A Differential Game with Investment in Transport and Communication R&D

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Abstract

We analyse R&D activity in transport and communication technology (TCRD) in a differential game where firms compete, alternatively, in quantities or prices. Transport and communication costs are of the iceberg type. Firms invest in TCRD to increase the net amount of the product that reaches consumers. We derive subgame perfect equilibria, and show that price competition yields the socially optimal investment, while Cournot competition involves excess investment and lower outputs.

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1 Introduction

Transport and communication technologies (TC) have always represented a relevant feature in the literature on trade (see Helpman and Krugman, 1985, *inter alia*). However, most of the works in this field tend to consider the transportation plan and its related technology as fixed characteristics. Even if firms devote substantial amounts on research and development (R&D), only process innovation and product innovation have been widely explored. The activity of product innovation consists in the development of technologies for producing new products or for increasing the quality of the existing ones. On the other hand, process innovation aims at decreasing the costs of producing existing products. Literature has considered the different degree of efficiency of process innovating R&D between the Cournot and the Bertrand setting. An established result states that there is an excess of process-innovating R&D under Cournot competition, while the opposite holds under Bertrand competition (Brander and Spencer, 1983; Dixon, 1985).

A more comprehensive analysis requires then the study of R&D activities that allows firms to reach markets in a more efficient way and be more competitive in serving their customers. Launhardt (1885), whose contribution has been recently acknowledged, proposed a simple spatial duopoly model with both horizontal and vertical product differentiation. Furthermore, he paid attention to the influence of differences in transportation costs. He thus recognized the possibility of different form of heterogeneity among firms, associated either to location or to transportation technology. Recently Thisse and Dos Santos Ferreira (1996) expanded Launhardt model by allowing firms to choose their transportation cost technologies.

So far, however, the topic of strategic investment to reduce the burden of TC costs has been rather neglected. In this paper we will then consider investments in R&D that concern mainly transport and communication needed to let the product reach the final buyer. The related R&D may be figured out as an expenditure that is going to improve the technology of the last stage of the production process (investment in the Internet, in advanced logistics, or in faster transport technology). We define this sort of activity transport and communication R&D (TCRD). Transport and communication costs are assumed to be of the 'iceberg' form invented by Samuelson (1954) and widely used in trade theory (Helpman and Krugman, 1985; Krugman, 1990). When a quantity q_i of product i is produced, yet only a portion q_i/s_i, $s_i > 1$ of the product reaches the consumer. By investing in communication and transport specific R&D, a firm may increase such a portion thus enlarging its market share.

Lambertini, Mantovani and Rossini (2001) analyse R&D activity in transport and communication technology (TCRD), in a static Cournot duopoly. They find that a variety of equilibria may appear as a result of the different levels of TCRD efficiency. If the analysis is extended to a continuous choice space equilibria exist only if production costs are low *vis à vis* market size and transport costs. Finally, Lambertini and Rossini (2001) investigate the role of TCRD in a Cournot duopoly with trade.

The aim of this paper is to extend the previous model to a dynamic setting. We propose a differential game where firms invest to increase the percentage of the product that arrives at destination. Most of the applications of differential games can be found in different fields of industrial organization.[1] Yet, as to our knowledge, the problem of TCRD investments has never been considered in such a framework.

[1] See Mehlmann (1988), Dockner *et al.* (2000) and Cellini and Lambertini (2003) for an exhaustive survey on the topic.

We present a dynamic model of oligopoly with differentiated products, where firms compete in the market and invest in TCRD. It is assumed, alternatively, that the two firms behave as quantity-setters or price-setters, hence both Cournot and Bertrand competition will be examined. Moreover, we will consider both open-loop and closed loop Nash equilibria, with a closer attention to closed loop ones. Finally, we will compare the solutions appearing in the previous cases with the social optimum and proceed with a welfare appraisal. This will allow us to draw some conclusions on the efficiency of Bertrand and Cournot oligopolies.

