Endogenous innovation waves and economic growth

Rainer Andergassen*, Franco Nardini**

* Department of Economics, University of Bologna,
E-mail: anderga@economia.unibo.it

** Department of Mathematics for Social Sciences, University of Bologna
E-mail: nardini@dm.unibo.it

We propose a simple model where large innovation waves arise from the endogenous propagation of information around sectors. Innovators of each sector invest in internal R&D and in local search for information. We show that depending on the structural parameters of the single sectors, some of the R&D sectors will engage in local search while others will not. Through localised search for information, technology adopted in certain sectors can be adopted also in other sectors, leading to a large technological correlation, and eventually to long ranged innovation waves. We characterise the endogenous balanced growth path of the economy, and the short run fluctuations around it. The model predicts a linear, positive relationship between the short run fluctuations and the long run growth rate. We test this latter relationship and find that we cannot reject the predictions of the model.

Keywords: innovation waves, balanced growth rate, aggregate fluctuations, technology diffusion, endogenous growth

J.E.L. Classification numbers: D92, E32, E23, O33, O41

This paper has been written parallel to a paper written by the present authors and Massimo Ricottilli on Innovation dynamics. The authors are so much indebted to Massimo Ricottilli for fruitful discussion. We thank also Christopher Laincz and seminar participants at the 2nd edition of the Summer School in Economic Theory, Venice 2002. The usual disclaimer applies.

It is widely recognised that technology spill-overs play a primary role in the dynamics of innovations. Rosenberg (1976) highlights their importance in the process of industrialisation. In particular, he emphasises the importance of the technological convergence which emerged in the process of industrialisation. Using Rosenberg's words: "... (the) industrialisation was characterized by the introduction of a relatively small number of broadly similar productive processes to a large number of industries (p. 15)". According to the author, its was this latter event that lead to the possibility of large spill-over effects. Each single firm tried to solve idiosyncratic problems, i.e. specific to her production. Once she succeeded in solving the problem, the innovation introduced was likely to be introduced also in other sectors due to the technological convergence which characterised different sectors. This latter phenomenon lead possibly to large innovation waves. Fai and Von Tunzelmann (2001) show that there exists also today this kind of technological convergence which characterised the process of industrialisation.

In order to study the dynamic effects of innovation waves, the idea of general purpose technologies (GPT) has been introduced[1]. GPT are major innovations such as steam engine, electric dynamo, laser or computer. The introduction of new innovations is costly since it requires a restructuring of the production process (David, 1990). Thus, Helpman and Trajtenberg (1994) argue that resources, such as labor for example, are withdrawn from the production process, and devoted to the R&D sector, and this latter generates a slowdown of aggregate production. Once the innovation has been successfully adopted by the firms, aggregate production increases. In this way there exists a positive relationship between long run growth and short run fluctuations. This framework has been enlarged by Aghion and Howitt (1998) p. 253 in order to study the diffusion of the adoption of GPT. The authors propose a simple model of social learning in order to capture the stylised fact that there exists an "...uneven transition path for aggregate output, ... where prolonged periods of relative stagnation are followed by an acceleration in the pace of technology diffusion" (Aghion and Howitt, 1998). The diffusion of the GPT occurs through the observation of other firms.

Assuming GPT implies superimposing exogenous technology paradigms. In this paper we are interested in how technology paradigms and innovation waves emerge endogenously from the interaction of single sectors. We consider innovations to be idiosyncratic, but through localised interaction of firms, the informational content of the innovation can be diffused. Aghion and Howitt state this problem in the following way: "... new technologies do not get implemented instantaneously throughout the economy. Instead, they diffuse gradually, through a process in which one sector gets ideas from the research and experience of others." (Aghion and Howitt, 1998 , p. 85). Our aim is to build a simple model where highly volatile large innovation waves emerge from localised spill-overs of information, leading to a large technological correlation, like the one Rosenberg (1976) highlighted as a crucial element for the process of industrialisation, and

[1] See for example David (1990) and Helpman (1994).

recently evidenced as a stylised fact today by Fai and Von Tunzelmann (2001).

The adoption of a new innovation usually requires, at least partially, an internal reorganisation (David, 1990). Thus, innovating firms face sunk costs of reorganisation, and the adoption of innovations is optimal only if the gain from the adoption is larger than its cost. This latter implies that innovations developed by R&D firms must reach a certain degree of perfection such that the gains from their introduction are at least as big as the sunk costs of reorganisation. This latter phenomenon has been highlighted also by Arrow (1974, p. 41) who pointed out that the problem of information channels resembles the problem of inventories under uncertainty. We are going to propose a multi-industry model, each industry possessing the Aghion and Howitt (1992) structure, where the single R&D firm of each industry has to complete an informational process, i.e. accumulate sufficient information, such that the adoption of the innovation by the final good firms of the same industry becomes fruitful. In this process of collection of information the single representative R&D firm invests in internal research, as well as in the observation of technologically related industries, i.e. industries facing similar technological problems. This constraint to the diffusion of information results in the clustering of all R&D sectors into neighbourhoods. Contrary to current economic literature, we are not going to model the localised informational spill-overs as a free lunch for each R&D sector. We are going to assume that specialised workers have to be allocated to the localised information search process, and that these latter can experience a reduced productivity in finding new information. In other words, we assume that once a technologically related industry introduces a new innovation, this latter has to be studied by specialised worker such that the informational content of this innovation can be extracted and eventually used for an innovation in this other industry. We assume that for some R&D sectors this activity is costly in the sense that it reduces the productivity of these workers in creating new information. This reduced productivity arises, for example, because of difficulties in communication between R&D sectors belonging to different, even though technologically correlated, industries, since different R&D sectors followed different directions of research. We will see how this influences the choice between investing in localised search for information or not.

