OPTIMAL OPPORTUNISTIC MONETARY POLICY IN A NEW-KEYNESIAN MODEL

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ABSTRACT. The present paper compares the performance in terms of second order accurate welfare of opportunistic non-linear Taylor rules and with respect to traditional linear Taylor rules. The macroeconomic model representing the benchmark for the analysis includes capital accumulation (with quadratic costs of adjustment), price rigidities (quadratic approach), along the standard New-Keynesian approach. The model is solved up to second order approximation and welfare is evaluated according to several criteria (conditional to the non-stochastic steady state, unconditional, and according to a linear *ad hoc* function). The results show that: (i) the opportunistic rule is a Pareto improvement with respect to other monetary policy rules traditionally considered in the literature; (ii) the computation of welfare costs reveals that the burden of adjustment is almost entirely on labor supply fluctuations; (iii) increasing the degree of price rigidities and the degree of competition in the final goods markets, makes the opportunistic rule even more preferable with respect to the alternatives. Business Cycle statistics for the model with opportunistic rule show a large volatility in labor supply, with a limited volatility for the nominal interest rate.

KEYWORDS: disinflation, monetary policy rules,nonlinear rules, Taylor rules

JEL CLASSIFICATION CODE: E31, E61

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1. INTRODUCTION

Modern central banks can adopt two approaches to pursue a disinflationary course of monetary policy. They can announce a final inflation target and set the policy rate accordingly. This has been called 'deliberate' approach to disinflation. The view advocated by Federal Reserve Governor Lawrence Meyer, instead, suggests that a central bank should wait for favorable unforeseen shocks to lower the inflation rate when inflation is moderate but above the central bank's target (see Meyer, 1996). The 'opportunistic' approach proposes that the central bank behaves in a nonlinear way. Until the desired reduction of inflation has occurred, the central bank should focus on counteracting any increase in inflation and on stabilizing the output gap. Once the disinflation has taken place, monetary policy should prevent the inflation rate from returning to the past levels.

Orphanides and Wilcox (2002) provide the theoretical foundations for the opportunistic approach. They show that a simple opportunistic rule for monetary arises from the optimizing behavior of a central bank with history-dependent intermediate target for inflation. The history-dependence in the loss function of the central bank provides the reason for path-dependence in the response of the policy rate to inflation. The loss function also includes absolute deviations of output from its natural level. The interaction between the objective on output and the objective on inflation generates nonlinearity in the optimal policy rule. Aksoy, Orphanides, Small, Wilcox, and Wieland (2003) consider the long-run implications of the opportunistic approach. They compare the stochastic distributions of inflation and the output gap arising from a linear and an opportunistic rule in an estimated rational-expectations model.

In this paper we focus on the business cycle properties of the opportunistic approach to disinflation. We set up a calibrated New Keynesian model with capital accumulation and nominal and real rigidities. The public sector is modeled as a simple rule for lump-sum taxes like in Leeper (1991). We include three sources of exogenous fluctuations in the form of stochastic shocks to productivity, firms markup and government spending. We solve the model through the second-order Taylor approach developed by Schmitt-Grohé and Uribe (2004). This allows us to compute optimized monetary policy rules by maximizing alternative measures of consumer welfare. We compare the impulse responses and business cycle statistics of the model economy under a set of maximized linear rules for monetary policy and under the opportunistic rule. The results indicate that the opportunistic rule allows to reach an higher level of welfare if compared with a traditional linear Taylor' type rule. Our optimal coefficients for the optimal opportunistic rule are not too dissimilar from those obtained by Aksoy, Orphanides, Small, Wilcox, and Wieland (2003), implying an high coefficient on the nonlinear part of the rule a strong interest rate inertia and no response to output fluctuations. The cost for the opportunistic rule is almost entirely in terms of higher labor supply fluctuations. On the other hand, by adopting a rule where monetary authority is committed to an explicit path of disinflation (deliberate disinflation rule in our language), the welfare gains are higher both with respect to traditional Taylor's type rule and with respect to the opportunistic rule itself. However, this occurs at the cost of an higher volatility of consumption and employment than would have been obtained under the other rules. The highest volatility of consumption is obtained under the optimal money growth rule. If we rank our rules according to an *ad hoc* loss function, imposing a linear trade-off between inflation and output stabilization, we find that the rankings for the various policy rules is completely different from the microfounded welfare function. Under this loss function, the opportunistic rule delivers a welfare value much higher than any other policy rules.

The robustness exercise conducted by raising the degree of price rigidity associated to a lower degree of monopoly power in the final goods markets, shows that the opportunistic rule allows to reach the highest possible welfare value among the other rules.

A final word is about impulse-response analysis. Nominal interest rate after a positive technology shock shows a moderate response, inserting an higher persistence in output response. After a government (positive) shock, the interest rate moderately falls, causing a modest increase in consumption (via money demand effect). With this response, our model supports the view by which the increase in public (non productive) government expenditure crowds out private consumption, but crowds in private investment.

A further point, albeit technical, of the present paper is about the computation of conditional moments. In the appendix we provide a closed form expression of the conditional moments based on the second order solution of the model. This result makes possible to compute the second order matrix, without using recursion, which could cause, instead, spurious higher order terms.

We believe that the present results are encouraging and calls for further generalization and extensions, given the empirical importance of non-linear rules, as possible explanation for the inertial of Central Banks.

The European Central Bank (ECB) is often supposed to follow a non-linear rule like the opportunistic one considered in the present paper. The intuition behind that relies on the prescription issued in the Stability and Growth Pact (SGP) which fixes the duties of the ECB. According to SGP, ECB should target inflation rate ahead of any other goals. Thus, after any inflationary shock, ECB should react in order to bring the inflation back to the target (prescribed as to be 2 per cent on an yearly basis). Since nothing else is specified with respect to either goals or situations where inflation is not increasing, this can be interpreted as a non-linear policy rules such that nominal rates are strongly raised if inflation raises, but they do not decrease proportionately to the inflation reductions. The opportunistic rule tries to formalize this inertial pattern of modern central banking.

This paper is organized as follows. Section 2 describes the model economy. Section 3 discusses the monetary policy rules that we study. Section 4 considers the equilibrium conditions. The calibration strategy is presented in section 5. Section 6 deals with the computational aspects of this work, including the approximation technique to the first-order conditions and the method for evaluating welfare. The results are discussed in section 7. Section 8 proposes some concluding remarks.

2. THE MODEL

The structure of the model economy is standard in the New-Keynesian tradition of Woodford (2002). We include money demand through the money-in-the-utility function approach studied by Feenstra (1986), and quadratic capital-adjustment costs like Kim (2000). Nominal price rigidity arises from quadratic-adjustment costs from changing prices.

2.1. *Households*

The model economy is populated by a large number of infinitely-lived agents indexed on the real line, $i \in [0, 1]$, each maximizing the following stream of utility

$$U_{it} = \sum_{t=0}^{\infty} \beta^t u(c_{it}, m_{it}, \ell_{it})$$

subjected to the following specification for the instantaneous utility function

$$u\left(c_{it}, m_{it}, \ell_{it}\right) = \frac{1}{1-\frac{1}{\sigma}} \left\{ \left[a c_{it}^{\frac{\mu-1}{\mu}} + (1-a) \left(\frac{M_{it}}{P_t}\right)^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}} \left(1-\ell_{it}\right)^{\xi} \right\}^{\left(1-\frac{1}{\sigma}\right)}$$

The utility function considers money in a weakly separable form with respect to consumption C_{it}. Basically, consumption and real money balances M_{it}/P_t are taken together via a CES aggregator type, as described by Chari, Kehoe, and McGrattan (2000). The advantage of such approach relies on the cross substitution effects between consumption and money derived from the weak separability between money and consumption. It is worth to note that the equivalence between money-in-the utility, transaction costs and cash-in-advance models has been proved by Feenstra (1986).

The i-th household budget constraint (in real terms) is given by

$$c_{it} + \frac{M_{it} - M_{it-1}}{P_t} + \frac{B_{it}}{P_t} + inv_{it}\left[1 + \frac{\phi_K}{2}\left(\frac{inv_{it}}{k_{it}}\right)^2\right] \leq \\ q_{it}k_{it} + w_{it}\ell_{it} + R_{t-1}\frac{B_{it-1}}{P_t} - \tau_t^{ls} + \int_0^1 \eta_i\left(j\right)\Omega_t\left(j\right)dj$$

Households' income derives from: i) labor income in the form of $w_{it}\ell_{it}$, with w_{it} real wage per unit of labor effort ℓ_{it}; ii) capital income $q_{it}k_{it}$, with q_{it} rental rate on capital stock k_{it}; (iii) proceedings from investment in government bonds $R_{t-1}B_{it-1}/P_t$, where R_t is the gross nominal rate, and B_{it} is the stock of government's bonds held by i-th household. Each agent participates in the profit of the firm producing good j via a constant share $\eta_i\left(j\right)$. We assume that this share is constant and out of the control of the single agent. Households also pay taxes in the form of lump-sum transfers τ_t^{ls} to the government.

Households allocate their wealth among money M_{it}, (nominal) bonds B_{it} and (real) investment inv_{it}. In order to reduce the high investment volatility typical of the RBC, we follow the suggestion of Kim (2000) and introduce an investment adjustment cost in the quadratic form. The assumption of quadratic cost of price adjustment simplifies algebra and delivers coherent results. The evolution of capital accumulation is governed by the following equation

$$k_{it+1} = (1-\delta)k_{it} + inv_{it} \tag{1}$$

Given the existence of differentiated goods and different labor inputs, there exists an intra-temporal optimization program on the final goods-sector.