The paper is organized as follows. The basic model is laid out in Section 2. Section 3 considers Bertrand competition while Section 4 deals with the Cournot setting. Section 5 deals with the social optimum and the welfare appraisal. Section 6 gives the conclusion.

2 The model

We employ a quadratic utility function for a representative consumer as in Bowley (1924), Dixit (1979) and Singh and Vives (1984):

$$U(t) = A\widehat{q}_1(t) + A\widehat{q}_2(t) - \frac{1}{2}\left[\widehat{q}_1(t)^2 + 2\gamma\widehat{q}_1(t)\widehat{q}_2(t) + \widehat{q}_2(t)^2\right] \tag{1}$$

whose maximization under the budget constraint $Y(t) \geq \sum_i p_i(t)\widehat{q}_i(t),\ i = \{1,2\}$ where $Y(t)$ is nominal income, yields demand functions:

$$p_i(t) = A - \widehat{q}_i(t) - \gamma\widehat{q}_j(t) \quad \forall i \neq j,\ i,\ j = \{1,2\} \tag{2}$$

where $\gamma \in [0,1]$ stands for the symmetric degree of substitutability between the two goods and A is the market-size, both supposed to be constant over time; $\widehat{q}_i(t) \equiv \dfrac{q_i(t)}{s_i(t)}$ represents the share of firm i's good that is available for consumption at price $p_i(t)$, i.e., $1 - 1/s_i\,(t)$, with $s_i\,(t) > 1 \;\forall t \in [0, \infty)$, indicates the fraction of firm i's good that is lost during the transportation phase (see Samuelson, 1954).

The direct demand functions can be obtained by solving (2) for $q_i(t)$:

$$q_i(t) = \frac{s_i(t)\left[A\,(\gamma - 1) + p_i(t) - \gamma\, p_j(t)\right]}{\gamma^2 - 1} \tag{3}$$

We analyse a duopoly where two firms compete in both a Cournot and a Bertrand setting. We make the assumption that production entails constant marginal costs, normalized to zero for the sake of simplicity. Instantaneous profits are then given by:

$$\pi_i(t) = \frac{q_i(t)}{s_i(t)} p_i(t) - \beta\,[k_i(t)]^2 \tag{4}$$

where $k_i(t)$ represents the amount of effort made by firm i at time t in order to reduce $s_i\,(t)$ and parameter β is an inverse measure of TCRD productivity. By reducing $s_i\,(t)$ through capital accumulation over time, firm i increases the fraction of $q_i(t)$ that reaches the market. We assume that $s_i\,(t)$ evolves over time according to the following kinematic equation:

$$\frac{\partial s_i(t)}{\partial t} = [\alpha k_i(t) - \delta s_i(t)]\,[1 - s_i(t)] \tag{5}$$

where δ denotes the depreciation rate, which is common to both firms and constant over time; α is a time-invariant parameter positively affecting the accumulation process. It is worth noting that (5) accounts for the fact that $s_i\,(t)$ has to be always greater than unity for our model to be meaningful: when $s_i\,(t) = 1$ what is produced corresponds to what is offered, and capital accumulation stops, otherwise an increase in the capital stock yields a decrease in $s_i(t)$ as long as $k_i(t) > \dfrac{\delta}{\alpha} s_i(t)$.

We assume that the two firms behave alternatively as either price or quantity setters. Each firm i aims at maximizing the discounted profit flow:

$$\Pi_i(t) = \int_0^\infty \pi_i(t)\, e^{-\rho t} dt \tag{6}$$

w.r.t. controls $k_i(t)$ and the market variable, either $p_i(t)$ or $q_i(t)$, under the constraint given by the state dynamics (5). The discount rate $\rho > 0$ is assumed to be constant and common to both firms. The corresponding current value Hamiltonian function is:

$$\mathcal{H}_i(t) = e^{-\rho t}\left[\pi_i(t) + \lambda_{ii}(t)\dot{s}_i(t) + \lambda_{ij}(t)\dot{s}_j(t)\right] \tag{7}$$

where $\lambda_{ii}(t) = \mu_{ii}(t)e^{\rho t}$ and $\lambda_{ij}(t) = \mu_{ij}(t)e^{\rho t}$, $\mu_{ii}(t)$ being the co-state variable associated to $s_i(t)$.