In this paper we build an endogenous growth model, where growth is driven by industry specific innovations, i.e. born in a given industry, and endogenous technology spill-overs between industries. We are going to characterise an economy whose aggregate production fluctuates due to the endogenous stochastic innovation waves. Further, we will see that the larger are the innovation waves, the larger will be the short run fluctuations of aggregate production, and the larger will be the long run growth rate of the economy. The empirical literature on the relationship between fluctuations and long run economic growth gives no clear answer, mainly because of a missing theoretical framework[2]. We are going to test the theoretical relationship which results from the theoretical model and

[2] See for example Ramey and Ramey (1995), Kormendi and Meguire(1985) and Elmer and Pedersen (1998).

find that we cannot reject the results of our model. Further, the results seem to be robust with respect to the specification of the model.

The model we are going to develope is consistent with the scale effects recently highlighted in the literature on endogenous growth. Akin to the paper by Peretto and Smulders (2002), our model captures the empirical evidence produced by Backus, et.al. (1992) that GDP growth is not related to the scale of the whole economy, while it is positively related to the scale of the single industry. Like Peretto and Smulders (2002), we obtain that the aggregate scale effect, which in our case is only positive, vanishes asymptotically as the number of industries increases, while the scale effect within the single industry is non-vanishing.

The remaining part of the paper is organised as follows. In Section 1 we solve the problem of allocating the specialised workforce between the internal and external search process. Further, we characterise the optimal informational content of innovations. In Section 2 we derive the aggregate innovation dynamics and the long and short run dynamics of aggregate production of the economy. In Section 3 we propose an empirical test for the model. Section 4 concludes.

1 The microfoundations

We assume that there are n industries in the economy, n very large, each composed of a final good sector, an intermediate good sector and an R&D sector. The intermediate good and the R&D sector are both specific to the final good produced in the same industry. The R&D sector discovers new ideas and innovations, which are used in the intermediate good sector of the same industry. Finally, the intermediate good which incorporates the technology is used by the final good sector in order to produce the output.

We assume that the final good sector is perfectly competitive, and that the intermediate good sector is made of a single monopolistic firm. Further, we assume that the R&D sector of each industry is composed of two firms, a leader and a follower. These two firms alternate in producing the innovation. Once a firm succeeds in introducing a new innovation, she has no incentive to invest further in a new innovation since the gain of a further innovation would be lower than its cost (Arrow's replacement effect). Thus, once a new innovation has been introduced this latter firm stops investing in R&D, while the follower starts investing in R&D and to search for the information necessary for the introduction of a new innovation.

We assume that firms producing the final good have to pay sunk costs of reorganisation if they want to introduce a new technology, and further, outsider firms have to pay sunk costs if they want to enter the market. If these sunk costs of reorganisation are sufficiently large, then it is no longer optimal to introduce an innovation if a small, idiosyncratic bit of information arrives at the R&D sector of the same industry, since the gains from the introduction this innovation would be lower than the costs of reorganisation. As a consequence, the R&D firm of industry i has to accumulate a sufficient number of informational bits such that the introduction of the new innovation at the final good sector level of industry i becomes profitable. As long as the accumulation of information continues, this latter remains tacit, and only once the new innovation has been introduced, other R&D sectors, i.e. R&D sectors belonging to technologically correlated industries, can infer its informational content, and can get in this way new information and ideas for their own.

The representative R&D firm faces the problem of allocating specialised workers to internal R&D and to localised search for information. These latter workers observe continuously a limited number of technologically related industries, i.e. industries facing similar technological problems, and contemporaneously they try to create new information for their own. We assume that one specialised worker observes at most one R&D sector belonging to another industry. The number of workers allocated to the localised search for information together with the technological correlation between industries defines the neighbourhood structure along which informational spill-overs occur. The constraints to the diffusion of information translates into a constraint on the productivity of workers allocated to the localised search; an upper bound on the number of these workers is the natural consequence of this formalisation.

Once a new innovation has been introduced in a particular industry, the information incorporated in this innovation can be observed and used by R&D firms of technologically correlated industries. We will assume that for some R&D sectors the spill-over effects come in as a free lunch, while for other R&D sectors we will assume that it is not. We will show that, under certain conditions, these latter will not engage in localised search since the marginal productivity of the workers allocated to the local search for information is lower than the marginal productivity of the workers allocated to the internal search for information. In this way, the spill-over dynamics and as a consequence the aggregate innovation waves will be reduced.

1.1 The representative final good sector

Consider a generic industry i. The final good i is produced using intermediate good i only. Neglecting for the moment being the problem of sunk costs, the problem of the representative firm is given by

$$\begin{array}{l} \max \left\{ p_\tau y_\tau - P\left(x_\tau\right) x_\tau - C \right\} \\ s.t.\ y_\tau = A\left(\tau\right) x_\tau^{1-\alpha} \end{array} \tag{1}$$

where C are fixed costs of production, τ indicates the number of innovations introduced, $A\left(\tau\right) = \left[1 + \gamma\left(s\right)\right]^\tau$, $\gamma\left(s\right)$ is the increment in productivity and $s \in \aleph$ indicates the informational content of the innovations. We assume that $\gamma'\left(s\right) > 0$ and $\gamma''\left(s\right) \leq 0$. From the first order condition of problem (1) we obtain the demand function of the intermediate good:

$$P\left(x_\tau\right) = p_\tau A\left(\tau\right)\left(1-\alpha\right) x_\tau^{-\alpha} \tag{2}$$

and the profit function

$$\pi_{y,\tau} = p_\tau A\left(\tau\right) \alpha x_\tau^{1-\alpha} - C$$

We are going to assume that there are entry costs for outsider firms and sunk costs of reorganisation for insiders if they want to upgrade the technology, each given by k. We assume that if firms introduce a new technology in τ, prices remain fixed at the previous level for a small time period[3] Δt, while after this prices decrease to the level $p_\tau = \frac{p_0}{A(\tau)}$, where $p_0 = \frac{C}{\alpha x^{1-\alpha}}$. Within this small time period firms makes profits in order to recover the sunk costs they face in the adoption of a new technology. After this, prices decrease because of the competition among final good producers.

