Optimal Fiscal Policy with Private and Public Investment in Education

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Abstract

This paper develops a three periods OLG growth model where agents accumulate human capital in the first period (benefiting from grandfathers savings, public expenditure in education, and human capital

level of their parents), work and save in the second period (earning a salary proportional to the human capital level accumulated), and retire in

the thid period, leaving bequest on for the young generation. Government raises taxes on labour and capital in order to finance public expenditure in education. We derive conditions for equilibrium and for the rate of growth, and then carry out welfare analysis in order to determine optimal taxation in a Nash policy setting.

JEL classification: H30, H52, H21

1 Introduction

This paper intends to contribute to the literature on optimal fiscal policy in economic growth by enriching the microfounded model underneath. In order to do so, we borrow and modify the framework from an apparently distant field, the literature on overlapping generation models with human capital accumulation and intergenerational transfer.The idea is to enrich the microfoundations of the model with more realistical features, such as the financing of education by older generations'lifetime savings, and the inclusion of taxation on both production factors, whose revenue is used by the government to raise the marginal utility of private investment in education[1] As a result of the welfare analysis, optimal tax rates on labour and capital are no longer zero (as in much of endogenous growth theory), and quantitative simulations lead to plausible values.

2 Review of the Literature

The importance of education and human capital in economic growth has been supported by a number of valuable contributions, starting from Romer (1986), Lucas (1988) and Barro, Sala-i-Martin (1995); all of them establish the fact that human capital accumulation is a major engine of long-run growth (for a good discussion on the issue, see Temple (1999,2000)). This represent the first, and fundamental, line of research which is relevant for the scope of the present paper.

A second line of research, instead, explores in depth the link between endogenous growth and human capital accumulation fully microfounded models with intergenerational links and schooling, and often in general equilibrium. Glomm and Ravikumar (1992) seems to be the first contribution in this stream; they present an overlapping generations model with heterogeneous agents in which human capital investment through schooling in the engine of growth; they analyze the evolution of growth and inequality (the latter is made possible by the assumption of lognormal distribution of initial skills) under both a public and private education scheme: in the former, investment in the quality of education is made through majority voting, whereas in the latter each household is free to choose (i.e. to buy) the quality of education she prefers. They conclude that public education reduces income inequality more quickly than private education, but private education yields higher per capita incomes (unless the initial income inequality is sufficiently large). Although this is not a general equilibrium model (there is no production), it is the first consistent and relevant attempt to address the issue of growth starting from a microfounded OLG model with intergenerational altruism (in fact, fathers care about the level of education achieved by their children); moreover, the analysis is enriched by the introduction of heterogeneity of agents, a feature that has not been so common in the later literature (

[1] This can be more broadly interpreted as public investment in education.

a similar analysis is conducted by Zhang 1996, who achieves not-so-different results in analysing the choice between provinding public education or subsidizing private one).

In an extension of their work, ten years later, Glomm and Ravikumar (2003) retain most of their previous framework, but focus on how an economy with public education behaves over time under different funding levels. In order to do so in a proper way, they abandon the logarithmic preferences and Cobb-Douglas technology forms (previously assumed for tractability) and use CRRA utility function and non-restricted human capital accumulation equations. As a result, time allocated to learning is no longer constant, and it is instead a non-trivial function of parental human capital and public expenditure on education, so that there are richer effects on distribution; the insertion of a CRRA functional form make the risk adversion parameter crucial, in comparision with the magnitude of the parameter governing the importance of parental human capital, in determing the convergence/ divergence: in one case, the growth rate of income is decreasing in the level of income (i.e. lower income families grow faster), in the other is increasing. They perform a set of simulation and conclude that the possibility of divergence, at least in the short run, cannot be ruled out by the comparative dynamics of the model, and thus public education may not always be the "great equalizer" as intended by its proponents.

For a more complete assessment of the issue, Brauninger and Vidal (2000) examines interactions between education policy and growth in a three periods-OLG models with two types of individuals (skilled and unskilled) and no population growth. Growth is determined by physical capital (with learning-by-doing) and human capital; public education spending takes the form of a education susbidy, who increases the proportion of skilled individuals (the choice of educating or not is exogenous and discrete, and the degree of intergenerational altruism is summarized by a i.i.d. random variable). They conclude that there are non-linear effects of an increased public expenditure on growth, since the design of the model (targeted at the effect of education policy on skills and saving) generates at the macroeconomic level a trade-off between the two sources of growth: on one hand, increasing the average level of skills in the economy increases the efficiency of labour and it is good for growth, on the other education expenditures crowd out saving and the formation of physical capital that is the other engine of growth through the learning-by-doing effect[2]. As a result, inducing more people to educate their offspring by increasing public education expenditure may have the above-mentioned non-linear effects on growth: starting from a skilled trapped economy (i.e. an economy in which almost no one is educated) harms growth and increases inequality: a former equal but unskilled society becomes unequal as some families educate their children and at the same time private education expenditures crowds out savings and results in a lower growth. On the other hand, increasing the subsidy from a sufficiently high level of education promotes growth and reduces inequality: the society, in fact, be-

[2] This latter effect is due to the fact that due to the increase in the public subsidy, some of the unskilled partents now decide to educate their offspring. These families now save less and there is, thus, a negative effect on the growth rate.

comes more equal as almost all individuals are skilled, and the adverse effect on savings is low as the increase in private education expenditures associated with a high level of the subsidy is moderate. These results conform with the stylized fact that more equal societies grow faster than unequal ones (see Persson and Tabellini 1994). It is important, for our scopes, to note that in this framework they assume non-distorsionary taxation (the specific focus being the trade off between resources devoted to savings and education).