The are j varieties of final goods produced that are aggregated according to the constant-elasticity of substitution technology proposed by Dixit and Stiglitz (1977)

$$c_{it} = \left[\int_{\omega_2} c_t^i\left(j\right)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}$$

where $\theta > 1$ is the elasticity of substitution between different varieties of goods produced by each j-th firm and $c_t^i\left(j\right)$ is the consumption of varieties j by i-th household. The constant elasticity of substitution inverse demand function for j-th variety expressed by i-th household is

$$\frac{c_t^i\left(j\right)}{c_{it}} = \left[\frac{P_t\left(j\right)}{P_t}\right]^{-\theta}$$

where $\mathcal{P}_t\left(j\right)$ is the price of variety j and P_t is the general price index defined as

$$P_t = \left[\int_0^1 P_t\left(j\right)^{1-\theta} dj\right]^{\frac{1}{1-\theta}}$$

Aggregate consumption is defined as $c_t = \int_{\omega_1} c_{it} di$, after aggregating over the $i \in \omega_1$ households.

The aggregate demand for variety j can be written as

$$c_t(j) + g_t(j) = y_t(j)$$

such that the individual demand curve takes the form

$$P_t(j) = \left[\frac{y_t(j)}{y_t}\right]^{-1/\theta} P_t \tag{2}$$

2.2. *Firms*

We assume the existence of a large number of firms indexed by $j \in \omega_2$, each producing a single variety. Each firm acts as a price taker with respect to the varieties supplied by other competitors.

The production function is given by

$$y_{jt} = z_t\,(k_{jt})^{\alpha}\,(\ell_{jt})^{1-\alpha} - \Phi_t \tag{3}$$

where k_{jt} and ℓ_{jt} are capital and labor inputs, respectively. We also introduce the exogenous shocks

$$\begin{aligned} \log z_t &= (1-\rho_z)\log z + \rho_z \log z_{t-1} + \varepsilon_t^z \\ \log \Phi_t &= (1-\rho_\Phi)\log \Phi + \rho_\Phi \log \Phi_{t-1} + \varepsilon_t^\Phi \end{aligned}$$

There are quadratic cost of price adjustment *à la* Rotemberg and Woodford (1992) specified as

$$AC_t^P(j) = \frac{\phi_P}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \pi\right)^2 y_t$$

The optimal choice of labor and capital to be hired is described as the maximization of the future stream of profit evaluated with the stochastic discount factor ρ_t

$$\max_{\{P_t(j),k_t(j),\ell_t(j)\}} E_0\left[\sum_{t=0}^{\infty}\lambda_t\Omega_t(j)\right]$$

$$\text{s. t.}\quad \Omega_t(j) = P_t(j)\,y_{jt} - W_t\ell_{jt} - P_t q_t k_{jt} - P_t AC_t^P(j)$$

given the demand for differentiated products in (2), and the production function (3).

2.3. *Government*

The government faces a standard flow budget constraint

$$B_{jt}dj + P_t\tau_t^{ls} + M_{jt} = R_{t-1}B_{jt-1} + P_t g_t + M_{jt-1}$$

Real total taxation is denoted as τ_t, and g_t indicates total government spending. The government issues one-period riskless (non-contingent) nominal bonds denoted by D_t. We also specify the intertemporal budget constraint of the government

$$R_t B_{jt} \le \sum_{p=0}^{\infty}\mathsf{E}_{t+p}\left(\frac{1}{R_{t+p}}\right)^p\left[M_{jt+p} - M_{jt-1+p} + P_{t+p}\tau_{t+p}^{ls} - P_{t+p}g_{t+p}\right]$$

As it is customary in the literature, we posit an exogenous path to public expenditure, by assuming an AR(1) described by the following equation

$$\log(g_t) = (1-\rho_g)\log(g) + \rho_g\log(g_{t-1}) + \varepsilon_t^g$$

with ε^g_{t+1} is i.i.d.$\sim N\left(0, \sigma_g^2\right)$.

The government flow budget constraint in equilibrium can be also re-written by defining the total amount of government's liabilities l_t

$$l_t := \frac{R_t B_t + M_t}{P_t}$$

In this case, the evolution of total liabilities is represented by the equation

$$l_t = \frac{R_t l_{t-1}}{\pi_t} + R_t \left(g_t - \tau_t^{ls}\right) - m_t \left(R_t - 1\right)$$

An important feature of the present model, shared by other contributions, consists in a feedback rule for tax revenues of the type suggested by Leeper (1991). In what follows, we introduce the fiscal rule

$$\tau_t^{ls} = \psi_0 + \psi_1 \left(l_{t-1} - l\right) + \psi_2 \left[g_t + \left(\frac{R_{t-1} - 1}{R_{t-1}}\right)\left(\frac{l_{t-1} - m_{t-1}}{\pi_t}\right)\right]$$

The economic interpretation is that the government set taxes in order to stabilize the level of total liabilities l_t in real terms. The particular functional form assumed allows to distinguish between two distinct forms of stabilization: a simple fiscal feedback rule *à la* Leeper (1991), obtained by setting $\psi_2 = 0$, and a balanced budget rule when $\psi_1 = 0$ and $\psi_2 = 1$. In other words, taxes can be adjusted to follow either a 'minimal' adjustment path, enough to avoid that the total amount of government's liabilities to explode, or a 'strong' stabilization path, when taxes are immediately adjusted according to a balanced budget rule.

3. Monetary policy rules

3.1. *Linear benchmarks*

We compare the macroeconomic performance of an array of simple rules for monetary policy that have become standard in the literature. The celebrated specification proposed by Taylor (1993) is the starting point for every study of monetary policy

$$\log\left[\frac{i_t}{\bar{i}}\right] = \alpha_\pi \left[\frac{\pi_t}{\bar{\pi}}\right] + \alpha_y \log\left[\frac{y_t}{\bar{y}}\right] + \alpha_R \log\left[\frac{i_t}{i_{t-1}}\right]$$

A critique of the previous rules is that their implementation requires an in-depth knowledge of the long-run state of the model economy (see Schmitt-Grohé and Uribe, 2003). To that hand, rules in first-differences have been proposed to cope the problem that the deterministic steady states are unobservable

$$\log\left[\frac{i_t}{i_{t-1}}\right] = \alpha_\pi \log\left[\frac{\pi_t}{\pi_{t-1}}\right] + +\alpha_y \log\left[\frac{y_t}{y_{t-1}}\right]$$

Clarida, Galí, and Gertler (2000) provide empirical evidence in favour of the proposition for which U.S. monetary policy is described appropriately by a simple rule with forward-looking variables. Woodford (2002) provides the conditions such that the forward-looking specification arises as the optimal rule from a set of microfounded assumptions. Hence, we introduce the following

$$\log\left[\frac{i_t}{\bar{i}}\right] = \alpha_\pi \mathsf{E}_t \log\left[\frac{\pi_{t+1}}{\bar{\pi}}\right] + \alpha_y \mathsf{E}_t \log\left[\frac{y_{t+1}}{\bar{y}}\right] + \alpha_R \log\left[\frac{i_t}{i_{t-1}}\right]$$

Since our model provides for a role for money demand, it is natural to consider also a simple rule for money growth with respect to the steady state

$$\log\left[\frac{m_t}{\bar{m}}\right] = \rho_m \log\left[\frac{m_{t-1}}{\bar{m}}\right]$$

The above equation introduces a persistent money supply rule, as it is customary in the current literature.

3.2. *Opportunistic monetary policy*

Orphanides and Wilcox (2002) formalize the idea underlying the 'opportunistic approach to disinflation' with a policy rule that is both time-dependent and nonlinear. The central bank pursues an intermediate target of inflation $\tilde{\pi}_t$ such that the closer current inflation to $\tilde{\pi}_t$, the stronger the defense of the lower inflation level against past inflation targets. The inflation target $\tilde{\pi}_t$ is a weighted average of long-run inflation $\bar{\pi}$ and inherited past inflation π_t^h

$$\tilde{\pi}_t := (1-\lambda)\log[\bar{\pi}] + \lambda \pi_t^h$$

The term π_t^h is the source of history dependence for monetary policy. Nonlinearity arises from the existence of a range of inflation deviations from the intermediate target within which output stabilization is the primary objective of monetary policy. The larger the deviation of inflation from $\tilde{\pi}_t$, the stronger the focus on price stability.