In the stationary state where $x_{\tau+1} = x_\tau = x$, once an innovation has been introduced, firms make profits equal to $\gamma(s)\, p_\tau A(\tau)\, \alpha x_\tau^{1-\alpha} \Delta t$. The condition such that introduction of the innovation is optimal, i.e. the profits from the introduction are larger than the costs of introduction, and the no entry condition require that

$$\gamma(s^*) = \frac{k}{\Delta t C} \tag{3}$$

where s^* indicates the optimal number of bits of information the representative R&D firm has to accumulate such that the introduction of the innovation by the final good sector becomes optimal. The optimal informational content of innovations will be $s^* = \Gamma\left(\frac{k}{\Delta t C}\right)$. We are going to assume that the R&D firms accumulate discrete units of informational bits, while s^* will be in general a real number. Thus, we will define probabilities p and $1-p$ such that $s^* = \underline{s}p + \overline{s}(1-p)$, where $\underline{s} = \text{integer}[s^*]$ and $\overline{s} = \text{integer}[s^*]+1$. We will assume that the representative firm R&D firm with probability p accumulates $\underline{s}$ informational bits, while with probability $1-p$ she accumulates $\overline{s}$ informational bits.

1.2 The representative intermediate good sector

Each industry i is endowed with $H^i = H$ specialised workers, where $i = 1, ..., n$. Within each industry, the specialised workers have to be allocated between the intermediate good sector (H_x) and the R&D sector (H_A), where $H = H_A + H_x$.

Given the demand function (2), we can now turn to the problem of the intermediate good producer. This latter produces the intermediate good using specialised workers H_x. Its problem can be stated as follows

$$\begin{array}{l} \max\left\{P(x_\tau)\, x_\tau - w_{x,\tau} H_{x,\tau}\right\} \\ s.t.\ x_\tau = \frac{1}{\eta} H_{x,\tau} \end{array}$$

Assuming that the intermediate good firm has a monopoly power and that there

[3] The small time period Δt and s^* are not perfect substitutes. To see this, consider the case where s^* is small, i.e. for example $s^* = 1$. In this case, if the sunk costs k are sufficiently large and $\gamma'(s)$ is sufficiently low, Δt has to be very large such that the final good producers are able to recover the sunk costs of reorganisation. But the lower is s^*, the earlier a new innovation arrives (see Section 1.3.3). Thus, it can happen that the final good producers are not able to recover fully the sunk costs of reorganisation before a new innovation arrives, and as a consequence these latter firms make a loss. In order to avoid this problem, we assume that Δt is exogenously determined, and arbitrarily small, while s^* is endogenously determined.

are labour turnover costs[4], we have that the wage $w_{x,\tau}$ and profits are given by

$$\begin{aligned} w_{x,\tau} &= p_0 \left(1-\alpha\right)^2 \tfrac{1}{\eta^{1-\alpha}} H_{x,\tau}^{-\alpha} \\ \pi_{x,\tau} &= p_0 \alpha \left(1-\alpha\right) \tfrac{1}{\eta^{1-\alpha}} H_{x,\tau}^{1-\alpha} \end{aligned} \tag{4}$$

1.3 The representative R&D sector

Innovations are produced using specialised workers H_A. The single representative R&D sector faces the problem of allocating optimally H_A between internal H_I and external H_E research (local search for information) activity.

If a firm succeeds in introducing a new innovation, then she fixes the price for this innovation equal to the profits of the intermediate good sector. Thus, the representative R&D firm has to determine optimally H_A in order to maximise the expected present value of profits.

Let us first calculate the innovation rate, and after this we will determine the optimal H_A.

1.3.1 The innovation rates

The representative R&D firm has to collect sufficient information such that the new innovation can be fruitfully adopted by the final good sector. The R&D firm can either be at the beginning, at the end, or in an intermediate phase of this information collection process.

We will make the following assumptions about the information collection activity. If a firm has no new information, then it is quite easy for this firm to get or create new information. On the other side, she is just at the beginning of the information collection process, and as a consequence the expected present value of investment in R&D is low. The more information a firm has already collected, the harder it is for this firm to get new information since the new information has to be compatible with the information already collected, i.e. the lower are the degrees of freedom. On the other side, the more information the firm has already accumulated, the sooner the new innovation can be introduced and, as a consequence, the larger will be the expected present value of investment in R&D. These postulates can be summarised in the following assumption:

Assumption 1. *The marginal incentive to invest in innovation is independent of the number of informational bits accumulated.*

We are now going to specify the production functions of information.

Nearest neighbouring R&D sectors are observed continuously. In particular, we will assume that the firms observe as many neighbouring firms as there are workers allocated to this activity. Since the number of observable R&D sectors is constrained by the technological heterogeneity, we assume that at most there

[4] Given that there are sufficiently large labour turnover costs, the intermediate good sectors will maintain the employment level constant during the time interval Δt where the price of the final good sector changes.

can be s^* workers allocated to this activity, where s^* indicates the number of bits of information which have to be accumulated. Further, we assume that the mean number of informational bits created by R&D firm i in a time unit is given by $\left(\vartheta^i H_I + \vartheta'^i H_E\right) \frac{\mu}{n}$, where ϑ^i is the productivity of the specialised workers allocated to internal research, ϑ'^i is the productivity of the specialised workers allocated to external research, μ is the aggregate stochastic arrival rate, n is the number of industries, whereas $\frac{\mu}{n}$ is the idiosyncratic stochastic arrival rate of each sector. Further, we assume that $\vartheta^i \geq \vartheta'^i$ indicating that the marginal productivity of workers allocated to the localised research is not larger than the marginal productivity of workers engaged in internal research. If $\vartheta^i = \vartheta'^i$, then we have that information spill-overs are free lunch, since the marginal productivity of those workers employed in localised search is the same as the marginal productivity of those workers employed in internal research. On the other side, if $\vartheta^i > \vartheta'^i$, then the spill-overs are costly in the sense that the marginal productivity of those workers engaged in localised search is reduced. For example, $\vartheta'^i \to 0$ implies that extrapolating the informational content of innovations introduced by neighbouring R&D sectors is so difficult, i.e. time consuming, such that the workers engaged in this activity are not able to create new information for their own. We will assume that there are m R&D sectors, where $1 \leq m \leq n$, which have $\vartheta^i > \vartheta'^i$, while the other $n - m$ R&D sectors are characterised by $\vartheta^i = \vartheta'^i$.