Blankenau and Simpson (2004), instead, explore the link between public education and growth in a similar framework (three periods-OLG general equilibrium model) but inserting distorsionary taxation. They build a model with homogenous agents in which human capital evolves according to private input, parental human capital and public expenditure on education, the latter being financed by taxation on consumption, on wage income and on capital in addiction to non-distorsionary taxation. They solve the model in steady-state and carry out a number of simulations by increasing the public expenditure on education but varying the source of financing, and conclude that the direct effect of this policy is an increase in the steady-state growth rate, but general equilibrium adjustments in other factors that affect growth may act in the opposite direction. Moreover, there are significant differences in this crowding-out process depending on whether the source of financing is mainly distorsionary or rather non-distorsionary taxation. In their opinion, these considerations can help explaining the nonmonotonic nature of the relationship public expenditure on education - growth[3], and can trigger a more cohmprensive analysis on how appropriate tax policies can help turn government education spending into a more efficient engine of growth.

A good contribution to the issue of effectiveness and financial sustainability of public policy when human capital accumulation process is specifically taken into account, comes from Glomm and Kaganovich (2003), who enriche the framework by studying the provision of public education in conjunction with the existence of a pension systems. The general equilibrium model focuses on the dynamic distributional effects of raising funding levels for public education in the presence of pay-as-you-go social security system, in a two-periods OLG economy with heterogenous agents and in which the government runs both programmes (education and pensions). Both public policy are financed by taxation on labour income but at different tax rates, and while education expenditure goes into human capital accumulation process (modelled in a similar fashion as Glomm, Ravikumar 1992 and 2003), pensions are paid to the old, entering second-period budget constraint. They study how an increase in the education tax rate affects the distribution of human capital in the economy, analysing the different co-movements of the two tax rates (namely, whether an increase in the tax rate on education is balanced by an analogous decrease in the tax rate used to finance pensions or rather the two fiscal policy instruments move independently). From these interesting fiscal policy considerations, the main conclusion

[3] The fact that we cannot find a clear and unambigous empirical validation that increasing public education expenditure also increases growth is one the major puzzle in this field.

is the arising of the possibility that raising funding for public education may generate higher inequality, since it may imply a reduction in social security, which in turn distorts the agents' incentives to invest private resources into the education of their children.

A third line of research focuses on the optimal taxation of human and physical capital in (neoclassical or endogenous) growth models. In the standard neoclassical growth model, where physical capital is the only factor that can be accumulated, the normative implication is that the optimal long-run tax on capital income should be zero, while the optimal tax rate on labour income should be positive[4] (Judd (1985, 1995), Chamley (1986), Diamond (1973)). As far as endogenous growth models are concerned, traditional literature (Bull (1993), Jones et al (1993,1997)) shows that optimal tax on both capital and labour income should be zero. Milesi-Ferretti and Roubini (1998) confirm this result in a three-sector general equilibrium endogenous growth model in continuos time where human capital accumulation depends also on physical as well as human capital, and when leisure is treated in a more sophisticated fashion (as a proper production sector). However, in this stream of literature (more resembling the first one we mentioned in this review), public expenditure takes the form of an exogenously-given path who does not enter the human capital production, and there is no kind of overlapping generations framework with intergenerational altruism and/or transfer.

At this point, the scope of the present paper are fairly clear. We intend to analyse the relationship between human capital accumulation and growth (first line of research above mentioned) using an OLG framework with intergenerational links (second line) with particular regard to tax policy and its effects on growth (third line).

3 The model

In order to do so, we innovate on the previous frameworks (mainly referring to the second line of literature) by building a two-period OLG model with intergenerational altruism and monetary bequest from the old generation to the young, which is directed to human capital formation.

3.1 Households

Households maximize the following utility function:

$$U_t = \ln c_{t,t} + \beta \ln c_{t,t+1} + \gamma \ln h_{t+1} \tag{1}$$

where:

[4] The underlying reason is that the distorsion created by capital income taxation is more severe: in fact, by reducing the return to savings, it taxes future consumption at an increasing rate, thereby inducing an *intertemporal* distorsion. On the other hand, a labour income tax affects only the labour/leisure choice, which is an *intratemporal* distorsion.

$c_{t,t}$ = consumption in period t
$c_{t,t+1}$ =consumption in period $t+1$
h_{t+1} = level of human capital of his offspring

$0 < \beta < 1$ is the discount factor, and $\gamma > 0$ measures the degree of altruism, telling us how much the agent cares about his childrens'education[5].

