Stackelberg Leadership in a Dynamic Duopoly with Capital Accumulation[1]

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Abstract

I propose a dynamic duopoly model where firms enter simultaneously but compete hierarchically *à la* Stackelberg at each instant over time. They accumulate capacity through costly investment, as in Solow's (1956) growth model. The main findings are the following. The leader invests more than the follower; as a result, in steady state the leader's capacity and profits are larger than the follower's. Therefore, the present analysis does not confim Gibrat's Law, since the individual growth rate is determined by the timing of moves.

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1 Introduction

For several decades, the established wisdom has maintained that expected firm growth rates are independent of firm size, a property known as Gibrat's Law (Gibrat, 1931).[1] However, recent empirical work has found a negative relation between firm growth and firm size, which can be explained on the basis of sunk costs (Cabral, 1995).[2] Likewise, on the theoretical side, the existing contributions provide heterogeneous answers to the question: how shall we expect market dynamics to unravel over time, if some initial asymmetry among firms is assumed?

Lucas and Prescott (1971) and Lucas (1978) investigate entry and exit decisions in a long-run competitive equilibrium in models where stochastic processes drive prices, outputs and investments. Jovanovic (1982), proposes a theory of noisy selection where firms enter over time and learn about their productive efficiency as they operate in the market. Those who are relatively more efficient grow and survive, while those who relatively less efficient decline and ultimately exit the industry. Hopenhayn (1992) analyses instead the case of individual productivity shocks and their effects on entry, exit and market dynamics in the long-run. He finds that the steady state equilibrium implies a size distribution of firms by age cohorts, and proves that the size distribution is stochastically increasing in the age of cohorts. Jovanovic's model is extended by Pakes and Ericson (1998) who consider two models of firm behavior, allowing for heterogeneity among firms, idiosyncratic (or firm-specific) sources of uncertainty, and discrete outcomes (exit and/or entry).

The overall appraisal of this literature leads one to think that 'older firms are bigger than younger firms'.[3] A related question is the following: is early entry a prerequisite (i.e., a necessary condition) for a firm to become larger than rivals, or is it only a sufficient condition?

In this paper, I propose a dynamic duopoly model where firms enter simultaneously but compete hierarchically *à la* Stackelberg at each instant over time, under perfect certainty. They accumulate capacity through costly in-

[1] For early empirical studies confirming Gibrat's law, see Hart and Prais (1956), Simon and Bonini (1958) and Hymer and Pashigian (1962).

[2] For an exhaustive overview of empirical findings, see Audretsch, Santarelli and Vivarelli (1999). For a comprehensive view of the links between theory and empirics, see Sutton (1991, 1997, 1998).

[3] Moreover, both size and age appear to be positively correlated with firm survival (Geroski, 1995, p.434). To this regard, see also Agarwal and Audretsch (2001).

vestment, as in Solow's (1956) growth model. Due to the formal properties of the model, the Stackelberg game is shown to produce a unique and time consistent open-loop Stackelberg equilibrium (see Xie, 1997). The main findings are the following:

- The leader invests more than the follower along the equilibrium path. As a result, in steady state the leader's capacity and profits are larger than the follower's.

- In comparison to the features of the Nash equilibrium path, the present analysis shows that the leader (resp., follower) (i) invests more (less) than in the Nash equilibrium; (ii) acquires a higher (lower) steady state capacity than in the Nash equilibrium; and (iii) obtains higher (lower) profits than in the Nash equilibrium. Hence, the duopoly model with capital accumulation *à la* Solow has a definite Cournot flavour.

- The above considerations holds independently of initial conditions, which can well be assumed to be symmetric across firms. Accordingly, in general, the present analysis does not confim Gibrat's Law, since the leader's strategic advantage entails a higher growth rate than that performed by the follower.

- Moreover, the Stackelberg model described in this paper shows that an industry equilibrium that is characterised by an uneven size distribution of firms may not necessarily be the outcome of the entry process, but rather the consequence of a strategic advantage of some firms over the others.