We are going to call ρ_c the stationary average density of R&D firms being in the state where they need just one more bit of information such that the introduction of the new innovation becomes optimal. Thus, we can write the average probability of innovating for a representative R&D firm i engaged and non engaged in localised search for information, δ_L^i and δ_{NL}^i respectively, as follows[5]

$$\begin{gathered}\delta_L^i = \max_{H_I^i, H_E^i} \rho_c \left[\left(\vartheta^i H_I^i + \vartheta'^i H_E^i\right) \tfrac{\mu}{n} + \delta H_E^i\right] \\ s.t.\ H_E^i \leq s^* \\ H_I^i + H_E^i = H_A^{L.i}\end{gathered} \tag{5}$$

$$\delta_{NL}^i = \rho_c \vartheta^i H_A^{NL,i} \frac{\mu}{n} \tag{6}$$

where s^* is the optimal informational content of innovations and δ is the average aggregate innovation rate, which is given by

$$\delta = \frac{1}{n} \sum_{i \in X_L} \delta_L^i + \frac{1}{n} \sum_{i \in X_{NL}} \delta_{NL}^i \tag{7}$$

From (5) we observe that, given that the R&D firm i decides to engage in localised search for information, if the marginal productivity of H_E^i is at least as large as the marginal productivity of H_I^i then the representative R&D sector has the incentive to allocate as much as possible specialised workers to the localised search.

[5] We use a mean-field in modelling the dynamic interaction among agents. See Section 3.1 for details.

Let us state the following result:

Proposition 1 *If $\vartheta^i - \vartheta'^i$ is sufficiently large, then those R&D firms where $\vartheta^i = \vartheta'^i$ will engage in local search for information, while those where $\vartheta^i > \vartheta'^i$ will not engage in local search for information.*

Proof. Note that R&D firms i where $\vartheta^i = \vartheta'^i$ find it always optimal to engage in local search for information since this latter activity is not costly. Let us assume that R&D firms where $\vartheta^i > \vartheta'^i$ do not engage in local search for information, and decide individually whether to engage in local search for information or not, taking δ as given. As long as

$$\delta < \left(\vartheta^i - \vartheta'^i\right)\frac{\mu}{n}$$

these latter firms will not engage in local search for information since the marginal productivity of H_E is lower than the marginal productivity of H_I. δ will be calculated explicitly in Section 1.3.3. ■

We will see in Section 2.1 that the stationary state density of firms being in state c is given by $\rho_c = \frac{1}{s^*}$. Substituting (5) and (6) in (7) we have that if the firm engages in localised search, i.e. $s^i = s^{*i}$, then its innovation rate will be given by

$$\begin{aligned}\delta_L^i = & \tfrac{\vartheta^i H_A^{L,i}}{s^*}\tfrac{\mu}{n} + \tfrac{1}{\varepsilon}\tfrac{1}{n}\textstyle\sum_{j\in X_L}\tfrac{\vartheta^j H_A^{L,j}}{s^*}\tfrac{\mu}{n} + \\ & + \tfrac{1}{\varepsilon}\tfrac{1}{n}\textstyle\sum_{j\in X_{NL}}\vartheta^j\tfrac{H_A^{NL,j}}{s^*}\tfrac{\mu}{n}\end{aligned} \tag{8}$$

where $\varepsilon = \frac{m}{n}$ and X_L and X_{NL} are the R&D firms (sectors) engaged and non engaged in local search for information, respectively. On the other side, the firm decides to not engage in local search for information, i.e. $s^i = 0$, then the innovation rate will be

$$\delta_{NL}^i = \vartheta^i\frac{H_A^{NL,i}}{s^*}\frac{\mu}{n} \tag{9}$$

1.3.2 Optimal innovation rates for firms engaged and non engaged in local search for information

For the following we are going to assume that all R&D firms are symmetric, i.e. $\vartheta^i = \vartheta$ and $\vartheta'^i = \vartheta'$. Given the innovation rate, the representative R&D firm maximises expected present value of profits. Consider first the problem of the R&D sectors investing in localised search for information. In this latter case, the problem can be stated as follows

$$\begin{aligned}&\max_{H_{A,\tau}^i}\left\{E\left[V_{\tau+1}^i\right] - w_{A,\tau}^L H_{A,\tau}^i\right\}\\ &s.t.\ E\left[V_{\tau+1}^i\right] = \left(\tfrac{\vartheta H_{A,\tau}^{L,i}}{s^*} + \delta\right)\tfrac{\mu}{n}\tfrac{1}{r+\delta_{L,\tau+1}^i}\pi_{x,\tau+1}\end{aligned} \tag{10}$$

where r is the nominal interest rate and $\pi_{x,\tau+1}$ is given by (4).

From the first order conditions of the problem we have that

$$w_{A,\tau}^{L}=\frac{1}{r+\delta_{L,\tau+1}}\left(1+\frac{1}{\varepsilon n}\right)\frac{\vartheta}{s^{*}}\frac{\mu}{n}\alpha\left(1-\alpha\right)p_{0}\frac{1}{\eta^{1-\alpha}}\left(H_{x,\tau+1}^{L}\right)^{1-\alpha} \tag{11}$$

R&D sectors where $\vartheta'<\vartheta$ will solve the following problem

$$\begin{gathered}\max_{H_{A,\tau}}\left\{E\left[V_{\tau+1}\right]-w_{A,\tau}H_{A,\tau}\right\}\\ s.t.\ E\left[V_{\tau+1}\right]=\vartheta\tfrac{H_{A}}{s^{*}}\tfrac{\mu}{n}\tfrac{1}{r+\delta_{NL,\tau+1}}\pi_{x,\tau+1}\end{gathered}$$

where $\pi_{x,\tau+1}$ is given by (4). From the first order condition of this problem we have that

$$w_{A,\tau}^{NL}=\frac{1}{r+\delta_{NL,\tau+1}}\frac{\vartheta}{s^{*}}\frac{\mu}{n}\alpha\left(1-\alpha\right)p_{0}\frac{1}{\eta^{1-\alpha}}\left(H_{x,\tau+1}^{NL}\right)^{1-\alpha}$$

