Give to Ceasar What Is Ceasar's. Or, Give to Launhardt What We Are Used to Think Is Bertrand's[1]

Luca Lambertini and Manuela Mosca
Dipartimento di Scienze Economiche
Università di Bologna
Strada Maggiore 45
40125 Bologna, Italy
fax: +39-051-2092664
lamberti@spbo.unibo.it
mosca@spbo.unibo.it

August 6, 2001

[1]We thank Gianpaolo Rossini and Riccardo Rovelli for helpful discussion and suggestions. The usual disclaimer applies.

Abstract

We revisit the Cournot-Bertrand debate in the light of Cournot (1863), Edgeworth (1881, 1897) and Launhardt (1885), tracing back to Launhardt the origin of price competition in duopoly models. Then, we discuss the formalisation of consumer utility function for differentiated products, first appearing in Launhardt (1885) and then in Bowley (1924). This allows us to point out that assuming that firms know the demand function(s) is equivalent to assuming that they know the structure of consumer preferences. Therefore, we argue that there is no role for the auctioneer, either in Cournot or in Walras.

Keywords: price competition, quantity competition, theory of markets, utility function, game theory

JEL Classification: B10, B20, B23, B31, B41, C70, L13

1 Introduction

This paper examines the origins of price competition models in the literature on oligopoly competition. In particular, we are interested in tracing the source of the now generalised use of the label *Bertrand competition*, as well as the emergence of what is now called *Bertrand paradox* or *pure Bertrand equilibrium*. This involves the analysis of the Bertrand-Cournot debate (Cournot, 1838; Bertrand, 1883), and its relation to the following contributions by Edgeworth (1881, 1897). All this literature makes use of demand functions as primitive notions, without tackling the issue of their derivation from a well defined utility function, which appears in Walras (1874, 1883) and Marshall (1890). The lack of an appropriate modelling of the utility function prevents all of the early scholars from producing a model of price competition with continuous reaction functions in partial equilibrium.

The *vulgata* of the Cournot-Bertrand debate, which is usually proposed in both undergraduate and postgraduate courses in microeconomics and industrial organization, maintains that the Cournot model needs an auctioneer, and therefore it is more sensible to think of firms as price-setters rather than quantity-setters, with the demand side of market (i.e., consumers) in charge of deciding equilibrium outputs. One of our aims is to clarify that Bertrand never proposed anything like that, notwithstanding that Bertrand's review of Cournot is carried out contextually with the review of Walras (1883). In connection with this point, we show that in full information models like those of Cournot, Walras and Edgeworth, there is no need of an auctioneer shouting a price list, and therefore the associated discussion of dynamic adjustments, popping up in both Walras and Cournot models, is ill-founded, as its correct (and formally equivalent) interpretation is rather that of equilibrium stability related to the Hessian matrix of the market problem.

The state of the art concerning the Cournot-Bertrand debate and the origins of the price competition model is the following. Magnan de Bornier (1992, p. 632) points out that "it seems wrong to say that Bertrand suggested that producers in an oligopoly use price as their strategic variable". Moreover, Magnan de Bornier (1992, p. 633) highlights that, most likely, Bertrand was not proposing an equilibrium with prices falling down to marginal cost. In Magnan de Bornier's view (1992, p. 638), the conjecture that Bertrand had in mind what is now labelled as the paradox is traced to Fisher's suggestion of a competitive outcome as the equilibrium of a duopoly competition with price reaction functions (Fisher, 1898). The conventional wisdom that

Bertrand had the idea of a price war (i.e., a price undercutting mechanism) can be subsequently found in Hotelling (1929) and Chamberlin (1929). In the latter, we can also find the argument explaining that this price war would ultimately lead to marginal cost (Chamberlin, 1929, p. 71). This view goes unchanged through Stackelberg (1934) and Stigler (1940)[1] to reach its first formalisation by a game theorist in Shubik (1959).

The discussion on the paternity of the price competition model proceeds in Morrison (1998), who claims that such paternity should go back to Cournot because, in Cournot (1838, ch. 9), Morrison (1998, p. 173) finds that "for a duopoly case, Cournot [...] states the proprietor profit functions concerning prices for both rivals and partially differentiates each of the profit functions with respect to their own price". However, as Morrison recognises in fn. 4 (p. 174), "Cournot is not discussing differentiated oligopoly but rather the case of a composite commodity whose components are supplied by rival monopolists". This view is adopted and reinforced in Dimand and Dore (1999) and Morrison (1999), where it is stressed that Fisher (1898) was inadvertently responsible for the diffusion of the idea that Bertrand competition was a synonymous for price competition.