For future reference, we also define consumer surplus $CS(t) \equiv U(t) - \sum_i p_i(t)\,\widehat{q}_i(t)$. Under the symmetry assumption $\widehat{q}_i(t) = \widehat{q}_j(t) = \widehat{q}(t) = \dfrac{q}{s}(t)$, consumer surplus writes as:

$$CS(t) = \widehat{q}(t)^2[1+\gamma] = \left[\frac{q(t)}{s(t)}\right]^2[1+\gamma] \tag{8}$$

Disregarding the issue of surplus distribution among agents, we can define social welfare as $SW(t) \equiv 2\pi(t) + CS(t)$, with $\pi(t) = q(t)\{(As(t) - q(t)[1+\gamma]\}/[s(t)]^2$:

$$SW(t) = 2A\widehat{q}(t) - \widehat{q}(t)^2[1+\gamma] = 2A\frac{q(t)}{s(t)} - \left[\frac{q(t)}{s(t)}\right]^2[1+\gamma] \tag{9}$$

Note that the above welfare function is decreasing in $s(t)$.

3 Bertrand competition

Let us move from the case of firms competing in prices. By substituting (3) in (6) we get the relevant objective function for firm i:

$$\Pi_i(t) = \int_0^\infty e^{-\rho t}\left\{\left[\frac{A(1-\gamma) - p_i(t) + \gamma p_j(t)}{1-\gamma^2}\right]p_i(t) - \beta\,[k_i(t)]^2\right\}dt \tag{10}$$

Using the Hamiltonian function (7), first order conditions on controls are (we omit the indication of time for brevity):[2]

$$\frac{\partial \mathcal{H}_i}{\partial p_i} = 0 \Rightarrow p_i = \frac{A(1-\gamma) + \gamma p_j}{2} \tag{11}$$

$$\frac{\partial \mathcal{H}_i}{\partial k_i} = 0 \Rightarrow \lambda_{ii} = \frac{2\beta k_i}{\alpha(1-s_i)} \tag{12}$$

Note, first, that (12) does not contain λ_{ij} since the present game features separated dynamics. Therefore, we set $\lambda_{ij} = 0$ for all $t \in [0, \infty)$. Secondly, (11) does not contain s_j, therefore the open-loop solution and the closed-loop memoryless solution coincide.[3]

3.1 Degenerate Markov Perfect Nash Equilibrium

According to the closed-loop memoryless solution concept, we specify the firm i's co-state equation as follows:

$$-\frac{\partial \mathcal{H}_i}{\partial s_i} = \lambda_{ii}(\alpha k_i + \delta - 2\delta s_i) = \dot{\lambda}_{ii} - \rho\lambda_{ii} \tag{13}$$

along with the transversality and initial conditions:

$$\lim_{t\to\infty} \mu_i s_i = 0, \quad s_i(0) > 1. \tag{14}$$

Now, by using (12), (11) and the co-state equation (13), we write:

$$\dot{k}_i = k_i(\rho + \delta - \delta s_i) \tag{15}$$

The steady state equilibrium requires $\left\{\dot{k}_i = 0, \dot{s}_i = 0\right\}$, yielding:[4]

$$k_i^{SS} = \frac{\rho+\delta}{\alpha} > 0; \quad s_i^{SS} = \frac{\rho+\delta}{\delta} > 1 \tag{16}$$

[2]Second order conditions are always met throughout the paper. They are omitted for brevity.