1.3.3 Allocation of specialised workforce

From the no-arbitrage condition we have that the wages paid in the representative intermediate good firm and in the representative R&D firm within in each industry have to be the same. Thus, using (8), (9), (4) and (11) and the stationary state condition that $H_{x,\tau}=H_{x,\tau+1}=H_{x}$ we obtain the following allocation of specialised workforce

$$\vartheta\frac{H_{A}^{L}}{s^{*}}\frac{\mu}{n}\frac{1}{\varepsilon}=\alpha\vartheta\frac{H}{s^{*}}\frac{\mu}{n}\frac{1+\alpha m}{m\left(1-\alpha\right)}\nu-\alpha\nu r \tag{12}$$

$$\vartheta\frac{H_{A}^{NL}}{s^{*}}\frac{\mu}{n}=\alpha\vartheta\frac{H}{s^{*}}\frac{\mu}{n}-r\left(1-\alpha\right) \tag{13}$$

where $\nu=\left(1+\frac{\alpha}{1+\alpha}\frac{m+1}{n}\right)^{-1}$. Note that $\nu\underset{\varepsilon\to 0}{\to}1$. The innovation rate of R&D firms engaged and non-engaged in localised search and the average innovation rate are, respectively

$$\delta_{L}=\vartheta\frac{H_{A}^{L}}{s^{*}}\frac{\mu}{m}+\vartheta\frac{H_{A}^{NL}}{s^{*}}\frac{\mu}{n}=\alpha\vartheta\frac{H}{s^{*}}\frac{\mu}{n}\left[\frac{1+\alpha m}{\left(1-\alpha\right)m}\nu+1\right]-\left(\alpha\nu+1-\alpha\right)r \tag{14}$$

$$\delta_{NL}=\alpha\vartheta\frac{H}{s^{*}}\frac{\mu}{n}-r\left(1-\alpha\right) \tag{15}$$

$$\delta=\alpha\vartheta\frac{H}{s^{*}}\frac{\mu}{n}\left[\frac{1+\alpha m}{\left(1-\alpha\right)m}\nu\left(1-\varepsilon\right)+1\right]-r\left[\alpha\nu\left(1-\varepsilon\right)+1-\alpha\right]$$

It is easy to see that the innovation rate of those R&D sectors engaged in localised search for information (14) is larger than the innovation rate of those R&D sectors not engaged in local search (15). Further, we can see that the innovation rate (14) is larger, the lower is the number of R&D firms not engaged in localised search (m) and/or the lower is the optimal informational content of innovations (s^{*}).

2 Aggregate innovation dynamics and economic growth

In this Section we focus on the aggregate innovation and growth dynamics of this economy. Given the spill-over dynamics outlined in the previous Sections, we obtain cross-sectional innovation dynamics due to the stochastic endogenous propagation of information once an innovation has been introduced. We will see in Section 2.1 how highly volatile endogenous innovation waves emerge, and in Section 2.2 how these latter lead to aggregate growth and aggregate fluctuations.

2.1 Cross-sectional innovation dynamics

We are going to study the cross-sectional spill-over dynamics of information. We cluster the R&D firms belonging to the different industries according to the number of bits of information accumulated.

We label the possible states as follows: $\Omega = \{0, 1, 2, ..., c, a\}$, where the cardinality of the state space Ω is $\overline{s} + 1$. Thus, $z_i \in \Omega$, for each $i = 1, ..., n$, where z_i indicates the state which characterises R&D firm of industry i. 0 indicates that the R&D firm has just arrived to industry i, and so she has no new accumulated information; 1 indicates that the R&D firm has accumulated one bit of information, ..., c indicate the state where the R&D firm need just one more bit of information such that the introduction of the innovation becomes optimal, respectively; a indicates the active state where the R&D firm introduces the new innovation.

As shown in the previous Section, with frequency $\vartheta H_A^L \frac{\mu}{n}$ and $\vartheta H_A^{NL} \frac{\mu}{n}$ a single R&D sector engaged and non-engaged in localised search succeeds in creating a new information, while $\tilde{\mu} = \varepsilon \vartheta H_A^{NL} \mu + (1 - \varepsilon) \vartheta H_A^L \mu = \alpha \vartheta \frac{H}{s^*} \frac{\mu}{n} \left[\frac{1+\alpha m}{(1-\alpha)m} \nu (1 - \varepsilon) + 1 \right] - r [\alpha \nu (1 - \varepsilon) + 1 - \alpha]$ is the aggregate frequency with which new information is created.

The dynamics of R&D firm i engaged in local search are as follows: given that she is in a state $z_i = k$, if she receives a bit of information (either exogenous or an endogenous one), she switches to state $z_i = k + 1$. If state $z_i = k + 1 < a$ nothing happens until the next bit of information arrives. On the other side, if $z_i = k + 1 = a$, then she introduces the new innovation, and transfers in this way a bit of information with probability $(1 - \varepsilon)$ to s^* neighbouring firms. In other words, $(1 - \varepsilon) s^*$ is the average number of R&D firms belonging to technologically correlated industries and observing the innovation introduced by R&D sector i. Once an R&D firm is in state a she exits the market, and the follower starts the information collection process, i.e. $z_i = 0$. Notice further that since the firms observe always the same neighbouring industries, nothing happens until a new innovation will be introduced in one of these industries or a new informational bit is created directely by the same industry. Thus, only once a new technology has been introduced, the information accumulated by the innovating firms will be freed. Thus, as long as the accumulation of information continues, this information will be tacit, and can not help other firms to introduce new innovations. On the other side, R&D firms not engaged in local search for information change state just if they succeed in creating a new information.

We take a mean-field approximation to the interaction between the single firms[6]. Thus, we cluster the firms according to their state. We call ρ_k the average density of firms being in state $k = 0, 1, ..., c, a$.