The remainder of the paper is structured as follows. Section two presents the general features of open-loop Stackelberg differential games. The specific duopoly model is then introduced in section 3. The open-loop Stackelberg equilibrium is derived in section 4. A comparative assessment between open-loop Stackelberg and Nash equilibria is carried out in section 5. Concluding remarks are in section 6.

2 Preliminaries: open-loop Stackelberg games

The game is played over continuous time, $t \in [0, \infty)$.[4] Define the set of players as $\mathbb{P} \equiv \{1, 2\}$. Moreover, let $x_i(t)$ and $u_i(t)$ define, respectively, the state variable and the control variable pertaining to player i. For simplicity, we consider the case where only one state and one control are associate to every single player. The dynamics of player i's state variable is described by the following:

$$\frac{dx_i(t)}{dt} \equiv \dot{x}_i(t) = f_i\left(\{x_i(t)\}_{i=1}^2, \{u_i(t)\}_{i=1}^2\right) \tag{1}$$

where $\{x_i(t)\}_{i=1}^2$ is the vector of state variables at time t, and $\{u_i(t)\}_{i=1}^2$ is the vector of players' actions at the same date, i.e., it is the vector of the values of control variables at time t. That is, in the most general case, the dynamics of the state variable associated to player i depends on all state and control variables associated to all players involved in the game. The value of the state variables at $t = 0$ is assumed to be known: $\{x_i(0)\}_{i=1}^2 = \{x_{0,i}\}_{i=1}^2$.

Each player has an objective function, defined as the discounted value of the flow of payoffs over time. The instantaneous payoff depends upon the choices made by player i as well as its rivals, that is:

$$\pi_i(t) = \pi_i\left(\{x_i(t)\}_{i=1}^2, \{u_i(t)\}_{i=1}^2\right). \tag{2}$$

Player i's objective is then

$$\max_{u_i(t)} J_i \equiv \int_0^\infty \pi_i(., t)e^{-\rho t}dt \tag{3}$$

subject to the dynamic constraint represented by the behaviour of the state variables (1) for $i = 1, ...N$. The factor $e^{-\rho t}$ discounts future gains, and the discount rate ρ is assumed to be constant and common to all players. In order to solve his optimisation problem, each player defines a strategy $u_i(t)$ at each t, for any admissible $u_j(t)$. If, in choosing $u_i(t)$, player i also takes into account the stock of state variables $\{x_i(t)\}_{i=1}^2$ (or their evolution up to time t), the game is solved in closed-loop strategies. Otherwise, it is solved by open-loop strategies.

[4] The game can be reformulated in discrete time without significantly affecting its qualitative properties. For further details, see Başar and Olsder (1982, 1995^2).

Now consider the Stackelberg differential game, and assume player i is the follower. Under the open-loop solution concept, the Hamiltonian of firm i writes as follows:

$$\begin{aligned}\mathcal{H}_i &\equiv e^{-\rho t}\Big[\pi_i\left(\{x_i(t)\}_{i=1}^2,\{u_i(t)\}_{i=1}^2\right)+\lambda_{ii}(t)\cdot f_i\left(\{x_i(t)\}_{i=1}^N,\{u_i(t)\}_{i=1}^N\right)+\\ &\quad+\lambda_{ij}(t)\cdot f_j\left(\{x_i(t)\}_{i=1}^N,\{u_i(t)\}_{i=1}^N\right)\Big]\ ,\end{aligned} \tag{4}$$

where $\lambda_{ij}(t)=\mu_{ij}(t)e^{\rho t}$ is the costate variable (evaluated at time t) associated with the state variable x_j. If the evolution of the state variable $x_i(t)$ depends only upon $\{x_i(t),u_i(t)\}$, i.e., it is independent of $u_j(t)$ and $x_j(t)$ and (1) simplifies as $\dot{x}_i(t)=f_i\left(x_i(t),u_i(t)\right)$, then one can set $\lambda_{ij}(t)=0$ for all $j\neq i$, which entails that the Hamiltonian of player i can be written by taking into account the dynamics of i's state variable only.