[3]There exist several classes of games where open-loop and closed-loop solutions coincide. For exhaustive expositions, see Mehlmann (1988, ch. 4) and Dockner *et al.* (2000, ch. 7), *inter alia*.

[4]There also exists another steady state, given by $k_i^{ss} = 0$, $s_i^{SS} = 1$, which is not taken into account since if $s_i = 1$, of course, no TCRD is undertaken. Such equilibrium is also unstable.

Proposition 1 *The steady state defined by* $\left\{k_i^{SS}, s_i^{SS}\right\}$ *is a saddle point.*

Proof. See the Appendix. ■

We are interested in the dynamics of the system (15) (5) in the $\{k_i, s_i\}$ space which can be represented in Figure 1:

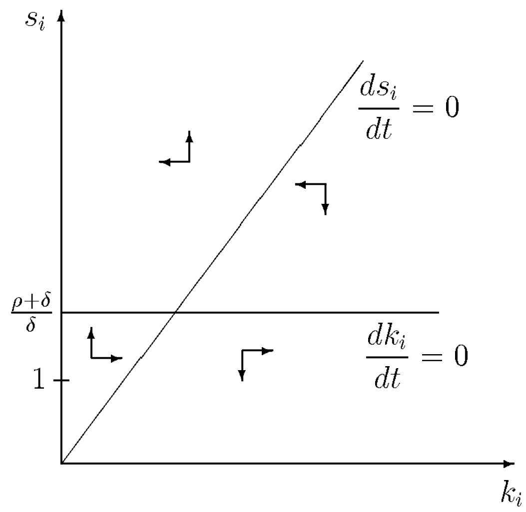

Figure 1 : Phase Diagram

>From the above phase diagram it can be easily seen that the equilibrium is a saddle, and it can be approached along the north-est arm of the path. It is worth noting that if $s_i(0) >> \frac{\rho + \delta}{\delta}$, then the system never converges to the equilibrium, the reason being that the effort required to increase the share of the good arriving at destination is too costly.

As to the comparative statics of the steady state w.r.t. all involved parameters, we have the following properties: $\frac{\partial s_i^{SS}}{\partial \rho} > 0$ and $\frac{\partial s_i^{SS}}{\partial \delta} < 0$. First, when the rate of time preference ρ increases, firms invest more at the steady state level but a lower fraction arrives at destination. Firms are impatient to deliver a higher fraction and spend substantial amounts on TCRD without waiting for the beneficial effect coming from the accumulation dynamics. As to δ, at the steady state a higher depreciation rate increases the level of capital and consequently the fraction of output that reaches the market. To verify the intuition behind this, observe that:

$$\frac{\partial s_i}{\partial \delta} = \frac{\partial s_i}{\partial k_i} \cdot \frac{\partial k_i}{\partial \delta}$$

where:

$$\frac{\partial s_i}{\partial k_i} < 0 \text{ and } \frac{\partial k_i}{\partial \delta} > 0.$$

4 Cournot Competition

The relevant objective function for firm i in case of quantity competition is:

$$\Pi_i = \int_0^\infty e^{-\rho t} \left\{ \frac{q_i}{s_i} \left[A - \frac{q_i}{s_i} - \gamma \frac{q_j}{s_j} \right] - \beta \, [k_i]^2 \right\} dt \tag{17}$$

Using the Hamiltonian function (7), first order conditions on controls are:

$$\frac{\partial \mathcal{H}_i}{\partial q_i} = 0 \Rightarrow q_i = \frac{s_i \left(A s_j - \gamma q_j \right)}{2 s_j} \tag{18}$$

$$\frac{\partial \mathcal{H}_i}{\partial k_i} = 0 \Rightarrow \lambda_{ii} = \frac{2 \beta k_i}{\alpha \left(1 - s_i \right)} \tag{19}$$

Note that, first, (19) is equivalent to (12), so it does not contain λ_{ij} (we set $\lambda_{ij} = 0$ for all $t \in [0, \infty)$). However, (18) contains s_j, i.e., the state variable of the rival, meaning that the open-loop solution and the closed-loop memoryless solution does not coincide anymore. As a consequence, we deal with the two solution concepts.

