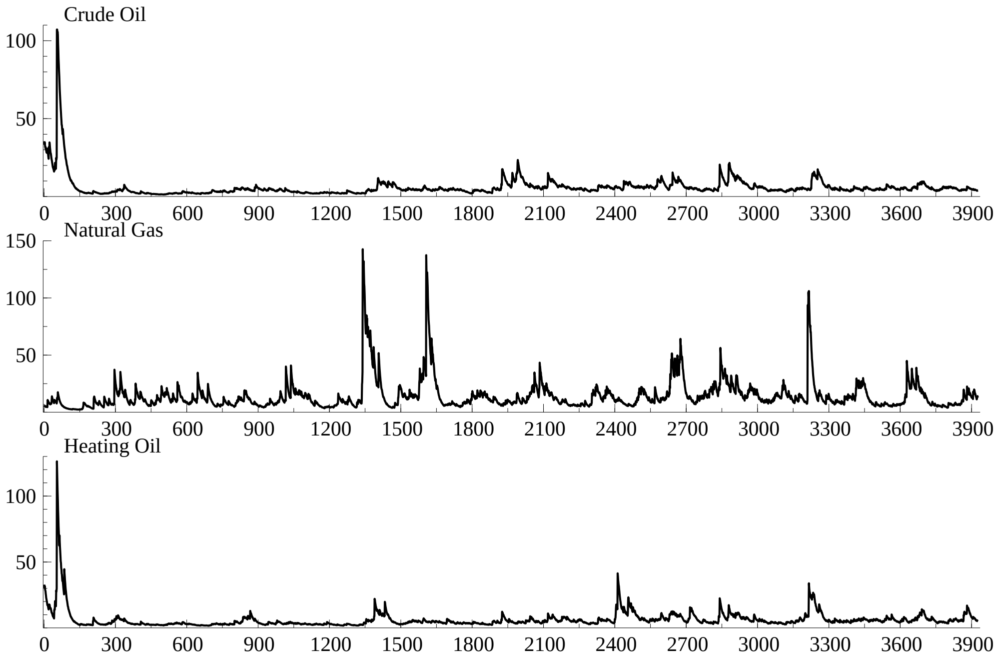

FIGURE 4—: Estimates of conditional variance

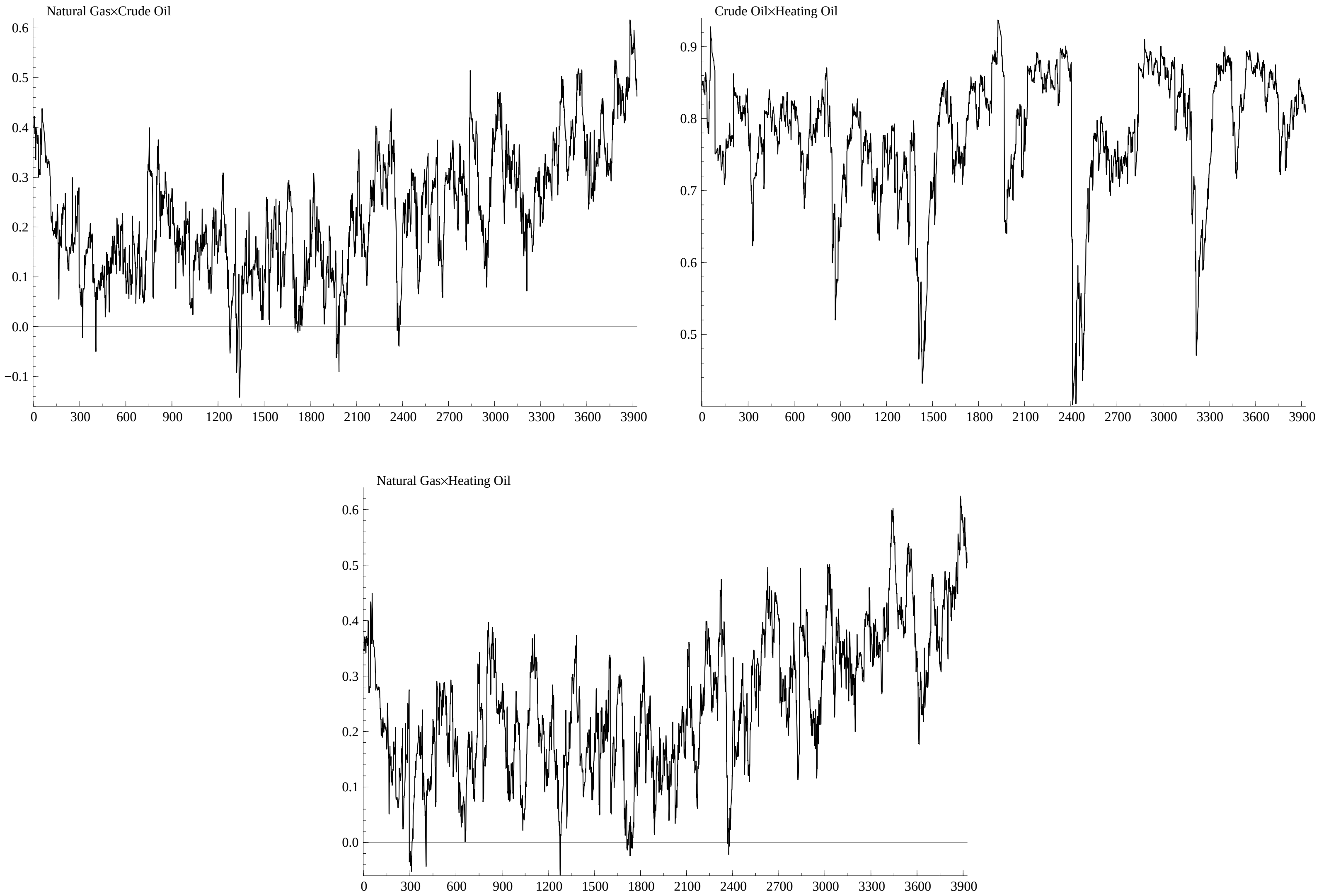

Figure 5—: Estimated conditional correlations

Table 1: Sample statistics

	Crude oil	Natural gas	Heating oil
Minimum	-40.04	-37.57	-39.09
Maximum	14.23	32.43	13.99
Mean	0.013	0.041	0.016
Stand. dev.	2.30	3.61	2.38
Skewness	-15.52	-35.64	-1.75
Kurtosis	29.85	13.65	27.40
Jarque-Bera	$1.2e5$	$1.9e4$	$9.9e4$
	[0.0]	[0.0]	[0.0]
Anderson-Darling	45.9413	n.a.	47.2272
	[0.0]		[0.0]
BDS(2)	5.7092	8.7307	6.7412
	[$1.13e{-8}$]	[0]	[$1.57e{-11}$]

	Correlations		
	Crude oil	Natural gas	Heating oil
Crude oil	1	0.21	0.75
Natural gas	0.21	1	0.26
Heating oil	0.75	0.26	1

Legend: p-values are in brackets. The BDS test was computed by setting the largest dimension to 2, and the length of the correlation integral to one times the standard deviation of the series. These values are chosen so that the first-order correlation integral estimate lies around 0.7

Table 2: Tests of Engle (1982) and Engle and Sheppard (2001)

Lag	Engle (1982)			Engle and Sheppard (2001)
	Crude oil	Natural gas	Heating oil	
1	37.79 [0.0]	12.20 [0.0]	7.54 [0.0]	28.53 [0.0]
2	54.20 [0.0]	19.79 [0.0]	15.46 [0.0]	33.96 [0.0]
3	162.23 [0.0]	98.04 [0.0]	56.79 [0.0]	45.49 [0.0]
4	164.56 [0.0]	125.59 [0.0]	56.74 [0.0]	45.52 [0.0]
