Cyclical Strikes and Human Capital Accumulation under Asymmetric Information

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Abstract

Strikes are totally inefficient from an economic point of view. They occur when the two parties that bargain over a contract do not find an agreement and the result is a loss of utility for both. In spite of their clear inefficiency in the real world strikes are very common both in the rich economies as well as in the poor countries. Moreover recent empirical literature found some regularities over time and over countries regarding the strike behavior of the Trade Unions. The aim of this paper is to develop a theory that could explain this apparent economic paradox as well as some of the most known regularities. At the same time we also aim to link the analysis of the strikes with the investment decisions of employers and workers in that particular kind of capital good known as Human Capital. This kind of approach can put under a new light the role played by the Trade Unions in the process of economic growth.

1 Introduction

Strikes are apparently not Pareto optimal because it means that the pie that has to be divided between employers and workers is reduced. Thus if the parties are rational it is difficult to provide an explanation regarding why they fail to negotiate a Pareto optimal outcome[1]. This paradox, defined by Kennan (1986) the "Hicks' paradox", represents probably the reason why

[1] For a more detailed discussion of bargaining processes and Pareto optimality see Harsany (1977) and Myerson (1984).

there is still no commonly accepted economic theory of strikes. Hicks (1963) proposed two lines of research to solve this dilemma: either the union is trying to maintain some form of "reputation for toughness", or there is private information on at least one side of the bargaining table.

This paper develops a model of repeated bargaining between employers and workers in which the employers have private information on the conditions of the market[2]. Basically we use the same methodology used by Green and Porter (1984) to model the occurrence of wars of prices in an oligopolist market. The idea is that when the …rms …re workers these are not able to distinguish if this decision is due to a negative shock on the Demand or to an attempt of employers of taking a bigger share of the "pie". Under these conditions in equilibrium the workers strike whenever they observe a reduction in the size of the "pie" that they receive and the employers act always fairly, so the strike occurs just when the economy is a¤ected by a negative shock. But, since the workers know that employers will redistribute less just as a consequence of a negative shock, why do they not disregard the strike and continue to work? The answer is that everyone understands the incentive properties of the equilibrium. If workers did not strike in response to the reduced share then …rms would have a strong incentive to redistribute a little amount to the workers even under good condition of the market.

It is important to underline that some comparative static exercises operated on the model replicate very well some qualitative features of the strikes found out by the empirical literature. Tracy (1984) found out countercyclical ‡uctuations of strikes in duration; in turns he claims that an economic recession leads to longer strikes and this feature is perfectly reproduced by our model: an increase in the parameter that represents the probability of occurrence of a negative shocks leads to longer periods of strike chosen by the workers[3]. Moreover, Kennan (1980) states that another determinant of the increase in strikes duration is an environment which reduces the losses induced by strikes; again, a reduction in the parameter that we choose to describe the cost of strikes leads to an increase in their duration in equilibrium.

[2] Fundemberg, Levine and Ruud (1983) and Hayes (1984) have shown that the theory of exchange with private information can provide a theoretically complete model of strike. The basic point of this literature is that although strikes are not Pareto optimal ex-post they may be Pareto optimal ex-ante, in the sense that every alternative leave either the union or the employer worse o¤ in some contingency which cannot be ruled out on the basis of the information which is common to both sides.

[3] For an analysis of the cyclicity of the strikes see also Ashfelter and Jhonson (1969).

In the second part of this paper we introduce another dimension of choice for the parties: investment in human capital. Recent research emphasized the fundamental role of human capital in economic growth (see, e.g. Lucas (1988) and Mankiw,Romer and Weil (1992) among the others), and the reason because of which people do invest in this particular kind of capital good is still an open question. We imagine that both parties can invest in human capital during the cooperation periods (i.e. the periods without strikes) and that the quantity of human capital accumulated by each party will a¤ect the new contract that they sign after the next strike. This mechanism is meant to model the fact that usually more skilled worker are able to obtain richer contracts. To model this particular dynamic optimization problem under uncertainty we use the technics developed by Calvo (1984) and applied in monetary economics models. We …nd that in equilibrium the quantity of human capital accumulated by each party is directly related with the effectiveness of the human capital (i.e. the way in which the level of human capital a¤ects the new terms of the contract) and with the expected value of the total revenue; in turns, our model states that richer countries will be characterized by higher level of human capital, a result that is in line with the empirical …ndings of the recent literature.

Finally, in the last section we propose a possible extension of the model. In particular we think that using the basic structure of this paper is possible to build up a growth model in which the engine of growth is the human capital accumulation and that generates growth cycles in correspondence of each strike period.

2 Cyclical Strikes and Demand Shocks

2.1 The model

The framework is that of a basic supergame. There are two parties Workers (from now on W) and Capitalists (from now on C) that divide between them the total revenues realized in each period. One can think to the Workers party as a Trade Union and to the Capitalists party as an organization of the owners of the …rms. In each period there are two possible states of nature which a¤ect the total revenue. With probability ® a negative shock on demand is realized and with probability 1 ¡ ® the demand is high; so that the total revenue in the good state is strictly larger than under the bad one:

$TR^H > TR^N$. For the moment let us imagine that there is a division rule which is considered as fair in the society: this rule is $¼^F = 1=2 ¤ TR$ for each party. Now, since in low demand state the total revenue is lower then the same fair sharing rule leads to $¼ = 1=2 ¤ TR^N < 1=2 ¤ TR^H = ¼^f$.

Moreover in each period each party can undertake two possible actions. The W can "work" or go to "strike". We assume that in a period of strike Capitalists can run the …rm at a "reduced" production level[4] while Workers can take part to di¤erent productive activities (like partial-time jobs,...); let's say that this "outside option" level for both parties is the same and equal to X (with $¼^f > X > 0$).The C can adopt the "fair" division rule or can take for them a bigger amount of the TR (let's say that if they take this second choice during an high demand period the workers are left with exactly a portion ¼). Let me assume that the realizations of demand are $i{:}i{:}d{:}$ over time and, for simplicity, that $¼ = X$.

The timing of the model is the following. First, the nature "chooses" the state: bad or good. Second, the Capitalists, once they observed the actually veri…ed state, propose to the Workers a remuneration: ¼ or $¼^f$. Finally the Workers move without knowing the state of nature: they can either accept the proposal and cooperate or refuse it and go to strike (in this case they get a payo¤ of X).

The point is that the W when receive $¼ < ¼^f$ at some date are not able to observe whether the reduction of their total wages is due to the realization to the shock or to an attempt of C of appropriating a larger share of total revenue.