The opportunistic policy rule proposed by Orphanides and Wilcox (2002) takes the form

$$\log[i_t/\bar{i}] = \kappa_0 \log[y_t/\bar{y}] + \mathcal{G}\left(\log[\pi_t] - \tilde{\pi}_t\right) + \kappa_3 \log[i_{t-1}/\bar{i}]$$

where hats denote deviations from the deterministic steady states, and $\mathcal{G}(\cdot)$ is represented by the discontinuos function

$$\mathcal{G}(\pi_t - \tilde{\pi}_t) := \begin{cases} \kappa_1(\log[\pi_t] - \tilde{\pi}_t - \kappa_2) & \text{if } \ (\log[\pi_t] - \tilde{\pi}) > \kappa_2 \\ 0 & \text{if } \ \kappa_2 \geq (\log[\pi_t] - \tilde{\pi}) \geq -\kappa_2 \\ \kappa_1(\log[\pi_t] - \tilde{\pi}_t + \kappa_2) & \text{if } \ (\log[\pi_t] - \tilde{\pi}) < -\kappa_2 \end{cases}$$

The set of deviations from the inflation target such that $\mathcal{G} = 0$ defines a 'zone of inaction'. If there is a drop in inflation below this zone, the central bank acts to prevent inflation from returning at the higher level of the past. In the intentions of the proponents of the opportunistic approach to disinflation, the zone of inaction defines the scope for opportunism. In the numerical solution of the model, we follow Aksoy, Orphanides, Small, Wilcox, and Wieland (2003), and use the following twice continuously-differentiable approximation of $\mathcal{G}$

$$\mathcal{G}(\cdot) \approx \kappa_1 \Bigg[0.05(\log[\pi_t] - \tilde{\pi}_t) + 0.475\left(-\kappa_2 + \log[\pi_t] - \tilde{\pi}_t + \left((-\kappa_2 + \log[\pi_t] - \tilde{\pi}_t)^2\right)^{0.51}\right) \\ +0.475\left(\kappa_2 + \log[\pi_t] - \tilde{\pi}_t - \left((\kappa_2 + \log[\pi_t] - \tilde{\pi}_t)^2\right)^{0.51}\right)\Bigg]$$

Since we are concerned with the U.S. economy, we assign the same parameter values to the approximated $\mathcal{G}$ that Aksoy, Orphanides, Small, Wilcox, and Wieland (2003) use. There is a slightly-positive slope even when inflation is within the zone of inaction. The numerical algorithm maximizes over a grid for κ_0, κ_1 and κ_2.

4. Equilibrium and Aggregation

DEFINITION 1: A symmetric monopolistically-competitive equilibrium consists of stationary sequences of prices $\{\mathbf{P_t}\}_{t=0}^{\infty} := \{\pi_t^*, R_t^*, w_t^*, r_t^*\}_{t=0}^{\infty}$, real quantities $\{\mathbf{Q_t}\}_{t=0}^{\infty} := \{\{\mathcal{Q}_t^h\}_{t=0}^{\infty}, \{\mathcal{Q}_t^f\}_{t=0}^{\infty}, \{\mathcal{Q}_t^g\}_{t=0}^{\infty}\}$, with $\{\mathcal{Q}_t^h\}_{t=0}^{\infty} := \{c_t^*, \ell_t^*, k_{t+1}^*, i_t^*, m_t^*, d_t^*\}_{t=0}^{\infty}$, $\{\mathcal{Q}_t^f\}_{t=0}^{\infty} := \{y_t^*, k_t^*, \ell_t^*\}_{t=0}^{\infty}$, $\{\mathcal{Q}_t^g\}_{t=0}^{\infty} := \{g_t^*, \tau_t^{ls*}, m_t^*, d_t^*\}_{t=0}^{\infty}$ and exogenous shocks $\{\mathcal{E}_t\}_{t=0}^{\infty} := \{\epsilon_t^z, \epsilon_t^g\}_{t=0}^{\infty}$ that aggregate over $\omega_1 = [0,1]$ and $\omega_2 = [0,1]$, that are bounded in a neighborhood of the steady state, and such that:

(i) given prices $\{\mathbf{P_t}\}_{t=0}^{\infty}$ and shocks $\{\mathcal{E}_t\}_{t=0}^{\infty}$, $\{\mathcal{Q}_t^h\}_{t=0}^{\infty}$ is a solution to the representative household's problem;

(ii) given prices $\{\mathbf{P_t}\}_{t=0}^{\infty}$ and shocks $\{\mathcal{E}_t\}_{t=0}^{\infty}$, $\{\mathcal{Q}_t^f\}_{t=0}^{\infty}$ is a solution to the representative firms' problem;

(iii) given quantities $\{\mathbf{Q_t}\}_{t=0}^{\infty}$ and shocks $\{\mathcal{E}_t\}_{t=0}^{\infty}$, $\{\mathbf{P_t}\}_{t=0}^{\infty}$ clears the market for goods, factors of production, money and bonds

$$y_t^* = c_t^* + inv_t^* + g_t^* + \mathrm{AC}_t^{k*} + \mathrm{AC}_t^{P*}$$

(iv) given quantities $\{\mathbf{Q_t}\}_{t=0}^{\infty}$, prices $\{\mathbf{P_t}\}_{t=0}^{\infty}$ and shocks $\{\mathcal{E}_t\}_{t=0}^{\infty}$, $\{\mathcal{Q}_t^g\}_{t=0}^{\infty}$ and satisfy the flow budget constraint of the government;

(v) fiscal policy is set according to a simple rule for lump-sum taxes;

(vi) the central bank sets the nominal interest rate according to a simple policy rule.

5. CALIBRATION

The parameters are calibrated on quarterly data for the US economy. We assume that households have an intertemporal discount rate of 0.996. They devote 1/4 of their time to labour activities at the steady state. The weight on the consumption objective in the consumption objective is 0.993. We assume that the intertemporal elasticity of substitution is 1/3, which lies on the boundary of the range of values used in the RBC literature (see Schmitt-Grohé and Uribe, 2003). The calibration of the other parameters in the utility function is consistent with a consumption-output ratio of 0.57, and a money-output ratio of 0.44 in the long run (see Table I). The nominal rate of interest is 5% a year, and the inflation rate is 4.2%. Both figures are consistent with the U.S. postwar experience.

We set the investment-output and capital-output ratios as 0.25 and 10.4, respectively. Capital depreciates for 10% a year. Both the parameter ϕ_K in the adjustment cost for capital, and the persistence of the markup shock are from Kim (2000). Capital income has a share of 1/3 in total output. The elasticity of substitution among intermediate goods generates a steady-state markup of approximately 10%. Stochastic productivity shocks are calibrated according to Chari, Kehoe, and McGrattan (2000).

We assume that government spending is 14.8% of GDP. The calibration for the public-spending shock is from Schmitt-Grohé and Uribe (2003). The steady-state ratio between public debt and output is 0.45. In the baseline calibration, we assume that lump-sum taxes evolve according to a simple rule with $\phi_2 = 0$, and with a strong feedback on total government liabilities ($\phi_1 = 0.8$). In the language of Leeper (1991), this gives rise to a regime with 'passive fiscal policy'. This assumption is relaxed at a later point to study the role of the policy mix.

6. COMPUTATIONAL ASPECTS

6.1. *Welfare evaluation*

Aggregate welfare is defined as the expected lifetime utility conditional on the initial distribution of the state varibles s_0

$$\mathcal{W}_0 := \mathsf{E}\left[\sum_{t=0}^{\infty} \beta^t u(s_t) \Big| s_0 \sim (\underline{s}, \underline{\Omega})\right]$$

Although unappealing from the point of view of statistical theory, this way of conditioning is based on the timing of events implicit in the New Keynesian model. Namely, the assumption that the stochastic shocks are realized at the beginning of every period. At the end of each period, economic decisions are taken following the optimality conditions. In order to obtain an accurate welfare evaluation, the conditional welfare function is approximated through a second-order Taylor expansion around the distorted steady state. This requires computing the second-order Taylor approximation to the system of nonlinear expectational equations from the first-order conditions. We use the algorithm of Schmitt-Grohé and Uribe (2004) for the solution of the model, and the formulas presented in Paustian (2003) for the approximation to intertemporal utility.

The welfare costs of alternative policies are measured as the permanent change in consumption, relative to the steady state, that yields the expected utility level of the distorted economy. Given steady states of consumption $\bar{c}^{\iota}$ and hours worked $\bar{\ell}$ of the model ι, this translates into the number Δ_c^{ι} such that

$$\sum_{t=0}^{\infty} \beta^t u\left(\left[1+\Delta_c^{\iota}\right]\bar{c},\ \bar{\ell}\right) = \mathcal{W}_0^{\iota} \tag{4}$$

where ι refers to the monetary-policy rule. Following Kollmann (2003), we decompose the conditional welfare cost Δ_c^{ι} into two components denoted as Δ_E^{ι} and Δ_V^{ι}. Given the second-order approximation to the utility function

$$u\left(\left[1+\Delta_c^{\iota}\right]\bar{c},\ \bar{\ell}\right) \approx u\left(\bar{c},\ \bar{\ell}\right) + (1-\beta)\sum_{t=0}^{\infty}\beta^t\left(\mathsf{E}\left[\hat{c}_t - \bar{\ell}\hat{\ell}_t|s_0\right] - \frac{1}{2}\mathsf{VAR}\left[\hat{c}_t|s_0\right]\right)$$

we compute the change in mean consumption Δ_E^{ι} that the household faces while giving up the total fraction of certainty-equivalent consumption Δ_c^{ι}

$$u\left(\left[1+\Delta_E^{\iota}\right]\bar{c},\ \bar{\ell}\right) = u\left(\bar{c},\ \bar{\ell}\right) + (1-\beta)\sum_{t=0}^{\infty}\beta^t\left(\mathsf{E}\left[\hat{c}_t|s_0\right] - \bar{\ell}\mathsf{E}\left[\hat{\ell}_t|s_0\right]\right)$$

Since the solution method is non-certainty equivalent, we can also calculate the change in conditional variance of consumption that is consistent with the total welfare cost of policies

$$u\left(\left[1+\Delta_V^{\iota}\right]\bar{c},\ \bar{\ell}\right) = u\left(\bar{c},\ \bar{\ell}\right) - (1-\beta)\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\mathsf{VAR}\left[\hat{c}_t|s_0\right] \tag{5}$$

where hats denote log-deviations from the deterministic steady states. Kollmann (2003) points out that the following relation holds

$$\left(1+\Delta_c^{\iota}\right) = \left(1+\Delta_E^{\iota}\right)\left(1+\Delta_V^{\iota}\right) \tag{6}$$

We calculate Δ_c^{ι} and Δ_V^{ι} through 4 and 5, and Δ_E^{ι} through 6. Since there is no closed-form solution for the infinite summations, we simulate the conditional moments for 2000 periods and compute the discounted sum. Appendix 3 provides analytical formulas for computing the moments conditional on an initial state vector.

