# Horizontal Mergers with Scale Economies

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**Abstract**

We examine the profitability and social efficiency of horizontal mergers in a Cournot oligopoly with decreasing average costs. Assuming the merger allows for a reduction in the total amount of fixed costs, we identify the conditions under which the merger is, respectively, profitable and socially desirable. There exists an admissible parameter range wherein the merger is socially convenient but not profitable. In such a case, the policy maker may induce firms to merge through subsidies financed via a lump sum tax.

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# 1 Introduction

The bearings of production technology on the profitability of horizontal mergers in a Cournot oligopoly has attracted wide attention ever since Salant *et al.* (1983). Under the assumption of constant average cost, they have shown that, unless a vast majority of the firms in the industry merge, the merger itself is not profitable, and therefore the scope for antitrust action is limited. This seminal contribution has generated a stream of literature investigating other cases, such as that of a quadratic cost function (Perry and Porter, 1985; McAfee and Williams, 1992).[1] To the best of our knowledge, the only contribution where the case of increasing returns (i.e., economies of scale) is illustrated is Farrell and Shapiro (1990), where, however, it is assumed that the amount of fixed costs retained by the firm resulting from the merger is exactly equal to the sum of the pre-merger fixed costs of the participating firms. The usual argument put forward to justify a merger is the increase in productive efficiency generated by the merger itself (see, e.g., Farrell and Shapiro, 2000). Therefore, an interesting perspective is that where such efficiency gain is the outcome of an adjustment in fixed costs via the merger. This is precisely the view that we adopt in the present paper. We model a situation where a synergy operates, in such a way that the overall fixed cost associated with the merger is a fraction of the sum of the fixed costs borne *ex ante* by the firms taking part into the merger. This setup allows us to draw several conclusions. The first is that, if the fixed cost savings generated by

[1] Other routes taken by the literature include the analysis of horizontal mergers under Bertrand competition (Davidson and Deneckere, 1985) or non-linear market demand functions (Hennessy, 2000), *inter alia*.

the merger are large enough, then the merger becomes profitable irrespective of the number of firms involved. Likewise, there exists a critical threshold of such efficiency gain above which the merger is socially desirable. The ranking of the two thresholds depend on the number of firms in the industry before the merger takes place. If the size of the merger (that is, the number of firms involved in it) is small enough, then we observe the counterintuitive and, thus far, neglected situation where the merger is socially convenient while being not profitable. If this is the case, then the policy maker may induce firms to merge by subsidising them (for instance, by using funds raised through taxes) provided that the representative shareholder is sufficiently atomistic to ensure that future dividends compensate current taxation.

## 2 The model

We examine a Cournot oligopoly where, *ex ante*, $n$ firms sell a homogeneous good whose demand function is $p = a - \sum_{i=1}^{n} q_i$. Each firm uses a technology characterised by increasing returns to scale, summarised by the cost function $C_i = cq_i + F$, where $c > 0$ is the marginal cost and $F > 0$ is a fixed cost.

Firms compete simultaneously. Therefore, individual outputs and profits at the symmetric Nash equilibrium are:

$$q^*(n) = \frac{a-c}{n+1}\,;\ \pi^*(n) = \frac{(a-c)^2}{(n+1)^2} - F \tag{1}$$

and the corresponding consumer surplus is:

$$CS^*(n) = \frac{[nq^*(n)]^2}{2} = \frac{n^2(a-c)^2}{2(n+1)^2}. \tag{2}$$

Hence, social welfare is $SW^*(n) = n\pi^*(n) + CS^*(n)$.

Now consider the situation where $m \in [2, n-1]$ firms evaluate the perspective of merging horizontally. We suppose that the merger gives rise to a restructuration of production plants within the resulting firm; in particular, we assume that the amount of fixed cost be equal to $\widehat{F} \equiv (1+b) F$, with $b \in [0, m-1]$. Therefore, for all $b < m-1$, the usual argument whereby a merger is justified on efficiency grounds applies in the form of a reduction of the total amount of fixed cost borne by society.