5	164.72 [0.0]	131.20 [0.0]	56.97 [0.0]	52.70 [0.0]

Legend: p-values are in brackets.

TABLE 3: Estimates of the VAR(2)

	Coefficient
Crude oil (t-1)	0.018 [0.735]
Crude oil (t-2)	−0.043 [−1.714]
Natural gas (t-1)	0.013 [1.171]
Natural gas (t-2)	0.036 [3.293]
Heating oil (t-1)	−0.061 [−2.499]
Heating oil (t-2)	−0.007 [−0.283]
Constant	0.018 [0.470]

Legend: t coefficients are in brackets.

Table 4: Maximized values of the likelihood

Distribution	$\hat{\mathcal{L}}$
Normal	-25053.87
Laplace	-27530.95
$t(1)$	n.a.
$t(2)$	n.a.
$t(3)$	n.a.
$t(4)$	-24245.36
$t(6)$	-24274.28
$t(6.5)$	-24287.34
$t(7)$	-24301.06
$t(7.5)$	-24315.07
$t(8)$	-24329.13

Table 5: Estimated coefficients of the DCC $t(6)$ model

	Coefficient estimates				
Parameter	$t(4)$	$t(6)$	$t(8)$	Normal	Laplace
ω(Crude oil)	0.0491	0.0419	0.0398	0.0405	0.0447
ν(Crude oil)	0.0522	0.0461	0.0451	0.0623	0.0534
γ(Crude oil)	0.9447	0.9439	0.9435	0.9317	0.9424
ω(Natural gas)	0.1718	0.1598	0.1616	0.2725	0.1846
ν(Natural gas)	0.0790	0.0712	0.0706	0.1037	0.0815
γ(Natural gas)	0.9165	0.9135	0.9114	0.8847	0.9112
ω(Heating oil)	0.0875	0.0749	0.0716	0.0751	0.0835
ν(Heating oil)	0.0651	0.0583	0.0576	0.0804	0.0677
γ(Heating oil)	0.9266	0.9250	0.9240	0.9088	0.9226
A	0.0248	0.0253	0.0256	0.0277	0.0257
B	0.9626	0.9624	0.9623	0.9558	0.9613