For the purpose of comparison with the literature, we also report the approximation to the expectation of the one-period (unconditional) utility around the distorted steady states

$$\mathcal{W} := \mathsf{E}\left[u(s_t)\right]$$

The steady-state jump Δ_U^ι that yields the unconditional utility level can be computed from

$$u\left(\left[1+\Delta_U^\iota\right]\bar{c},\ \bar{\ell}\right) = \mathcal{W}^\iota$$

It is well known since Kim and Kim (2003) that this welfare index produces incorrect rankings of the policy rules, for it disregards the transition costs of moving from the deterministic to the stochastic steady states.

6.2. *Local validity of approximate solutions*

Second-order perturbation methods are defined only around small neighbourhoods of the approximation points, unless the approximated function is globally analytic (see Anderson, Levin, and Swanson, 2004). Since the conditions for an analytic form of the policy function are hardly establishable, the problem of validity of the Taylor expansion remains. To that end, we impose an *ad hoc* bound that restricts the stochastic steady state of the nominal interest rate to be arbitrarily close to its deterministic counterpart

$$\ln\left(\bar{R}\right) > \kappa\sigma_{\hat{R}_t} \tag{7}$$

with a constant κ, and $\sigma_{\hat{R}_t}$ as the unconditional variance of $\hat{R}$. This constraint rules out policies that are excessively aggressive. The reason is that large deviations of the nominal rate of interest from the deterministic steady state are likely to prescribe violations of the zero bound at some point in time. In what follows, we set $\kappa = 2$.

7. Quantitative results

Tables II-IV report the specifications of the optimized monetary and fiscal policy rules. The feedback coefficients on the inflation targets for all the linear rules are higher than one, and larger than the coefficients on the output objective (see Table II). Panel (a) shows that the welfare loss is due to the variability of consumption. The mean dynamics of consumption and labour, instead, produce gains with respect to the deterministic steady state. The forward-looking rule achieves the same conditional welfare level as the standard rule. However, the prescribed policy mix is very different. The optimized coefficient on inflation is less than half than for the standard rule, whereas the feedback coefficient identifies a fiscal policy rule more passive than under the standard Taylor rule. The money-growth rule improves on the welfare level (see panel (a) of Table IV). This is at the cost of a large variability in consumption. The optimal opportunistic rule is similar to that computed by Aksoy, Orphanides, Small, Wilcox, and Wieland (2003). It entails a large coefficient on the nonlinear part of the rule, strong interest-rate inertia and no response to output fluctuations (see panel (a) of Table III).

The impulse responses for the optimized standard rule depict well-known results. A positive productivity shock raises real output and lowers inflation (see figure 1(a)). Consumption and investment rise with respect to the deterministic steady state. There is a liquidity effect, namely a fall in the short-term interest rate triggers an increase in money holdings. In the literature there is a debate on the reaction of the fraction of time devoted to work activities in the post-Volcker period. Our model supports the idea that hours worked fall after a positive productivity shock. Figure 1(b) shows that a government-spending shock leads to an increase in output. However, public (non-productive) spending crowds out private consumption, and crowds in private investment. This indicates that our model subscribes to the interpretation of government spending shocks as productivity shocks. Finally a markup shock causes inflation to rise and output to fall, and triggers a contractionary response of monetary policy (see figure 1(c)).

With the optimal money-growth rule, the impulse responses from government-spending shocks change considerably (see figure 2(b)). In agreement with the findings of Blanchard and Perotti (2002), there is a rise in both consumption and output. The fact that the deviation of investment from its steady state widens indicates that the model is incapable of generating a realistic mechanism for crowding out. The noticeable aspect of the responses under a money-growth rule is that the changes in money holdings are very small. This does not prevent the responses from showing an adequate level of persistence.

The opportunistic monetary policy rule makes the short-term rate rise after a positive productivity shock (see figure 3(a)). The initial increase in the interest rate is lower than both the increse in output and the fall of the inflation rate. This causes output to fall more slowly than under the standard Taylor rule. Inflation converges to its deterministic steady state more quickly though. Since bond holding increase, the rental rate of capital falls. The nonlinear response of price investment contemplates an increase more gradual than under the standard rule followed by a persistent decline. After a government-spending shock, the short-term interest rate falls by less than the increase in output (see figure 3(b)). This pushes the demand for money upward, and prevents consumption from keeping on growing through the wealth effect after the initial increase. The opportunistic central bank lowers the policy rate also in response to a positive markup shock (see figure 3(c)). In this case, however, both real wages fall and consumption fall.

Table V reports some selected statistics for the model economy subject to all the type of shocks. The opportunistic rule for monetary policy leads to very large swings especially for the supply of labour (see panel (a)). This does not happen at the cost of large volatility of the short-term interest rate. The interest rate is the only variable with a negative correlation with output (see panel (b)). Differently from the all the linear rules, the inflation rate is pro-cyclical under the opportunistic monetary policy.

Our numerical results also include the optimized policy rules based on unconditional welfare (see panel (b) of Tables II-IV). The welfare rankings of unconditional welfare are different from those of conditional welfare. Ignoring the transitional effects of the monetary-policy rules leads to a large overestimation of the welfare costs, as the index Δ_U becomes very large for some of the rules.

7.1. *Opportunistic vs Deliberate Disinflation*

Bomfim and Rudebusch (2000) formalize the distinction between opportunistic and deliberate disinflation. Whereas in the former the central bank waits for exogenous shocks to bring about a favourable output-inflation tradeoff, the latter prescribes an explicit path of disinflation. In practice, also the standard Taylor rule can can be argued to embody the idea of explicit disinflation, as the policy rate is adjusted to deviation of current inflation from the target. However, comparing between the standard and the opportunistic rule requires taking into consideration two different inflation targets. In light of this, we formalize the deliberate approach to disinflation as a modification of the standard Taylor rule

$$\log\left[\frac{i_t}{\bar{i}}\right] = \alpha_\pi \left[\frac{\pi_t}{\tilde{\pi}_t}\right] + \alpha_y \log\left[\frac{y_t}{\bar{y}}\right] + \alpha_R \log\left[\frac{i_t}{i_{t-1}}\right]$$

where the inflation target takes the form

$$\tilde{\pi}_t := (1-\lambda)\bar{\pi} + \lambda\pi_{t-1}$$

The weight λ on inherited inflation is calibrated to 0.5 like for the opportunistic rule.

The qualitative responses of the model economy under deliberate disinflation are very similar to those emerging under the standard Taylor rule. The main difference is that, with deliberate disinflation, monetary policy responds more strongly to all types of shocks than with the standard rule. This causes both investment and capital to fall in the cases of a productivity shock (see figure 4(a)) and a government-spending shock (see figure 4(b)). Table II shows that the policy rule for deliberate disinflation achieves a conditional welfare level higher than that of the rest of the linear rules. However that happens at the cost of a large variability in consumption (see panel (a)). A comparison with panel (a) of Table III shows that deliberate disinflation is a preferable strategy with respect to the opportunistic approach. This result can be motivated in the following.

Our model is calibrated in such a way that the stochastic fluctuations around the deterministic steady state is small. This implies that there is a limited scope for exploiting the favourable exogenous conditions in the reduction of the inflation rate. As a consequence, the benefits from the nonlinearity in the opportunistic rule are not as large as in models like those proposed by Aksoy, Orphanides, Small, Wilcox, and Wieland (2003).

7.2. *A comparison with ad-hoc loss functions*

A natural benchmark for comparing policy rules that maximize conditional welfare is the optimization of ad-hoc loss functions for the central. We start out by considering a one-period loss function whereby the central bank aims to minimize a weighted average of unconditional standard deviations of selected macroeconomic variables

$$\mathcal{L}_t^{op} = \phi_\pi \mathsf{VAR}\left[\hat{\pi}_t\right] + (1-\phi_\pi)\mathsf{VAR}\left[\hat{y}_t\right] + \phi_i \mathsf{VAR}\left[\hat{i}_t\right] \tag{8}$$

We also compute loss-minimizing policies according to an intertemporal loss function

$$\mathcal{L}_t^{int} = \sum_{t=0}^{\infty}\left(\phi_\pi \mathsf{VAR}\left[\hat{\pi}_t|s_0\right] + (1-\phi_\pi)\mathsf{VAR}\left[\hat{y}_t|s_0\right] + \phi_i \mathsf{VAR}\left[\hat{i}_t|s_0\right]\right) \tag{9}$$

that depends on the second moments $\mathsf{VAR}\left[\cdot|s_0\right]$ conditional on the initial state vector s_0.[1] Following Rudebusch and Svensson (1999), we assume $\phi_i = 0.2$.

Since there is no closed-form solution for the infinite summation, we apply the computational strategy outlined in section 6.1 and approximate the summations in two steps. First, the model is solved over each point of a grid including the parameters of both the policy rules and the inflation weight ϕ_π. Then, the second-order solution of the model is simulated for 2000 periods, and the discounted sum of the per-period losses is calculated.

Tables VI and VII reports the optimized policy rules for the loss function 8 and 9, respectively. Two points emerge. First, the rankings of the policy rules according to the minimized losses are different from those of the microfounded welfare function. Second, differently from the maximization of the intertemporal utility, the opportunistic rule for monetary policy produces welfare tens of times larger than for the linear Taylor rules.

