Natural Advantage, Location and Trade Patterns in Increasing Returns to Scale Industries *

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Abstract

In a two sectors, two regions economy I show that the higher increasing returns to scale of an industry, the easier it will concentrate in response to natural advantage. To this end, one sector is assumed to be perfectly competitive and the other is monopolistically competitive, with a region's firms producing at a lower marginal cost than the others in the monopolistic sector (or equivalently producing varieties more intensely demanded by consumers). If capital is mobile between regions in the long run, I analytically characterize the process of industrial location of the imperfectly competitive sector in the region with the comparative advantage.

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1 Introduction

This paper follows a well established tradition in economic theory by studying the interaction between increasing returns and local market conditions in the joint determination of trade patterns and firms' location. The main issue this paper addresses is to establish how exogenous variations in increasing returns to scale map into changes of location equilibria and, consequently, trade patterns in a two regions world, where one of them has a comparative advantage over the other in the production of imperfectly competitive commodities. Since the comparative advantage descends from an exogenous technological difference in the increasing returns to scale industry, I will refer interchangeably to the terms natural advantage and comparative advantage. I will argue, in general terms speaking, that the higher market power of firms in a given industry, the more sensitive they will be to local market conditions. Local markets' attractiveness can be defined with respect to several dimensions: for instance, it could be defined in terms of the size of local demand, as in Krugman (1980) and Venables (1987) trade models, or in terms of a local cost advantage, within a one-factor-of-production Ricardian model, as in Venables (1987) again. In investigating how location and trade depend on the intensity of scale economies, when a region has a natural advantage that leads to a comparative advantage, I then extend Venables research.

Let us set up the model. Economic space is made of two regions hosting two sectors which differ in the underlying market structure. The first is monopolistically competitive and produces an array of horizontally differentiated varieties, while the other is a residual sector, characterized by perfect competition, representing the rest of the economy. I assess the emergence of different patterns of trade and location equilibria in the monopolistic sector as a consequence of: *a)* a differential in marginal cost among firms according to the region they belong to; *b)* a differential in the degree of competition in each region, due to the difference in the number of firms in each local market; *c)* different degrees of overall competitive pressure, measured by the total number of firms in the economy, determined in turn by the degree of increasing returns to scale; *d)* the abandoning of the CES (Dixit-Stiglitz) monopolistic competition model, in favour of a linear demand specification. Due to the linearity of the demand for differentiated varieties, we will show that point *a)* is amenable to an interpretation in terms of different intensities of demands for the differentiated products, according to the region where they are manufactured.

The perfectly competitive sector is characterized by constant returns to scale. The monopolistically competitive sector is modelled according to a quadratic specification cast in an economic geography setting by Ottaviano, Tabuchi, and Thisse (2002). This sector can be thought to be manufacturing. The absence of strategic interaction among firms makes monopolistic competition particularly suited to capture market structure prevailing in traditional sectors as textiles, clothing, and food processing.

After having derived short-run trade equilibria, we make the hypothesis that capital is mobile between regions in the long run, and flows where the rental rate is higher. I build on Ottaviano et al. (2002) and Behrens (2004, 2005) modelling. While their papers are full-fledged core-periphery (CP henceforth) models (what is mobile there are workers that locate where indirect utility is higher) I assume the mobility of capital towards locations where the rental rate is higher. The main contribution of the present paper is to add to the picture exogenous asymmetries between the two regions, and to see how they interact with increasing returns to scale in shaping the space economy. I model asymmetries as stemming either from a cost advantage of producing in region A over region B, or from a demand premium that varieties manufactured in A enjoys with respect to B products. The linearity of the demand functions then ensures that, from the point of view of the individual firm, profit functions in the two circumstances are analytically equivalent, giving to our problem a twofold interpretation.

The basic set up employed in this paper then follows what is known in the economic geography literature under the headings of *footloose capital* model (Martin and Rogers, 1995, Ottaviano, 2001, Baldwin et al., 2003), FC model hereafter. The model choice is dictated by the need of building a framework not too far from standard trade models, such that labour is immobile but capital and firms are mobile, thus easing the comparison of our results to those of the trade literature. The differences between CP and FC models are thoroughly analyzed in Baldwin et al. (2003). For our purposes it suffices to stress the following. In a CP setting the mobile factor is labour, so that its migration induces also an expenditure shifting in the region where migration occurs, because people spend their earnings where they live. On the contrary, in a FC model the mobile factor is capital, whose rewards are repatriated to capital owners who are immobile in the two regions. Due to this fundamental difference, the FC model does not show circular causality (self-reinforcement) in agglomeration, and is more tractable analytically.[1]

As already shown in Behrens (2005), the linear demand model gives rise to asymmetric trade patterns when the two regions differ in the number of firms located, with one region hosting significantly more firms than the other. The idea is that when many firms are located in a region it is difficult to penetrate its market. The possibility of asymmetric trade patterns is a realistic feature being inevitably lost under the CES Dixit-Stiglitz specification when both regions host a positive share of firms. Introducing cost (or demand) asymmetries among the two regions, so that the more crowded region produces at a lower cost, strengthens the tendency of asymmetric patterns to arise. If they want to export to the other region, firms in the high cost one have to overcome the cost disadvantage as well. This makes the range of two-way trade smaller.

As argued in Behrens (2004), the choice of including or not in the computation of the price index of a given region varieties not traded in equilibrium is sometimes essential for the resulting equilibrium. This is precisely the case under the weighted quadratic utility specification of Tabuchi and Thisse (2002). If different ways of computing the price index in a given region yield different results, the equivalency between price and quantity competition should *a fortiori* be invalid in trade models with non-traded varieties, and this is a striking difference with respect to baseline specifications of monopolistic competition models. I argue that, in Behrens model, price competition *and* a special

[1] Another possible comparison could be carried out between the FC model and the *footloose entrepreneur* model (see Baldwin *et al.*, chapter 4). I prefer to stick to the comparison with the CP model since previous literature on asymmetric trade in economic geography was about CP models.

assumption he makes about prices of non-traded commodities are indeed equivalent to assume quantity competition. This feature gives to Behrens conjecture more strength with respect to competing hypotheses about the formation of the price index. All subsequent calculations in this paper are derived consistently with quantity setting, since it simplifies computations.

The paper is organized as follows. In section 2 we present the model. In section 3 we compute the short-run equilibrium of the economy (for a fixed spatial distribution of firms) distinguishing among different trade patterns. Afterwards (section 4) we let capital going where the rental rate is higher. In some cases, which we analytically characterize, full agglomeration of the manufacturing sector in one region will be the long-run equilibrium. Section 5 focuses on the link between location, trade surplus, and scale economies in the presence of natural advantage.

2 The model

The model developed in this paper is in various manners linked to other works in the economic geography literature. The closest relatives are Belleflamme et al. (2000), Ottaviano et al. (2002), and Behrens (2004, 2005). The economy is made of two regions $s = \{A, B\}$ of equal size, and two sectors: a monopolistically competitive sector, producing an array of differentiated varieties, and a perfectly competitive sector producing a homogeneous good 0, which we may think of as a composite commodity summarizing the rest of the economy.

2.1 Consumer's behaviour

The representative consumer in the two regions shares the same preferences and maximizes the following utility function:

$$U(q_0, x(j)) = \xi \int_{j\in N} x(j)dj - \frac{1-\omega}{2}\int_{j\in N}[x(j)]^2 dj - \frac{\omega}{2}\left[\int_{j\in N} x(j)dj\right]^2 + q_0 \qquad (1)$$

Parameters in the utility function are $\xi > 0$ and $0 \leq \omega \leq 1$. The set of varieties is $S = \{j | j \in [0, N]\}$. They are uniformly distributed on $[0, N]$, with N being the total mass of the monopolistic sector.[2] The parameter ξ is a proxy for the intensity of preference for the differentiated good. The higher ξ, the higher this preference. The parameter ω represents the degree of product differentiation among varieties. When ω approaches zero varieties are so much differentiated that they can be thought to belong to completely different sectors (total utility is simply additive in the utility derived from each good and the inverse demand function of each variety, derived later, only depends on the quantity demanded of that same variety), while ω equal to 1 represents perfectly homogeneous products.

We assume that the representative consumer in region s is endowed with K_s units of capital and L units of labour, with labour supply L being equal in the two regions. Income comes from the rental rate of capital and wage.