The first order condition on the control variable $u_i(t)$ is:

$$\frac{\partial\mathcal{H}_i\left(.,t\right)}{\partial u_i(t)}=0 \tag{5}$$

and the adjoint equations concerning the dynamics of state and costate variables are as follows:

$$-\frac{\partial\mathcal{H}_i\left(.,t\right)}{\partial x_j(t)}=\frac{\partial\lambda_{ij}(t)}{\partial t}-\rho\lambda_{ij}\ ,\forall\, j=1,2. \tag{6}$$

They have to be considered along with the initial conditions $\{x_i(0)\}_{i=1}^N=\{x_{i0}\}_{i=1}^N$and the transversality conditions, which set the final value (at $t=\infty$) of the state and/or co-state variables. In problems defined over an infinite time horizon, one sets:

$$\lim_{t\to\infty}\ \lambda_{ij}(t)\cdot x_j(t)=0,\ j=1,2. \tag{7}$$

From (5) one obtains the instantaneous best reply of player i, which can be differentiated with respect to time to yield the kinematic equation of the control variable $u_i(t)$. Moreover, given (1), the first order condition (5) will contain the co-state variable $\lambda_{ii}(t)$ associated with the kinematic equation of the state variable $x_i(t)$. Therefore, (5) can be solved w.r.t. $\lambda_{ii}(t)$ so as to yield the optimal value of the co-state variable of the follower. If such expression contains the leader's control variable $u_j(t)$, the open-loop

Stackelberg strategies are bound to be time inconsistent, in that the leader can control the follower's state dynamics by manoeuvring $u_j(t)$.[5] If, instead, $\lambda_{ii}(t)$ does not depend upon $u_j(t)$, then the game is *uncontrollable* by the leader, and the resulting open-loop Stackelberg equilibrium strategies are time consistent (Xie, 1997).[6]

The remainder of the analysis illustrates a hierarchical capital accumulation model where this property holds, and uses such a setting to investigate the features of the dynamic performance of firms over time.

3 The setup

Two firms, labelled as 1 and 2, operate over $t \in [0, \infty)$ in a market for a homogeneous good, whose demand function at any t is:

$$p(t) = a - q_1(t) - q_2(t) \,. \tag{8}$$

In order to supply the final good, firms must build up capacity (i.e., physical capital) $k_i(t)$ through intetemporal investment:

$$\frac{\partial k_i(t)}{\partial t} \equiv \dot{k}_i = I_i(t) - \delta k_i(t) \,, \tag{9}$$

where $\delta \in [0, 1]$ is the depreciation rate, constant and equal across firms. The model where has been investigated extensively in the previous literature (Spence, 1979; Fudenberg and Tirole, 1983; Fershtman and Muller, 1984; Reynolds, 1987; Cellini and Lambertini, 2001; Calzolari and Lambertini, 2002) and can be ultimately traced back to Solow (1956).

For the sake of simplicity, in the remainder I assume that $q_i(t) = k_i(t)$, i.e., both firms operate at full capacity at any instant, *à la* Kreps and Scheinkman (1983). At any t, firm i bears the following total costs:

$$C_i(t) = c q_i(t) + \frac{[I_i(t)]^2}{2} \,. \tag{10}$$

[5] In such a case, the game is *controllable* by the leader, who cannot resist the temptation to renege any initial plans later on during the game. This is precisely what happens in models dealing with the time inconsistency of the optimal economic (monetary or fiscal) policy in macroeconomic settings (see Kydland and Prescott, 1977; Calvo, 1978, Turnovsky and Brock, 1980; Lucas and Stokey, 1983; Persson, Persson and Svensson, 1987).

[6] For a detailed discussion of the issue of time consistency and subgame perfection in differential games, see Başar and Olsder (1982, 1995^2), Mehlmann (1988), Dockner *et al.* (2000) and Cellini and Lambertini (2001).