We consider an in…nitely repeated version of this game and we look at a Subgame Perfect Nash Equilibrium with the following strategies:

-Workers' Strategy: they act alternating "working phases" to "strike phases". The W start acting fairly, that is they do not strike until they receive a payo¤ of ¼. The occurrence of ¼ triggers a "strike phase": they strike for exactly T periods (where T can be …nite or in…nite) getting a payo¤ of X and obliging also the C to receive such a payo¤ (remember that when they strike the production is reduced). At the end of this phase they restart to work as long as they receive $¼^f$;

-Capitalists' Strategy: they start applying the fair division rule and they redistribute ¼ to the W just when the negative shock is realized. But the

[4] This assumption is particularly e¤ective for …rms with an high capital-labor share: see Tracy (1984) for a discussion.

occurrence of a “strike phase” generate an hostile mood of C so that they apply the unfair division rule for all the phase, i.e. T periods

Let us now check if these strategies constitute an equilibrium, i.e. if along the equilibrium path they are optimal from the point of view of each party. Then we will …nd the value of T that support such an equilibrium.

2.2 Equilibrium Analysis

We start checking for the optimality of the Workers strategy we characterize in the previous section. The strategy is always optimal along the “strike phase”. Given that C impose $¼ = X$ no matter what during T periods, the W cannot improve from the strike position.

Let now V^{+} (respectively V^{i}) denote the present discounted value of workers’ earnings from date t on, assuming that at date t the game is in a “working phase” (respectively that starts the “strike phase”). By stationarity V^{+} and V^{i} do not depend on the time. By de…nition we have:

$$V^{+} = (1 ¡ ®)(¼^{f} + ±V^{+}) + ®(±V^{i} + X) \quad (1)$$

$$V^{i} = ±^{T}V^{+} + \sum_{s=0}^{T\,1} ±^{s}X \quad (2)$$

Equation (1) says that in a “working phase” Workers do not go to strike and Capitalists apply the “fair rule”. With probability $1 ¡ ®$ Demand is high, Workers (as well as Capitalists) receive $¼^{f}$ and the game remains in such a phase. With probability ® there is low demand today and the game will be in a “strike phase” tomorrow. Equation (2) yields the present discounted value of earnings at the begin of the “strike phase” (note now that this value functions are exactly the same also for the capitalists).

Since $¼^{f} > X$ by assumption it is possible to show that (see Appendix I):

$$V^{+} > V^{i}$$

So W do not want to go to strike during the “working phase” and we have proved that the Workers’ strategy is optimal in each one of the game phases.

Now we have to analyze the behavior of the Capitalists. The strategy is clearly optimal in the “strike phase”: given that W go to strike no matter what during the T periods, the C cannot improve the X payo¤ that they get running the …rm at a reduced speed.

Now we have to add an "Incentive Constraint" (IC) which state that the Capitalists do not have any advantages undercutting the "working phase" through the application of the "unfair" division rule in an high demand period. That is,

$$\pi^f + \delta V^+ \geq TR^H - \pi + \delta V^-$$

we can rewrite this constraint as follows

$$V^+ \geq (1-\alpha)(TR^H - \pi + \delta V^-) + \alpha(\delta V^- + X)$$

and since we have assumed that $1/2 \cdot TR^H = \pi^f$ and $\pi = X$ we can express the IC as follows,

$$V^+ \geq (1-\alpha)(2\pi^f - X + \delta V^-) + \alpha(\delta V^- + X) \quad (3)$$

This equation expresses the trade off that capitalists face. If the apply the "unfair" sharing rule, they get $2\pi^f - X > \pi^f$. However applying such a rule automatically triggers the "strike phase", which yields valuation V^- instead of V^+. Thus to deter the application of the unfair rule V^- should be sufficiently lower than V^+. This means that the strike must last long enough. When this constraint is satisfied also the Capitalists' strategy is optimal.

2.3 Equilibrium length of strikes

So far we have shown that the pair of strategies we were interested in represents a Nash equilibrium for our game if and only if the length of the strike periods is big enough. But does this value of T which supports an equilibrium always exists?

Note that because strikes are costly and occur with positive probability, T should be chosen (by the Trade Unions) as small as possible given that equation (3) must be satisfied.

Now, plugging (1) in (3) we find:

$$(1-\alpha)(\pi^f + \delta V^+) + \alpha(\delta V^- + X) \geq (1-\alpha)(2\pi^f - X + \delta V^-) + \alpha(\delta V^- + X)$$

And, after some algebra we get:

$$\delta(V^+ - V^-) \geq \pi^f - X \quad (4)$$

on the other hand, plugging (2) in (1) I get:

$$V^+ = (1-\alpha)(\pi^f + \delta V^+) + \alpha\left(\delta^{T+1}V^+ + \delta\sum_{s=0}^{T-1}\delta^s X + X\right)$$

Now let's define,

$$\sum_{s=0}^{T-1}\delta^s X \equiv \overline{X}$$

So that:

$$V^+ = (1-\alpha)(\pi^f + \delta V^+) + \alpha(\delta^{T+1}V^+ + \delta\overline{X} + X)$$

Therefore,

$$V^+ = \frac{(1-\alpha)\pi^f + \alpha(\delta\overline{X} + X)}{1-(1-\alpha)\delta-\alpha\delta^{T+1}} \tag{5}$$

Working now with equations (5), (2) and (4) it is possible to express the "Incentive Constraint" as follows:

$$\delta\left(\frac{(1-\alpha)(1-\delta^T)\pi^f + \alpha(\delta\overline{X}+X)(1-\delta^T)}{1-(1-\alpha)\delta-\alpha\delta^{T+1}} - \overline{X}\right) \geq \pi^f - X$$

Thus the problem that we have to solve to find the optimal T (that is the problem that the Trade Union face when it has to decide the length of the strike) is the following,

$$\underset{T\in[0;+\infty[}{Max}\; V^+ = \frac{(1-\alpha)\pi^f + \alpha(\delta\overline{X}(T) + X)}{1-\delta+\alpha\delta-\alpha\delta^{T+1}}$$

over the set of T such that:

$$\delta\left(\frac{(1-\alpha)(1-\delta^T)\pi^f + \alpha(\delta\overline{X}(T)+X)(1-\delta^T)}{1-\delta+\alpha\delta-\alpha\delta^{T+1}} - \overline{X}\right) \geq \pi^f - X$$

where,

$$\overline{X}(T) = \sum_{s=0}^{T-1}\delta^s X = X\cdot\frac{1-\delta^T}{1-\delta}$$

so,

$$\overline{X}(T) = X(1 - \delta^T)(1 - \delta)^{-1}$$

Solving this problem is quite difficult but for our ends it will be enough to show that the value function V^+ is strictly decreasing in T while the constraint is strictly increasing so that the optimal T is exactly the more little value of this variable that satisfies the constraint..