The state space dynamics are given by the following master equations,

$$\begin{aligned}
\dot{\rho}_a &= -\rho_a + [\tilde{\mu} + s^* (1-\varepsilon)\rho_a]\rho_c \\
\dot{\rho}_c &= -[\tilde{\mu} - s^* (1-\varepsilon)\rho_a]\rho_c + [\tilde{\mu} + s^* (1-\varepsilon)\rho_a]\rho_{c-1} + \\
&\quad + p[\tilde{\mu} + s^* (1-\varepsilon)\rho_a]\rho_{c-2} \\
\dot{\rho}_{c-1} &= -[\tilde{\mu} - s^* (1-\varepsilon)\rho_a]\rho_{c-1} + (1-p)[\tilde{\mu} + s^* (1-\varepsilon)\rho_a]\rho_{c-2} \\
\dot{\rho}_{c-2} &= -[\tilde{\mu} - s^* (1-\varepsilon)\rho_a]\rho_{c-2} + [\tilde{\mu} + s^* (1-\varepsilon)\rho_a]\rho_{c-3} \\
&\quad ... \\
\dot{\rho}_1 &= -[\tilde{\mu} - s^* (1-\varepsilon)\rho_a]\rho_1 + [\tilde{\mu} + s^* (1-\varepsilon)\rho_a]\rho_0 \\
\dot{\rho}_0 &= -[\tilde{\mu} - s^* (1-\varepsilon)\rho_a]\rho_0 + \rho_a
\end{aligned}$$

The average densities ρ_k, for $k = 0, 1, ..., c, a$, have to satisfy also the normalisation condition:

$$\rho_a + \rho_c + ...\rho_1 + \rho_0 = 1 \tag{16}$$

It can be shown that the stationary state is asymptotically stable[7]. Thus, we concentrate our analysis on the stationary state dynamics. In particular, we are interested in the average number of R&D sectors innovating, given that new information has been discovered by a single R&D sector, and how this average number changes over time. We call z the number of R&D sectors introducing a new innovation, given that a single informational bit has been created by a single R&D sector. We are able to state the following proposition:

Proposition 2 *The average number of R&D sectors innovating in the limit of vanishing exogenous driving force μ is given by*

$$\lim_{\mu \to 0} \frac{\partial}{\partial \tilde{\mu}} \rho_a = E(z) = \frac{1}{\varepsilon s^*} \tag{17}$$

while the second moment of the number of R&D sectors innovating is given by

$$\lim_{\mu \to 0} \frac{\partial^2}{\partial \tilde{\mu}^2} \rho_a = E(z^2) = 2\left(\frac{1}{\varepsilon s^*}\right)^2 \tag{18}$$

[6] Vespignani and Zapperi [18] show through numerical simulations that the stationary state properties of the mean-field approximation are the same as the one of the deterministic interaction model.

[7] See Vespignani and Zapperi (1998).

Proof. First we have to calculate the stationary average density of R&D sectors being in the active innovating state. Solving the model for the stationary state we obtain the following distribution of average densities:

$$\rho_0 = \rho_1 = ... = \rho_{c-2} = \rho_c = \frac{\rho_a}{(1+\lambda)\eta\mu + s^*(1-\varepsilon)\rho_a}, \quad \text{while} \quad \rho_{c-1} = (1-p)\,\rho_c$$

Using the normalisation condition of average densities (16) we obtain the following.

$$s_L^* \frac{\rho_a}{\tilde{\mu} + s^*(1-\varepsilon)\,\rho_a} + \rho_a = 1$$

For a vanishing probability, i.e. $\mu \to 0$ we obtain that the distribution of the average density of firms being in each state is the following

$$\rho_a = \frac{\tilde{\mu}}{\varepsilon s^*}$$

$$\rho_0 = \rho_1 = ... = \rho_{c-1} = \rho_c = \frac{1}{s^*}$$

After this we can use the results stated in Vespignani and Zapperi [18] and Andergassen [3]. ■

A vanishing probability of exogenous information corresponds to a slow driving [8]. In other words, we are looking for a situation where the system evolves through endogenous forces such as the endogenous propagation of information. In this latter case technological paradigms are emergent features.

Using the fact that $\varepsilon = \frac{m}{n}$ and Proposition 2 we see that $\frac{1}{n}E(z) = \frac{1}{ms^*}$. Thus, if $m \propto n$, and where $n \to \infty$ we see that the average fraction of sectors innovating is vanishing small. In this latter case the spill-over dynamics are too small, and thus the propagation of information will come to a stop soon because of the dissipation of information due to firms not investing in local search for information. In particular, a law of large numbers applies and the fluctuations average out in the process of aggregation. On the other side, if $m << n$, such that if $n \to \infty$, m remains finite, we will have that the average fraction of sectors innovating will remain non-negligible. In this latter case the propagation will be strong enough and further the probability of observing an innovation wave which is of the same size of the system is no longer equal to zero. Further, the system is driven by large endogenous fluctuations, which do not cancel out in the process of aggregation.

(17) indicates the average number of innovations introduced in the economy, given that a single bit of information has been created by a single R&D sector. In the same way, the second moment of the average number of sectors introducing innovations is given by (18). Thus, the variance of the number of sectors introducing an innovation is given by $Var(z) = \left(\frac{1}{s^*\varepsilon}\right)^2$ and the volatility of the growth rate will be $Sd(z) = \frac{1}{s^*\varepsilon} = \frac{n}{ms^*} = E(z)$, where $Sd(z)$ is the standard deviation of z. Thus the larger are the fluctuations in the introduction of innovations, the larger will be the long run growth rate of these latter. Growth occurs through large fluctuations.

[8] For a similar assumption see Aghion and Howitt (1998), p. 253.

For the following we are going to assume that n is very large, while m remains small compared to n, i.e. $n \to \infty$ and $\varepsilon = \frac{m}{n} \to 0$.