The instantaneous profit of firm i is:

$$\pi_i(t) = [p(t) - c]\, q_i(t) - \frac{[I_i(t)]^2}{2}\,. \tag{11}$$

For each firm i, the instantaneous investment effort $I_i(t)$ is the control variable, while capacity $k_i(t)$ is the state variable. The value of the state variables at $t = 0$ is $k_i(0) = k_{i0}$. The solution concept is the open-loop Stackelberg equilibrium. The aim of firm i consists in:

$$\max_{I_i(t)} J_i \equiv \int_0^\infty \pi_i(t) e^{-\rho t} dt \tag{12}$$

subject to the relevant dynamic constraint(s). The factor $e^{-\rho t}$ discounts future gains, and the discount rate $\rho > 0$ is assumed to be constant and common to all players.

4 Equilibrium analysis

The Stackelberg game is assumed to be solved by firms in open-loop strategies. Consider fist the optimum problem for the follower, firm 2. Her Hamiltonian function is:

$$\begin{aligned} \mathcal{H}_2(t) &= e^{-\rho t} \left\{ [a - k_1(t) - k_2(t) - c]\, k_2(t) - \frac{[I_2(t)]^2}{2} + \lambda_{22}(t)\, [I_2(t) - \delta k_2(t)] \right. \\ &\quad \left. + \lambda_{21}(t)\, [I_1(t) - \delta k_1(t)] \right\}, \end{aligned} \tag{13}$$

where $\lambda_{2j}(t) = \mu_{2j}(t) e^{\rho t}$ is the costate variable (evaluated at time t) associated with state variable $k_j(t)$, $j = 1, 2$. The first order conditions are (exponential discounting is omitted for brevity):

$$\frac{\partial \mathcal{H}_2(t)}{\partial I_2(t)} = -I_2(t) + \lambda_2(t) = 0\,; \tag{14}$$

$$-\frac{\partial \mathcal{H}_2(t)}{\partial k_2(t)} = \frac{\partial \lambda_{22}(t)}{\partial t} - \rho \lambda_{22}(t) \Rightarrow \tag{15}$$

$$\frac{\partial \lambda_{22}(t)}{\partial t} = \lambda_{22}(t)\, (\rho + \delta) - a + c + k_1(t) + 2k_2(t)\,; \tag{16}$$

$$-\frac{\partial \mathcal{H}_2(t)}{\partial k_1(t)} = \frac{\partial \lambda_{21}(t)}{\partial t} - \rho \lambda_{21}(t) \Rightarrow \tag{17}$$

$$\frac{\partial \lambda_{21}(t)}{\partial t} = \lambda_{21}(t)\,(\rho + \delta) + k_2(t)\,, \tag{18}$$

together with the initial conditions $k_i(0) = k_{i0}$ and the transversality condition:

$$\lim_{t \to \infty} \lambda_2(t) \cdot k_2(t) = 0\,. \tag{19}$$

From (14), one obtains:

$$\lambda_{22}(t) = I_2(t)\,;\ \frac{\partial \lambda_{22}(t)}{\partial t} = \frac{\partial I_2(t)}{\partial t}\,. \tag{20}$$

Since the follower's co-state variable is independent of the leader's control path, (20) proves the following result (see Xie, 1997):

Lemma 1 *The follower's investment effort $I_2(t)$ is non-controllable by the leader. Therefore, the open-loop Stackelberg equilibrium is time consistent.*

Before approaching the leader's problem, it is worth observing, again from (20), that the evolution of firm 2's investment does not depend on $\lambda_{21}(t)$ either. This redundancy of the dynamics of the leader's state variable as to the follower's decisions is going to become even clearer in the remainder.

Now I can characterise the leader's problem. Firm 1's Hamiltonian function is:

$$\mathcal{H}_1(t) = e^{-\rho t}\left\{ [a - k_1(t) - k_2(t) - c]\,k_1(t) - \frac{[I_1(t)]^2}{2} + \lambda_{11}(t)\,[I_1(t) - \delta k_1(t)] + \right.$$

$$\left. + \lambda_{12}(t)\,[\lambda_{22}(t) - \delta k_2(t)] + \theta_1(t)\left[\frac{\partial \lambda_{21}(t)}{\partial t}\right] + \theta_2(t)\left[\frac{\partial \lambda_{22}(t)}{\partial t}\right]\right\} \tag{21}$$