The budget constraint of the representative individual in region A can be written as

$$\int_{j \in n_A} p(j)x(j)dj + \int_{j \in n_B} p(j)x(j)dj + p_0 q_0 = w_A L + \max\{r_A, r_B\} K_A \tag{2}$$

where $p(j)$ is the price of a variety, $x(j)$ is the quantity demanded, w_A is wage in region A, r_A is the rental rate of capital in region A, and r_B rental rate in region B. Consumers will allocate capital in the region where the prevailing return is higher. We distinguish between varieties produced in region A (whose mass is n_A), and varieties produced in region B (whose mass in n_B). The quasilinear structure of $U(\cdot)$ implies that consumption of commodity 0 is the residual of what is spent on the monopolistic sector. Consequently, provided income is high enough so to allow a positive consumption of good 0 in equilibrium, every further increase in income corresponds to an equal increase in the consumption of the agricultural commodity, not affecting the demand for the differentiated varieties in manufacturing.

[2]In Vives (1990) and Belleflamme et al. (2000) the total mass of the monopolistically competitive sector N is normalized to 1. We do not use this normalization because N will turn to be one of the key parameters of the model.

After having plugged the budget constraint in the utility function, maximization yields inverse demand functions. Inverse demand for a variety $j \in n_A$ produced in A and sold in A is

$$\frac{p_{AA}(j)}{p_0} = \xi - (1-\omega)x_{AA}(j) - \omega X_A \tag{3}$$

where $x_{AA}(j)$ is demand for variety j and

$$X_A = \int_{j \in n_A} x_{AA}(j)dj + \int_{j \in n_B} x_{AB}(j)dj$$

is total demand for the monopolistic sector from consumers located in region A, consisting of varieties manufactured both in region A and in region B. A variety $j \in n_B$ produced in B but sold in A has an inverse demand equal to

$$\frac{p_{AB}(j)}{p_0} = \xi - (1-\omega)x_{AB}(j) - \omega X_A \tag{4}$$

where variables have the same interpretation as above. Similar expressions can be derived for products sold in market B.

2.2 Labour market

L is employed as a variable input either in manufacturing or in agriculture, and the supply of labour is perfectly elastic between sectors. In manufacturing, c_s units of labour are needed for each unit of output, and this labour requirement differs in the two regions, that is $c_A \neq c_B$. This assumption wants to capture the fact that there are locations where productivity of labour in manufacturing is higher. Turning to the production of the homogeneous good, it is carried out under constant returns to scale, with unit labour requirement equal across the two regions and set equal to one by an appropriate choice of scale. A positive amount of labour is employed in sector 0 because labour supply is assumed to be high enough so to cover all input requirements of the differentiated commodity sector. Constant returns to scale ensure that wage in sector 0 is equal to the exogenously fixed price p_0. Since labour market is assumed to be in equilibrium, $w_A = p_0$. If $w_A \gtrless p_0$ workers would move from one sector to the other until equality in the wage rates is reached due to perfect elasticity of supply. Total numeraire production Q_0 in region A is then

$$Q_0 \equiv L - \int_{j \in n_A} c_A x(j) dj.$$

2.3 Firms' behaviour

Firms play a two-stages game. In the first stage they establish in each region, up to the point clearing capital market. Each plant requires ϕ units of capital for functioning. By an appropriate choice of scale, each plant's capital requirement can indeed be normalized to one (entry cost). In this case, the mass of firms will exactly equal the mass of capital available in that region at a given moment.[3] In the second stage there is market competition. As said earlier, c_s units of labour are needed to produce one unit of the differentiated output, and this marginal cost differs across regions. Markets are segmented, so that each firm sets the strategic variable (price or quantity) in each regional market in which it operates. Notice that exporting in the foreign region requires t units of good 0 for each unit of output. Total profits of a representative A firm are then:

$$\frac{\Theta_A(i)}{p_0} = \left[\frac{p_{AA}(i)}{p_0} - c_A\right] x_{AA}(i) + \left[\frac{p_{BA}(i)}{p_0} - c_A - t\right] x_{BA}(i) - \frac{r_A}{p_0}$$

We substitute inverse demands (3) and (4) in the profit function. We do so because we assume that firms maximize profits with respect to quantities. As claimed by Vives (1990), in a model of monopolistic competition maximization with respect to prices or quantities brings the same results, since the individual firm behaves as a monopolist on the residual demand. The equivalency could possibly fail in a trade model like ours if trade does not take place actually, because transport costs are too high, and a positive foreign demand does not correspond to a price greater or equal to costs. In this case firms may be thought to set a *fictional* price abroad, even if demand at this price is zero, and this

[3] This is the same assumption made in Martin and Rogers (1995). As they do, we will introduce two different time horizons. In the short run capital available in each region equals the capital endowment of the representative consumer in that region ($K_s = n_s$). In the long run capital flows freely from one region to the other, so that the only equality that has to hold is $K_A + K_B = N$.

could prevent the equivalency. In Behrens (2004), when a firm does not sell in the foreign market, because transport costs are too high, it will "*set the lowest possible price for which this* [foreign] *demand is zero*".[4] These fictional prices enter the foreign region's price index, even if the corresponding varieties are not actually traded. Including or not such prices affect the resulting equilibrium. If one were to include in the price index only the prices of varieties actually exported in equilibrium,[5] the equilibrium would in general be different.[6] In Appendix 7.1 I show that, employing Behrens fictional pricing rule, price and quantity setting are perfectly equivalent. On this ground, his conjecture about the formation of the price index is to be preferred because it preserves the distinctive property of monopolistic competition models, that is the equivalency of price and quantity competition. Using a quantity index, the output of a firm enters the index only if it is a strictly positive quantity (i.e. only if the variety is effectively traded).

Solving the model with respect to quantities, and substituting inverse demand functions, profits for a firm in A are:

$$\frac{\Theta_A(i)}{p_0} = \left[\xi - (1-\omega)x_{AA}(i) - \omega X_A - c_A\right] x_{AA}(i) + \\ + \left[\xi - (1-\omega)x_{BA}(i) - \omega X_B - c_A - t\right] x_{BA}(i) - \frac{r_A}{p_0} \quad (5)$$

Given that each firm is negligible with respect to aggregate quantities, a change in output in one of them leaves unchanged the output index X_A. A similar expression can be derived for profits of a firm in B.

3 Short-run equilibrium

In the short run capital is immobile so that the number of firms located in each region is fixed and equal to the capital endowment of residents. In the first stage of the game, free entry and exit imply that there is a bidding process for available capital in both

[4] See Appendix A in Behrens (2004).

[5] This is done by Tabuchi and Thisse (2002).

[6] See Appendix B in Behrens (2004).

regions so that the rental rate equalizes operating profits. We now introduce the following assumption.

Assumption 1. *Throughout the paper, $c_A < c_B$, and $\theta \equiv c_B - c_A$.*

This is the formalization of the idea of comparative advantage of region A in the production of varieties belonging to the monopolistic sector, while region B has a comparative advantage in the production of the homogeneous good. The parameter θ is the fixed cost differential. As to the number of firms located in the short run in the two regions, we restrict to the case $n_A > n_B$. This is not restrictive at all: as it will be clear below, given the properties of the model, in the long-run equilibrium region A will always end up with more varieties (i.e. firms) than region B. Notice that assuming that low cost region A hosts less firms than B would not be interesting because producing in A would be unconditionally more profitable in the short run.[7]

In addition, as mentioned earlier, there is another interpretation to our set up. Due to the linearity of demand, the cost disadvantage of region B is equivalent to an upward shift of the intercept of the demand function for region A products with respect to region B, with firms in both regions incurring the same marginal cost of production, as an inspection of the objective function (5) shows. Recovering the underlying preference structure, it is

$$U(q_0, x(j)) =$$
$$\xi \int_{j \in n_A} x(j)dj + (\xi - \theta) \int_{j \in n_B} x(j)dj - \frac{1-\omega}{2} \int_{j \in N} [x(j)]^2 dj - \frac{\omega}{2} \left[\int_{j \in N} x(j)dj \right]^2 + q_0$$

Asymmetric trade patterns are then the by-product of an asymmetry in tastes, with the region having a stronger preference for its products hosting at the same time more firms.

Remark. *In the profit function (5), the cost differential θ is analytically equivalent to an upward shift equal to θ of the intercept of demand functions for A varieties with respect to B.*

[7] This is true in terms of operating profits. It applies also to total profits if overhead and setup production costs, others than the fixed cost for capital, are null or equal across the two regions.