The remainder of the paper is structured as follows. In section 2, we reinterpret the Cournot-Bertrand debate in the light of Cournot (1863), Edgeworth (1881, 1897) and Launhardt (1885). Following Sonnenschein (1968), we also assess Cournot's model of price-setting complementary monopolists (1838, ch. 9), to claim that it cannot be considered as the proper source of price competition, contrary to what is instead maintained by Morrison (1998). Our reading of the seminal literature on oligopoly leads us to claim that the paternity of price duopoly models is to be attributed to Edgeworth and Launhardt. In particular, Lauhardt appears to be the first to offer a thourough analysis of pricing behaviour in a model with product differentiation and continuous demand (and therefore reaction) functions, producing a price equilibrium which encompasses the so-called Bertrand paradox as a special case when, in the limit, products become perfect substitutes. Section 3 contains a discussion concerning the formalisation of consumer preferences into a utility function. Again, the first formalisation of consumer preferences in a duopoly model is due to Launhardt (1885). Then, we trace back to

[1] Fellner's (1949) discussion of Cournot and Stackelberg is revealing of the misperception of strategic interaction before game theory assumed a central role in the theory of industrial organization. See Leonard (1994).

Bowley (1924) and Shubik (1959, 1980) two isomorphic formulations of a utility function defined over a set of substitute or complement goods. This allows us to point out that assuming that firms know the demand function(s) is equivalent to assuming that they know the structure of consumer preferences. Therefore, choosing a price or a quantity strategy is just a matter of taste or convenience in the process of profit maximisation, but does not imply at all that firms need an auctioneer to set prices or consumers to decide on quantities, as the hypothesis of full information involves that firms know in advance both. Section 4 contains concluding remarks.

2 The Cournot-Bertrand debate revisited

In the well known ch. 7 of his *Principles*,[2] Cournot states that

"Proprietor (1) can have no direct influence on the determination of D_2: all that he can do, when D_2 has been determined by proprietor (2), is to choose for D_1 the value which is the best for him. *This he will be able to accomplish by properly adjusting his price*, except as proprietor (2), who, seeing himself forced to accept his price and this value of D_1, may adopt a new value for D_2, more favourable to his interests than the preceding one." (p. 64; the emphasis is ours)

The sentence in italic is inconsistent with the quantity-setting model that Cournot is presenting in this chapter. In particular, it is the only sentence where Cournot refers to a price adjustment throughout the treatment of the duopoly in quantities. In fact, the idea that either producer may unilaterally change his price is unfounded in this setting, as with homogeneous products it is just impossible to observe two different prices in a competition in quantities. Of course, this sounds obvious in retrospect, from the standpoint of our current understanding of oligopoly theory.[3] Cournot (1863, §62) himself drops the reference to a unilateral price adjustment.

[2] Throughout the paper, we refer to the reprint of Cournot (1838) in Daughety (1989).

[3] This is the case, for instance, of Martin (2001, ch. 2): "Despite having adopted output as the decision variable, and explicitly using a notation that makes price a function of total output, Cournot writes of proprietor 1 adjusting *his price*. This is inexact on two counts, first because it speaks of proprietor 1 adjusting price and second for the reference to 'his' price, since with a homogeneous product there is only one price (and it is $p = f(D)$)."

The sentence at stake appears to trigger the following overshooting critique by Bertrand:[4]

"Cournot conjectures that *one of the competitors will lower his price to attract buyers*, and that the other, in order to bring them back, will lower his more. They will continue until each of them will no loner gain anything more by lowering his price. A peremptory objection arises: With this hypothesis a solution is impossible; the price reduction would have no limit. In fact, whatever jointly determinated price where adopted, if only one of the competitors lowers his, he gains, disregarding all unimportant exceptions, all the sales, and he will double his returns if his competitors allows him to do so. If Cournot's formulas mask this result, it is because through a peculiar oversight, he introduces under the names D and D' the quantities sold by the two competitors, and treating them as independent variables, he assumes that the one quantity happening to change through the will of one owner, the other would remain constant. The contrary is obviously true." (p. 77; the emphasis is ours).