In period t, the consumer works (earning a wage proportional to her human capital level) and divides her net disposable income across consumption and savings. The corresponding period budget constraint is therefore:

$$c_t + s_t = (1 - \tau_w) w_t h_t \tag{2}$$

where:
s_t = savings
$w_t h_t$ = labour income (with w_t the wage, and $h_t$the human capital level)
τ_w =tax rate on labour

In period $t+1$, the consumer receives yields on period t savings but pays taxes on this financial income; then she divides the resulting net income across consumption and savings, devoting the latter to the human capital accumulation of the young, as bequest.

$$c_{t+1} + s_{t+1} = (1 + (1 - \tau_k) r) s_t \tag{3}$$

where:
r = interest rate
τ_k = taxation on capital income

Since population grows according to:

$$N_{t+1} = (1+n) N_t \tag{4}$$

Each "old" saves s_{t+1} and gives it back to young people, so that the percapite variable is $\frac{s_{t+1}}{(1+n)^2}$

Human capital evolves according to:

$$h_{t+1} = \frac{s_{t+1}}{(1+n)^2} E^{\mu} h_t^{1-\mu} \tag{5}$$

where:

[5] Here altruism is introduced (as in Glomm and Kaganovich 2002) via the inclusion of the childrens'achievements in terms of human capital level, which is directly linked to children's utility since the more they study, the more they earn and hence the more they can consume. In the literature, bequest motive has assumed the form of childrens'income (Brauninger and Vidal 2000), the quality of childrens'education (Glomm and Ravikumar 2000), or childrens'utility (Zhang 1996).

E_t = public expenditure on education
h_t =parental human capital
$\frac{s_{t+1}}{(1+n)^2}$ = private input to education

The related literature has always assumed a similar specification for the learning technology. Regarding the arguments of the function, whereas there is common consensus on the importance of public expenditure[6] and of parental human capital[7], many have focused on time devoted to learning (Glomm and Ravikumar (2002,2003),(Blankenau andSimpson (2004)) rather than physical resources in monetary terms[8]. Regarding the functional form, specifications vary only in terms of the degree of homogeneity of (5), establishing degree one (and thus constant returns to scale, as in Blankenau and Simpson (2004)), or choosing not to restrict parameters (as in other studies). A notable exception in both dimensions is the work of Galor et al.,, who assume an implicit functional form $h_{t+1} = f(e_t)$ where e_t is alternatively public funding of education (as in Galor, Moav and Vollrath (2003) and Galor and Moav (2006)) or private funding (Galor and Moav (2004))[9].

Maximization problem is:

$$\max_{s_t, s_{t+1}} (1)$$
$$s.t.(2), (3), (5)$$

and it has to be solved via backward induction, following the temporal sequence in the reverse order: so first we maximize with respect to s_{t+1}, and then with respect to s_t , to get the optimal combination of the two choice variables:

$$s_t^* = \frac{(\beta + \gamma)}{1 + \beta + \gamma}(1 - \tau_w)w_t h_t \tag{6}$$

$$s_{t+1}^* = \frac{\gamma}{1 + \beta + \gamma}(1 + (1 - \tau_k)r))(1 - \tau_w)w_t h_t \tag{7}$$

Exploiting budget constraints (2) and (3) we can also get expressions for optimal consumption in both periods:

$$c_t = \left(\frac{1}{1 + \beta + \gamma}\right)(1 - \tau_w)w_t h_t \tag{8}$$

$$c_{t+1} = \frac{\beta}{1 + \beta + \gamma}(1 + (1 - \tau_k)r))(1 - \tau_w)w_t h_t \tag{9}$$

[6] For example, in OECD countries, public school enrollment exceeds 70 per cent (source:OECD).

[7] See for example Coleman et al (1966) and Oreopoulos et al (2003).

[8] One relevant exception is Glomm and Kaganovich (2003), who adopt a specification in which private and public funding, plus time devoted by parents to childrens'education, coexist.

[9] Galor et al analyze this issue in the context of a different stream of research, regarding the process of development and the class structure from Industrial Revolution onwards.

Optimal accumulation of human capital results to be:

$$h_{t+1} = \frac{\gamma}{(1+\beta+\gamma)}(1-\tau_w)w_t E^{\mu}(1+(1-\tau_k)r))\ h_t^{2-\mu} \tag{10}$$

3.2 Firms

Consistently with historical evidence (Goldin and Katz (1998)), the economy is characterized by capital-skill complementarity:

$$Y_t = AK_t^{\alpha} H_t^{1-\alpha} \tag{11}$$

With the assumption of perfectly competitive markets:

$$r = \alpha \frac{Y}{K} \tag{12}$$

$$w = (1-\alpha)\frac{Y}{H} \tag{13}$$

Production function in units of efficient labour, calling $y_t = \frac{Y_t}{H_t}$ and $k_t = \frac{K_t}{H_t}$, is:

$$y_t = Ak_t^{\alpha} \tag{14}$$

And also:

$$r = \alpha A k_t^{\alpha-1} \tag{15}$$

$$w = (1-\alpha)Ak^{\alpha} \tag{16}$$

3.3 Government

Government raises taxes on wage and capital income and uses it to finance public expenditure on education. Balanced budget constraint is assumed, so that no deficit can arise:

$$E_t = \tau_w w_t h_t + \tau_k r k_t \tag{17}$$

Note that in the welfare analysis we will first analyse the simplest case, where only taxation on labour exists.