where $\theta_j(t)$ is the additional co-state variable attached by the leader to the follower's co-state equations, and the expressions $\partial \lambda_{2j}(t)/\partial t$ are given by (16-18). The first order conditions are:

$$\frac{\partial \mathcal{H}_1(t)}{\partial I_1(t)} = -I_1(t) + \lambda_{11}(t) = 0\,; \tag{22}$$

$$-\frac{\partial \mathcal{H}_1(t)}{\partial k_1(t)} = \frac{\partial \lambda_{11}(t)}{\partial t} - \rho \lambda_{11}(t) \Rightarrow \tag{23}$$

$$\frac{\partial \lambda_{11}(t)}{\partial t} = \lambda_{11}(t)(\rho+\delta) - a + c + 2k_1(t) + k_2(t) - \theta_2(t)\,; \tag{24}$$

$$-\frac{\partial \mathcal{H}_1(t)}{\partial k_2(t)} = \frac{\partial \lambda_{12}(t)}{\partial t} - \rho\lambda_{12}(t) \Rightarrow \tag{25}$$

$$\frac{\partial \lambda_{12}(t)}{\partial t} = \lambda_{12}(t)(\rho+\delta) + k_1(t) - \theta_1(t) - 2\theta_2(t) \tag{26}$$

$$-\frac{\partial \mathcal{H}_1(t)}{\partial \lambda_{21}(t)} = \frac{\partial \theta_1(t)}{\partial t} - \rho\theta_1(t) \Rightarrow \frac{\partial \theta_1(t)}{\partial t} = -\delta\theta_1(t)\,; \tag{27}$$

$$-\frac{\partial \mathcal{H}_1(t)}{\partial \lambda_{22}(t)} = \frac{\partial \theta_2(t)}{\partial t} - \rho\theta_2(t) \Rightarrow \frac{\partial \theta_2(t)}{\partial t} = -\lambda_{12}(t) - \delta\theta_1(t)\,. \tag{28}$$

The above conditions are accompanied by the initial conditions $k_i(0) = k_{i0}$ as well as the transversality condition:

$$\lim_{t\to\infty} \lambda_1(t)\cdot k_1(t) = 0\,. \tag{29}$$

From (22) one immediately gets:

$$\lambda_{11}(t) = I_1(t)\,;\ \frac{\partial I_1(t)}{\partial t} = \frac{\partial \lambda_{11}(t)}{\partial t} \Rightarrow \tag{30}$$

$$\frac{\partial I_1(t)}{\partial t} = \lambda_{11}(t)(\rho+\delta) - a + c + 2k_1(t) + k_2(t) - \theta_2(t)\,. \tag{31}$$

Moreover, from (27), we observe that $\frac{\partial \theta_1(t)}{\partial t} = 0$ iff $\theta_1(t) = 0$. Proceeding likewise, note that from (28), we have:

$$\frac{\partial \theta_2(t)}{\partial t} = 0 \Leftrightarrow \theta_2(t) = -\frac{\lambda_{12}(t)}{\delta} \tag{32}$$

and, from (26):

$$\frac{\partial \lambda_{12}(t)}{\partial t} = 0 \Leftrightarrow \lambda_{12}(t) = -\frac{\delta k_1(t)}{2+\delta(\rho+\delta)}\,. \tag{33}$$

Plugging these expressions into (31), one obtains the following dynamic equation for the leader's investment (henceforth, I omit the indication of time for the sake of brevity):

$$\frac{\partial I_1}{\partial t} \propto (\rho+\delta)\left[2+\delta(\rho+\delta)\right] I_1 - k_1 - (a - c - 2k1 - k2)\left[2+\delta(\rho+\delta)\right] \tag{34}$$

which is nil at

$$I_1^* = \frac{1}{2}\left[\frac{2(a-c) - 3k_1 - 2k_2}{\rho + \delta} - \frac{\delta k_1}{2 + \delta(\rho + \delta)}\right] \tag{35}$$