In the model we work with, production side parameters have an equivalent interpretation in terms of preference parameters. This is so because in both ways we measure the incentives of producing a good (either in the sense that making it wastes few resources or in the sense that utility consumers derive from it is high). Without loss of generality, we make the following substitutions:

$$\eta \equiv \xi - c_A, \qquad \eta - \theta = \xi - c_B$$

Firms in A maximize profits with respect to $x_{AA}(j)$ and $x_{BA}(j)$, taking as given quantity indices X_A and X_B. The adoption of the Nash equilibrium solution implies that each firm takes as given individual output of rival firms and consequently total market output X_s. Equilibrium quantity of a firm located in A and selling to consumers in A is

$$x^*_{AA} = \frac{\eta - \omega X_A}{2(1-\omega)} \tag{6}$$

As to the quantity sold by firms located in A to consumers in B we have

$$x^*_{BA} = \frac{\eta - t - \omega X_B}{2(1-\omega)} \tag{7}$$

Expressions pertaining to firms in B can be derived similarly.

Let us initially consider the case where the two markets overlap, with firms in A exporting to B, and firms in B exporting to A (two-way trade). Total output in market A at equilibrium is:

$$X_A = n_A x^*_{AA} + n_B x^*_{AB}$$

where we used the symmetry of the model to say that

$$x_{AA}(i) = x_{AA}(j) \quad \forall i, j \in n_A, \quad i \neq j$$

$$x_{AB}(i) = x_{AB}(j) \quad \forall i, j \in n_B, \quad i \neq j$$

The equilibrium values are:

$$\begin{aligned} x^*_{AA} &= \frac{2\eta(1-\omega) + \omega n_B(\theta + t)}{2(1-\omega)(2 - 2\omega + \omega N)} \\ x^*_{AB} &= \frac{2(\eta - \theta - t)(1-\omega) - \omega n_A(\theta + t)}{2(1-\omega)(2 - 2\omega + \omega N)} \end{aligned}$$

At equilibrium, quantities sold in market B are:

$$x^*_{BB} = \frac{2(\eta-\theta)(1-\omega)-\omega n_A(\theta-t)}{2(1-\omega)(2-2\omega+\omega N)}$$
$$x^*_{BA} = \frac{2(\eta-t)(1-\omega)-\omega n_B(t-\theta)}{2(1-\omega)(2-2\omega+\omega N)}$$

We see the role of the comparative cost (or demand) advantage and competitive pressure (proxied by n_A and n_B) computing the relative share of export from a firm in A with respect to a firm in B,

$$\frac{x^*_{BA}}{x^*_{AB}} = \frac{2(\eta-t)(1-\omega)-\omega n_B(t-\theta)}{2(\eta-\theta-t)(1-\omega)-\omega n_A(\theta+t)} > 1$$

meaning that firms located in region A export more than their counterparts in region B. There are two elements bringing this result: the cost advantage, and the fact that $n_A > n_B$. Let us consider the case where $\theta = 0$, and $n_A = n_B$. The ratio x^*_{BA}/x^*_{AB} would be then equal to 1. If $n_A > n_B$, a firm in A exports more: even if firms in A have no a priori advantage, they face less competition when exporting to market B because less firms are located there.[8] As soon as there is a comparative advantage of A firms ($\theta > 0$), the ratio x^*_{BA}/x^*_{AB} gets even bigger. Turning to equilibrium delivered prices, p^*_{BA} and p^*_{AB}, after some straightforward algebra it is possible to show that

$$\frac{p^*_{BA}}{p^*_{AB}} > 1$$

meaning that prices charged by firms located in A for market B are higher than those charged by B firms for market A. The explanation is that even though the quantity sold by each exporting firm is higher in market B than in market A ($x^*_{BA} > x^*_{AB}$), market A as a whole is bigger than market B ($X_A > X_B$ in equilibrium), this depressing prices in A with respect to B. As a result, region A is a *net exporter* of the differentiated commodity ($n_A p^*_{BA} x^*_{BA} > n_B p^*_{AB} x^*_{AB}$).

[8] When a firm located in A exports to B it has to face a *greater* number of competitors from the same region ($n_A > n_B$) than a firm in B which exports to market A. However, due to transport costs, the pressure exerted by domestic competitors in the export region (i.e. B) is relatively more important in determining firms' relative profitability.

Focusing on the ratio of domestic production of each firm in the two regions,

$$\frac{x^*_{AA}}{x^*_{BB}} = \frac{2\eta(1-\omega) + \omega n_B(\theta + t)}{2(\eta - \theta)(1-\omega) - \omega n_A(\theta - t)}$$

we have that this ratio could be higher, lower, or equal to 1 according to the parameters of the model. When $\theta = 0$, and $n_A = n_B$, then $x^*_{AA}/x^*_{BB} = 1$. If $n_A > n_B$, and $\theta = 0$ then $x^*_{AA}/x^*_{BB} < 1$ because firms in A face more domestic competitors than firms in B. The same holds if θ is sufficiently small. On the contrary, when θ is high enough, the competition effect is offset by the technological (or demand) effect, and domestic output of a firm in A will be higher than output of a firm in B.

3.1 Trade patterns

We now determine the different trade patterns that arise in our linear demand monopolistic competition model. The analysis of trade patterns other than two-way trade has already been carried out in a core-periphery setting by Behrens (2004, 2005), *without* asymmetries. The aim of this section is to stress the role played by the total mass of the monopolistic sector N, which is an endogenous parameter determined by the availability of capital in the economy on the one side, and increasing returns to scale on the other. The total mass of firms can be written as $N = (K_A + K_B)/\phi$. The higher the total mass N, the tougher competition will be. Since we normalized the fixed capital requirement of firms ϕ to 1, a higher value for N stands for less capital required to set up a firm, so that returns to scale are less intense.

We proceed imposing the non-negativity of equilibrium quantities x^*_{AA}, x^*_{BA}, x^*_{BB}, and x^*_{AB}, and then we consider all possible configurations to get the full characterization of short-run trade patterns. The share of firms in region A is $\lambda \equiv n_A/N$.

Firms in A always produce a positive quantity of the monopolistic good for their domestic market, because it is always true that $x^*_{AA} > 0$.

We concentrate now on exports of A firms to B. If $t > \eta$, $x^*_{BA} < 0$ for every λ. When $\theta < t < \eta$, $x^*_{BA} > 0$ if the share of B firms is small enough relatively to the total:

$$\lambda > 1 - \frac{2(\eta - t)(1-\omega)}{\omega N(t - \theta)} \equiv \nu_{BA} \tag{8}$$

When the share of firms in B is high enough, export to region B is blockaded. By assumption, the number of firms in region A is greater than the number of firms in B, so that admissible values are $\lambda \in (1/2, 1)$. Hence ν_{BA} will be binding if it belongs to $(1/2, 1)$. This is verified when the total mass N is

$$N > \frac{4(\eta - t)(1 - \omega)}{\omega(t - \theta)} \equiv N_{BA}$$

On the contrary, for $N < N_{BA}$, the threshold ν_{BA} is less than $1/2$, it is not binding, and export to B is always possible under our assumptions. Finally, when $t < \theta$, A firms export for every λ.

We pass on to deriving conditions for firms located in region B. They always sell a positive quantity in their domestic market if $t > \theta$. If transport costs are greater than the cost differential, firms in B, producing under less favourable terms, will be protected in their domestic market, because the disadvantage they have is more than offset by barriers to trade. If $t < \theta$, $x^*_{BB} > 0$ if

$$\lambda < \frac{2(\eta - \theta)(1 - \omega)}{\omega N(\theta - t)} \equiv \nu_{BB} \tag{9}$$

that is relatively few firms are located in A. The intuition is that when many competitors have a cost advantage (many firms are located in A) the competitive pressure exerted by A firms hinders domestic production in region B. Conversely, if λ is small, there are few competitors producing at a lower cost. The threshold ν_{BB} belongs to the interval $(1/2, 1)$ when the total mass of firms is

$$N_{BB} < N < 2N_{BB}$$

where

$$N_{BB} \equiv \frac{2(\eta - \theta)(1 - \omega)}{\omega(\theta - t)}$$

with $\nu_{BB} > 1$ for $N < N_{BB}$ ($\nu_{BB} > 1/2$ for $N < 2N_{BB}$). If the total mass of firms N is less than N_{BB}, competitive pressure is softened, allowing a positive (i.e. profitable) production of B firms in their domestic market whatever the spatial distribution is. If N

exceeds $2N_{BB}$, the fact that x_{BB} be positive is not compatible with the assumption that $\lambda > 1/2$.