3.4 Growth

Growth is given by:

$$\frac{Y_{t+1}}{Y_t} = \left(\frac{K_{t+1}}{K_t}\right)^{\alpha} \left(\frac{H_{t+1}}{H_t}\right)^{1-\alpha} \tag{18}$$

We assume that first-period savings go to capital formation, so $s_t = k_t$; we thus plug the equation for first-period optimal saving (6) into (18):

$$\frac{Y_{t+1}}{Y_t} = \left(\frac{w_{t+1}}{w_t}\right)^{\alpha} \left(\frac{h_{t+1}}{h_t}\right) \tag{19}$$

Using the FOC for wages and simplifying:

$$\frac{Y_{t+1}}{Y_t} = \left(\frac{\frac{Y_{t+1}}{H_{t+1}}}{\frac{Y_t}{H_t}}\right)^{\alpha} \left(\frac{h_{t+1}}{h_t}\right)$$

$$\frac{Y_{t+1}}{Y_t} = \left(\frac{Y_{t+1}}{H_{t+1}}\frac{H_t}{Y_t}\right)^{\alpha} \left(\frac{h_{t+1}}{h_t}\right)$$

$$\left(\frac{Y_{t+1}}{Y_t}\right)^{1-\alpha} = \left(\frac{h_{t+1}}{h_t}\right)^{1-\alpha} \tag{20}$$

Equation (20) tells us that the **rate of growth of output is the same as the rate of growth of human capital.**

Let us now compute it, taking (5) and dividing by h_t:

$$g = \frac{h_{t+1}}{h_t} = \frac{s_{t+1}}{(1+n)^2} E^{\mu} h_t^{-\mu}$$

Plugging in the expression for s_{t+1} (7):

$$g = \frac{\gamma}{(1+\beta+\gamma)(1+n)^2}(1+(1-\tau_k)r))(1-\tau_w)w_t E^{\mu} h_t^{1-\mu} \tag{21}$$

Equation (21) tells us that the rate of growth is positively affected by:
- an increase in public expenditure in education (and its efficiency)
- the degree of altruism

and it is negatively affected by:
- the increase in population
- taxation on labour and capital

3.5 Optimal taxation

This section is concerned with welfare analysis in order to define optimal taxation. Since utility is logaritmic, in order to conduct welfare analysis we can simply maximize the individual utility function with respect to the tax rates. Let us first analyse the case with only one tax rate (on labour), and then insert the one on capital income.

3.5.1 Welfare analysis with only one tax rate

Social planner problem is thus:

$$\begin{aligned}
\max_{\tau} U_t &= \ln c_{t,t} + \beta \ln c_{t,t+1} + \gamma \ln h_{t+1} \qquad (22)\\
& \quad s.t. \\
E &= \tau W h \\
c_t &= \left(\frac{1}{1+\beta+\gamma}\right)(1-\tau_w) w_t h_t \\
c_{t+1} &= \frac{\beta}{1+\beta+\gamma}(1+r)(1-\tau_w) w_t h_t \\
h_{t+1} &= \frac{\gamma}{(1+\beta+\gamma)}(1-\tau_w) w_t E^{\mu}(1+r)\ h_t^{2-\mu}
\end{aligned}$$

Making use of the firms FOC (12) and (13), and dragging along only the terms in τ[10],we get to the following unconstrained maximization problem (details of the calculations can be found in Appendix A):

$$\max_{\tau} U = \frac{1}{1-\alpha}\ln(1-\tau) + \beta\left[\frac{\alpha}{1-\alpha}\ln(1-\tau)\right] + \gamma\left[\frac{1}{1-\alpha}\ln(1-\tau) + \mu\ln\tau + \mu\frac{\alpha}{1-\alpha}\ln(1-\tau)\right]$$

In the proceeding of the maximization, we distinguish three cases, according to the assumption we make about β(the discount factor) and γ (the degree of altruism):

- assuming $\beta = \gamma = 1$
- assuming $\beta, \gamma \neq 1$
- assuming $\beta = 1$ and $\gamma \neq 1$

3.5.2 Assuming $\beta = \gamma = 1$

Social planner problem becomes:

$$\max_{\tau} 2\ln(1-\tau) + \frac{3\alpha}{1-\alpha}\ln(1-\tau) + \mu\ln\tau + \frac{\mu\alpha}{1-\alpha}\ln(1-\tau) \qquad (23)$$

which leads to:

$$\tau^* = \frac{\mu(1-\alpha)}{\alpha+\mu+2} \qquad (24)$$

[10] We can do this thanks to the assumption of logaritmic utility, which greatly simplifies the calculations.

Qualitative analysis: derivative analysis *Derivative of (24)with respect to* μ

$$\frac{\partial\tau^*}{\partial\mu} = \frac{(1-\alpha)(\alpha+2)}{(\alpha+\mu+2)^2} > 0 \tag{25}$$

So $\frac{\partial\tau^*}{\partial\mu} > 0$, that is, raising the efficiency of public expenditure on education raises the optimal tax rate.

Derivative of (24) with respect to α :

$$\frac{\partial\tau^*}{\partial\alpha} = -\frac{\mu(3+\mu)}{(\alpha+\mu+2)^2} \quad < 0 \tag{26}$$

So, raising the share of output going to capital (=decreasing the share of output going to human capital), decreases optimal tax rate.