Now, let us define,

$$1 - \delta = \varepsilon \tag{8}$$

$$d(T) = 1 - \delta^T$$

So that

$$\overline{X}(T) = \varepsilon^{-1} X d(T)$$

And note that the function $d(T)$ is strictly increasing, infect:

$$\frac{d}{dT} d(T) = -\log_e \delta \cdot \delta^T > 0 \tag{6}$$

Moreover, let us define

$$(1 - \alpha)\pi^f = a \tag{7}$$

Now simply plug (5), (7) and (8) in (1) and (2). Our problem can now be stated as follows:

$$\underset{T \in [0;+\infty[}{\mathrm{Max}} \; f(T) = \frac{a + \alpha X(\delta \varepsilon^{-1} d(T) + 1)}{\varepsilon + \alpha \delta d(T)} \tag{9}$$

over the set of T such that

$$\delta f(T) d(T) + X - \delta \varepsilon^{-1} X d(T) \geq \pi^f \tag{10}$$

Let us now study the value function $f(T)$. It easy to show that the following proposition is verified (see Appendix II).

Proposition 1 for $X < \pi^f$, that is always true by hypothesis, we have that:

$f(T)' < 0 \, \forall T$ and so the function $f(T)$ is strictly decreasing in T

Let us now study the constraint (10). To this end let us define,

$$h(T) = \delta f(T)d(T) + X - \delta \varepsilon^{-1} X d(T) \tag{11}$$

One can show that the following Proposition is true (see Appendix III),

Proposition 2 for $X < \pi^f$ we have that:

$$h(T)' > 0 \ \forall T \text{ and so the function } h(T) \text{ is strictly increasing in } T$$

So far we have shown that $h(T)' > 0 \ \forall T$ and so that the function $h(T)$ is strictly increasing. Now, since the function $h(T)$ is strictly increasing in T and is unbounded then it will exist a unique T^* such that $h(T) = \pi^f$. Moreover, since again the function $h(T)$ is strictly increasing, it will be: $h(T) \geq \pi^f$ if and only if $T \geq T^*$.

Therefore the set of T such that the constraint (10) is satisfied is the interval $[T^*; +\infty[$.

Recall now that we already shown in Proposition 1 that the value function $f(T)$ is strictly decreasing in its argument so that the maximum of this function over the interval $[T^*; +\infty[$ will be exactly T^*.

2.4 Discussion of the Equilibrium

There are now two important observations regarding the equilibrium we found so far. First, on the equilibrium path the capitalists never try to cheat the workers since the gains generated by the application of the "unfair" sharing rule are exactly compensated by the future costs associated with such an action, that is the reduction in production due to the strike. Second, despite the fact that workers know that low wages reflect demand conditions rather than cheating attempts by employers, it is rational for them to participate in strikes. Why do we get such a paradoxical result? The answer is that the workers perfectly understand the incentive properties of the equilibrium; if they would not go to strike in response to low wages the "incentive constraint" (3) would not hold the rest of the time, so "fair" behavior would cease to be optimal for the capitalists. In a sense, the strikes implemented by the workers are a form of "insurance" against the potential cheating behavior of the capitalists: the fact that workers actually strike each time they receive low wages oblige the employers try to avoid, whenever it is possible, this event. In this light we can say that in explaining the occurrence of strikes an important role is played by the fact that unions try to maintain some form of "reputation for toughness" like in the Hick's analysis.

2.5 Comparative Statics Exercises

Now let us study how changes in some of the parameters of our model (namely changes in the values of X and α) will affect the duration of the cyclical strike period T^*. The basic idea here is to test if our model fit some of the regularities presented in the empirical literatures on strikes and unions behavior.

2.5.1 Changes in the outside-option's value

The idea here is to study how the duration of strikes is affected by changes in the environment. There is some empirical evidence on the fact that reduction in the costs of strikes for both parties leads to an increase in strike duration[5]. Kennan (1980), for example, uses data on the strike duration in US for the period 1955-1980 provided by the Bureau of Labor Statistics and finds out that the probability of settling a strike over a given period of time depends on the total cost of strike to both parties over that period. For example he finds that Unemployment Insurance payments to strikers will tend to prolong strikes if these payments constitute a net subsidy to the firm and its workers.

In the present section we are interested in studying the response of the equilibrium length of strike to an exogenous reduction of the monetary losses related to strikes (in terms of our model we are going to study how an increase of the parameter X affects the value of T^*). We will see how this simple exercise of comparative statics will lead us to the same conclusions reached by the empirical analysis for a sufficiently big range of the parameters values.

To develope this exercise we have now to study the behavior of the function $T^* = T(X)$, in particular we are interested in studying the sign of the derivative $\frac{\partial}{\partial X}T(X)$. To do this let us start considering the function $h(X;T)$ instead of the function $h(T)$. From the previous analysis we know that:

$$\forall X : h(X;T(X)) = \pi^f$$

and therefore, the function

$$X \to h(X;T(X))$$

[5] From a theoretical point of view Reder and Neumann (1980) and Kennan (1980a) have proposed the theory that strike activity is inversely related to its cost. In their analysis what matters is the sum of the costs to both parties, since costs which are incurred by one side can be shifted to the other side by making a more generous bargaining proposal..

is constant of constant value π. So,

$$\frac{d}{dX}h(X;T(X)) = 0 \ \forall X$$

that is,

$$\forall X: \ \frac{\partial}{\partial X}h(X;T(X))\cdot\frac{\partial}{\partial X}X + \frac{\partial}{\partial T}h(X;T(X))\cdot\frac{\partial}{\partial X}T = 0$$

$$\Rightarrow \ \forall X: \ \frac{\partial}{\partial X}h(X;T(X)) + \frac{\partial}{\partial T}h(X;T(X))\cdot\frac{\partial}{\partial X}T = 0$$

$$\Rightarrow \ T(X)' = \frac{\partial}{\partial X}T(X) = -\frac{\frac{\partial}{\partial X}h(X;T(X))}{\frac{\partial}{\partial T}h(X;T(X))} \qquad (12)$$

but we have already shown that

$$\frac{\partial}{\partial T}h(X;T(X)) > 0 \qquad (13)$$

so to study the sign of $T(X)'$ we just need to study the sign of:

$$\frac{\partial}{\partial X}h(X;T(X)) = \frac{\partial}{\partial X}(\delta f(T)d(T) + X - \delta\varepsilon^{-1}Xd(T))$$

In particular we have that (see Appendix 2),

$$\frac{\partial}{\partial X}h(X;T(X)) = \delta d\frac{\alpha(\delta\varepsilon^{-1}d + 1)}{\varepsilon + \alpha\delta d} - \delta\varepsilon^{-1}d + 1$$