Firms in B cannot export a positive quantity to A as long as $t > \eta - \theta$. If $t < \eta - \theta$, $x^*_{AB} > 0$ provided

$$\lambda < \frac{2(\eta - \theta - t)(1 - \omega)}{\omega N(\theta + t)} \equiv \nu_{AB} \tag{10}$$

We have that $\nu_{AB} \in (1/2, 1)$ when

$$N_{AB} < N < 2N_{AB}$$

where

$$N_{AB} \equiv \frac{2(\eta - \theta - t)(1 - \omega)}{\omega(\theta + t)}$$

with $\nu_{AB} > 1$ for $N < N_{AB}$ ($\nu_{AB} > 1/2$ for $N < 2N_{AB}$).

Two cases should be distinguished at this point: the cost advantage of region A could be high (respectively low), if $\theta > \eta - \theta$ (respectively $\theta < \eta - \theta$). In terms of asymmetries of the demand functions, the intercept of the demand for B products $\eta - \theta$ could be smaller (respectively bigger) than the difference θ between the two intercepts. In what follows we stick to the following assumption.

Assumption 2. *The cost advantage θ is such that $\theta < \eta - \theta$.*

The analysis could be carried out without substantial changes to the results for the case $\theta > \eta - \theta$ as well, but for simplicity it is carried out only in one case. Moreover, if we interpret the model as one featuring taste asymmetries, it is preferable to assume that the difference between the intercepts of the demand functions (θ) be smaller than the smallest intercept ($\eta - \theta$).

First we consider autarchy, the case involving no-trade among the two regions.

Lemma 1. *Autarchy constitutes the short-run equilibrium if one of the following conditions is satisfied:*

i) $t > \eta$;

ii) $\theta < t < \eta$, with $N > N_{BA}$ and $\lambda \leq \nu_{BA}$.

Proof. Point *i)* is easily derived. As to point *ii)*, autarchy is the short-run equilibrium only if $N > N_{BA}$, that is only when x^*_{BA} could be zero. Both for $\theta < \eta - \theta < t < \eta$, and for $\theta < t < \eta - \theta < \eta$, this is true if $\lambda \leq \nu_{BA}$ and $N > N_{AB}$. Since $2N_{AB} < N_{BA}$, when $N > N_{BA}$ x^*_{AB} is zero. ■

Under one-way trade firms in A supply domestic and foreign markets, while firms in B supply their domestic market only. There is an asymmetry in trade relations.

Lemma 2. *One-way trade constitutes the short-run equilibrium if one of the following conditions is satisfied:*

i) for $\eta - \theta < t < \eta$, $N < N_{BA}$*; or* $N > N_{BA}$ *and* $\lambda > \nu_{BA}$*;*

ii) for $\theta < t < \eta - \theta$, $N_{AB} < N < 2N_{AB}$ *and* $\lambda \geq \nu_{AB}$*; or* $2N_{AB} < N < N_{BA}$*; or* $N > N_{BA}$ *and* $\lambda > \nu_{BA}$*;*

iii) for $t < \theta$, $N_{AB} < N < 2N_{AB}$ *and* $\nu_{AB} \leq \lambda < \nu_{BB}$*; or* $2N_{AB} < N < 2N_{BB}$ *and* $\lambda < \nu_{BB}$*.*

Proof. Let us start from $t > \theta$. Remember again that $N_{BA} > 2N_{AB}$. Then simply consider all the combinations of N and λ ensuring that $x^*_{BA} > 0$ and $x^*_{AB} = 0$.

When $t < \theta$, it is possible to show that $N_{BB} > N_{AB}$. Nothing can be said about the ordering among N_{BB} and $2N_{AB}$ and the threshold ν_{BB} becomes redundant when it is greater than 1 (think for example of a case where $2N_{AB} < N < N_{BB}$). ■

We now characterize two-way trade, when both regions trade with each other.

Lemma 3. *Two-way trade constitutes the short-run equilibrium for* $t < \eta - \theta$*, and* $N < N_{AB}$*; or* $N_{AB} < N < 2N_{AB}$ *and* $\lambda < \nu_{AB}$*.*

Proof. We derive conditions making the quantity sold abroad by B firms, x^*_{AB}, positive either for $\theta < t < \eta - \theta$, or $t < \theta < \eta - \theta$. Two-way trade is possible for every admissible λ when $N < N_{AB}$. When the total mass is $N_{AB} < N < 2N_{AB}$, for two-way trade to be possible it has to be $\lambda < \nu_{AB}$. As to the quantity x^*_{BB}, it will be always positive under the conditions stated in the proposition: it suffices to remind that $N_{AB} < N_{BB}$ (implying trivially $2N_{AB} < 2N_{BB}$) and $\nu_{AB} < \nu_{BB}$. ■

As mentioned earlier, the role played by N, and scale economies has often been neglected in the literature. Notice that when $N_{AB} < N < 2N_{AB}$, the share of firms located in A should not exceed the threshold ν_{AB} if we want two-way trade to be feasible. If this were not the case, then it would be prohibitive to export to region A for B firms, due to toughness of competition. If the total mass of the monopolistic sector exceeds $2N_{AB}$, two-way trade is impossible, since profits' margins will be compressed by the large number of firms, and B firms will not be able to export under the assumption that $\lambda > 1/2$.[9] Belleflamme et al. (2000) in their paper restrict attention to two-way trade. The only condition imposed concerns the level of transport costs t, that should be sufficiently low, and they normalize the total mass N to 1. By Lemma 3, the relative share λ could be ignored only if $N < N_{AB}$ (which corresponds to $N_{AB} > 1$ in their setting). If $N_{AB} < N < 2N_{AB}$, as the agglomeration process of firms in region A unfolds, when λ reaches ν_{AB} two-way trade is no longer sustainable, and the short-run equilibrium consists of one-way trade.[10]

We say that in region B a process of deindustrialization has occurred when it is not possible for a firm operating in B to make non-negative profits.

Lemma 4. *Short-run equilibrium involves deindustrialization of region B for $t < \theta$, $N_{BB} < N < 2N_{BB}$ and $\lambda \geq \nu_{BB}$; or $N \geq 2N_{BB}$.*

To appreciate the economic meaning, let us focus on the cost of products that could be sold in market B. The fact that $t < \theta$, means that the total cost for A firms ($c_A + t$) of a product sold in region B is lower than the cost incurred by B firms themselves (c_B). If $N \geq 2N_{BB}$, only A firms are capable of being in the market, assuming a spatial distribution $\lambda \in (1/2, 1)$.

[9]When $N \geq 2N_{BB}$, firms in B cannot profitably produce even for their domestic market, see below Lemma 4.

[10]Even if our model is not equivalent to Belleflamme *et al.*, because we assume that the cost differential is fixed, what they do is hence to assume implicitly that $N \equiv 1 < N_{AB}$,

$$N_{AB} > 1 \Leftrightarrow 2(\eta - \theta - t)(1 - \omega) > \omega(\theta + t)$$

which is an additional parameters' restriction that should be indicated explicitly.

Equilibrium prices can be derived simply substituting equilibrium quantities in (3) and (4). Conditions for the non-negativity of mark-ups $(p^*_{AA} - c_A)$ and $(p^*_{BA} - c_A - t)$ coincide with those for equilibrium quantities x^*_{AA}, x^*_{BA} so that non-negativity of mark-ups is implied by non-negativity of quantities (the same applies for firms in B).

4 Long-run equilibrium

Bidding for available capital determines the equality between equilibrium operating profits, Π^*_s, and the rental rate in the short run, $r^*_s/p_0 = \Pi^*_s$, for a given spatial distribution of firms.[11] In the long run capital is mobile between regions so that the spatial distribution of firms is no longer equal to the initial endowment of capital in A and B. Capital flows occur in response to the differential in the equilibrium rental rate $r^*_A(\lambda) - r^*_B(\lambda)$, determined in the short-run. When the differential is positive capital goes from region B to region A. Viceversa, when the differential is negative, capital flows out of A into B.

For every trade pattern we identified, we argue about existence, uniqueness and convergence to the equilibrium distribution λ^*, so that $r_A(\lambda^*) = r_B(\lambda^*)$, in the sections below. If an interior equilibrium is reached, operating profits in the two regions will be equalized as well, $\Pi^*_A = \Pi^*_B$. In other terms, individuals look first for investment opportunities in the local market (which fixes the rental rate at operating profits in that region), then they look abroad, causing the exit of firms in the local market and the subsequent entry in the foreign one if the rental rate obtained there is higher (which determines equality of rental rates and operating profits across regions).

In what follows we compare operating profits in the two regions under different trade patterns. Our goal is to establish which firms are performing better, whether those located in A or in B, as a function of the total mass of the monopolistic sector N, and the spatial distribution λ. Once having obtained these results, it is possible to determine the long-run outcome of the economy, under the assumption that capital is invested where interest rate

[11]Competition in capital market drives the rental rate down to operating profits because of free entry of firms hiring capital.

is higher.