2.2 Long and short run dynamics of economic growth

We are going to assume that the representative consumer maximises the following utility function

$$U\left(c^1, ..., c^n\right) = \int_0^\infty e^{-\rho t} \sum_{i=1}^{n} \frac{\left(c^i\right)^{1-\sigma} - 1}{1-\sigma} dt$$

From the first order conditions we obtain that

$$\frac{1}{c^i} \frac{d}{dt} c^i = \frac{r-\rho}{\sigma} - \frac{1}{\sigma} \frac{\dot{p}^i}{p^i}$$

Since the innovation growth rate of those industries, whose R&D firms are engaged in localised search for information is larger than the one of those industries, whose R&D firms are not engaged in localised search, the relative weight of the production of the former industries will be increasing while the relative weight of the latter will be decreasing. In the asymptotically limit we will have that the relative weight of these latter industries will be vanishing and the real GDP growth rate will be only given by the average growth rate of those industries whose R&D sector is engaged in localised search for information.

From this latter reasoning we have that the long run growth rate of aggregate consumption is given by

$$\gamma_C = \frac{r - \rho + \gamma \delta_L}{\sigma}$$

Given the assumptions about the production of final goods, we have that the growth rate of aggregate output is the same as the aggregate technology growth rate.

The technology growth rate is given by the average number of industries succeeding in introducing a new innovation, times the frequency with which idiosyncratic information is born in those sectors engaged in localised search and times the increment in productivity. Formally, we have that

$$E\left(\gamma_Y\right) = Sd\left(\gamma_Y\right) = \gamma \delta_L \tag{19}$$

where this latter property follows straightforwardly from the properties of the innovation avalanche dynamics. Thus, the larger are the short run fluctuations, the larger will be also the long run growth rate. In Section 3 we are going to test this latter relationship.

Since we are interested in stationary balanced growth paths where $E\left[\gamma_Y\right] = E\left[\gamma_C\right] = g^e$, we have that

$$r = \rho + (1-\sigma)\gamma \delta_L$$

In order to simplify the exposition, we are going to assume that $\sigma \to 1$. From this latter expression we obtain that $r = \rho$. Thus, using (19), (17), (18), and (14) we have that the long run growth rate of aggregate output and its short run fluctuations are given by

$$E(\gamma_Y) = Sd(\gamma_Y) = \bar{\gamma}\left(\tilde{k}\right)\alpha\vartheta H\frac{\mu}{n}\frac{1+m}{(1-\alpha)m} - \tilde{\gamma}\left(\tilde{k}\right)\rho \tag{20}$$

where $\tilde{k} = \frac{k}{\Delta tC}$, $\tilde{\gamma}\left(\tilde{k}\right) = \gamma\left(\Gamma\left(\tilde{k}\right)\right)$, $\bar{\gamma}\left(\tilde{k}\right) = \frac{\gamma(\Gamma(\tilde{k}))}{\Gamma(\tilde{k})}$, and $\tilde{\gamma}'(\cdot) > 0$, while $\bar{\gamma}'(\cdot) \leq 0$. Thus, the lower is the number of R&D sectors not engaged (m) in localised search for information, the higher is the long run growth rate and its short run fluctuations. Note also that for each $m < n$ the growth rate (20) is always larger than the one we observe in the case where no firm engages in local search for information.

The growth path of the economy (20) depends on structural parameters characterising the economy. For example, the larger are the sunk costs of reorganisation, the lower will be the long run growth rate and its short run fluctuations.

We can also make some few considerations about the scale effects. H in (20) is the number of specialised workers of each single industry. We observe a scale effect within each industry, like the one highlighted by recent empirical studies (see Backus et al., 1992). On the other side, defining H_T the total amount of specialised worker in the economy, and given the assumption of symmetry used in the paper, we have that $H = \frac{1}{n}H_T$. Thus, there will be positive aggregate scale effects, but these latter are asymptotically vanishing as n, the number of industries, diverges towards infinity.

3 Testing the model

In this Section we are going to test the prediction of the theoretical model that the long run growth rate of the economy is equal to the size of its short run fluctuations. We are going to test this relationship using data on growth rates of the real GDP per capita of 21 OECD countries[9], over the time period 1960 - 1990 (see Figure 1). The countries are Canada, Usa, Japan, Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, UK and Australia.

We calculate the average growth rate for each country over the time period and its standard deviation. After this, we run the regression

$$E\left[\gamma_Y^i\right] = \beta_0 + \beta_1 Sd\left[\gamma_Y^i\right] + \epsilon^i \tag{21}$$

where ε^i indicates the error term of the regression, i.e. the random deviation from the relationship, where we assume that ϵ^i is i.i.d. normally distributed across the countries. We are going to estimate parameters β_0 and β_1 using

[9] Data are taken from the Penn World Tables.

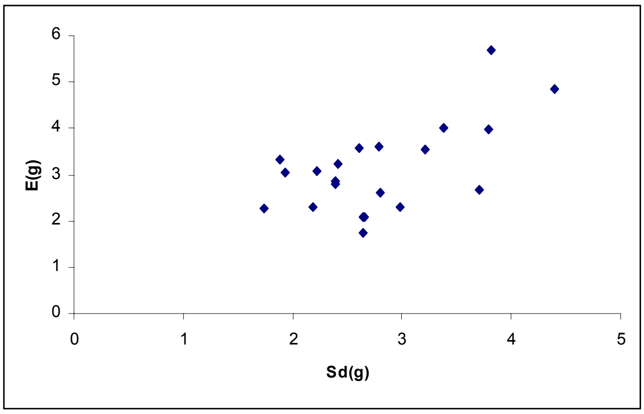

Figure 1: Long run growth rate vs. short run fluctuations for 21 Oecd countries

OLS, and after this we are going to test the hypothesis that $\beta_1 = 1$ and $\beta_0 = 0$, which are the theoretical predictions of our model. Our estimates are (standard deviation in parenthesis)

$$\underset{(0.7121929)}{\beta_0 = 0.8072508} \quad and \quad \underset{(0.2478091)}{\beta_1 = 0.832128}$$

Thus, we find that we cannot reject the hypothesis that β_1 is different from zero, and further we cannot reject the hypothesis that β_1 is equal to one. Further, we find that β_0 is not statistically different from zero. We performed also a Ramsey-type test of omitted variables and a heteroschedasticity test, and we reject both hypotheses of omitted variables and of heteroschedasticity. The R^2 of the regression is equal to 0.3724.