Quantitative analysis :simulation of τ^* holding $\alpha = 0.3$ If we take for granted the value that literature has traditionally assigned to α,we can carry out a number of simulations varying the value of μ:

The table:

μ	$\tau^*_{\alpha=0.3}$
0.1	0.0250
0.2	0.0483
0.3	0.0700
0.4	0.0903
0.5	0.1094
0.6	0.1273
0.7	0.1441
0.8	0.1600
0.9	0.1750
1	0.1892

Whereas the graph is:

FIGURE 1 HERE

If we want to do a comparision using different values of α :

$$\left\{\begin{array}{cccc} \mu & \tau^*_{\alpha=0.2} & \tau^*_{\alpha=0.3} & \tau^*_{\alpha=0.4} \\ 0.1 & 0.0276 & 0.0250 & 0.0240 \\ 0.2 & 0.0533 & 0.0483 & 0.0462 \\ 0.3 & 0.0774 & 0.0700 & 0.0667 \\ 0.4 & 0.1000 & 0.0903 & 0.0857 \\ 0.5 & 0.1212 & 0.1094 & 0.1034 \\ 0.6 & 0.1412 & 0.1273 & 0.1200 \\ 0.7 & 0.1600 & 0.1441 & 0.1355 \\ 0.8 & 0.1778 & 0.1600 & 0.1500 \\ 0.9 & 0.1946 & 0.1750 & 0.1636 \\ 1 & 0.2105 & 0.1892 & 0.1765 \end{array}\right\}$$

3.5.3 Assuming $\beta, \gamma \neq 1$

In that case, the maximization leads to:

$$\tau^* = \frac{\gamma\mu(1-\alpha)}{1+\beta\alpha+\gamma+\mu\gamma} \tag{27}$$

from which we can see that if optimal taxation increases with the degree of altruism, in fact:

$$\frac{\partial\tau^*}{\partial\gamma} = \frac{\mu(1-\alpha)(1+\beta\alpha)}{[1+\beta\alpha+\gamma+\mu\gamma]^2} > 0 \tag{28}$$

3.5.4 Assuming $\beta = 1$and $\gamma \neq 1$

In that case:

$$\tau^* = \frac{\gamma\mu(1-\alpha)}{1+\alpha+\gamma+\mu\gamma} \tag{29}$$

3.5.5 Optimal rate of growth

Let us retain the assumption of one tax rate on labour.

Then equation (21) becomes:

$$g = \frac{\gamma}{(1+\beta+\gamma)(1+n)^2}(1+r)(1-\tau)w_t E^\mu h_t^{1-\mu} \tag{30}$$

which, plugging in the previous expressions and the optimal tax rate (24), we get an expression for the optimal rate of growth:

$$g = \left(\frac{\mu(1-\alpha)}{\alpha+\mu+2}\right)^\mu \left(\frac{\alpha(1+\mu)+2}{(\alpha+\mu+2)(1+\beta+\gamma)}\right)^{\frac{\alpha(1+\mu)}{1-\alpha}} (1-\alpha)^{\frac{\mu+\alpha}{1-\alpha}} (\beta+\gamma)^{\frac{\alpha(2+\mu)-1}{1-\alpha}} \frac{\alpha\gamma}{(1+n)^2} A^{\frac{1+\mu}{1-\alpha}} h_t \tag{31}$$

3.5.6 Welfare analysis with two tax rate

Now we consider, as in the original setting in the first part of the paper, the existence of both tax rates, $\tau_w$and τ_k.

Maximization problem is now:

$$\begin{aligned} & \max_{\tau_k,\tau_w} \ln c_{t,t} + \beta \ln c_{t,t+1} + \gamma \ln h_{t+1} \qquad (32) \\ & s.t. \\ E &= \tau_w w_t h_t + \tau_k r_t s_t \\ c_t &= \left(\frac{1}{1+\beta+\gamma}\right)(1-\tau_w) w_t h_t \\ c_{t+1} &= \frac{\beta}{1+\beta+\gamma}(1+(1-\tau_k)r)(1-\tau_w) w_t h_t \\ h_{t+1} &= \frac{\gamma}{(1+\beta+\gamma)}(1-\tau_w) w_t E^{\mu}(1+(1-\tau_k)r)\ h_t^{2-\mu} \end{aligned}$$

The problem reduces to:

$$\begin{aligned} \max_{\tau_k,\tau_w} U &= \frac{1}{1-\alpha}\ln(1-\tau_w) + \beta\left[\ln(1-\tau_k) + \frac{\alpha}{1-\alpha}\ln(1-\tau_w)\right] + \qquad (33) \\ & +\gamma\left[\frac{\alpha(1+\mu)}{1-\alpha}\ln(1-\tau_w) + \ln(1-\tau_k) + \mu\ln\left(\tau_w + \tau_k\frac{\alpha}{1-\alpha}\right)\right] \end{aligned}$$

As we did in the one-tax case,let us assume for simplicity that $\beta = \gamma = 1$,so (33) becomes:

$$U = \left(\frac{1+2\alpha+\alpha\mu}{1-\alpha}\right)\ln(1-\tau_w) + 2\ln(1-\tau_k) + \mu\ln\ \left(\tau_w + \tau_k\frac{\alpha}{1-\alpha}\right) \qquad (34)$$

Let us first maximize wrt τ_w :

$$\tau_w^* = \frac{-\alpha(1+\alpha(2+\mu))}{\mu(1-\alpha)+\alpha(1-2\alpha)+1}\tau_k + \frac{\mu(1-\alpha)^2}{\mu(1-\alpha)+\alpha(1-2\alpha)+1} \qquad (35)$$

And then wrt τ_k :

$$\tau_k^* = \frac{-2(1-\alpha)}{\alpha(2+\mu)}\tau_w + \frac{\mu\alpha}{\alpha(2+\mu)} \qquad (36)$$