4.1 Autarchy

Under autarchy, operating profits of a variety $i \in [0, n_s]$ produced in $s \in \{A, B\}$ are

$$\Pi_s(i) = [\xi - (1-\omega)x_s(i) - \omega X_s - c_s]x_s(i)$$

where total output under autarchy in region s is equal to

$$X_s = \int_{j \in n_s} x(j)dj$$

Equilibrium output x_s^* is:

$$x_s^* = \frac{\xi - c_s}{2(1-\omega) + \omega n_s} \tag{11}$$

and correspondingly equilibrium price is

$$\frac{p_s^*}{p_0} = \frac{(1-\omega)(\xi + c_s) + c_s n_s \omega}{2(1-\omega) + \omega n_s}$$

Profits are finally

$$\Pi_s^* = \frac{(1-\omega)(\xi - c_s)^2}{[2(1-\omega) + \omega n_s]^2} \tag{12}$$

Proposition 1. *When differentiated varieties are not traded (see Lemma 1),* $\Pi_A^* > \Pi_B^*$ *if:*

i) $N < N_U$*, where*

$$N_U \equiv \frac{2\theta(1-\omega)}{\omega(\eta - \theta)}$$

and every λ*;*

ii) $N > N_U$ *and* $\lambda < \lambda_U$*, where*

$$\lambda_U \equiv \frac{2\theta(1-\omega) + \omega\eta N}{\omega N(2\eta - \theta)}$$

with λ_U *being the long-run spatial equilibrium.*

Proof. Solving the inequality $\Pi_A^* > \Pi_B^*$ leads to the condition

$$\omega[(\eta - \theta)n_A - \eta n_B] < 2\theta(1-\omega)$$

that, after having substituted $n_A \equiv \lambda N$, and $n_B \equiv (1-\lambda)N$, is equivalent to

$$\lambda < \frac{2\theta(1-\omega) + \omega\eta N}{\omega N(2\eta - \theta)} \equiv \lambda_U \tag{13}$$

When $\lambda = \lambda_U$ it immediately descends that $\Pi_A^* = \Pi_B^*$. It is easy to argue about stability of the long-run equilibrium, since (12) is monotonically decreasing in n_s. Assuming that $N < N_U$, where

$$N_U = \frac{2\theta(1-\omega)}{\omega(\eta - \theta)}$$

makes λ_U bigger than 1, so that $\Pi_A^* > \Pi_B^*$ for every λ and the equilibrium involves full agglomeration of the manufacturing sector in region A, provided conditions in Lemma 1 are fulfilled. ■

The proposition says that a long-run equilibrium involving partial agglomeration exists only if the total mass of the monopolistic sector is greater than N_U. The economic intuition of this result is the following. When the total mass of firms N is small, and scale economies are strong, firms in A will make higher profits for every admissible λ, because of the cost advantage θ: region A is sufficiently attractive to host the whole manufacturing sector. On the other hand, if N is large, then the whole manufacturing sector could not locate entirely in A, still doing better than an isolated firm in B. In this case the actual spatial distribution of firms will matter for profitability, and the fraction of firms in A should be small enough to get $\Pi_A^* > \Pi_B^*$.

Two components related to the degree of competition affect profitability: the first is overall competitive pressure, measured by N, being a measure of increasing returns to scale; the second is local competitive pressure, measured by λ. The bigger the cost advantage θ, the larger the values of N, and the lower scale economies intensity, under which we get full agglomeration. Similarly to N_U, also λ_U increases as θ rises. A partially agglomerated stable spatial equilibria under autarchy is also found in Behrens (2004).

We now proceed to show how the same kind of reasoning applies to the other trade regimes.

4.2 One-way trade

Under one-way trade, firms located in region B do not make positive export to A. The output index in region A is then $X_A = \int_{j \in n_A} x_{AA}(j)dj$. Substituting in (6), and employing the symmetry of the model, the equilibrium quantity x^*_{AA} is

$$x^*_{AA} = \frac{\eta}{2(1-\omega) + \omega n_A}$$

equal to (11), the quantity sold in region A under autarchy. As in a situation without trade at all, firms in A are protected against competition coming from foreign firms, and they behave in the same way of autarchy in the local market. This makes the home component of profits equal to autarchy profits. Profits of A firms are also made of a component coming from abroad, making total profits equal to:

$$\Pi^*_A = \Pi^h_A + \Pi^f_A = \frac{(1-\omega)\eta^2}{[2(1-\omega) + \omega n_A]^2} + \frac{[2(\eta - t)(1-\omega) + \omega n_B(\theta - t)]^2}{4(1-\omega)(2 - 2\omega + \omega N)^2}. \quad (14)$$

This is the sum of home profits (Π^h_A) and foreign profits (Π^f_A). Profits of firms in B are

$$\Pi^*_B = \Pi^h_B = \frac{[2(\eta - \theta)(1-\omega) - \omega n_A(\theta - t)]^2}{4(1-\omega)(2 - 2\omega + \omega N)^2} \quad (15)$$

corresponding just to the home component.

A sufficient condition for A profits to be greater than B profits is $t < \theta$, which turns to be true under case *iii)* of Lemma 2. If the cost (or demand intensity) advantage of region A is greater than transport costs, markets are relatively well integrated and location in A allows higher profits regardless of the spatial distribution.

When $t > \theta$, I am not able to provide a closed-form solution for λ_O, the long-run spatial equilibrium such that $\Pi^*_A(\lambda_O) = \Pi^*_B(\lambda_O)$, and $\Pi^*_A(\lambda) \gtreqless \Pi^*_B(\lambda)$ for every $\lambda \lesseqgtr \lambda_O$. Nonetheless in Appendix 7.2 I prove that, whenever this value exists, it is unique. Results are summarized in the proposition that follows.

Proposition 2. *When one-way trade is established (see Lemma 2),* $\Pi^*_A > \Pi^*_B$ *if* $t < \theta$. *If* $t > \theta$, *we have one of the following cases:*

i) If $N < N_O$, *where*

$$N_O \equiv \frac{2(1-\omega)}{\omega(t-\theta)} \left(\sqrt{\eta^2 + (\eta - t)^2} - \eta + \theta \right)$$

$\Pi_A^* > \Pi_B^*$ *for every* λ.

ii) If $N > N_O$*, and the long-run spatial equilibrium* λ_O *exists, then there exists a unique* λ_O*, and* $\Pi_A^* \gtreqless \Pi_B^*$ *for* $\lambda \lesseqgtr \lambda_O$.

iii) If $N > N_O$*, and* λ_O *does not exist, then* $\Pi_A^* < \Pi_B^*$.

Proof. See Appendix 7.2. ■

Under one-way trade and $t > \theta$, several configurations are possible in the short run, with firms in A performing better than firms in B, or, viceversa, firms in B doing better than in A. If λ_O does not exist, and in the Appendix I state when this is the case, one-way trade cannot be a stable long-run outcome of the economy: we have either full agglomeration (point *i)* in the proposition) or transition to another trade pattern (point *iii)*).

If λ_O exists, then the interior equilibrium will be stable. Behrens (2005) shows that partial agglomeration is unstable as soon as one-way trade emerges. This contrasts our result about the stability of λ_O, provided its existence. The reason lies in the fact that in this paper the only mobile factor is capital, while consumers-workers stay put in their regions of residence. In other words, our model lacks the strong agglomeration forces typical of the core-periphery setting.[12] In the CP model of Behrens (2005) migration of workers generates both *demand-linked* circular causality (migration generates expenditure shifting by workers, which generates in turn production shifting, and this determines more migration to fulfill firms' fixed costs requirements) and *cost-linked* circular causality (a higher mass of differentiated products is available where production is concentrated, and workers find more convenient to locate there to save on trade costs), whereas in our FC model, these effects are not present.

The existence of a stable λ_O in our framework highlights once more that the type of spatial equilibrium depends on the type of factor that moves across regions, and it makes a difference assuming that capital, instead of labour, is mobile.

[12]See Chapter 3 in Baldwin *et al.* (2003) for a comparison of FC and CP models.