We have so to study the following disequation,

$$\delta d\frac{\alpha(\delta\varepsilon^{-1}d + 1)}{\varepsilon + \alpha\delta d} - \varepsilon^{-1}\delta d + 1 \geq 0$$

This implies since $\varepsilon + \alpha\delta d > 0$

$$\delta d\alpha(\delta\varepsilon^{-1}d + 1) + (\varepsilon + \alpha\delta d) - \varepsilon^{-1}\delta d(\varepsilon + \alpha\delta d) \geq 0$$

$$\Rightarrow \ \alpha\varepsilon^{-1}(\delta d)^2 + \delta d\alpha + \varepsilon + \alpha\delta d - \delta d - \alpha\varepsilon^{-1}(\delta d)^2 \geq 0$$

$$\Rightarrow 2\delta d\alpha + \varepsilon - \delta d \geq 0$$

$$\Rightarrow 2\alpha + \frac{\varepsilon}{\delta d} - 1 \geq 0$$

$$\Rightarrow \alpha \geq \frac{1}{2} - \frac{\varepsilon}{2\delta d}$$

moreover it is now worthwhile to note that

$$\frac{1}{2\delta}\frac{\varepsilon}{d} = \frac{1}{2\delta}\frac{1-\delta}{(1-\delta^T)} = \frac{1-\delta}{2\delta - 2\delta^{T+1}} = \frac{1}{2T} \text{ for } \delta \to 1$$

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$$\lim_{\delta \to 1} \frac{1-\delta}{2\delta - 2\delta^{T+1}} \underset{\text{by l'Hopital}}{=} \lim_{\delta \to 1} \frac{-1}{2 - 2(T+1)\delta^T} = \frac{1}{2T}$$

Thus we have that

$$\text{for } \delta \to 1 \text{ and } \alpha < \frac{1}{2} - \frac{1}{2T}: \ \frac{\partial}{\partial X} h(X; T(X)) < 0. \tag{14}$$

Now by (12), (13) and (14) obviously we get the following,

Proposition 3 for $\delta \to 1$ and $\alpha < \frac{1}{2} - \frac{1}{2T}$: $T(X)' > 0$

2.5.2 Changes in the probability of shock realization

In this section we are going to study how variation in the probability of shock realization affects the duration of strikes. Empirical studies found out that there are statistically significant countercyclical variations in the duration of strikes. A very early peace of evidence on this can be found in Bevan (1880). During the boom years 1872 and 1873 the duration of average strike was 20 days in 1872 and 21 days in 1873; a severe recession occurred in 1879 and in this year the average strike duration was 27 days. More recently Kennan (1985) and Tracy (1984) found results that confirm the countercyclical behavior of the duration of the strikes.

In our model the parameter α should be interpreted as an idiosyncratic shock which affect the firm. An high α means infect that we are in recession (i.e. there is an high probability that a negative shock a¤ects the …rm) while a low probability of shock means that we are in an expansion (i.e. there is a low probability that a negative shock a¤ects the …rm). What we have to do is therefore to study how an increase in this parameter a¤ects the duration of the strike. This simple comparative statics exercise will show how the prediction of our model are again perfectly in line with the empirical …ndings.

The next step is therefore to study the behavior of the function $T^{¤} = T(®)$, in particular we are interested in studying the sign of the derivative $\frac{@}{@®}T(®)$. To do this let us start considering the function $h(®; T(®))$ instead of the function $h(T)$. Using the same reasoning used in the previous section (see Appendix IV for the details) one can show that,

Proposition 4 The function $T(®)$ is strictly increasing in ® infect: $T(®)^{0} > 0$ 8®

3 An extension: Human Capital Accumulation

Now we let the agents invest in human capital. The idea is that the level of human capital accumulated by each one of the party during the cooperation phase will a¤ect the sharing rule chosen at the end of the following strike period (that is the division rule seen as fair in the society can change over the time and it depends on the relative amount of human capital owned by the two parties). One can think in the investment in graduate education did by the managers (like an MBA) and in the courses that increase the skills of the workers. We assume that both C and W when take the decision about how much to invest in education just consider the e¤ect that the accumulation of human capital will have on the new sharing rule and that the total revenue does not depend on the level of human capital achieved[6].

We will show that the equilibrium is characterized by a perfect symmetry in investment decisions. Therefore, since the parties have rational expectations, they know that on the equilibrium path it will always be $¼_{C;t} = ¼_{W;t} = \frac{1}{2} ¤ TR_t$: The consequence of this result is that the intro-

[6] It seems natural to extend the present analysis to the case in which the human capital accumulation has e¤ects on the total revenues of the future periods.

duction of this "investment stage" will not affect the strike decisions and so the cyclical equilibrium we previously found will remain unaffected.

To model this problem we also assume the parties choose how much to invest in Human Capital (HC) in each moment of the "cooperation phase" at the beginning of this phase. In turns, after the conclusion of a strike and the settlement of a new contract the parties have to choose how much to invest in education, this decision will last till the next cooperation phase and will affect all the periods of the present phase.

The resource constraint of each party when they take the HC investment decision is:

$$HC_{i,t} + c_{i,t} \leq \pi_{i,t} \qquad i = C, W$$

Moreover we consider Risk Neutral agents, so that the intertemporal utility function can be express as follows,

$$U_i = \sum_{t=0} \delta^t u(c_{i,t}) dt$$

where,

$$u(c_{i,t}) = c_{i,t}$$

And we consider that the sharing rule applied after each strike period (we call this function the Conflict Technology) is the following

$$\pi_{i,t} = \frac{f(\overline{HC}_{i,t})}{f(\overline{HC}_{i,t}) + f(\overline{HC}_{-i,t})} TR_t$$

where $\overline{HC}_{i,t}$ is the total human capital accumulated by the party i in the previous cooperation phase, that is the sum of human capital accumulate in each period of the cooperation phase. Calling K the generic length of a cooperation phase then $\overline{HC}_{i,t}$ can be express as follows,

$$\overline{HC}_{i,t} = \sum_{k=0} HC_{i,t-T-k} = K \cdot (HC_{i,t-T-K})$$

Infect the amount of money invested in human capital in each period of the cooperation phase is constant, that is $HC_{i,t-T-k} = HC_{i,t-T-K} \; \forall k \in [0, K]$. So we have

$$\pi_{i,t} = \frac{f(K \cdot HC_{i,t-T-K})}{f(K \cdot HC_{i,t-T-K}) + f(K \cdot HC_{-i,t-T-K})} TR_t$$

and the function $f(\cdot)$ is a concave and increasing function. In particular let us consider the following case:

$$\pi_{i,t} = \frac{(K \cdot HC_{i,t-T-K})^{\beta}}{(K \cdot HC_{i,t-T-K})^{\beta} + (K \cdot HC_{-i,t-T-K})^{\beta}} TR_t$$

therefore,

$$\pi_{i,t} = \frac{K^{\beta}(HC^{\beta}_{i,t-T-K})}{K^{\beta}(HC^{\beta}_{i,t-T-K}) + K^{\beta}(HC^{\beta}_{-i,t-T-K})} TR_t$$

and then,

$$\pi_{i,t} = \frac{(HC^{\beta}_{i,t-T-K})}{(HC^{\beta}_{i,t-T-K}) + (HC^{\beta}_{-i,t-T-K})} TR_t$$