4.1 Open-Loop Nash Equilibrium

Under the open-loop solution concept, we can specify the firm i's co-state equation as follows:

$$-\frac{\partial \mathcal{H}_i}{\partial s_i}=\frac{2q_i^2}{s_i^3}+\frac{q_i\left(\gamma q_j-As_j\right)}{s_i^2 s_j}-\lambda_{ii}\left(\alpha k_i+\delta-2\delta s_i\right)=\dot{\lambda}_{ii}-\rho\lambda_{ii} \tag{20}$$

along with the transversality and initial conditions:

$$\lim_{t\to\infty}\mu_i s_i=0,\ \ s_i(0)>1. \tag{21}$$

Now, by using (19), (18) and the co-state equation (20), we write:

$$\dot{k}_i=k_i\left(\rho+\delta-\delta s_i\right) \tag{22}$$

The steady state equilibrium requires $\left\{\dot{k}_i=0,\dot{s}_i=0\right\}$, yielding:

$$k_i^{OL}=\frac{\rho+\delta}{\alpha};\quad s_i^{OL}=\frac{\rho+\delta}{\delta} \tag{23}$$

It is straightforward to see that (23) are the same as (16), albeit it may be quickly checked that equilibrium profits are different. Nonetheless, such a comparison has a limited interest in that it involves open loop solutions, which are only weakly time consistent.

4.2 Closed-Loop Nash Equilibrium

In order to perform a meaningful comparison between market regimes, we need to solve the game in closed-loop, taking into account the feed-back between player i's strategy and player j's state variable. This will lead to an equilibrium characterized by subgame perfection.

We specify the firm i's co-state equation:

$$-\frac{\partial \mathcal{H}_i}{\partial s_i}-\frac{\partial \mathcal{H}_i}{\partial q_j}\frac{\partial q_j^*}{\partial s_i}\equiv\Phi=\dot{\lambda}_{ii}-\rho\lambda_{ii} \tag{24}$$

with

$$\Phi=\frac{2q_i^2}{s_i^3}+\frac{q_i\left(\gamma q_j-As_j\right)}{s_i^2 s_j}-\lambda_{ii}\left(\alpha k_i+\delta-2\delta s_i\right)-\left(-\frac{\gamma q_i}{s_i s_j}\right)\left(\frac{\gamma q_i s_j}{2s_i^2}\right)$$

along with the transversality and initial conditions:

$$\lim_{t\to\infty} \mu_i s_i = 0, \;\; s_i(0) > 1. \tag{25}$$

Now, by using (19), (18) and the co-state equation (24), we write:

$$\dot{k}_i = \frac{-A^2\gamma^2 (s_i - 1)\alpha + 4\beta (2+\gamma)^2 k_i s_i (\rho - \delta s_i + \delta)}{4\beta (2+\gamma)^2 s_i} \tag{26}$$

The steady state conditions $\left\{\dot{k}_i = 0, \, \dot{s}_i = 0\right\}$ do not yield tractable solutions.[5] Therefore, we proceed as follows. We impose $\dot{k}_i = 0$ to determine an equilibrium relation between k_i and s_i:

$$k_i^{CL}(s_i) = \frac{A^2\gamma^2 (s_i - 1)\alpha}{4\beta (2+\gamma)^2 s_i (\rho - \delta s_i + \delta)} \tag{27}$$

We are now in a position to compare the optimal open-loop and closed-loop level of R&D investments under Cournot competition. This is equivalent to compare the closed-loop solution arising in Cournot *vis à vis* Bertrand's, given that the open-loop solution with quantities as a control variable corresponds to the closed-loop solution with prices as a control variable.