The follower's optimal investment is:

$$I_2^* = \lambda_2^* = \frac{a - c - k_1 - 2k_2}{\rho + \delta} \tag{36}$$

Finally, from the kinematic equations of state variables (9), one can compute the steady state capacity levels of both firms:

$$k_1^L = \frac{(a-c)\left[1 + \delta(\rho + \delta)\right]}{2 + \delta(\rho + \delta)\left[4 + \delta(\rho + \delta)\right]}\ ; \tag{37}$$

$$k_2^F = \frac{(a-c)\left[1 + \delta(\rho + \delta)(3 + \delta(\rho + \delta))\right]}{\left[2 + \delta(\rho + \delta)\right]\left[2 + \delta(\rho + \delta)(4 + \delta(\rho + \delta))\right]}\ . \tag{38}$$

Expressions (37-38) can be used to write the equilibrium expressions for $\left\{I_1^L, I_2^F\right\}$.[7]

Now assess the difference between steady state capital endowments:

$$k_1^L - k_2^F = \frac{a - c}{\left[2 + \delta(\rho + \delta)\right]\left[2 + \delta^2\left(4 + \delta^2\right) + \delta\rho\left(4 + 2\delta^2 + \delta\rho\right)\right]} > 0 \tag{39}$$

and

$$\begin{aligned} I_1^L - I_2^F &= \frac{(a-c)\,\delta}{\left[2 + \delta(\rho + \delta)\right]\left[2 + \delta^2\left(4 + \delta^2\right) + \delta\rho\left(4 + 2\delta^2 + \delta\rho\right)\right]} \\ &= \delta\left(k_1^{SS} - k_2^{SS}\right) > 0 \text{ for all } \delta \in (0, 1]\ . \end{aligned} \tag{40}$$

Moreover, $\pi_1^L - \pi_2^F > 0$.[8] Hence:

Proposition 2 *Along the optimal Stackelberg open-loop path:*

(i) for all $\delta \in (0, 1]$, the leader invests more than the follower. As a result, the leader's steady state capacity and profits are larger than the follower's.

(ii) if $\delta = 0$, leader and follower produce the same investment effort. However, the leader's capacity and profits are larger than the follower's.

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[7] It can also be easily shown that the equilibrium is stable, in that the pairs $\left\{I_1^L, I_2^F\right\}$ and $\left\{k_1^L, k_2^F\right\}$ identify a saddle point. The stability analysis is omitted for the sake of brevity. For the detailos concerning the stability of the open-loop Nash equilibrium, see Cellini and Lambertini (2001).

[8] The detailed expression of profits are omitted for brevity.

5 A comparison with the Nash solution

Now briefly consider the performance of the two firms when they play simultaneously. As shown by Cellini and Lambertini (2001), in the simultaneous game the open-loop Nash equilibrium path coincides with the closed-loop one, and therefore the former is strongly time consistent and qualifies as a subgame perfect equilibrium.

The symmetric steady state capacity and investment levels associated with the Nash equilibrium are (for the computational details, see Cellini and Lambertini, 2001):

$$k^N = \frac{a-c}{3+\delta(\rho+\delta)} \; ; \; I^N = \frac{(a-c)\,\delta}{3+\delta(\rho+\delta)} = \delta k^N \tag{41}$$

while the Nash equilibrium profits are:

$$\pi^N = \frac{(a-c)^2\,[2+\delta(\rho+\delta)]}{2\,[3+\delta(\rho+\delta)]} \,. \tag{42}$$

Using (37-38) and (41), one obtains:

$$k_1^L - k^N = \frac{a-c}{[3+\delta(\rho+\delta)]\left[2+\delta^2\left(4+\delta^2\right)+\delta\rho\left(4+\delta\left(2\delta+\rho\right)\right)\right]} > 0 \tag{43}$$

$$k^N - k_2^F = \frac{k_1^L - k^N}{2+\delta(\rho+\delta)} > 0 \tag{44}$$

which also entails $I_1^L \geq I^N \geq I_2^F$ for all $\delta \in [0,1]$. Finally, $\pi_1^L > \pi^N > \pi_2^F$, with $2\pi^N > \pi_1^L + \pi_2^F$.