7.3. *Robustness exercise*

In what follows, we propose two robustness checks of the previous results. We consider optimized monetary policy for the utility-based criterion with price rigidity higher than baseline ($\phi_P = 90$), and with a lower degree of monopolistic competition ($\theta = 2$). As prices become more rigid, the welfare levels achieved by linear rules show no substantial change. The interesting point is that there is a sizeable improvement in the welfare level achieved by the opportunistic monetary policy. A comparison between Tables VIII and IX shows that the less monopolistically-competitive the

[1]It is interesting to consider the case where the weight on the output objective is different from $1-\phi_\pi$. We impose the restriction $1-\phi_\pi$ on the output objective for the mere purpose of limiting the computational costs.

economy, the more desirable is the adoption of opportunistic disinflation with respect to the linear Taylor rules. Finally, the optimized opportunistic rule is robust to increases in the degree of history dependence (λ) of the intermediate inflation target.

8. Final remarks

In the present paper we studied the operational performance of a specific class of non-linear rules, here shortly defined as opportunistic, in comparison with traditional linear Taylor's type monetary policy rule. Our results show that opportunistic rules allow to reach a policy configuration which is - in terms of both conditional and unconditional welfare measure - better with respect to the traditional approaches. The disadvantage of the opportunistic rule is given by the large labor supply fluctuations, which remains lower than other linear rules where central bank commits to a pre-specified path for disinflation (deliberate disinflation).

Overall, our results suggest that the opportunistic/nonlinear approach to monetary policy offers a better control on the distortions derived from price rigidities, without inducing a large depressionary effect on aggregate activity. Under this view, the opportunistic approach can be thought as a rationale for the 'flexibility with judgment' approach to monetary policy suggested by Svensson (2005). However, we are perfectly aware that this approach should be further qualified by extending to a richer framework with additional shocks and source of stickiness, both on the real and the nominal side.

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APPENDIX 1: MODEL EQUATIONS

The system of first-order conditions is

$$\lambda_t = \overline{u}_t^{-1} (1-\ell_t)^{\xi\left(1-\frac{1}{\sigma}\right)} ac_{it}^{-\frac{1}{\mu}}$$

$$\lambda_t w_t = \overline{u}_{it} (1-\ell_t)^{\xi\left(1-\frac{1}{\sigma}\right)-1}$$

$$\frac{\lambda_t}{P_t} = \overline{u}_t^{-1} (1-\ell_t)^{\xi\left(1-\frac{1}{\sigma}\right)} (1-a)\, m_{it}^{-\frac{1}{\mu}} \frac{1}{P_t} + \beta \mathsf{E}_t \frac{\lambda_{t+1}}{P_{t+1}}$$

$$\left[q_t + \phi_K \left(\frac{inv_t}{k_t}\right)^3\right] \lambda_t = \mu_t - \beta (1-\delta) E_t \mu_{t+1}$$

$$\beta E_t \mu_{t+1} = \lambda_t \left[1 + \frac{3\phi_K}{2}\left(\frac{inv_{it}}{k_{it}}\right)^2\right]$$

$$\lambda_t = \beta R_t \mathsf{E}_t \frac{\lambda_{t+1}}{\pi_{t+1}}$$

$$(1-\alpha)\mathrm{mc}_t \frac{y_t + \Phi_t}{\ell_t} = w_t$$

$$\alpha \mathrm{mc}_t \frac{y_t + \Phi_t}{k_t} = q_t$$

$$(1-\theta) y_t - \phi_P (\pi_t - \pi) \frac{y_t}{\pi_{t-1}} + \theta \mathrm{mc}_t y_t + \beta \mathsf{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[\phi_P (\pi_{t+1} - \pi) \pi_{t+1} y_{t+1}\right] = 0$$

$$\overline{u}_t := \left[ac_t^{\frac{\mu-1}{\mu}} + (1-a)(m_t)^{\frac{\mu-1}{\mu}}\right]^{\left(\frac{\mu}{\mu-1}\right)\left(1-\frac{1}{\sigma}\right)}$$

The model is closed with the simple rules for monetary and fiscal policy, the goverment flow budget constraint, the laws of motion for the exogenous shocks, and the aggregate resource constraint for the economy.

APPENDIX 2: STATE-SPACE FORM

Suppose that the first-order conditions of a model economy can be arranged as

$$\mathsf{E}_t \mathcal{H}(e_{t+1}, e_t, x_{t+1}, x_t | \sigma) = 0 \tag{A2.1}$$

where y is a vector of co-state variables. The state variables are collected in x

$$x_t := \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix}$$

with vectors of endogenous state variables $x_{1,t}$, and exogenous state variables $x_{2,t}$

$$x_{2,t+1} = \Lambda_1 x_{2,t} + \Lambda_2 \sigma \epsilon_{t+1}$$

with matrices Λ_1 and Λ_2. The scalar $\sigma > 0$ is known.

Schmitt-Grohé and Uribe (2004) show that the coefficients of the solutions on the terms linear and quadratic of the state vector are certainty-equivalent. As a result, the approximate solution is

$$\hat{e}_t = D\hat{x}_t + \frac{1}{2} G \hat{x}_t \otimes \hat{x}_t + \frac{1}{2} H \sigma^2$$

$$\hat{x}_{t+1} = D^*\hat{x}_t + \frac{1}{2}G^*\hat{x}_t \otimes \hat{x}_t + \frac{1}{2}H^*\sigma^2 + \sigma N\sigma\epsilon_{t+1}$$

where hats denote deviations from the deterministic steady state.

We define

$$x_{1,t} = [k_t \;\; R_{t-1} \;\; d_{t-1} \;\; m_{t-1} \;\; y_{t-1} \;\; \pi_{t-1}]'$$

$$x_{2,t} = [z_t \;\; \Phi_t \;\; g_t]'$$

$$e_t = \left[y_t \;\; R_t \;\; d_t \;\; \mathrm{mc}_t \;\; inv_t \;\; c_t \;\; m_t \;\; \pi_t \;\; \ell_t \;\; q_t \;\; w_t \;\; \lambda_t \;\; \mu_t \;\; \tau_t^{ls}\right]'$$

APPENDIX 3: COMPUTING CONDITIONAL MOMENTS

Kim, Kim, Schaumburg, and Sims (2003) suggest that using the expressions of the full second-order approximation for computing conditional moments recursively introduces spurious higher-order terms. This problem can be avoided by exploiting the linear (first-order) part of the solution. Let $\hat{e}_t^{(2)}$ denote the full second-order solution, and $\hat{e}_t^{(1)}$ denote the linear part. Following Paustian (2003), we can re-write the system of solutions

$$\begin{bmatrix} \hat{e}_t^{(2)} \\ \hat{e}_t^{(1)} \otimes \hat{e}_t^{(1)} \end{bmatrix} = M_1 \begin{bmatrix} \hat{x}_t^{(2)} \\ \hat{x}_t^{(1)} \otimes \hat{x}_t^{(1)} \end{bmatrix} + K_1 \tag{A3.1}$$

$$\begin{bmatrix} \hat{x}_{t+1}^{(2)} \\ \hat{x}_{t+1}^{(1)} \otimes \hat{x}_{t+1}^{(1)} \end{bmatrix} = M_2 \begin{bmatrix} \hat{x}_t^{(2)} \\ \hat{x}_t^{(1)} \otimes \hat{x}_t^{(1)} \end{bmatrix} + K_2 + u_{t+1} \tag{A3.2}$$

Define

$$X_t = \begin{pmatrix} \hat{x}_t^{(2)} \\ \hat{x}_t^{(1)} \otimes \hat{x}_t^{(1)} \end{pmatrix}$$

$$Y_t = \begin{pmatrix} \hat{e}_t^{(2)} \\ \hat{e}_t^{(1)} \otimes \hat{e}_t^{(1)} \end{pmatrix}$$

Equations A3.2 and A3.1 can be re-written by repeated substitution as

$$X_{t+k} = M_2^k X_t + \sum_{i=0}^{k-1} M_2^i (K_2 + u_{t+k-i})$$

$$Y_{t+k} = M_1 X_{t+k} + K_1 = K_1 + M_1 M_2^k X_t + \sum_{i=0}^{k-1} M_1 M_2^i (K_2 + u_{t+k-i})$$

The expectation conditional on an initial state vector takes the form

$$E\left(Y_{t+k}|X_t\right) = K_1 + M_1 M_2^k X_t + \sum_{i=0}^{k-1} M_1 M_2^i K_2$$

The conditional variance can be computed from

$$Y_{t+k} - E\left(Y_{t+k}|X_t\right) = \sum_{i=0}^{k-1} M_1 M_2^i u_{t+k-i}$$

$$Cov(Y_{t+k}|X_t) = E\left\{\left[Y_{t+k} - E\left(Y_{t+k}|X_t\right)\right]\left[Y_{t+k} - E\left(Y_{t+k}|X_t\right)\right]'|X_t\right\} = \sum_{i=0}^{k-1} M_1 M_2^i \Sigma_u \left(M_1 M_2^i\right)'$$

where $\Sigma_u := E(u_t u_t')$.