First of all, in the above argument, the only statement that can be attributed to Cournot is what we have italicized. The following implication, that the rival will lower his price even more to regain demand, belongs to Bertrand, while there is no trace of that in Cournot, who claims that proprietor 2 is forced to accept that price. Second, the price undercutting mechanism is envisaged by Bertrand as if that was indeed what Cournot was talking about, while it should be clear by now that this is not the case as a price undercutting story cannot be observed in a quantity setting duopoly. Third, Bertrand claims that there should be no limit to the price undercutting, and, in so doing, overlooks the existence of what, in retrospect, should be called a fixed point (or Nash equilibrium) towards which the prices should converge, namely, the marginal cost (this, in the case under examination, is zero; however, as long as it is assumed to be constant and equal across firms, the normalisation to zero is just immaterial). Therefore, the alleged *Bertrand paradox* cannot be traced back to Bertrand. Moreover, the common wisdom that Bertrand proposes a model of price competition as an alternative to Cournot's quantity competition is ill-founded, as Bertrand attributes to

[4]We quote from the first English translation of Bertrand's review, published in Daughety (1989).

Cournot a confusion between quantity and price competition and criticises Cournot on these grounds.[5]

Concerning the position taken by Morrison (1998) as to Cournot's description of two monopolists offering complement goods and optimizing in prices, it suffices to point out that this issue has already been dealt with by Sonnenschein (1968), who shows that "Cournot's theories of oligopoly and complementary monopoly are formally identical" (p. 316). The following remark is particularly relevant to our present purpose:

"It is immediately clear how one can be obtained from the other by a simple reinterpretation of symbols. A consequence of the equivalence is that a theorem for one theory is a theorem for the other; for example, the well-known result that the quantity supplied under duopoly is greater than the quantity supplied under pure monopoly may be translated into the proposition that the price charged under complementary monopoly is higher than the price charged under pure monopoly." (pp. 316-317)

Now, if this were a prototype of current price competition models with substitute goods, increasing the number of suppliers would entail decreasing price, and therefore it can hardly be interpreted as the first instance of such literature.

Having dealt with Bertrand's discussion on Cournot, we now proceed to examine the approach to price competition that is adopted by Edgeworth (1897). In 1897, Edgeworth solves a model of duopoly in prices with homogeneous goods and capacity constraints, while he had previously proposed a price duopoly without constraints but with increasing marginal costs (Edgeworth, 1881). The conventional wisdom in the current theory of industrial organization about the work of Edgeworth maintains that he proposed the first solution to the Bertrand paradox by using capacity constraints, which could prevent firms to flood the market with the competitive output.[6] Yet,

[5] Bertrand also criticises Cournot because the latter briefly considers, and quickly dismisses, the cooperative solution along the industry profit possibility frontier. Bertrand, instead, claims that this should be the most sensible solution. However, also on this ground, he makes a false step, in that Cournot is aware of the instability of collusion, as he says "this condition is not one of stable equilibrium; and, although the most favourable for both producers, it can only be maintained by means of a formal engagement" (p. 67).

[6] This perception of the Edgeworth contribution can be found in Tirole (1988, ch. 5), *inter alia*, and is a consequence of the papers by Levitan and Shubik (1972) and Kreps and Scheinkman (1983), who refer to Edgeworth (1897) to produce a two-stage subgame

as we have just shown, at that time there was no conscience of any such paradox, simply because the competitive outcome of a homogenous duopoly in price is not formalised or even suggested by Bertrand. Accordingly, our view is that Edgeworth should be credited the paternity of the first model of price competition with homogeneous goods. However, given the homogeneous good assumption, his model is not strategic in the same sense as the model of quantity competition presented by Cournot. This is due to the fact that perfect substitutability between goods entails that any unilateral price change produces a discontinuity in demands and therefore a discontinuity in best replies.