>From a direct comparison between (23) and (27), we have:

Proposition 2 $k_i^{CL} > k_i^{OL} \Rightarrow s_i^{CL} < s_i^{OL}$. *The optimal effort in TCRD is higher under Cournot competition than under Bertrand competition.*

Proof. $k_i^{CL}(s_i) = \dfrac{A^2\gamma^2 (s_i - 1)\alpha}{4b (2+\gamma)^2 s_i (\rho - \delta s_i + \delta)} > 0$ *to be acceptable. Since the numerator is always positive by definition, it has to be true that* $\rho - \delta s_i + \delta > 0 \Rightarrow s_i < \dfrac{\rho+\delta}{\delta} \equiv s_i^{OL}$. *This amounts to saying that* $k_i^{CL} > k_i^{OL}$. ■

[5] We find three solutions, only one being real. Calculations are available upon request.

This result is in line with the kind of R&D activity under consideration, which aims at increasing the percentage of the produced good that reaches the market. Moreover, we confirm the conventional wisdom that firms invest more when using closed-loop decision rules than open-loop ones.

5 Social Optimum and Welfare Appraisal

The aim of this section is threefold: (i) characterizing the first best solution; (ii) comparing the private optima with the social optimum; (iii) comparing the welfare generated by Bertrand competition with that generated by Cournot competition. The objective function of an hypothetical benevolent planner is:

$$SW(t) = \int_0^\infty e^{-\rho t} \left\{ 2A\frac{q(t)}{s(t)} - \left[\frac{q(t)}{s(t)}\right]^2 (1+\gamma) - 2\beta\, [k(t)]^2 \right\} dt \tag{28}$$

to be maximized w.r.t. $q(t)$ and $k(t)$ under the dynamic constraint:

$$\frac{\partial s(t)}{\partial t} = [\alpha k(t) - \delta s(t)]\,[1 - s(t)] \tag{29}$$

The current value Hamiltonian function writes:

$$\mathcal{H}^{SP}(t) = e^{-\rho t}\,[SW(t) + \lambda(t)\dot{s}(t)] \tag{30}$$

First-order conditions on controls are (we omit the indication of time for brevity):

$$\frac{\partial \mathcal{H}^{SP}}{\partial q} = 0 \Rightarrow q = \frac{As}{1+\gamma} \tag{31}$$

$$\frac{\partial \mathcal{H}^{SP}}{\partial k} = 0 \Rightarrow \lambda = \frac{4\beta k}{\alpha(1-s)} \tag{32}$$

$$-\frac{\partial \mathcal{H}^{SP}}{\partial s} = \frac{q\,[2As - 2q(1+\gamma)] + \lambda s^3\,[\delta(1-2s) + \alpha k]}{s^3} = -\rho\lambda + \dot{\lambda} \tag{33}$$

By differentiating (32) w.r.t. time we obtain:

$$\dot{\lambda} = \frac{4\beta\left(\dot{k}\,(1-s) + k\dot{s}\right)}{\alpha\,(1-s)^2} \tag{34}$$

By plugging (34), (32) and (31) into (33) and by using (29) we have:

$$\dot{k} = k_i\,(\rho + \delta - \delta s_i) \tag{35}$$

The steady state equilibrium requires $\left\{\dot{k} = 0, \dot{s} = 0\right\}$, yielding:

$$k^{SP} = \frac{\rho + \delta}{\alpha}; \quad s^{SP} = \frac{\rho + \delta}{\delta} \tag{36}$$

On the basis of the steady state solutions previously obtained, we can write:

Proposition 3 *Consider closed-loop memory less equilibria. Under price competition, the amount of effort in TCRD is socially optimal; under quantity competition, the amount of effort in TCRD is socially excessive.*