In what follows, we report the computations of Σ_u. Paustian (2003) shows that u_t takes the form

$$u_t = \begin{pmatrix} \sigma N \epsilon_t \\ \sigma^2 \left(N \otimes N\right)\left(vec(I) - \epsilon_t \otimes \epsilon_t\right) \end{pmatrix}$$

The mean of u_t is $\mathsf{E}u_t = 0$. Since $\epsilon_t \backsim N(0, I)$, we have the following

$$\mathsf{E}\epsilon_{it}^3 = 0$$

$$\mathsf{E}\epsilon_{it}\epsilon_{jt}\epsilon_{kt} = 0$$

if any of the indices i, j, k are different. This gives

$$\sigma^2 \mathsf{E}\left(N\epsilon_t\epsilon_t' N'\right) = \sigma^2 NN'$$

$$\sigma^3 \mathsf{E}\left\{\left(N\epsilon_t\right)\left[\left(N \otimes N\right)\left(vec(I) - \epsilon_t \otimes \epsilon_t\right)\right]'\right\} =$$

$$= \sigma^3 \mathsf{E}\left\{\left(N\epsilon_t\right)\left(vec(I)' - \epsilon_t' \otimes \epsilon_t'\right)\left(N' \otimes N'\right)\right\} = 0$$

Finally

$$\sigma^4 \mathsf{E}\left\{\left[\left(N \otimes N\right)\left(vec(I) - \epsilon_t \otimes \epsilon_t\right)\right]\left[\left(N \otimes N\right)\left(vec(I) - \epsilon_t \otimes \epsilon_t\right)\right]'\right\} =$$

$$= \sigma^4 \left(N \otimes N\right) \mathsf{E}\left\{\left[\left(vec(I) - \epsilon_t \otimes \epsilon_t\right)\right]\left[\left(vec(I) - \epsilon_t \otimes \epsilon_t\right)\right]'\right\}\left(N \otimes N\right)'$$

where

$$\mathsf{E}\left\{\left[\left(vec(I) - \epsilon_t \otimes \epsilon_t\right)\right]\left[\left(vec(I) - \epsilon_t \otimes \epsilon_t\right)\right]'\right\} =$$

$$= \mathsf{E}\left\{vec(I)vec(I)' + \epsilon_t\epsilon_t' \otimes \epsilon_t\epsilon_t' - \left(\epsilon_t \otimes \epsilon_t\right) vec(I)' - vec(I)\left(\epsilon_t \otimes \epsilon_t\right)\right\} =$$

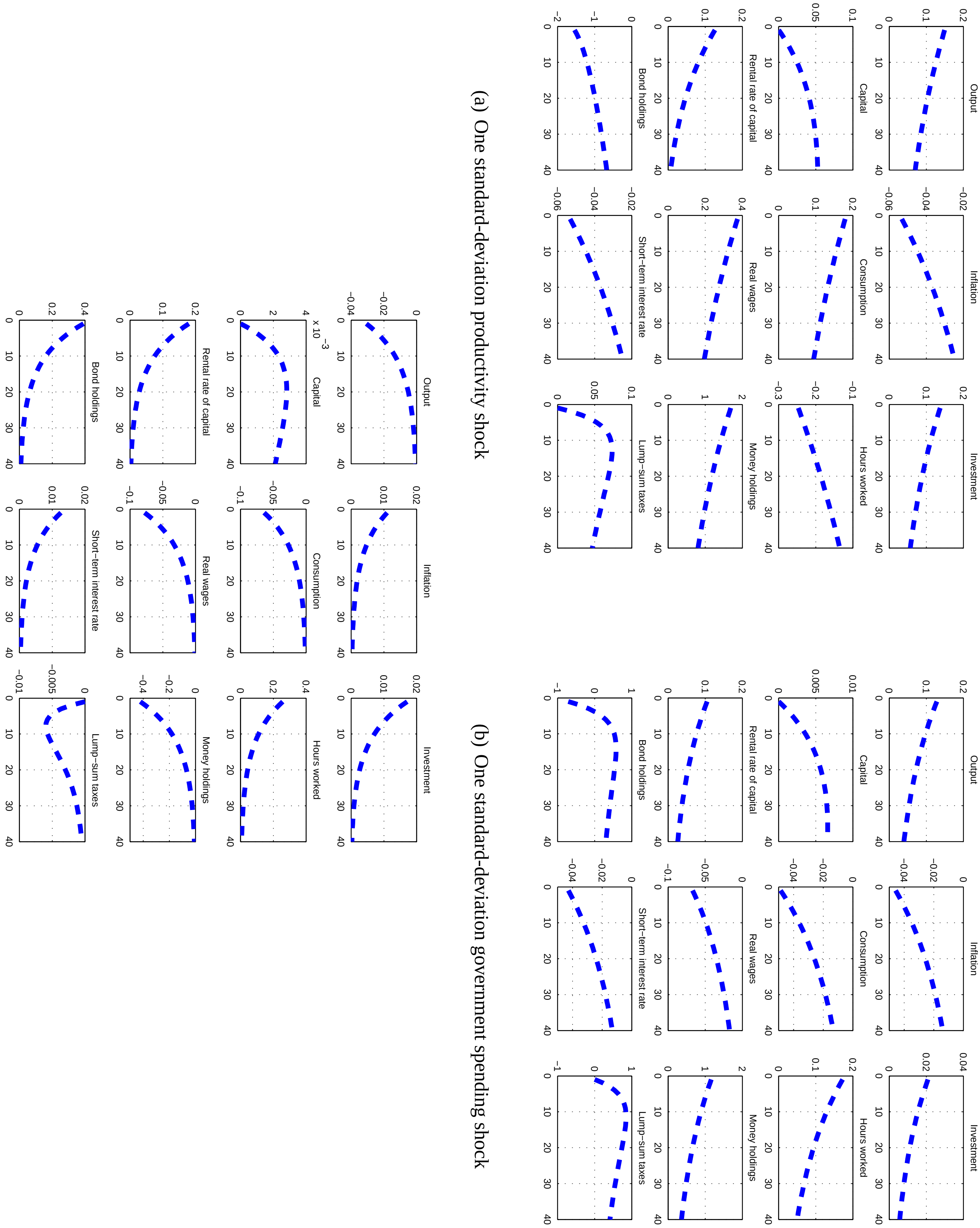
$$= \mathsf{E}\left(\epsilon_t\epsilon_t' \otimes \epsilon_t\epsilon_t'\right) - vec(I)vec(I)' =$$

$$= 2vec(I)vec(I)'$$

The variance matrix Ξ of ξ_t is

$$\mathsf{E}\xi_t\xi_t' = M_1 \mathsf{E}\begin{pmatrix} \sigma N\epsilon_t \\ \sigma^2\left(N \otimes N\right)\left(vec(I) - \epsilon_t \otimes \epsilon_t\right) \end{pmatrix}\begin{pmatrix} \sigma N\epsilon_t \\ \sigma^2\left(N \otimes N\right)\left(vec(I) - \epsilon_t \otimes \epsilon_t\right) \end{pmatrix}' M_1' =$$

$$= M_1 \begin{pmatrix} \sigma^2 NN' & 0 \\ 0 & 2\sigma^4\left(N \otimes N\right) vec(I)vec(I)'\left(N \otimes N\right)' \end{pmatrix} M_1'$$