This is precisely the departure point taken by Hotelling (1929):

"After the work of the late Professor F.Y. Edgeworth one may doubt that anything further can be said on the theory of competition among a small number of entrepreneurs. However, one important feature of actual businesses seems until recently to have escaped scrutiny. This is the fact that of all the purchasers of a commodity, some buy from one seller, some from another, in spite of moderate differences of price. If the purveyor of an article gradually increases his price while his rivals keep theirs fixed, the diminution in volume of his sales will in general take place continuously rather than in the abrupt way which has tacitly been assumed." (p. 41)

It is rather peculiar that, after discussing the price undercutting mechanism with homogeneous products (which he attributes to Bertrand through the intermediation of Edgeworth, p. 42), Hotelling builds a model of spatially differentiated duopoly where each seller's demand is a function of both prices, and, using this, intends to illustrate a case where prices changes give rise to small variations in individual firms' demands. In the modern jargon, this amounts to saying that his aim is to construct a price duopoly where demands and reaction functions are continuous. After having solved price competition for given locations, Hotelling proceeds to identify the optimal locations. This yields the so-called *minimum differentiation principle* where products are homogeneous and, nevertheless, prices are above marginal cost at equilibrium. Unfortunately, however, Hotelling overlooks that his result

perfect equilibrium in capacities and prices for a homogeneous duopoly. These papers have generated a stream of literature dealing with the same topic (Davidson and Deneckere, 1986; Osborne and Pitchik, 1986; Deneckere and Kovenock, 1996; Kovenock and Roy, 1998, *inter alia*).

is undermined by the same undercutting argument that he is trying to do away with in building up a product differentiation model. Presumably, if he were aware of anything called the Bertrand paradox, then he would conclude that, once firms supply sufficiently close substitutes, the unique equilibrium involves both prices falling down to marginal cost. This mistake has been amended fifty years later by d'Aspremont, Gabszewicz and Thisse (1979).

However, the Hotelling model of horizontal differentiation with transportation costs was anticipated by Launhardt (1885). To the best of our knowledge, this is first pointed out by Anderson, de Palma and Thisse (1992, p. 2, fn. 1) who remark that "[Launhardt's book] has not received its due recognition among non-German-speaking economists". One would rather be tempted to say "even among German-speaking economists", since Hotelling was German, although working at Stanford University.[7] In his Introduction to the English edition of Launhardt's book, Creedy (1993, p. 1) states that "[Launhardt's book] may claim to be one of Germany's most important contributions to neoclassical economic theory during the last quarter of the nineteenth century". Creedy (1993, p. 2) also quotes Blaug (1986), who points out that

"to anyone interested in the fascinating topic of multiple discoveries in science, and the associated questions of why some figures are systematically neglected, Launhardt's case affords a rich example." (p. 123)

Likewise, Schumpeter (1954, p. 851) comments that "it is curious to observe - and characteristic of the conditions in our field - that a type of research may be present and in full view and yet pass unnoticed". These opinions concern Launhardt's work on the pure theory of exchange and general equilibrium. We are about to show that completely analogous considerations also apply to Launhardt's contribution to the understanding of price competition. To see this, one has to read Part 3 of his book ("The Transport of Goods", pp. 139-189 of the English edition), where Launhardt presents his model of spatial pricing. After analysing the monopoly case (pp. 141-146), he passes on to the duopoly case, where international trade is considered. At p. 151 he derives the following price equilibrium (what he defines 'a basis of

[7] Much more than the anticipation of the theory of spatial competition can be found in Launhardt. A modern revisitation of Launhardt's model where his approach to vertical differentiation is also highlighted, can be found in Dos Santos Ferreira and Thisse (1996).

peace'):

$$\begin{aligned} g' &= \{(p'' - p') + (2f'' + f')\, h\}/3 \\ g'' &= \{(p' - p'') + (2f' + f'')\, h\}/3 \end{aligned} \tag{1}$$

where g' (respectively, g'') is the unit profit of firm 1 (firm 2), which is increasing in (i) the difference between what he calls production prices p'' and p' (while we would define them as constant marginal costs) and (ii) the "line which measures h in length and which and which connects the two neighbouring market places" (p. 148); that is, the degree of horizontal differentiation. With f' and f'', Launhardt indicates the unit transportation cost rates ("the rate of freight for a given unit", p. 141). Now, it suffices to observe that, if firms have the same marginal cost and their products are homogeneous (i.e., $h = 0$), then the unit profit is $g' = g'' = 0$, and therefore price is equal to marginal cost. Hence, Launhardt must be credited for the introduction into the economic literature of the first formal treatment of price competition with constant marginal cost where equilibrium prices may fall to marginal cost and consequently profits may fall to zero when firms are completely symmetric and products are perfect substitutes. It is also worth stressing that Launhardt is the first to formalise market demand functions starting from consumer utility, and therefore he's first in treating a price game with product differentiation and continuous demand and reaction functions.[8] We will come back to this in the next section.