4.3 Two-way trade

From Lemma 3, two-way trade is possible only if $t < \eta - \theta$. At the same time, the total number of firms in the economy has to satisfy the conditions $N < N_{AB}$; or $N_{AB} < N < 2N_{AB}$, and $\lambda < \nu_{AB}$. Profits of firms in A are

$$\Pi_A^* = \Pi_A^h + \Pi_A^f = \frac{[2\eta(1-\omega) + \omega n_B(\theta + t)]^2 + [2(\eta - t)(1-\omega) + \omega n_B(\theta - t)]^2}{4(1-\omega)(2 - 2\omega + \omega N)^2} \tag{16}$$

while profits of firms in B are

$$\Pi_B^* = \Pi_B^h + \Pi_B^f = \frac{[2(\eta-\theta)(1-\omega) - \omega n_A(\theta - t)]^2 + [2(\eta - \theta - t)(1-\omega) - \omega n_A(\theta + t)]^2}{4(1-\omega)(2 - 2\omega + \omega N)^2} \tag{17}$$

made up of a home component and a foreign component. The following proposition explains the relative profitability of firms in the two regions as a function of the total mass N and the share λ.

Proposition 3. *When two-way trade is established (see Lemma 3),* $\Pi_A^* > \Pi_B^*$ *if* $t < \theta$*. If* $\theta < t < \eta - \theta$ *one of the following conditions has to be satisfied:*

i) $N < N_T$*, where*

$$N_T = \frac{2\theta(1-\omega)(2\eta - t - \theta)}{\omega(\theta^2 + t^2)}$$

and every λ*;*

ii) $N > N_T$ *and* $\lambda < \lambda_T$*, where*

$$\lambda_T \equiv \frac{1}{2} + \frac{1}{2}\frac{N_T}{N}$$

with λ_T *being the long-run spatial equilibrium of the economy.*

Proof. If $t < \theta$, it is easy to see that $\Pi_A^* > \Pi_B^*$. If $\theta < t < \eta - \theta$, comparing (16) and (17), A profits are greater than B profits if

$$n_A - n_B < \frac{2\theta(1-\omega)(2\eta - t - \theta)}{\omega(\theta^2 + t^2)} \equiv N_T \tag{18}$$

which could be expressed in terms of λ and N as

$$\lambda < \frac{1}{2} + \frac{1}{2}\frac{N_T}{N} \equiv \lambda_T$$

The threshold λ_T is lower than one as long as $N > N_T$. The stability of the long-run equilibrium in this case is ensured by the fact that profits are monotonic in λ under two-way trade. When $N < N_T$ we have full agglomeration in A, provided conditions in Lemma 3 are fulfilled. ■

If the total mass of firms is big enough, this guarantees the existence of a spatial distribution making better off firms in B in the short run. The long-run behaviour of the economy depends as usual on the assumption that capital flows where the interest rate is higher, with the interest rate equal to operating profits, and a long-run equilibrium with partial agglomeration and bilateral trade exists. This is not the case in the CP setting of Ottaviano et al. (2002), where the equilibrium is either symmetric or involves full agglomeration in one region. As in the previous trade patterns, the partially agglomerated nature of the equilibrium does not depend on the cost asymmetry only. Keeping fixed θ, depending on N, so on the strength of returns to scale, we have different spatial configurations.

The effect played by N is similar to the one present in CP models: a lower N (higher increasing returns to scale) makes full agglomeration more likely. The explanation is partially different. In CP models stronger scale economies (higher fixed costs) imply that firms' relocation will involve a considerable amount of demand shifting, due to workers' migration, this enhancing circular causality. Since in a FC model there is no demand shifting, our context retains just one effect of returns to scale: the higher fixed costs, the lower the total mass of firms, the lower overall competitive pressure, the higher mark-ups and profits, so that firms more easily establish in the region with the location advantage because they are influenced less intensely by the presence of the other competitors.

4.4 Full versus partial agglomeration in the long run

We now characterize in terms of the parameters' values, and in terms of the total mass of the monopolistically competitive sector the emergence of full agglomeration of manufacturing in region A. We give conditions so that, starting from a short-run equilibrium involving a positive share of firms in B, capital eventually becomes employed solely in region A.

Lemma 5. *Full agglomeration of the manufacturing sector in region A is the long-run equilibrium of the economy whenever one of the following conditions is met:*

i) $t > \eta$ and $N < N_U$;

ii) $\eta - \theta < t < \eta$, and $N < N_O$;

iii) $\theta < t < \eta - \theta$, and $N < N_{AB}$; or $N_{AB} < N < N_T < 2N_{AB}$ (equivalently $N_{AB} < N < 2N_{AB} < N_T$), and $N < N_O$; or $\lambda_T > \nu_{AB}$ if $N_T < N < 2N_{AB}$, and $N < N_O$;

iv) $t < \theta$.

Proof. See Appendix 7.3. ■

Summarizing the results, we can say that when $t > \theta$, that is transport costs are greater than the cost advantage of A, full agglomeration in the long run of the manufacturing sector requires that the total mass of firms in the economy be sufficiently small. If this is not the case then the long-run equilibrium of the economy involves partial agglomeration only. Analytically this requires that N be less than N_U and N_O under autarchy and one-way trade respectively. When transport costs allow the emergence of two-way trade (point *iii)*) the conditions are more elaborated, essentially due to the fact that full agglomeration can be reached either directly ($N < N_{AB}$) or transiting across one-way trade first. In the latter case, the conditions in the proposition guarantee two things: that two-way trade be not the long-run equilibrium of the economy, and that once one-way trade is reached as a result of migration of firms to region A it cannot be a long-run equilibrium either (which is the case if $N < N_O$). Finally, if $t < \theta$, comparative advantage of region A is strong and full agglomeration of the manufacturing sector will be the long-run equilibrium whatever the total mass of the monopolistically competitive sector is. We have then proved the following proposition.

Proposition 4. *Full agglomeration of manufacturing in a single region can be the outcome of two situations: i) strong natural advantage of a region over the other; ii) moderate* *natural advantage but strong increasing returns to scale in the sector undergoing agglomeration.*

In addition, a clear-cut implication this paper shares with other economic geography models is that sufficiently low transport costs foster full agglomeration of the monopolistically competitive sector in the more productive region ($\partial N_O/\partial t < 0$ and $\partial N_{AB}/\partial t < 0$).

5 Natural Advantage Effect and Home Market Effect

The analysis of location equilibria of the mobile manufacturing industry in relation to the degree of increasing returns to scale can be easily carried out under partial agglomeration either. We state as follows a property of interior equilibria common to all the trade patterns we identified.

Proposition 5. *When trade is allowed between regions, region A is a net exporter of the differentiated commodity. If the long-run equilibrium involves only partial agglomeration of firms in the region with the natural advantage, the equilibrium share of firms in region A is a decreasing function of increasing returns to scale.*

Proof. We have already shown that region A is a net exporter of the differentiated commodity under two-way trade (see section 3). The statement is also trivially true under one-way trade.

Then we need to show that $\partial \lambda_p/\partial N < 0$, with $p = \{U, O, T\}$. While the proof is straightforward in the case of λ_U and λ_T, since we managed to express them in closed form, it is slightly more complicated for the equilibrium under one-way trade. Considering the definition of λ_O,

$$\Pi^*_A(\lambda_O, N) = \Pi^*_B(\lambda_O, N)$$

and totally differentiating, one gets

$$\left.\frac{\mathrm{d}\lambda}{\mathrm{d}N}\right|_{\lambda=\lambda_O} = \frac{\partial \Pi^*_B(\lambda_O, N)/\partial N - \partial \Pi^*_A(\lambda_O, N)/\partial N}{\partial \Pi^*_A(\lambda_O, N)/\partial \lambda - \partial \Pi^*_B(\lambda_O, N)/\partial \lambda}$$

The numerator of this fraction is positive (see the expressions of Π_A^* and Π_B^* under one-way trade) while the denominator is negative (see the characterization of Π_A^* and Π_B^* contained in Appendix 7.2). ∎

Natural advantage makes region A more attractive than B, and since a higher mass of firms is located in A, it becomes tougher for B firms to export in A, this leading to a trade surplus for region A. Moreover, the lower increasing returns to scale, the higher the endogenous equilibrium number of firms in the economy N, the lower the market power enjoyed by firms, the more sensitive they will be to the presence of other competitors in the same region, due to the presence of transport costs. In other words, when increasing returns to scale are strong (low N), firms will care little about the presence of other firms in region A, and this facilitates the process of spatial clustering in that region. When N is high they will be more sensitive to competition and a higher share of them will cluster in B, even at the expense of higher production costs. I call the interaction among imperfect competition and natural advantage *Natural Advantage Effect* (NAE). The fact that cost advantage in a region makes that region a net exporter of differentiated products in a Ricardian framework has been previously analyzed, within a CES Dixit-Stiglitz monopolistic competition model, by Venables (1987). The result of Proposition 5 adds to Venables paper by working out, in a different monopolistic competition model, how variations in scale economies affect the equilibrium distribution of firms between the two regions, something that in the context of Venables model is a difficult task.[13]

The reasoning about NAE shares a lot with the well-known Home Market Effect (HME) by Krugman (1980).