In both cases there is a negative relationship between tax rates, so the reaction functions are both negatively sloped. The system is thus:

$$\tau_w^* = \frac{-\alpha(1+\alpha(2+\mu))}{\mu(1-\alpha)+\alpha(1-2\alpha)+1}\tau_k + \frac{\mu(1-\alpha)^2}{\mu(1-\alpha)+\alpha(1-2\alpha)+1} \quad (37)$$
$$\tau_k^* = \frac{-2(1-\alpha)}{\alpha(2+\mu)}\tau_w + \frac{\mu\alpha}{\alpha(2+\mu)}$$

The "Nash solution" is:

$$\tau_w^* = \frac{-2\mu\alpha+2-5\alpha+\mu}{(1-\alpha)(\mu+3)} \quad (38)$$

$$\tau_k^* = \frac{\mu^2\alpha+7\mu\alpha+10\alpha-2\mu-4}{\alpha(2+\mu)(\mu+3)} \quad (39)$$

Some quantitative simulations on τ_w:

μ	$\tau^*_{w,\alpha=0.2}$	$\tau^*_{w,\alpha=0.3}$	$\tau^*_{w,\alpha=0.4}$
0.1	0.4274	0.2488	0.0108
0.2	0.4375	0.2589	0.0208
0.3	0.4470	0.2684	0.0303
0.4	0.4559	0.2773	0.0392
0.5	0.4643	0.2857	0.0476
0.6	0.4722	0.2937	0.0556
0.7	0.4797	0.3012	0.0631
0.8	0.4868	0.3083	0.0702
0.9	0.4936	0.3150	0.0769

Summary 1 ***Proposition 2*** *In a Nash-setting, the tax rate on labour is always positive.*
Proof. *Take (38):* ■

$\tau_w^* > 0$ if $-2\mu\alpha + 2 - 5\alpha + \mu > 0$
$-2\mu\alpha + \mu > 5\alpha - 2$
$\mu(1-2\alpha) > 5\alpha - 2$
If $\alpha < 1/2$
$\mu > \frac{5\alpha-2}{1-2\alpha}$
For $0 < \alpha < 1$ this is always satisfied.

Proposition 3 *In a Nash-setting, the tax rate on capital is positive for plausible values of parameters α and μ.*

Proof. Take (39): ■

$\tau_k^* > 0$ if $\mu^2\alpha + 7\mu\alpha + 10\alpha - 2\mu - 4 > 0$
It looks like τ_k^* is positive for plausible values of α and μ.

$$\left\{\begin{array}{cccc} \mu & \tau_k^*(\alpha=0.3) & \tau_k^*(\alpha=0.35) & \tau_k^*(\alpha=0.4) \\ 0.1 & -0.98 & -0.45 & 0.08 \\ 0.2 & -0.96 & -0.39 & 0.17 \\ 0.3 & -0.94 & -0.33 & 0.27 \\ 0.4 & -0.91 & -0.26 & 0.38 \\ 0.5 & -0.87 & -0.18 & 0.50 \\ 0.6 & -0.83 & -0.10 & 0.62 \\ 0.7 & -0.78 & -0.01 & 0.75 \\ 0.8 & -0.72 & 0.08 & 0.89 \\ 0.9 & -0.66 & 0.18 & 1.04 \\ 1 & -0.60 & 0.3 & 1.2 \end{array}\right\}$$

For $\alpha = 0.3$ no value of μ satisfies the equation.
For $\alpha = 0.4$ all values of μ satisfy the equation (so $\tau_k^* > 0$).
For $\alpha = 0.35$ μ has to be greater than 0.8.

To evaluate welfare, plug (38) and (39) into the social welfare function to get:

$$U = \left(\frac{1+2\alpha+\alpha\mu}{1-\alpha}\right)\ln\left(\frac{\mu\alpha+2\alpha+1}{(1-\alpha)(\mu+3)}\right)+2\ln\left(\frac{2(1-\alpha)}{\alpha(\mu+3)}\right)+\mu\ln\left(\frac{\mu(1-\alpha)(\mu+2)+8}{(\mu+2)\,(\mu+3)(1-\alpha)}\right) \tag{40}$$

4 Conclusions

This paper attempted to analyse the problem of optimal fiscal policy in an endogenous growth model by enriching the microfoundation of the model itself; in order to do so, we built a three-period overlapping generation model in which accumulation of human capital depends on private investment in education (the bequest left by the oldest generation to young people), human capital of the previous generation (following empirical evidence) and public investment in education (financed by taxation on labour and capital). Welfare analysis is conducted both with one and then two tax rates, and in either case our findings are that optimal taxation in production factors, for plausible values of structural parameters is no longer zero.

Possible extensions of this work include the insertion of endogenous labour supply and more detailed analysis of human capital accumulation that overcomes the *ad-hoc* assumption of the current functional form. Moreover, we would like to develop a richer demographic structure, capable to reproduce more efficiently stylized facts in advanced countries, which are currently observing aging population with increasing pressure on welfare states and growth.