Note that the expected length of a cooperation period is:

$$\alpha\cdot 1 + 2\cdot(1-\alpha)\cdot\alpha + 3\cdot(1-\alpha)^2\cdot\alpha + 4\cdot(1-\alpha)^3\cdot\alpha \ldots = \alpha \sum_{k=0}^{\infty} (1-\alpha)^k (k+1) = \alpha \sum_{k=1}^{\infty} k(1-\alpha)^{k-1}$$

It is straightforward to show that this series is convergent and let us call the value of this series μ_{α}.

Since the individuals are risk neutral when they discount they consider as a data this expectation. Moreover the agents are aware that the length of the strike phase is T^*.

In the light of what we said the problem that each agent will face at the beginning of each cooperation phase is the following:

$$\underset{HC_{i,t};c_{i,t}}{Max} \sum_{k=0}^{\infty} \delta^k (1-\alpha)^k [c_{i,t}] + \sum_{j=0}^{\infty} \delta^{\mu_{\alpha}+T^*+j}(1-\alpha)^j \left[\frac{HC^{\beta}_{i,t}}{HC^{\beta}_{i,t} + HC^{\beta}_{-i,t}} E_t(TR_{t+\mu_{\alpha}+T}) \right]$$

$$s.t. \; c_{i,t} = \pi_{i,t} - HC_{i,t}$$

But since we have assumed that the total revenue is unaffected by the human capital investment decisions, this means that when the parties maximize they consider the total revenue of the next cooperation phase as given and in particular their expectation is $E_t(TR_{t+\mu_{\alpha}+T}) = TR_t$. Moreover the agents are Risk Neutral, therefore we can express the maximization problem as follows:

$$\underset{HC_{i,t}}{Max} \underbrace{\sum_{k=0}^{\infty} \delta^k (1-\alpha)^k [\pi_{i,t} - HC_{i,t}]}_{\text{Expected costs of HC investment}} + \underbrace{\sum_{j=0}^{\infty} \delta^{\Psi\alpha+T^*+j} (1-\alpha)^j \frac{HC_{i,t}^{\beta}}{HC_{i,t}^{\beta}+HC_{-i,t}^{\beta}} TR_t}_{\text{Expected gains of HC investment}}$$

This maximization problem has a very straightforward interpretation: the agents, both workers and capitalists, try to maximize the sum of the discounted gains derived from human capital investment (given by its effect on the new contract) and of its costs (given by the amount of money spent in education).

The maximization problem can be rewritten as follows:

$$\underset{HC_{i,t}}{Max} \sum_{k=0}^{\infty} \left\{ \delta^k (1-\alpha)^k [\pi_{i,t} - HC_{i,t}] + \delta^{\Psi\alpha+T^*+k} (1-\alpha)^k \left[\frac{HC_{i,t}^{\beta}}{HC_{i,t}^{\beta} + HC_{-i,t}^{\beta}} TR_t \right] \right\}$$

And thus,

$$\underset{HC_{i,t}}{Max} \sum_{k=0}^{\infty} (1-\alpha)^k \cdot \delta^k \cdot \left[\pi_{i,t} - HC_{i,t} + \delta^{\Psi\alpha+T^*} \frac{HC_{i,t}^{\beta}}{HC_{i,t}^{\beta} + HC_{-i,t}^{\beta}} TR_t \right]$$

The FOC for this problem is $\frac{\partial F}{\partial HC_{i,t}} = 0$. Therefore, recalling that in this problem $\pi_{i,t}$ is given when the agents take the decision and so we can treat it as a constant, we get the following condition:

$$\sum_{k=0}^{\infty} (1-\alpha)^k \cdot \delta^k \cdot \left[-1 + \delta^{\Psi\alpha+T^*} \cdot \frac{TR_t \cdot \beta \cdot HC_{i,t}^{\beta-1} \cdot (HC_{i,t}^{\beta} + HC_{-i,t}^{\beta}) - TR_t \cdot \beta \cdot HC_{i,t}^{\beta-1} \cdot HC_{i,t}^{\beta}}{\left(HC_{i,t}^{\beta} + HC_{-i,t}^{\beta}\right)^2} \right]$$

This condition implicitly defines the Best Reply functions for both parties: the equilibrium is given by the intersections of these functions. In particular, it is easy to show that the Nash equilibrium for this game is symmetric. Moreover the equilibrium level of investment in human capital for the two players is positively related with the total revenue, the parameter β and the intertemporal discount factor and negatively related, since $\delta < 1$, with the strike length and the expected length of a cooperation phase (i.e. the more distant in time they will be the advantages derived by the investment the less parties will invest). This equilibrium level is (see Appendix V for the derivation):

$$HC^*_{i;t} = \frac{1}{4}\cdot\delta^{a\alpha+T^*}\cdot TR_t\cdot\beta \tag{15}$$

There are now some important observations to do regarding this result. First, equation (15) states that the amount of money that the parties choose to invest in human capital in equilibrium is related with the total revenue of the current period; this means that, ceteris paribus, in a richer economy (with larger demand and higher revenue) there will be more investment in human capital than in a poor country. This theoretical result is perfectly in line with recent theoretical and empirical works on human capital (see, e.g. Lucas (1988), Romer (1990) and Ciccone and Peri (2000)).

Second, in equilibrium the investment in human capital is, of course, linearly related with the effectiveness of the Conflict Technology. This means that, ceteris paribus, a country in which the contracts are more "sensible" with respect to the skills of the individuals should be characterized by higher levels of human capital investments with respect to a country in which the contracts are related with other variables. This result is perfectly in line with the theoretical analysis of Skaperdas and Syropoulos (1999).

Finally, note that in our framework the "engine" of the investment in human capital is the existence of a repeated bargaining over the contracts. The verification of the strikes is a precondition for this continuous bargaining and therefore one direct implication of this result is that strikes and Trade Unions play an important role in explaining the accumulation of this kind of capital good in the modern economies.