The above discussion can be summarised as follows:

Proposition 3 *In comparison to the features of the Nash equilibrium path, in the Stackelberg equilibrium the leader (resp., follower) (i) invests more (less) than in the Nash equilibrium; (ii) acquires a higher (lower) steady state capacity than in the Nash equilibrium; and (iii) obtains higher (lower) profits than in the Nash equilibrium.*

The foregoing discussion shows that the capital accumulation game *à la* Solow has some typical properties of a Cournot game with substitute goods (see Dowrick, 1986, *inter alia*). In particular, the Stackelberg leader (follower) is better off (worse off) than in the Nash equilibrium. Moreover, as in any static Cournot game with substitutes, it can be easily shown that social welfare is higher in the Stackelberg equilibrium than in the Nash equilibrium. This fact has some relevant consequences as to the evaluation of the growth process experienced by firms from a social (or policy) standpoint. Is it really relevant whether firms' growth rates coincide with each other, and eventually lead to a convergence of equilibrium sizes? That is, is the set of questions usually associated with Gibrat's law appropriate? If we assess the conclusions drawn from the model presented above, the answer is definitely negative, because an uneven distribution of growth rates generated by sequential play, and the associated uneven size distribution of firms in steady state, imply a higher social welfare than the one generated by simultaneous play. It is true that the follower is damaged and the Stackelberg equilibrium industry profits are lower as compared to the Nash equilibrium, yet a planner should evaluate the market mechanism and the capital accumulation process with a view to social welfare rather than the performance of firms only.

6 Concluding remarks

The open-loop Stackelberg differential game analysed in this paper predicts that earlier movers will perform better in the steady state equilibrium than later movers, although all firms enter the market at the same time. On the one hand, if we take the first mover advantage as a substitute for an earlier entry, this implies that the present model does not confirm Gibrat's law. On the other hand, from a welfare standpoint, the above analysis suggests that there may not be any scope for a policy in support of smaller firms,[9] as the steady state of the Stackelberg game has the same qualitative properties of a static game in output levels, where sequential play produces a higher equilibrium welfare than simultaneous play.

[9] To this regard, see Lotti, Santarelli and Vivarelli (2001).

References

[1] Agarwal, R. and D.B. Audretsch (2001), "Does Entry Size Matter? The Impact of the Life Cycle and Technology on Firm Survival", *Journal of Industrial Economics*, **49**, 21-43.

[2] Audretsch, D.B., E. Santarelli and M. Vivarelli (1999), "Does Start-Up Size Influence the Likelihood of Survival?", in D. Audretsch and R. Thurik (eds.), *Innovation, Industry Evolution and Employment*, Cambridge, Cambridge University Press, 280-96.

[3] Başar, T. and G.J. Olsder (1982, 1995^2), *Dynamic Noncooperative Game Theory*, San Diego, Academic Press.

[4] Cabral, L. (1995), "Sunk Costs, Firm Size and Firm Growth", *Journal of Industrial Economics*, **43**, 161-72.

[5] Calvo, G. (1978), "On the Time Consistency of Optimal Policy in a Monetary Economy", *Econometrica*, **46**, 1411-28.

[6] Calzolari, G. and L. Lambertini (2002), "Tariffs vs Quotas in a Model of Trade with Capital Accumulation", in R. Neck (ed.), *Proceedings of the IFAC Symposium on Modelling and Control of Economic Systems (SME 2001)*, Pergamon Press, forthcoming.

[7] Cellini, R. and L. Lambertini (2001), "Differential Oligopoly Games where the Closed-Loop Memoryless and Open-Loop Equilibria Coincide", working paper no. 402, Dipartimento di Scienze Economiche, Università degli Studi di Bologna.

[8] Dockner, E.J, S. Jørgensen, N. Van Long and G. Sorger (2000), *Differential Games in Economics and Management Science*, Cambridge, Cambridge University Press.

[9] Dowrick, S. (1986), "von Stackelberg and Cournot Duopoly: Choosing Roles", *RAND Journal of Economics*, **17**, 251-60.

[10] Fershtman, C. and E. Muller (1984), "Capital Accumulation Games of Infinite Duration", *Journal of Economic Theory*, **33**, 322-39.