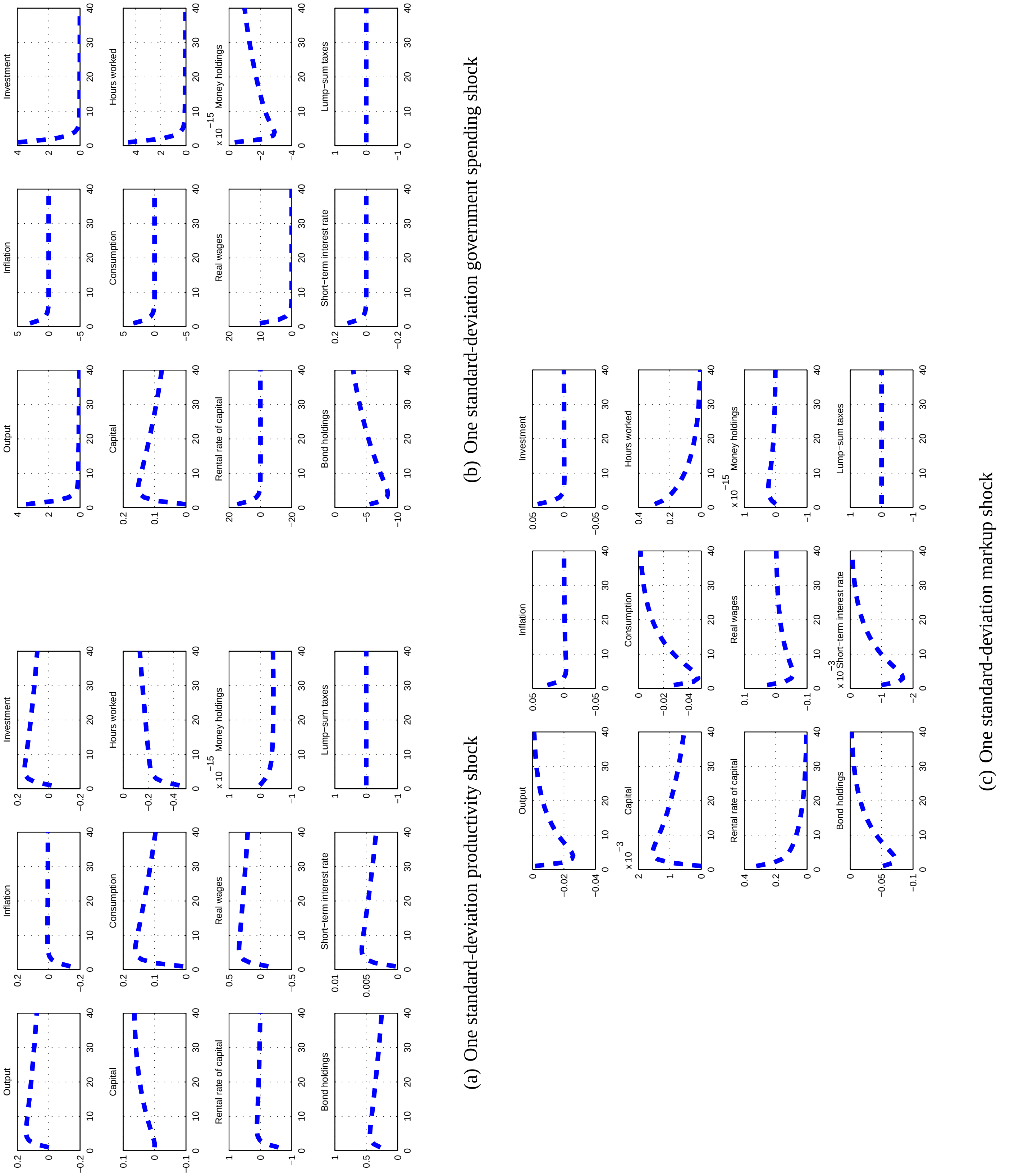


(a) One standard-deviation productivity shock

(b) One standard-deviation government spending shock

(c) One standard-deviation markup shock

FIGURE 1—: Impulse responses for the standard Taylor rule

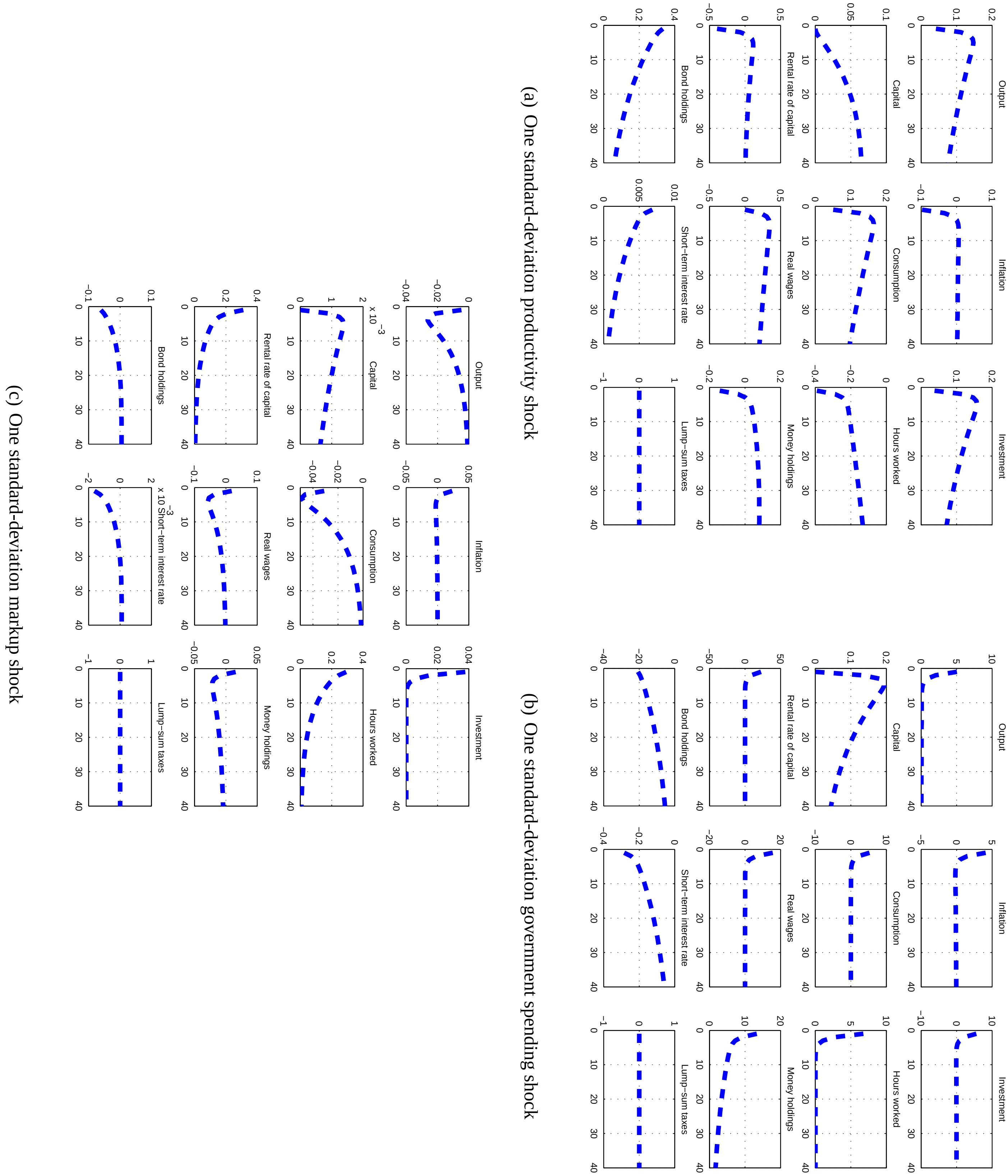


(a) One standard-deviation productivity shock

(b) One standard-deviation government spending shock

(c) One standard-deviation markup shock

FIGURE 2—: Impulse responses for the money-growth rule

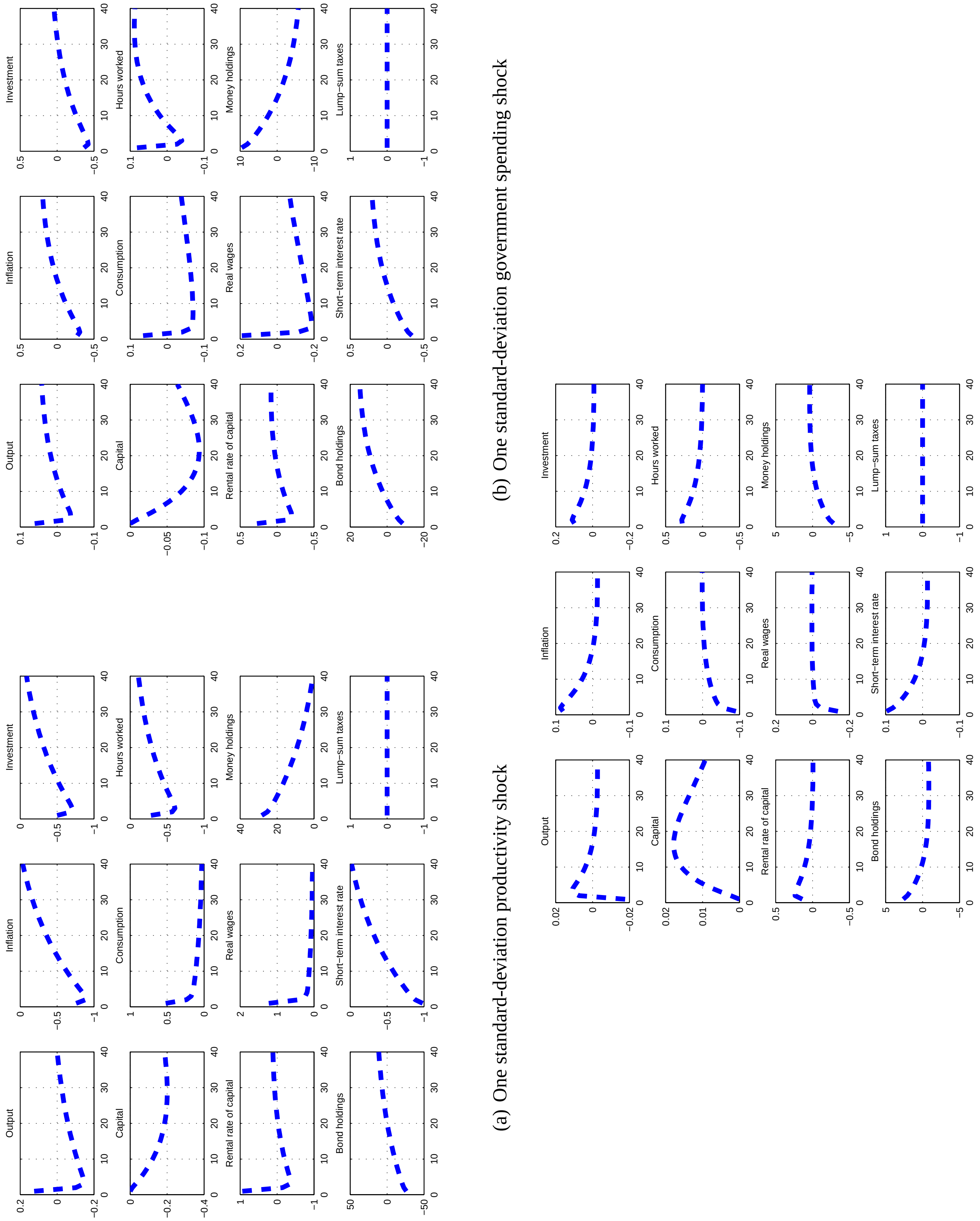


(a) One standard-deviation productivity shock

(b) One standard-deviation government spending shock

(c) One standard-deviation markup shock

FIGURE 3—: Impulse responses for the monetary policy rule with opportunistic disinflation



(a) One standard-deviation productivity shock

(b) One standard-deviation government spending shock

(c) One standard-deviation markup shock

FIGURE 4—: Impulse responses for the monetary policy rule with deliberate disinflation