4 Conclusions

In this paper a simple two-players repeated game theoretic model is developed. It allows to analyze the interaction between workers and employers and the occurrence of cyclical periods of strikes. The main result is that we provide a theory of strikes consistent with some important empirical findings; this theory is based on the assumption of asymmetric information between the parties on the conditions of the market. Workers use strikes as a form of insurance against the potential cheating of the employers: the fact that workers actually strike each time they receive low wages oblige the employers try to avoid, whenever it is possible, this event. As a consequence employers reduce wages just in case of negative demand shock and only in this case strikes happen.

We also provide an extension allowing the parties to invest in human capital. The incentive for this kind of investment is given by an hypothesis on the bargaining power of the parties. We assume the bargaining power as a function of the human capital accumulated by each party. This assumption is meant to capture the fact that relative more skilled individuals are usually able to obtain better contracts in the bargaining phase. The basic result is that, once allowed to take this kind of decisions, both workers and employers invest in human capital an amount of money proportional to the total revenues and to the effectiveness of the conflict technology. This result put in a new light the role that strikes and Trade Unions play in the process of development of a modern economy, infect they are in a sense the cause of human capital investment.

Finally, it is worthwhile to note that in the present paper is not developed a complete growth model. This choice was made to better evidence the reasons that lead to the strikes and the key role of repeated bargaining over the contracts to explain the human capital investment decisions. I guess that developing a growth model on the basis of the present model could give rise to an equilibrium characterized by growth cycles starting when negative shocks affect the economy and lasting till the end of the consequent strike. In this case the source of growth would be the human capital accumulation while the source of the cycles would be the negative shocks.

5 Appendixes

5.1 Comparison of Value Functions

We have to show that $V^{+} > V^{-}$. By (2) and (5) we know that we can express these value functions as follows:

$$V^{+} = \frac{(1-\alpha)\pi^{f} + \alpha(\delta\overline{X} + X)}{1-(1-\alpha)\delta - \alpha\delta^{T+1}}$$

$$V^{-} = \frac{(1-\alpha)\pi^{f}\delta^{T} + \alpha(\delta\overline{X} + X)\delta^{T}}{1-(1-\alpha)\delta - \alpha\delta^{T+1}} + \overline{X}$$

Therefore we remain to prove that the following disequation is always verified,

$$(1-\delta^{T}) \cdot \frac{(1-\alpha)\pi^{f} + \alpha(\delta\overline{X} + X)}{1-(1-\alpha)\delta-\alpha\delta^{T+1}} \geqslant \overline{X}$$

Using (6), (7) and (8) we can rewrite this expression as follows,

$$d \cdot \frac{a + \alpha x(\delta\varepsilon^{-1}d + 1)}{\varepsilon + \alpha\delta d} \geqslant \varepsilon^{-1}xd$$

And rearranging terms we easily get

$$(1-\alpha)x + \alpha\delta d(1-\varepsilon^{-1})x \leqslant (1-\alpha)\pi^{f}$$

Note now that

$$\varepsilon^{-1} = \frac{1}{1-\delta} > 1 \Rightarrow (1-\varepsilon^{-1}) < 0 \Rightarrow \alpha\delta d(1-\varepsilon^{-1})x < 0$$

Thus

$$(1-\alpha)x + \alpha\delta d(1-\varepsilon^{-1})x \leqslant (1-\alpha)x$$

And since $\pi^{f} > x$ by assumption we have that

$$(1-\alpha)x + \alpha\delta d(1-\varepsilon^{-1})x \leqslant (1-\alpha)x < (1-\alpha)\pi^{f}$$

That means $V^{+} \geqslant V^{-}$.

5.2 Proof of proposition 1

To study the value function $f(T)$ we have to consider its first derivative,

$$\frac{d}{dT}f(T) = f(T)' = \frac{\alpha X\delta\varepsilon^{-1}d'(\varepsilon + \alpha\delta d) - \alpha\delta d'(a + \alpha X(\delta\varepsilon^{-1}d + 1))}{(\varepsilon + \alpha\delta d)^{2}} \qquad (*)$$

Since $(\varepsilon + \alpha\delta d)^{2} > 0$, to study the sign of this first derivative is enough to study the sign of its numerator. So we have to study the sign of

$$\alpha X\delta\varepsilon^{-1}d'(\varepsilon + \alpha\delta d) - \alpha\delta d'(a + \alpha X(\delta\varepsilon^{-1}d + 1)) =$$

$$= \alpha X\delta\varepsilon^{-1}d'\varepsilon + (\alpha X\delta\varepsilon^{-1}d')(\alpha\delta d) - a\alpha\delta d' - \alpha^2\delta d' X - (\alpha\delta d)(\alpha X\delta\varepsilon^{-1}d') =$$

$$= X\alpha\delta d'(1-\alpha) - a\alpha\delta d'$$

Therefore:

$$f(T)' < 0 \text{ if and only if } X(1-\alpha) - a < 0$$

that is:

$$f(T)' < 0 \text{ if and only if } X(1-\alpha) - (1-\alpha)\pi^f < 0 \qquad (**)$$

but this conditions means $X < \pi^f$ that is always true by hypothesis.

5.3 Proof of proposition 2

We have to study the following function,

$$h(T) = \delta f(T)d(T) + X - \delta\varepsilon^{-1}Xd(T)$$

In particular we want to analyze how this function varies when T varies; so let us consider its first derivative

$$\frac{d}{dT}h(T) = h(T)' = \delta(f'\cdot d + d'\cdot f) - \delta\varepsilon^{-1}Xd'$$

so

$$\frac{1}{\delta}h(T)' = f'\cdot d + d'\cdot f - \varepsilon^{-1}Xd'$$

thus by (*) we get

$$\frac{1}{\delta}h(T)' = \frac{\alpha X\delta\varepsilon^{-1}d'(\varepsilon+\alpha\delta d) - \alpha\delta d'(a+\alpha X(\delta\varepsilon^{-1}d+1))}{(\varepsilon+\alpha\delta d)^2}\cdot d + d'\cdot f - \varepsilon^{-1}Xd'$$

$$\frac{1}{\delta}h(T)' = d'\cdot\left[f - \varepsilon^{-1}X + \frac{\alpha X\delta\varepsilon^{-1}(\varepsilon+\alpha\delta d) - \alpha\delta(a+\alpha X(\delta\varepsilon^{-1}d+1))}{(\varepsilon+\alpha\delta d)^2}\cdot d\right]$$

$$\frac{1}{\delta}h(T)' = d' \cdot \left\{ -\varepsilon^{-1}X + \frac{\alpha\delta\varepsilon^{-1}X\varepsilon + (\alpha\delta)^2\varepsilon^{-1}Xd - a\alpha\delta - (\alpha\delta)^2\varepsilon^{-1}Xd - \alpha^2X\delta}{(\varepsilon + \alpha\delta d)^2} \cdot d \right\}$$