Now, we proceed with a comparison between market regimes in terms of equilibrium welfare. Steady state welfare levels, gross of investment efforts, are:

$$SW^C = A^2\frac{3+\gamma}{(2+\gamma)^2} \tag{37}$$

$$SW^B = A^2\frac{3-2\gamma}{(1+\gamma)\,(-2+\gamma)^2} \tag{38}$$

where superscript C and B stand for Cournot and Bertrand, respectively. Note that the state variable does not enter the above welfare expressions, in that it cancels out once plugged equilibrium quantities into (9). From a direct comparison between (37) and (38) it is straightforward to conclude that $SW^B > SW^C$ always in the relevant parameter range, since the quantity firms decide to produce under Bertrand competition is always higher than the one they decide to produce under Cournot's, no matter the share of the good that reaches the market. *A fortiori*, taking into account that, as we know from Proposition 2, $k^{CL} > k^{OL}$, the welfare performance of the Bertrand game is superior to that of the Cournot game, net of steady state investments.

One could ask himself whether the fact that a planner would prefer firms to be price setters depends upon the fact that in this regime market coverage is larger. Reasoning for given quantities produced, meaning that production plans are independent of market regimes, one can investigate upon the effect of TCRD investments on welfare under both kinds of competition. Since (9) is decreasing in the state variable, and provided that firms invest more in TCRD under quantity competition, it is now true that $SW^C > SW^B$. Net of investment efforts, this inequality may indeed take either sign. Hence, our final result is the following:

Proposition 4 *Consider a given amount of production that must reach the final market. The welfare generated by Cournot competition may be higher than the one generated by Bertrand competition.*

The above Proposition states that a planner willing to ask firms to produce a given amount of substitute goods and ship it to the final market for consumption, is not indifferent over the choice of variable, rather, he might have a strict preference towards quantity competition. The reason of this result lies in the incentives for firms to invest in TCRD, which are higher under Cournot than under Bertrand competition. As a consequence, under the former, a larger mass of consumers is served and welfare might improve.

6 Concluding remarks

An established result states that there is an excess of process-innovating R&D under Cournot competition, while the opposite holds under Bertrand competition (Brander and Spencer,1983; Dixon, 1985). In this paper we have taken a differential game approach to investments in transport and communication technology, confirming the acquired wisdom. Comparing the closed loop private optima with the social optimum, we have shown that the unique distortion that arises under price competition involves equilibrium prices, above marginal cost due to the presence of market power, while under Cournot competition, together with the usual downward market distortion, the amount of effort in TCRD turns out to be upward distorted. Finally, dealing with a welfare appraisal, we have shown that a planner willing to ask firms to produce a given amount of substitute goods and ship it to the final market for consumption, might have a strict preference towards quantity competition. Once accounted for different production levels, no matter the share of the good that reaches the market, the same planner would opt for price competition.

Appendix

Proof of Proposition 1.

We consider first the system composed by (5) in combination with the appropriate kinematics of the control variable k_i, that is, (15) The system can be written in matrix form as follows:

$$\begin{bmatrix} \dot{s}_i \\ \dot{k}_i \end{bmatrix} = \begin{bmatrix} -\delta + 2\delta s_i - \alpha k_i & -\alpha\left(-1+s_i\right) \\ -\delta k_i & \left(\rho + \delta - \delta s_1\right) \end{bmatrix} \begin{bmatrix} s_i \\ k_i \end{bmatrix}$$

Now, we evaluate the 2×2 matrix in the steady state:

$$\begin{bmatrix} \rho & -\dfrac{\alpha\rho}{\delta} \\ -\dfrac{\delta\left(\rho+\delta\right)}{\alpha} & 0 \end{bmatrix}$$

Since the determinant of the above 2×2 matrix is $-\rho\left(\rho+\delta\right) < 0$, the equilibrium we have obtained is a saddle. From the phase diagram, it is clear that this saddlepoint equilibrium can be approached only along the north-est arm of the saddle path.

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