As a consequence of the merger, outputs, profits and consumer surplus modify as follows:

$$\begin{aligned} q^* (n-m+1) &= \frac{a-c}{n-m+2}; \\ \widehat{\pi}^* (n-m+1) &= \frac{(a-c)^2}{(n-m+2)^2} - (1+b) F \\ \pi^* (n-m+1) &= \frac{(a-c)^2}{(n-m+2)^2} - F \\ CS^* (n-m+1) &= \frac{(n-m+1)^2 (a-c)^2}{2 (n-m+2)^2} \end{aligned} \tag{3}$$

where $\widehat{\pi}^* (n-m+1)$ is the profit of the firm resulting from the merger while $\pi^* (n-m+1)$ is the profit accruing to each of the $n-m$ firms that have remained independent. The associated social welfare is

$$SW^* (n-m+1) = \widehat{\pi}^* (n-m+1) + (n-m) \pi^* (n-m+1) + CS^* (n-m+1). \tag{4}$$

Comparing *ex ante* and *ex post* profits and welfare, we may prove the following:

**Proposition 1** *For all admissible levels of* $\{b, n, m\}$, *there exist the threshold values of fixed costs* $F_\pi$ *and* $F_{SW}$ *above which the merger involving* $m$

*firms is, respectively, privately and socially desirable. If $n = 3$ and $m = 2$, then $F_\pi < F_{SW}$. If instead $n \geq 4$, then (i) $F_\pi > F_{SW}$ if $m$ is sufficiently small, and (ii) $F_\pi < F_{SW}$ if $m$ is sufficiently large.*

**Proof.** The private (profit) incentive to carry out the merger requires:

$$\frac{\widehat{\pi}^* (n - m + 1)}{m} > \pi^* (n) \Leftrightarrow \tag{5}$$

$$F > F_\pi \equiv \frac{m (a - c)^2}{m - 1 - b} \left[ \frac{1}{(n + 1)^2} - \frac{1}{m (n - m + 2)^2} \right]$$

while the social incentive requires:

$$SW^* (n - m + 1) > SW^* (n) \Leftrightarrow \tag{6}$$

$$F > F_{SW} \equiv \frac{(a - c)^2}{2 (m - 1 - b)} \left[ \frac{1}{(n - m + 2)^2} - \frac{1}{(n + 1)^2} \right].$$

Comparing $F_\pi$ and $F_{SW}$, we obtain $F_\pi = F_{SW}$ at

$$m_1 = \frac{5}{4} + n - \frac{1}{4}\sqrt{24n + 33}; m_2 = \frac{5}{4} + n + \frac{1}{4}\sqrt{24n + 33}. \tag{7}$$

Clearly, $m_2 > n$ and can be disregarded; $m_1$ is always smaller than $n$, but $m_1 \geq 2$ if and only if $n \geq 4$. If $n = 3$, we have a special case where $m_1 = 1.688$. Therefore, we have two cases:

(i) for $n = 3$ and $m = 2$

$$F_\pi = \frac{(a - c)^2}{72 (1 - b)} < F_{SW} = \frac{7 (a - c)^2}{288 (1 - b)}; \tag{8}$$

(ii) for all $n \geq 4$,

$$\begin{aligned} F_\pi > F_{SW} \text{ for all } m \in [2, m_1); \\ F_\pi < F_{SW} \text{ for all } m \in (m_1, n - 1]. \end{aligned} \tag{9}$$

This concludes the proof. ■

Now take the special case where $n = 3$ and $m = 2$. Here, given that the *a priori* degree of concentration of the industry is very high, the critical threshold for $F$ above which the antitrust agency permits the merger is necessarily higher than that above which firms find it profitable.

The perspective changes significantly for all $n \geq 4$. Provided that, as $n$ increases, the market becomes progressively more competitive, then $m_1$ becomes relevant and determines the presence of an interval where relatively small mergers are socially desirable. Note that $m_1$ is monotonically increasing in $n$. To this regard, it is worth stressing that, for all $F \in (F_{SW}, F_\pi)$, any merger involving $m \in [2, m_1)$ is socially convenient while it is not privately so. That is, we have the seemingly counterintuitive result whereby the antitrust agency would like the $m$ firms to merge, whereas they prefer to remain independent. To the best of our knowledge, this possibility has been overlooked so far in the existing literature. The reason of this result is that $m$ is small. This entails two related consequences: the first is that, for any given $b$, the reduction in fixed costs enjoyed by the firms involved in the merger is limited; the second is that the reduction in the overall number of firms in the industry after the merger has taken place is also limited. The first fact makes the merger unattractive to firms, while the second makes it appealing for the regulator. When this is the case, the policy maker may design an income transfer from consumers to the merging firms by means of a lump sum tax that becomes a subsidy to firms, so as to make the merger attractive to them. The total subsidy amounts to

$$\left(\pi^*(n) - \frac{\widehat{\pi}^*(n - m + 1)}{m} + \varepsilon\right) m \tag{10}$$

where $\varepsilon$ is positive and arbitrarily small. This of course has no redistributive effects provided that all agents holds symmetric shares of the stock of those firms that are to carry out the merger.

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