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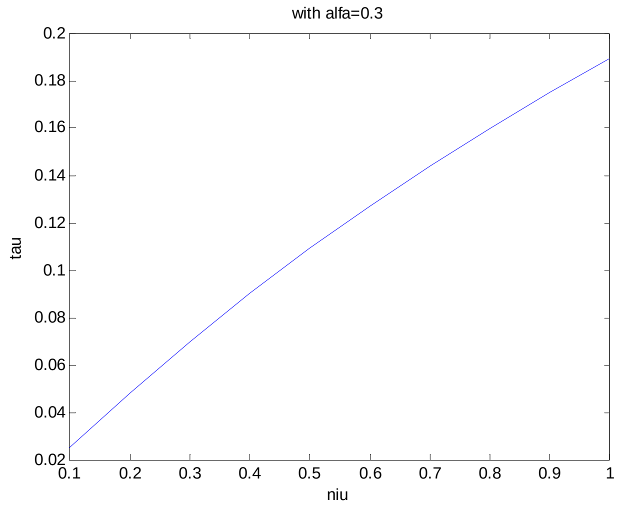

Figure 1:

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6 Tables of figures

Optimal tax rate (τ) according to the efficiency of public expenditure in education (μ):

7 Appendix A

7.1 Social planner problem with one tax rate

Utility function is:

$$U_t = \ln c_{t,t} + \beta \ln c_{t,t+1} + \gamma \ln h_{t+1} \tag{A1}$$

Optimal choice of c_t, c_{t+1}, h_{t+1} are:

$$c_t = \left(\frac{1}{1+\beta+\gamma}\right)(1-\tau_w)w_t h_t \tag{A2}$$

$$c_{t+1} = \frac{\beta}{1+\beta+\gamma}(1+r)(1-\tau_w)w_t h_t \tag{A3}$$

$$h_{t+1} = \frac{\gamma}{(1+\beta+\gamma)}(1-\tau_w)w_t E^{\mu}(1+r)\ h_t^{2-\mu} \tag{A4}$$

Exploiting government balanced budget constraint $E = \tau W h$, equation (A4) becomes:

$$h_{t+1} = \frac{\gamma}{(1+\beta+\gamma)}(1-\tau_w)w_t(\tau W h)^{\mu}(1+r)\ h_t^{2-\mu} \tag{A5}$$

From firms'FOC:

$$1+\ r = \alpha A k_t^{\alpha-1} \tag{A6}$$

$$w = (1-\alpha)Ak^{\alpha} \tag{A7}$$

From maximization we also know that:

$$k_t = \frac{K_t}{h_t} = \frac{s_t}{h_t} = \frac{\beta+\gamma}{1+\beta+\gamma}(1-\tau)w_t \tag{A8}$$

Put (A8) into (A7):

$$w = \left[(1-\alpha)\,A\left(\frac{\beta+\gamma}{1+\beta+\gamma}\right)^{\alpha}(1-\tau)^{\alpha}\right]^{\frac{1}{1-\alpha}} \tag{A9}$$

Put (A9) and (A8) into (A6):

$$1+r = \frac{\alpha}{\left(\frac{\beta+\gamma}{1+\beta+\gamma}\right)(1-\tau)(1-\alpha)} \tag{A10}$$

So the relevant logarithms (considering that we are maximizing w.r.t. τ,so we need to drag along only those terms) of (A9) and (A10) are:

$$\ln w = \frac{\alpha}{1-\alpha}\ln(1-\tau) \tag{A11}$$

$$\ln(1+r) = -\ln(1-\tau) \tag{A12}$$

From (A2),(A3), (A4), the relevant logarithms for maximization turn out to be:

$$\ln c_t = \ln(1-\tau) + \ln w \tag{A13}$$

$$\ln c_{t+1} = \ln(1+r) + \ln(1-\tau) + \ln w \tag{A14}$$

$$\ln h_{t+1} = \ln(1-\tau) + \ln w + \mu[\ln\tau + \ln w] + \ln(1+r) \tag{A15}$$

Using (A11) and (A12):

$$\ln c_t = \ln(1-\tau) + \frac{\alpha}{1-\alpha}\ln(1-\tau) \tag{A16}$$

$$\ln c_{t+1} = \frac{\alpha}{1-\alpha}\ln(1-\tau) \tag{A17}$$

$$\ln h_{t+1} = \ln(1-\tau) + \frac{\alpha}{1-\alpha}\ln(1-\tau) + \mu\ln\tau + \mu\frac{\alpha}{1-\alpha}\ln(1-\tau) \tag{A18}$$

Putting (A16), (A17), (A18) into (A1):

$$U = \frac{1}{1-\alpha}\ln(1-\tau) + \beta\left[\frac{\alpha}{1-\alpha}\ln(1-\tau)\right] + \gamma\left[\frac{1}{1-\alpha}\ln(1-\tau) + \mu\ln\tau + \mu\frac{\alpha}{1-\alpha}\ln(1-\tau)\right] \tag{A19}$$

which is the social welfare function after substitution of the constraints.

8 Appendix B

8.1 Social planner problem with two tax rates

Utility function is always:

$$U_t = \ln c_{t,t} + \beta\ln c_{t,t+1} + \gamma\ln h_{t+1} \tag{B1}$$

Optimal choices of c_t, c_{t+1}, h_{t+1} are:

$$c_t = \left(\frac{1}{1+\beta+\gamma}\right)(1-\tau_w)w_t h_t \tag{B2}$$

$$c_{t+1} = \frac{\beta}{1+\beta+\gamma}(1+(1-\tau_k)r)(1-\tau_w)w_t h_t \tag{B3}$$

$$h_{t+1} = \frac{\gamma}{(1+\beta+\gamma)}(1-\tau_w)w_t E^{\mu}(1+(1-\tau_k)r)\ h_t^{2-\mu} \tag{B4}$$