TABLE I:
CALIBRATION OF THE MODEL

Description	Parameter	Value
Subjective discount factor	β	0.9966
Weight on leisure objective	ξ	0.001
Share of consumption objective	a	0.99
Interest elasticity	μ	0.39
Coefficient of relative risk aversion	$1/\sigma$	3
Share of labour effort	ℓ	1/4
Investment-output ratio	$\bar{i}/\bar{y}$	0.25
Capital-output ratio	$\bar{k}/\bar{y}$	10.4
Money-output ratio	$\bar{m}/\bar{y}$	0.44
Steady-state inflation	$\bar{\pi}$	$1.042^{(1/4)}$
Adjustment cost of prices	ϕ_P	60
Adjustment cost of capital	ϕ_K	433
Capital depreciation rate	δ	0.024
Capital elasticity of intermediate output	α	0.33
Elasticity of substitution of interm. goods	θ	10
Persistence of productivity shock	ρ_z	0.98
Steady state of productivity shock	z	1
Standard dev. of productivity shock	σ_z^2	0.055
Persistence of markup shock	ρ_Φ	0.911
Standard dev. of markup shock	σ_Φ^2	0.141
Persistence of government-spending shock	ρ_G	0.97
Standard dev. of government-spending shock	σ_G	0.1
Public spending-output ratio	$\bar{g}/\bar{y}$	0.148
Weight on objective of inherited inflation	λ	0.5
Baseline parameter on fiscal-policy rule	ψ_1	0.8
Baseline parameter on fiscal-policy rule	ψ_2	0

TABLE II:
OPTIMAL LINEAR TAYLOR RULES

	(a) Conditional Welfare							
	α_π	α_y	α_R	ψ_1	$\mathcal{W}_0$	$\%\Delta_c$	$\%\Delta_E$	$\%\Delta_V$
Standard rule	2.8	1.0	1.0	0.2	-91.4098*	3.3318	-9.1087	13.6871
Rule in first difference	2.8	1.0	1.0	0.2	-91.4098*	3.3318	-9.1087	13.6871
Forward-looking rule	1.2	0.2	0.4	0.8	-91.4096	3.3315	-9.1085	13.6866
Deliberate disinflation	1.2	1.0	0.2	0.0	-86.99	-6.41	-98.62	670

	(b) Unconditional Welfare					
	α_π	α_y	α_R	ψ_1	$\mathcal{W}$	$\%\Delta_U$
Standard rule	1.2	1.0	0.0	0.2	-3.35	$1.2e4$
Rule in first difference	1.2	0.4	0.0	1.0	-0.12	-84.81
Forward-looking rule	3.0	0.0	0.2	0.2	-3.378	$1.2e4$
Deliberate disinflation	1.4	1.0	0.4	0.2	0.001	-99.99

Legend: *The welfare levels achieved by the standard rule and the rule in first difference differ only at the seventh digit.

TABLE III:
OPTIMAL OPPORTUNISTIC POLICY RULES

(a) Conditional Welfare

κ_0	κ_1	κ_3	ψ_1	$\mathcal{W}_0$	$\%\Delta_c$	$\%\Delta_E$	$\%\Delta_V$
0.0	5.0	1.0	0.0	-89.26	-1.48	-70.45	233

(b) Unconditional Welfare

κ_0	κ_1	κ_2	ψ_1	$\mathcal{W}$	$\%\Delta_U$
0.0	3.2	0.0	0.0	-3.41	$1.2e4$

TABLE IV:
OPTIMAL MONEY-GROWTH RULE

(a) Conditional Welfare

ρ_m	ψ_1	$\mathcal{W}_0$	$\%\Delta_c$	$\%\Delta_E$	$\%\Delta_V$
0.6	0.0	-90.21	0.64	-51.23	106

(b) Unconditional Welfare

ρ_m	ψ_1	$\mathcal{W}$	$\%\Delta_U$
0.7	0.0	-3.40	1.2e4

TABLE V:
DESCRIPTIVE STATISTICS

	(a) Standard deviation (%)					
	y_t	π_t	R_t	k_t	c_t	ℓ_t
Standard rule	2.22	0.28	0.28	0.73	3.30	7.25
Rule in first difference	2.22	0.28	0.28	0.73	3.30	7.25
Forward-looking rule	2.23	0.26	0.26	0.75	3.30	7.25
Deliberate disinflation	1.46	46.44	43.94	7.45	2.38	17.01
Money-growth rule	17.25	10.82	0.02	17.67	17.04	33.59
Opportunistic rule	32.58	19.65	0.85	1.69	35.56	60.93
	(b) Correlation with output					
	y_t	π_t	R_t	k_t	c_t	ℓ_t
Standard rule	1	-1.00	-0.99	0.72	0.89	-0.80
Rule in first difference	1	-1.00	-0.99	0.72	0.89	-0.80
Forward-looking rule	1	-0.99	-0.99	0.72	0.89	-0.79
Deliberate disinflation	1	0.91	0.88	0.34	-0.21	0.66
Money-growth rule	1	0.92	0.99	0.35	0.99	0.68
Opportunistic rule	1	0.93	-0.47	0.26	0.99	0.83

TABLE VI:
OPTIMAL POLICY RULES FOR THE AD-HOC LOSS FUNCTION 8

	α_π	α_y	α_R	ψ_1	ϕ_π	$\mathcal{L}^{op}$
Standard rule	2.8	0.6	1.0	0.2	1.0	0.0013
Rule in first difference	3.0	0.2	0.8	0.6	1.0	-0.0043
Forward-looking rule	1.6	0.2	0.4	0.6	1.0	0.0012
Deliberate disinflation	1.8	0.2	1.0	1.0	1.0	0.0014
	κ_0	κ_1	κ_2	ψ_1	ϕ_π	$\mathcal{L}^{op}$
Opportunistic rule	0.6	3.0	0.4	0.0	0.0	0.093
		ρ_m	ψ_1	ϕ_π	$\mathcal{L}^{op}$	
Money-growth rule		0.0	0.0	1.00	0.11	

TABLE VII:
OPTIMAL POLICY RULES FOR THE AD-HOC LOSS FUNCTION 9

	α_π	α_y	α_R	ψ_1	ϕ_π	$\mathcal{L}^{int}$
Standard rule	1.1	0	0	0.2	0	1.5177
Rule in first difference	1.2	0.1	0.3	0.1	0.3	2.4755
Forward-looking rule	1.1	0	0	0.2	0	4.2512
Deliberate disinflation	1.3	0.2	0	0.5	0.5	3.2257
	κ_0	κ_1	κ_2	ψ_1	ϕ_π	$\mathcal{L}^{int}$
Opportunistic rule	0	3.0	0	0.4	1.0	38.16
		ρ_m	ψ_1	ϕ_π	$\mathcal{L}^{op}$	
Money-growth rule		0	0	1.0	39.81	

TABLE VIII:
ROBUSTNESS ANALYSIS ON OPTIMAL LINEAR TAYLOR RULES

	(a) $\phi_P = 90$							
	α_π	α_y	α_R	ψ_1	$\mathcal{W}_0$	$\%\Delta_c$	$\%\Delta_E$	$\%\Delta_V$
Standard rule	2.8	1.0	1.0	0.2	-91.407	3.335	-9.110	13.6923
Rule in first difference	2.8	1.0	1.0	0.8	-91.405	3.332	-9.114	13.6941
Forward-looking rule	1.2	0.2	0.4	0.8	-91.409	3.337	-9.111	13.6958

	(a) $\theta = 2$							
	α_π	α_y	α_R	ψ_1	$\mathcal{W}_0$	$\%\Delta_c$	$\%\Delta_E$	$\%\Delta_V$
Standard rule	1.2	1.0	0.2	0.6	-58.745	-3.522	-13.662	11.7445
Rule in first difference	1.2	1.0	0.2	0.6	-58.744	-3.521	-13.668	11.7535
Forward-looking rule	1.2	0.8	0.2	0.4	-58.740	-3.540	-13.645	11.7017

TABLE IX:
ROBUSTNESS ANALYSIS ON OPTIMAL OPPORTUNISTIC POLICY RULES

	κ_0	κ_1	κ_3	ψ_1	$\mathcal{W}_0$	$\%\Delta_c$	$\%\Delta_E$	$\%\Delta_V$
$\phi_P = 90$	0.0	5.0	1.0	0.0	-88.48	-3.20	-77.68	333
$\theta = 2$	0.0	5.0	1.0	0.0	-45.06	-43.25	-97.67	234
$\lambda = 0.2$	0.0	5.0	1.0	0.0	-89.26	-1.48	-70.45	233
$\lambda = 0.3$	0.0	5.0	1.0	0.0	-89.26	-1.48	-70.45	233
$\lambda = 0.4$	0.0	5.0	1.0	0.0	-89.26	-1.48	-70.45	233
$\lambda = 0.6$	0.0	5.0	1.0	0.0	-89.26	-1.48	-70.45	233
$\lambda = 0.7$	0.0	5.0	1.0	0.0	-89.26	-1.48	-70.45	233
$\lambda = 0.8$	0.0	5.0	1.0	0.0	-89.26	-1.48	-70.45	233
$\lambda = 0.9$	0.0	5.0	1.0	0.0	-89.26	-1.48	-70.45	233

TABLE X:
ROBUSTNESS FOR THE MONEY-GROWTH RULE

(a) $\phi_P = 90$

ρ_m	ψ_1	$\mathcal{W}_0$	$\%\Delta_c$	$\%\Delta_E$	$\%\Delta_V$
0.7	0.0	-89.79	-30.01	-71.58	146

(b) $\theta = 2$

ρ_m	ψ_1	$\mathcal{W}_0$	$\%\Delta_c$	$\%\Delta_E$	$\%\Delta_V$
0.0	0.0	-52.44	-23.14	-87.48	514