[11] Fudenberg, D. and J. Tirole (1983), "Capital as a Commitment: Strategic Investment to Deter Mobility", *Journal of Economic Theory*, **31**, 227-50.

[12] Geroski, P.A. (1995), "What Do We Know About Entry?", *International Journal of Industrial Organization*, **13**, 450-6.

[13] Gibrat, R. (1931), *Les Inegalites Economiques*, Paris, Librairie du Recueil Sirey.

[14] Hart, P.E. and S.J. Prais (1956), "The Analysis of Business Concentration: A Statistical Approach", *Journal of the Royal Statistical Society*, **119**(A), 150-91.

[15] Hopenhayn, H.A. (1992), "Entry, Exit, and Firm Dynamics in Long Run Equilibrium", *Econometrica*, **60**, 1127-50.

[16] Hymer, S. and P. Pashigian (1962), "Firm Size and the Rate of Growth", *Journal of Political Economy*, **70**, 556-69.

[17] Jovanovic, B. (1982), "Selection and the Evolution of Industry", *Econometrica*, **50**, 649-70.

[18] Kreps, D. and J. Scheinkman (1983), "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes", *Bell Journal of Economics*, **14**, 326-37.

[19] Kydland, F. and E. Prescott (1977), "Rules rather than Discretion: The Inconsistency of Optimal Plans", *Journal of Political Economy*, **85**, 473-92.

[20] Lotti, F., E. Santarelli and M. Vivarelli (2001), "Is It Really Wise to Design Policies in Support of New Firm Formation?", in Lambertini, L. (ed.), *Antitrust, Regulation and Competition: Theory and Practice*, special issue of *Rivista di Politica Economica*, **41**, 145-62.

[21] Lucas, R.E. Jr. (1978), "On the Size Distribution of Business Firms", *Bell Journal of Economics*, **9**, 508-23.

[22] Lucas, R.E. Jr. and E. Prescott (1971), "Investment under Uncertainty", *Econometrica*, **39**, 659-81.

[23] Lucas, R.E. Jr. and N.L. Stokey (1983), "Fiscal and Monetary Policy in an Economy without Capital", *Journal of Monetary Economics*, **12**, 55-93.

[24] Mehlmann, A. (1988), *Applied Differential Games*, New York, Plenum Press.

[25] Pakes, A, and R. Ericson (1998), "Empirical Implications of Alternative Models of Firm Dynamics", *Journal of Economic Theory*, **79**, 1-45.

[26] Persson, M., T. Persson and L.E.O. Svensson (1987), "Time Consistency of Fiscal and Monetary Policy", *Econometrica*, **55**, 1419-31.

[27] Reynolds, S.S. (1987), "Capacity Investment, Preemption and Commitment in an Infinite Horizon Model", *International Economic Review*, **28**, 69-88.

[28] Simon, H.A. and C.P. Bonini (1958), "The Size Distribution of Business Firms", *American Economic Review*, **48**, 607-17.

[29] Solow, R. (1956), "A Contribution to the Theory of Economic Growth", *Quarterly Journal of Economics*, **70**, 65-94.

[30] Spence, A. M. (1979), "Investment Strategy and Growth in a New Market", *Bell Journal of Economics*, **10**, 1-19.

[31] Sutton, J. (1991), *Sunk Costs and Market Structure: Price Competition, Advertising, and the Evolution of Concentration*, Cambridge, MA, MIT Press.

[32] Sutton, J. (1997), "Gibrat's Legacy", *Journal of Economic Literature*, **35**, 40-59.

[33] Sutton, J. (1998), *Technology and Market Structure: Theory and History*, Cambridge, MA, MIT Press.

[34] Turnovsky, S.J. and W.A. Brock (1980), Time Consistency and Optimal Government Policies in Perfect Foresight Equilibrium", *Journal of Public Economics*, **13**, 183-212.

[35] Xie, D. (1997), "On Time Consistency: A Technical Issue in Stackelberg Differential Games", *Journal of Economic Theory*, **76**, 412-30.