Exploiting government balanced budget constraint $E = \tau_w w_t h_t + \tau_k r_t s_t$, equation (B4) becomes:

$$h_{t+1} = \frac{\gamma}{(1+\beta+\gamma)}(1-\tau_w)w_t(\tau_w Wh + \tau_k r_t s_t)^{\mu}(1+(1-\tau_k)r)\ h_t^{2-\mu} \tag{B5}$$

From firms'FOC:

$$r = \alpha A k_t^{\alpha-1} \tag{B6}$$

$$w = (1-\alpha)Ak^{\alpha} \tag{B7}$$

From maximization we also know that:

$$k_t = \frac{K_t}{h_t} = \frac{s_t}{h_t} = \frac{\beta+\gamma}{1+\beta+\gamma}(1-\tau_w)w_t \tag{B8}$$

Put (B8) into (B7):

$$w = \left[(1-\alpha)\, A \left(\frac{\beta+\gamma}{1+\beta+\gamma}\right)^{\alpha}(1-\tau_w)^{\alpha}\right]^{\frac{1}{1-\alpha}} \tag{B9}$$

Put (B9) and (B8) into (B6):

$$r = \frac{\alpha}{\left(\frac{\beta+\gamma}{1+\beta+\gamma}\right)(1-\tau_w)(1-\alpha)} \tag{B10}$$

With the capital taxation, it becomes:

$$1+(1-\tau_k)r = 1+(1-\tau_k)\frac{\alpha}{\left(\frac{\beta+\gamma}{1+\beta+\gamma}\right)(1-\tau_w)(1-\alpha)} \tag{B11}$$

So the relevant logarithms (considering that we are maximizing w.r.t. τ,so we need to drag along only those terms) of (B9) and (B10) are:

$$\ln w = \frac{\alpha}{1-\alpha}\ln(1-\tau_w) \tag{B12}$$

$$\ln[1+(1-\tau_k)r] = \ln(1-\tau_k) - \ln(1-\tau_w) \tag{B13}$$

From (B2), (B3), (B4), the relevant logarithms for maximization turn out to be:

$$\ln c_t = \ln(1-\tau_w) + \ln w \tag{B14}$$

$$\ln c_{t+1} = \ln(1+(1-\tau_k)r) + \ln(1-\tau_w) + \ln w \tag{B15}$$

$$\ln h_{t+1} = \ln(1-\tau_w) + \ln w + \mu \ln[\tau_w wh + \tau_k rs] + \ln(1+(1-\tau_k)r) \quad \text{(B16)}$$

Using (B12) and (B13):

$$\ln c_t = \frac{1}{1-\alpha} \ln(1-\tau_w) \quad \text{(B17)}$$

$$\ln c_{t+1} = \ln(1-\tau_k) + \frac{\alpha}{1-\alpha} \ln(1-\tau_w) \quad \text{(B18)}$$

$$\ln h_{t+1} = \frac{\alpha}{1-\alpha} \ln(1-\tau_w) + \ln(1-\tau_k) + \mu \ln[\tau_w wh + \tau_k rs] \quad \text{(B19)}$$

Let us take the term $\mu \ln[\tau_w wh + \tau_k rs]$ and further expand it by plugging the optimal expression for r and s :

$$\begin{aligned}
\mu \ln[\tau_w wh + \tau_k rs] &= \mu \ln[\tau_w wh + \tau_k \frac{\alpha}{\left(\frac{\beta+\gamma}{1+\beta+\gamma}\right)(1-\tau_w)(1-\alpha)} \frac{(\beta+\gamma)}{1+\beta+\gamma}(1-\tau_w) w_t h_t \\
\mu \ln[\tau_w wh + \tau_k rs] &= \mu \ln[\tau_w wh + \tau_k \frac{\alpha}{1-\alpha} wh] \\
\mu \ln[\tau_w wh + \tau_k rs] &= \mu \ln[wh(\tau_w + \tau_k \frac{\alpha}{1-\alpha}] \\
\mu \ln[\tau_w wh + \tau_k rs] &= \mu \ln wh + \mu \ln \left(\tau_w + \tau_k \frac{\alpha}{1-\alpha}\right) \\
\mu \ln[\tau_w wh + \tau_k rs] &= \mu \ln w + \mu \ln h + \mu \ln \left(\tau_w + \tau_k \frac{\alpha}{1-\alpha}\right) \\
\mu \ln[\tau_w wh + \tau_k rs] &= \mu \frac{\alpha}{1-\alpha} \ln(1-\tau_w) + \mu \ln \left(\tau_w + \tau_k \frac{\alpha}{1-\alpha}\right)
\end{aligned}$$

Putting it into (B19):

$$\ln h_{t+1} = \frac{\alpha(1+\mu)}{1-\alpha} \ln(1-\tau_w) + \ln(1-\tau_k) + \mu \ln \left(\tau_w + \tau_k \frac{\alpha}{1-\alpha}\right) \quad \text{(B20)}$$

And plugging (B17), (B18),and (B20) into (B1):

$$\begin{aligned}
U &= \frac{1}{1-\alpha} \ln(1-\tau_w) + \beta \left[\ln(1-\tau_k) + \frac{\alpha}{1-\alpha} \ln(1-\tau_w)\right] + \quad (41) \\
&\quad \gamma \left[\frac{\alpha(1+\mu)}{1-\alpha} \ln(1-\tau_w) + \ln(1-\tau_k) + \mu \ln \left(\tau_w + \tau_k \frac{\alpha}{1-\alpha}\right)\right]
\end{aligned}$$

which is the social welfare function after substitution of the constraints.