and by (9)

$$\frac{1}{\delta}h(T)' = d' \cdot \left\{ \frac{a + \alpha X(\delta\varepsilon^{-1}d + 1)}{\varepsilon + \alpha\delta d} - \varepsilon^{-1}X + \frac{\alpha\delta\varepsilon^{-1}X\varepsilon - a\alpha\delta - \alpha^2X\delta}{(\varepsilon + \alpha\delta d)^2} \cdot d \right\}$$

$$\frac{1}{\delta}h(T)' = d' \cdot \left\{ \frac{(a + \alpha X(\delta\varepsilon^{-1}d + 1))(\varepsilon + \alpha\delta d)}{(\varepsilon + \alpha\delta d)^2} - \varepsilon^{-1}X + \frac{\alpha\delta\varepsilon^{-1}X\varepsilon - a\alpha\delta - \alpha^2X\delta}{(\varepsilon + \alpha\delta d)^2} \cdot d \right\}$$

that is

$$\frac{1}{\delta}h(T)' = d' \cdot \left\{ \frac{(a + \alpha X\delta\varepsilon^{-1}d + \alpha X)(\varepsilon + \alpha\delta d)}{(\varepsilon + \alpha\delta d)^2} - \frac{\varepsilon^{-1}X(\varepsilon + \alpha\delta d)^2}{(\varepsilon + \alpha\delta d)^2} + \frac{\alpha\delta\varepsilon^{-1}X\varepsilon - a\alpha\delta - \alpha^2X\delta}{(\varepsilon + \alpha\delta d)^2} \right.$$

We have to study whether or not $h(T)' > 0$ so, since $(\varepsilon + \alpha\delta d)^2 > 0$ and $d' > 0$, we have to analyze the simply the following disequation

$$(a+\alpha X\delta\varepsilon^{-1}d+\alpha X)(\varepsilon+\alpha\delta d) - \varepsilon^{-1}X(\varepsilon+\alpha\delta d)^2 + (\alpha\delta\varepsilon^{-1}X\varepsilon - a\alpha\delta - \alpha^2X\delta)d \geq 0$$

so,
$a\varepsilon+\alpha X\delta\varepsilon^{-1}d\varepsilon+\alpha X\varepsilon+a\alpha\delta d+(\alpha\delta d)^2X\varepsilon^{-1}+\alpha^2X\delta d+\alpha X\delta\varepsilon^{-1}d\varepsilon - a\alpha\delta d - \alpha^2 dX\delta - \varepsilon^{-1}X\varepsilon^2 - \varepsilon^{-1}X(\alpha\delta d)^2 -$
$-2\varepsilon^{-1}X\varepsilon\alpha\delta d \geq 0$, $a\varepsilon + \alpha X\varepsilon - \varepsilon^{-1}X\varepsilon^2 \geq 0$, (since $\varepsilon > 0$) , $a + \alpha X - \varepsilon^{-1}X\varepsilon \geq 0$

Thus, for $h(T)'$ be positive it should be:

$$(1-\alpha)\pi^f + \alpha X - X \geq 0$$

Therefore,

$$h(T)' > 0 \text{ if and only if } (1-\alpha)\pi^f - (1-\alpha)X > 0$$

but this conditions means $\pi^f > X$ that is always true by hypothesis.

5.4 Proof of proposition 4

We are interested in studying the sign of the derivative $\frac{\partial}{\partial \alpha}T(\alpha)$. To do this let us start considering the function $h(\alpha; T(\alpha))$ instead of the function $h(T)$. We know that:

$$\forall \alpha : h(\alpha; T(\alpha)) = \pi^f$$

and therefore, the function

$$\alpha \rightarrow h(\alpha; T(\alpha))$$

is constant of constant value π^f.So,

$$\frac{d}{d\alpha}h(\alpha; T(\alpha)) = 0 \ \forall \alpha$$

that is,

$$\forall \alpha : \frac{\partial}{\partial \alpha}h(\alpha; T(\alpha)) \cdot \frac{\partial}{\partial \alpha}\alpha + \frac{\partial}{\partial T}h(\alpha; T(\alpha)) \cdot \frac{\partial}{\partial \alpha}T = 0$$

$$\Rightarrow \forall \alpha : \frac{\partial}{\partial \alpha}h(\alpha; T(\alpha)) + \frac{\partial}{\partial T}h(\alpha; T(\alpha)) \cdot \frac{\partial}{\partial \alpha}T = 0$$

$$\Rightarrow T(\alpha)' = \frac{\partial}{\partial \alpha}T(\alpha) = -\frac{\frac{\partial}{\partial \alpha}h(\alpha; T(\alpha))}{\frac{\partial}{\partial T}h(\alpha; T(\alpha))} \qquad \text{(i)}$$

recall that studying the effects of changes in X we have already shown that

$$\frac{\partial}{\partial T}h(\alpha; T(\alpha)) > 0 \qquad \text{(ii)}$$

so to study the sign of $T(\alpha)'$ we just need to study the sign of:

$$\frac{\partial}{\partial \alpha}h(\alpha; T(\alpha)) = \frac{\partial}{\partial \alpha}(\delta f(T)d(T) + X - \delta \varepsilon^{-1} X d(T)) =$$

$$= \delta d \frac{\partial}{\partial \alpha} f(T)$$

And by (9),

$$\frac{\partial}{\partial \alpha} h(\alpha; T(\alpha)) = \delta d \frac{\partial}{\partial \alpha} \frac{a + \alpha X(\delta \varepsilon^{-1} d + 1)}{\varepsilon + \alpha \delta d}$$

$$= \delta d \frac{\partial}{\partial \alpha} \frac{(1 - \alpha)\pi^f + \alpha X(\delta \varepsilon^{-1} d + 1)}{\varepsilon + \alpha \delta d}$$

that is

$$\frac{\partial}{\partial \alpha} h(\alpha; T(\alpha)) = \delta d \frac{[X(\delta \varepsilon^{-1} d + 1) - \pi^f](\varepsilon + \alpha \delta d) - \delta d[(1 - \alpha)\pi^f + \alpha X(\delta \varepsilon^{-1} d + 1)]}{(\varepsilon + \alpha \delta d)^2}$$