Definition. *The Home Market Effect implies that the region with the larger number of consumers of an industry's goods will be a net exporter of those goods. Keeping constant the size of the Home market compared to the Foreign one, the share of firms located in the Home market decreases as the intensity of increasing returns to scale diminishes.*

[13] I tried to analytically characterize in Venables (1987) how variations in increasing returns to scale, captured by the parameter ϵ, modified the relative share of firms located in the two regions, but I was not successful.

The firs part of the definition is formalized in Krugman (1980). It is also straightforward to derive the second part of the definition from Krugman paper.[14] The channel through which HME operates is precisely the same of NAE, that is market power, so that monopolistically competitive industries are sensitive to local market conditions, and the sensitiveness rises along with the strength of increasing returns. The only difference is that while HME is defined with respect to the local market demand size, NAE is defined with respect to the local market natural advantage.[15]

5.1 Discussion of testable implications

A testable implication of this paper is that variations in the degree of increasing returns to scale in some industries should map into changes of the concentration of those industries in the regions with a natural advantage. From a cross-sectional perspective, industries with higher increasing returns to scale will be more concentrated, keeping fixed the cost differential among two regions. Since we have proved that the marginal effect of θ is increased by a lower N, in a regression explaining the share of industry located in a region we would interact the variable capturing natural advantage with a proxy measuring scale economies. If our model is correct, we expect that the coefficient on this interacted variable be negative.

6 Conclusion

This work focused on cost and competitive asymmetries among regions shaping different trade patterns at various levels of transport costs. We derived analytical conditions

[14]Let us consider expression (25), p. 957, in Krugman (1980). The relative number of varieties produced in Home region with respect to those produced in the Foreign one, n/n^*, equivalent in our paper to $\lambda/(1-\lambda)$, is equal to $(L/L^* - \sigma)/(1 - \sigma L/L^*)$. L is the dimension of Home market, while L^* is the dimension of Foreign market, $L > L^*$. The parameter σ is an index of the importance of scale economies: the lower σ, the lower the importance of scale economies. It is easy to show that $\mathrm{d}(n/n^*)/\mathrm{d}\sigma > 0$, when both regions host a positive share of the monopolistic sector.

[15]One may wonder at this point whether the peculiar monopolistic competition model I considered exhibits HME itself. The answer is affirmative, and this is proved in Head et al. (2000).

ensuring full agglomeration in the long run of the monopolistically competitive sector in the region where production is cheaper or, alternatively, where more intensely demanded varieties are manufactured. We proved that if scale economies are sufficiently strong a single region hosts the whole sector, because firms take advantage of the better production conditions while not suffering excessively from the presence of the other competitors. When scale economies are weak, both regions host a positive share of the industry, and we derived several possible trade patterns as a function of the parameters. The share of firms in the region with the natural advantage is increasing in scale economies. Finally, our model yields a testable implication that could undergo a falsification test through appropriate empirical work.

7 Appendix

7.1 Equivalency of quantity and price setting under the pricing rule in Behrens (2004)

In Appendix A in Beherens (2004) it is shown that in order to achieve the equivalency between the *perceived* demand function and the *realized* demand function in the linear demand model it is sufficient to assume a particular pricing rule for varieties not traded in equilibrium.

The *perceived* demand function is the solution to maximization of utility function (1) under the budget constraint, that we call maximization problem $\mathcal{P}_Q$:

$$(\mathcal{P}_Q)\begin{cases} \max & U(q_0, x(j)) \\ \text{s.t.} & \int_0^N p(j)x(j)dj + q_0 = \phi_0 \end{cases}$$

Substituting directly the equality constraint and computing the first order conditions yield the following system of equations for the differentiated varieties:

$$\xi - (1-\omega)x(i) - \omega \int_0^N x(j)dj - p(i) = 0, \quad i \in [0, N]. \tag{19}$$

The system is solved giving

$$x^*(i) \equiv \frac{\xi}{1+\omega(N-1)} - \frac{1}{1-\omega}p(i) + \frac{\omega}{(1-\omega)[1+(N-1)\omega]}\int_0^N p(j)dj \tag{20}$$

For sufficiently high values of $p(i)$ the term $x^*(i)$ could be negative. Since the quantity demanded of a variety is non-negative by definition, we get that the *perceived* demand function of variety i, $\tilde{x}^*(i)$, is $\tilde{x}^*(i) = \max\{0, x^*(i)\}$. Demand is indeed zero when $x^*(i) \leq 0$, that is

$$p(i) > \bar{p}(i) \equiv \frac{\xi(1-\omega)}{1+(N-1)\omega} + \frac{\omega}{1+(N-1)\omega}\int_0^N p(j)dj$$

and $\tilde{x}^*(i) = x^*(i) = 0$ for $p(i) = \bar{p}(i)$. Hence $\bar{p}(i)$ is the smallest price making the *perceived* demand equal to zero (see Figure 1).

[Figure 1 about here]

It can be shown that the solution to $\mathcal{P}_Q$ is in general different from the solution to the following maximization problem, taking into account explicitly non-negativity constraints on the quantity consumed of each variety:

$$(\mathcal{P}_{QE}) \begin{cases} \max & U(q_0, x(j)) \\ \text{s.t.} & \displaystyle\int_0^N p(j)x(j)dj + q_0 = \phi_0, \quad x(i) \geq 0, \quad i \in [0, N] \end{cases}$$

Following Behrens (2004), the lagrangian associated to this optimization problem gives the Karush-Kuhn-Tucker conditions

$$\xi - (1-\omega)x(i) - \omega\int_0^N x(i)di - p(i) + \mu(i) = 0, \quad i \in [0, N] \tag{21}$$

$$\mu(i) \geq 0, \quad x(i) \geq 0, \quad i \in [0, N] \quad \text{and} \quad \int_0^N \mu(i)x(i)di = 0 \tag{22}$$

and *realized* demand functions are then

$$x^*(i) = \frac{\xi}{1+\omega(N-1)} - \frac{1}{1-\omega}[p(i) - \mu(i)] + \frac{\omega}{(1-\omega)[1+(N-1)\omega]}\int_0^N [p(j) - \mu(j)]dj$$

where $\mu(i)$ are the multipliers.

Behrens demonstrates that if firms set $\bar{p}(i)$ abroad whenever they do not export, perceived and realized demands coincide. Our purpose it to show that this equivalency could be achieved directly making the behavioural assumption that firms set quantities. Actually in this case the dependent variable in the demand functions has to be $p(i)$. The *perceived* demand is

$$p(i) = \xi - (1-\omega)x(i) - \omega \int_0^N x(i)di$$

while the *realized* demand is

$$p(i) = \xi - (1-\omega)x(i) - \omega \int_0^N x(i)di + \mu(i)$$

It is straightforward to see that the two demand functions always coincide as long as $\mu \equiv 0$, which is a necessary condition to get equivalency between the two optimization problems (if $\mu(i) \neq 0$ for some i the equivalency never holds). Moreover when $\mu \equiv 0$ conditions (21) and (22) reduces to (19).

7.2 Proof of Proposition 2

Step 1 (Non-monotonicity of $\Pi_A^*(\lambda)$*).* We substitute in (14) and (15) the expressions $n_A \equiv \lambda N$, and $n_B \equiv (1-\lambda)N$. First of all we determine whether, under one-way trade and $t > \theta$, $\Pi_A^*(\lambda)$ and $\Pi_B^*(\lambda)$ are strictly monotonic in λ. It is easy to see that Π_B^* is increasing in λ.

The function $\Pi_A^*(\lambda)$ is non-monotone. First of all notice that $\partial\Pi_A^h/\partial\lambda < 0$. Then we have that $\partial\Pi_A^f/\partial\lambda \geq 0$ (both quantity x_{BA}^* and price p_{BA}^* are non-decreasing in λ). Moreover $\partial\Pi_A^f(\lambda,t)/\partial\lambda = 0$ when $t = \{\theta, t_{\text{sup}}\}$, where t_{sup} is the maximum value of transport costs compatible with one-way trade for a given λ (derived making explicit in (8) transport costs t). The function $\partial\Pi_A^f(\lambda,t)/\partial\lambda$ has a unique maximum in t, computed equalizing to zero its derivative, let it be $t_{\max}$. Then if

$$\frac{\partial\Pi_A^h(\lambda)}{\partial\lambda} + \left.\frac{\partial\Pi_A^f(\lambda,t)}{\partial\lambda}\right|_{t=t_{\max}} < 0, \tag{23}$$

$\partial\Pi_A^*(\lambda,t)/\partial\lambda < 0$ for every admissible t. Actually it turns out that (23) is less than zero if and only if the following condition is verified:

$$\frac{\eta^2}{[2(1-\omega)+\omega\lambda N]^3} > \frac{(\eta-\theta)^2}{4(2-2\omega+\omega N)^2[2-2\omega+\omega(1-\lambda)N]} \tag{24}$$

Consequently (24) does not hold when λ is sufficiently close to one and θ is small. In such a case profits of firms located in region A increase as the share of firms in A increases, because the rise in profits coming from the foreign region more than offset the fall in the home component. The function Π_A^* can be first decreasing and then increasing in λ as it tends to 1, provided that t is in a neighborhood of t_{max}.