We performed also some specification tests. In particular, following the idea of Levine and Renelt (1992) we introduce additional explanatory variables. We introduce so the average population growth rate, the initial human capital level and the average investment fraction of real GDP

$$E\left[\gamma_Y^i\right] = \beta_0 + \beta_1 Sd\left[\gamma_Y^i\right] + \beta_2 h60^i + \beta_3 n^i + \beta_4 I^i + \epsilon^i \tag{22}$$

where $h60^i$ indicates the average years of schooling for individuals taken from the total population over age 25 years in the year 1960[10] for country i, n^i indicates the average population growth rate over the time period for country i and I^i is the average investment fraction of real GDP for country i. We obtain the following estimates

$$\underset{(0.9972457)}{\beta_0 = 0.308047} \quad \underset{(0.1956116)}{\beta_1 = 0.7514994} \quad \underset{(0.0000601)}{\beta_2 = -0.0001547}$$

[10] Data are taken from Barro and Lee (1993).

$\beta_3 = -0.7328333$ $\beta_4 = 0.0855499$
(0.2375725) (0.0297373)

Notice first that all the estimates but the constant β_0 are significantly different from zero. Further, the signs of the estimated values are consistent with theoretical predictions. The results confirm our previous analysis: β_0 is not significantly different from zero, while we cannot reject the hypothesis $\beta_1 = 1$. As a consequence, our previous estimates are robust against the introduction of additional explanatory variables. The adjusted R^2 of the regression is 0.6970.

If we introduce the initial real GDP per capita in regression (22) we observe that the estimates of β_1 become somehow worse. While the estimated value $\beta_1 = 0.3970314$ (0.1786774) is still significantly different from zero, it is also significantly different from 1. This latter effect is mainly due to the large negative correlation between the long run growth rate and the initial level of real GDP per capita. This latter leads us to the conclusion that the exogenous arrival rate of information depends negatively on the technology level. This latter could be, for example due to international technology spill-overs and imitation. Thus, countries which have a lower technology will benefit from countries with a higher level through imitation. In this way, the arrival rate in countries with lower technology will have a larger innovation rate due to international informational spill-overs. The aspect of international technology spill-overs has not been addressed in this paper and will be object of future research.

4 Conclusions

We proposed a simple model where large, highly volatile, aggregate innovation waves emerge endogenously from the propagation of information around the single industries. The single R&D sector face the problem of engaging in localised search for information or not. Those R&D sectors, for whom the spill-over effects are not completely free lunch face the problem of a reduced productivity in the of local search for information. We show that under certain conditions, these latter will not invest in local search for information. The more are those firms not engaging in localised search, the less will be the endogenous propagation of information, and so the less will be the aggregate innovation rate. Growth in our model occurs through large highly volatile innovation waves: the larger are these waves, the higher will be the long run growth rate of the economy. Thus, growth occurs in the model through large short run fluctuations.

We showed how the aggregate GDP growth path depends on structural parameters, such as, for example, the sunk costs the final good sectors face in introducing new innovations. We showed that the larger are these costs, the lower will be the aggregate growth rate.

References

[1] Aghion P. and Howitt P. (1998) *Endogenous Growth Theory*, Massachusetts Institute of Technology.

[2] Aghion P. and Howitt P. (1992) A model of Growth through Creative Destruction, *Econometrica* 60: 323 - 51.

[3] Andergassen R. (2001): Investment, growth and economic fluctuations, University of Siena, working paper n. 318.

[4] Arrow K. (1974) *The limits of organization*, W W Norton & Company, New York, London.

[5] Backus D., Kehoe P., and Kehoe T. (1992) In search of scale effects in trade and growth, Journal of Economic Theory, vol. 57, p. 377 - 40.

[6] Barro, R. J. and Lee, J.-W. (1993) International Comparisons of Educational Attainment, Journal of Monetary Economics, December, 32 (3), p. 363-394.

[7] David, P. (1990) The Dynamo and the Computer: An Historical Perspective on the Modern Productivity Paradox, *American Economic Review* 80: 355 - 361.

[8] Elmer A. M. and Pedersen T. M. (1998) International Evidence on the Connection between Business Cycles and Economic Growth, Discussion Paper n. 23, Institute of Economics, University Copenhagen.

[9] Fai F. and von Tunzelmann N. (2001) Industry-specific competencies and converging technological systems: evidence from patents, *Structural Change and Economic Dynamics*, 12, p. 141-170.

[10] Helpman, E. (1998) *General Purpose Technologies and Economic Growth*, MIT press, Cambridge MA.

[11] Helpman, E. and Trajtenberg, M. (1994) A Time to Sow and a Time to Reap: Growth Based on General Purpose Technologies, Centre for Economic Research Policy, working Paper no. 1080.

[12] King R. and Rebelo S.T. (1999) Resuscitating Real Business Cycles, in *Handbook of Macroeconomics*, eds. Woodford M. and Taylor J., Elsevier.

[13] Kormendi R. C. and Meguire P. G. (1985) Macroeconomic Determinants of Growth, *Journal of Monetary Economics*, 16, p. 141-163.

[14] Peretto P. and S. Smulders (2002) Technological distance, growth, and scale effects, forthcoming in *Economic Journal*, July 2002.9.

[15] Phelan C. and Trejs A. (2000), The aggregate effects of sectoral reallocations, *Journal of Monetary Economics* 45, p. 249-268.

[16] Ramey G. and Ramey V. A. (1995) Cross-Country Evidence on the Link Between Volatility and Growth, *American Economic Review*, vol. 85, no. 5, p. 1138-1151.

[17] Rosenberg, N. (1976) *Perspectives on Technology*, Cambridge University Press.

[18] Vespignani A., Zapperi S. (1998) How Self-organized criticality works: A unified mean-field picture, *Physical Review E*, vol. 57 n.6, p. 6345.