Now, since $\frac{\delta d}{(\varepsilon + \alpha \delta d)^2} > 0$, to study the sign of $\frac{\partial}{\partial \alpha} h(\alpha; T(\alpha))$ we have to study the sign of the numerator of the previous expression. That is

$$\varepsilon X(\delta \varepsilon^{-1} d + 1) - \pi^f \varepsilon + \alpha X \delta d(\delta \varepsilon^{-1} d + 1) - \pi^f \alpha \delta d - \delta d(1 - \alpha)\pi^f - \delta d \alpha X(\delta \varepsilon^{-1} d + 1) \geq 0$$

$$\Rightarrow \varepsilon X(\delta \varepsilon^{-1} d + 1) - \pi^f \varepsilon - \pi^f \delta d \geq 0$$

$$\Rightarrow \varepsilon X \delta q d + \varepsilon X - \pi^f \varepsilon - \pi^f \delta d \geq 0$$

$$\Rightarrow \varepsilon(X - \pi^f) + \delta d(\varepsilon^{-1} \varepsilon X - \pi^f) \geq 0$$

so we have

$$(\varepsilon + \delta d)(X - \pi^f) \geq 0$$

Now, since $\varepsilon + \delta d > 0$ and since $X < \pi$ by assumption, we get that

$$\frac{\partial}{\partial \alpha} h(\alpha; T(\alpha)) < 0 \ \forall \alpha \qquad \text{(iii)}$$

And finally by (i), (ii) and (iii)

$$T(\alpha)' > 0$$

5.5 Human Capital investment equilibrium strategies

We know that the FOC for the problem of the two parties leads to the following condition:

$$\sum_{k=0}^{\infty}(1-\alpha)^k\left[-1+\delta^{[\alpha+T]}\cdot\frac{TR_t\cdot\beta\cdot HC_{i;t}^{\beta-1}\cdot(HC_{i;t}^{\beta}+HC_{-i;t}^{\beta})-TR_t\cdot\beta\cdot HC_{i;t}^{\beta-1}\cdot HC_{i;t}^{\beta}}{\left(HC_{i;t}^{\beta}+HC_{-i;t}^{\beta}\right)^2}\right]=$$

that is,

$$-1+\delta^{[\alpha+T]}\cdot\frac{TR_t\cdot\beta\cdot HC_{i;t}^{\beta-1}\cdot(HC_{i;t}^{\beta}+HC_{-i;t}^{\beta})-TR_t\cdot\beta\cdot HC_{i;t}^{\beta-1}\cdot HC_{i;t}^{\beta}}{\left(HC_{i;t}^{\beta}+HC_{-i;t}^{\beta}\right)^2}=0$$

to find the equilibrium we have thus to solve the following system of two equations in two unknowns given by the best reply functions of the two parties:

$$BR_1(HC_{2;t}):\ -1+\delta^{[\alpha+T]}\cdot\frac{TR_t\cdot\beta\cdot HC_{1;t}^{\beta-1}\cdot(HC_{1;t}^{\beta}+HC_{2;t}^{\beta})-TR_t\cdot\beta\cdot HC_{1;t}^{\beta-1}\cdot HC_{1;t}^{\beta}}{\left(HC_{1;t}^{\beta}+HC_{2;t}^{\beta}\right)^2}=0 \qquad \text{(a)}$$

$$BR_2(HC_{1;t}):\ -1+\delta^{[\alpha+T]}\cdot\frac{TR_t\cdot\beta\cdot HC_{2;t}^{\beta-1}\cdot(HC_{1;t}^{\beta}+HC_{2;t}^{\beta})-TR_t\cdot\beta\cdot HC_{2;t}^{\beta-1}\cdot HC_{2;t}^{\beta}}{\left(HC_{1;t}^{\beta}+HC_{2;t}^{\beta}\right)^2}=0 \qquad \text{(b)}$$

So we have to solve:

$$-1+\delta^{[\alpha+T]}\cdot\frac{TR_t\cdot\beta\cdot HC_{1;t}^{\beta-1}\cdot(HC_{1;t}^{\beta}+HC_{2;t}^{\beta})-TR_t\cdot\beta\cdot HC_{1;t}^{2\beta-1}}{\left(HC_{1;t}^{\beta}+HC_{2;t}^{\beta}\right)^2}=0$$

$$-1+\delta^{[\alpha+T]}\cdot\frac{TR_t\cdot\beta\cdot HC_{2;t}^{\beta-1}\cdot(HC_{1;t}^{\beta}+HC_{2;t}^{\beta})-TR_t\cdot\beta\cdot HC_{2;t}^{2\beta-1}}{\left(HC_{1;t}^{\beta}+HC_{2;t}^{\beta}\right)^2}=0$$

By (a) rearranged we get:

$$HC_{1;t}^{2¯} + HC_{2;t}^{2¯} + 2HC_{1;t}^{¯}HC_{2;t}^{¯} = ±^{ª®+T¤}¢TR_t¢¯¢(HC_{2;t}^{2¯\,i\,1} + HC_{2;t}^{¯\,i\,1}HC_{1;t}^{¯} ¡ HC_{2;t}^{2¯\,i\,1})$$

$$)\ HC_{1;t}^{2¯} + HC_{2;t}^{2¯} + 2HC_{1;t}^{¯}HC_{2;t}^{¯} = ±^{ª®+T¤}¢TR_t¢¯¢HC_{2;t}^{¯\,i\,1}HC_{1;t}^{¯} \quad \text{(a1)}$$

Equivalently by (b) rearranged we get:

$$HC_{1;t}^{2¯} + HC_{2;t}^{2¯} + 2HC_{1;t}^{¯}HC_{2;t}^{¯} = ±^{ª®+T¤}¢TR_t¢¯¢HC_{1;t}^{¯\,i\,1}HC_{2;t}^{¯} \quad \text{(b1)}$$

Therefore by (a1) and (a2) since they have the same LHS:

$$TR_t¢¯¢HC_{2;t}^{¯\,i\,1}HC_{1;t}^{¯} = TR_t¢¯¢HC_{1;t}^{¯\,i\,1}HC_{2;t}^{¯}$$

$$)\ HC_{2;t}^{¯\,i\,1}¢HC_{1;t}^{¯} = HC_{1;t}^{¯\,i\,1}¢HC_{2;t}^{¯}$$

and dividing both sides by $HC_{1;t}^{¯}¢HC_{2;t}^{¯}$ we end with,

$$)\ HC_{2;t}^{i\,1} = HC_{1;t}^{i\,1}\ )\ HC_{1;t} = HC_{2;t}$$

Substituting this value in (a1) we have,

$$HC_{1;t}^{2¯} + HC_{1;t}^{2¯} + 2HC_{1;t}^{2¯} = ±^{ª®+T¤}¢TR_t¢¯¢HC_{1;t}^{2¯\,i\,1}$$

$$)\ HC_{1;t} = \frac{1}{4}¢±^{ª®+T¤}¢TR_t¢¯ = HC_{2;t}$$

that is,

$$HC_{i;t}^{¤} = HC_{i\,i;t}^{¤} = \frac{1}{4}¢±^{ª®+T¤}¢TR_t¢¯$$

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