Step 2 (Uniqueness of λ_O). We demonstrate the following two properties. They turn to be useful when dealing with existence and uniqueness of λ_O.

Property P_1. The first is that

$$\left.\frac{\partial \Pi_B^*(\lambda, t)}{\partial \lambda}\right|_{\lambda=1} - \left.\frac{\partial \Pi_A^*(\lambda, t)}{\partial \lambda}\right|_{\lambda=1} > 0. \tag{25}$$

Computing (25), we get the following condition:

$$\frac{N\omega\phi(\omega)}{2(1-\omega)(2-2\omega+\omega N)^3} > 0,$$

where $\phi(\omega)$ is a parabola with upward concavity and imaginary roots, so that it is always positive.

Property P_2. The second property we are interested in is that $\partial^2\Pi_A^*/\partial\lambda\partial\lambda > 0$, meaning that Π_A^* is a convex function.

Taken together, these two properties ensure that whenever $\Pi_B^*(\lambda)$ crosses $\Pi_A^*(\lambda)$ it will do it only once: provided λ_O exists in an admissible range of λ, it will be unique.

Step 3 (Cases of non-existence of λ_O). $\Pi_A^*(\lambda)$ and $\Pi_B^*(\lambda)$ are continuous functions on $\lambda \in (1/2, 1]$, but existence of λ_O is not always guaranteed. The first case of non-existence is when $\Pi_A^*(1) > \Pi_B^*(1)$. Given properties in Step 2, this is also a necessary and sufficient condition for $\Pi_A^*(\lambda)$ to be greater than $\Pi_B^*(\lambda)$ for every admissible λ. Solving the inequality, $\Pi_A^*(1) > \Pi_B^*(1)$ if $N < N_O$, where

$$N_O \equiv \frac{2(1-\omega)}{\omega(t-\theta)}\left(\sqrt{\eta^2+(\eta-t)^2}-\eta+\theta\right).$$

Other cases of non-existence are when $\Pi_B^*(\lambda)$ lies above $\Pi_A^*(\lambda)$ for every admissible λ. In particular, it could be the case that, even though $\Pi_A^*(\lambda)$ and $\Pi_B^*(\lambda)$ intersect at some

$\lambda \in (1/2, 1]$, this point does not satisfy constraints ν_{AB}, or ν_{BA} under points *i)* and *ii)* in Lemma 2. When this is the case, in the long run we have transition from one-way trade to two-way trade ($\lambda < \nu_{AB}$) or autarchy ($\lambda \leq \nu_{BA}$).

7.3 Proof of Proposition 5

I prove separately each point in the statement of the proposition.

Point i). The proof descends from Lemma 1 and Proposition 1 and corresponds to full agglomeration with non-tradeable varieties.

Point ii). If $\theta < t < \eta$ and we are in the short-run autarchic equilibrium (point *ii)* in Lemma 1), full agglomeration is not possible because no-trade requires that $\lambda \leq \nu_{BA} < 1$, while full agglomeration obviously entails $\lambda = 1$. Full agglomeration cannot be reached unless transiting across the one-way trade short-run equilibrium.

With one-way trade and $\eta - \theta < t < \eta$ (point *i)* of Lemma 2), Proposition 2 requires that $N < N_O$. Notice that $N_O < N_{BA}$. This can be checked solving the corresponding inequality, and arriving at a point where it is straightforward to see that

$$\sqrt{\eta^2 + (\eta - t)^2} < 2\eta - t < 3\eta - t - \theta$$

When the total mass of firms is less than N_O then full agglomeration takes place.

Point iii). When $\theta < t < \eta - \theta$, we could be either in a one-way or a two-way short-run equilibrium. Two-way short-run equilibrium occurs under conditions in Lemma 3. By Proposition 3, if $N < N_T$ then $\Pi_A^* > \Pi_B^*$ for every λ. If $N < N_{AB}$, two-way trade is the short-run equilibrium for every share λ. We prove that whenever $N < N_{AB}$ we get full agglomeration, since $N_T - N_{AB} > 0$. Solving this inequality is equivalent to solve $f(\theta) > 0$, where $f(\theta)$ is equal to

$$f(\theta) = (\eta - t)\theta^2 + 2t\eta\theta - t^2(\eta - t) \tag{26}$$

The function $f(\theta)$ is a parabola in θ with upward concavity, with two negative roots. Since all admissible values of θ are greater than zero, $f(\theta)$ will be positive in this range, implying that $N_T - N_{AB} > 0$.

For $N_{AB} < N < 2N_{AB}$, two-way trade arises only if $\lambda < \nu_{AB}$, and full agglomeration in the long run can be reached only through transition to short-run one-way trade equilibrium ($\lambda \geq \nu_{AB}$, see point *ii)* in Lemma 2). With one-way trade, we recall that a necessary condition for complete agglomeration is $N < N_O$.

Transition to one-way trade happens if two-way trade is not a long-run equilibrium, which turns to be true in the following cases. The first case is when $N_{AB} < N < N_T < 2N_{AB}$ (equivalently $N_{AB} < N < 2N_{AB} < N_T$), this making profits in A greater than in B for every λ under two-way trade. Consequently, λ rises until the economy experiences one-way trade. The second case is when $N_{AB} < N_T < N < 2N_{AB}$, so that an equilibrium distribution λ_T exists. In such a case two-way trade equilibrium is impossible only if $\lambda_T \geq \nu_{AB}$. Again, there will be a switching to one-way trade before the equilibrium share λ_T could be reached.

Point iv). When $t < \theta$, the short-run equilibrium depends on the total mass of firms N. Let us first consider short-run one-way trade (*iii)* in Lemma 2). Profits in A are higher than in B by Proposition 2. As capital moves to region A (λ rises) we have deindustrialization of B provided $N > N_{BB}$. Actually when λ becomes greater or equal to ν_{BB} profits of firms in B are non-positive. Consequently all the residual capital in B is suddenly diverted towards region A, and this ensures complete agglomeration in A. If $N < N_{BB}$, firms in B make positive profits for every λ but profits made in A are higher and full agglomeration is attained again.

Consider short-run two-way trade of Lemma 3. If $N_{AB} < N < 2N_{AB}$, as the fraction of firms in A rises, we go back to one-way trade ($\nu_{AB} \leq \lambda < \nu_{BB}$), and so it applies what we said earlier, that is full agglomeration is always the long-run outcome. If $N < N_{AB}$, being in A is always more profitable, and in the long-run the whole manufacturing sector will be concentrated in this region.

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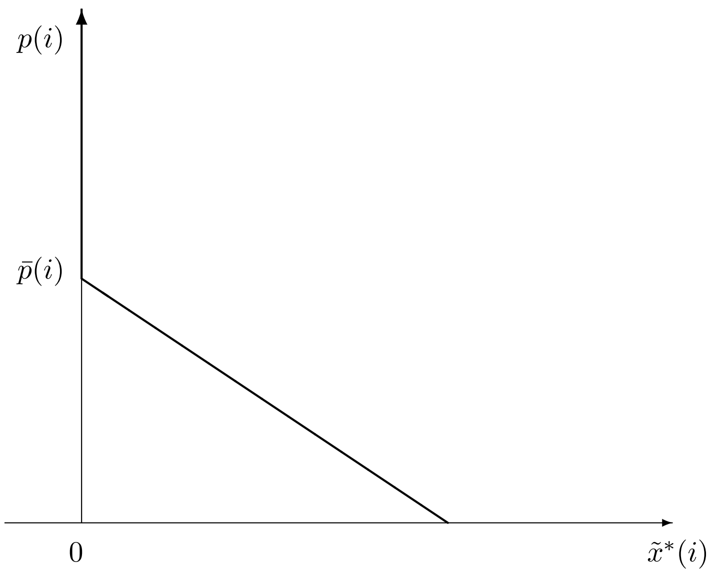

Figure 1: The perceived demand function $\tilde{x}^*(i)$.