The *trait d'union* between the theories of price competition before and after John Nash (1950)[9] is to be found in Shubik (1955). He sets out by taking Edgeworth (1881, 1897) as the reference for price competition, without mentioning either Bertrand or Hotelling at all.[10] Then, he describes a price duopoly under complete information, where

"In order to simplify matters as much as possible, let us imagine that the method of marketing for the duopolists is such that each phones in his

[8] On the figure of Launhardt as a forerunner, see also Pinto (1977), Dos Santos Ferreira (1998), Perreur (1998) and Backhaus (2000). However, none of these authors dwells upon this specific aspect of Launhardt's contribution to economic theory.

[9] The investigation of the relevance of the Nash equilibrium concept in the development of mathematical economics in general and industrial economics in particular, is beyond the scope of the present paper. For a discussion of this matter, see Weintraub (1992) and Leonard (1994).

[10] He refers to Cournot as the origin of the quantity game investigated by Mayberry, Nash and Shubik (1953).

strategy to a marketing board which is in touch with all the customers and knows their individual demand schedules. This board sells to the customers and remits to the duopolists ... i.e., the firms only produce to advance orders from the marketing board after having stated their price and production limits." (p. 418)

Two crucial points arise from Shubik's words. The first is that the game unravels under full information; this notwithstanding, Shubik introduces the figure of the marketing board, that closely recalls the Walrasian auctioneer. The job of both agents consists in matching demand and supply. On this particular issue, we will dwell extensively in section 3. The second element is that, following Edgeworth, Shubik assumes that firms are capacity constrained. Indeed, in the remainder of the paper, he proves that Edgeworth cycles can be reproduced as the outcome of a non-cooperative simultaneous game. Shubik concludes that "the price variation game appears to be more 'competitive' than the Cournot game" (p. 431), but, having assumed increasing marginal costs with rationing, his treatment of price competition cannot produce the so-called Bertrand paradox as an equilibrium outcome.

The first systematical assessment of the theory of markets from a game theorist's standpoint, is in Shubik (1959).[11] Shubik introduces the analysis of price behaviour in duopoly as follows:

"Bertrand objected to Cournot's analysis of the duopoly problem in terms of quantity as the strategic variable. He suggested a solution that depends upon price variation. As with Cournot's model, the method offered was apparently dynamic although it can be cast in static terms. Bertrand considered two producers with no costs of production and wished to demonstrate that the profits of both would be wiped out by their competition. He purported to show that the two competitors would keep on undercutting each other until they reached the competitive equilibrium." (p. 80)

To the best of our knowledge, this is the first publication where the name of Bertrand is explicitly associated with the perfectly competitive outcome with zero profits of a price game with homogeneous goods (see also pp. 100-109).

[11] According to Shubik himself, "the primary purpose of this book is to begin to develop a unified approach to the various theories of competition and markets" (p. xi).

From this source, the subsequent literature on industrial organization absorbs the notion that the price game with homogeneous goods is the Bertrand game. This holds, e.g., for Baumol, Panzar and Willig (1982), looking for predecessors of the contestability theory:

"In addition to the oral tradition that, in Chicago, two is taken to be a large number, the most notable of similar results is found in the work of Bertrand (1883). There too, the oligopolistic (Nash) equilibrium attained when two or more *price*-setting firms have constant marginal costs involves price equal to marginal cost." (p. 44),

it holds for Friedman (1983) reviewing the literature on oligopoly theory:

"He [Bertrand] presents an analysis of duopoly using Cournot's famous mineral spring example in which costs are zero, and he switches to price as the firm's decision variable. ... Positive prices for the firms cannot be in equilibrium, and $p_1 = p_2 = 0$ is a noncooperative equilibrium." (pp. 46-47),

as well as for Tirole (1988):

"Chapter 5 deals with short-run price competition, examines the Bertrand paradox (in which two or more identical firms producing a homogeneous good with a constant-returns-to-scale technology in equilibrium sell at marginal cost and make no profit)..." (pp. 205-206),

and, more recently, for Anderson and Renault (1999):

"We study price competition in the presence of search costs and product differentiation. The limit cases of the model are the 'Bertrand Paradox', the 'Diamond Paradox', and Chamberlinian monopolistic competition." (p. 719)

Summing up, the current perception of price games as the presumed heritage of Bertrand is that Bertrand has introduced strategic consumers into a picture where Cournot initially considered only firms as strategic agents.[12] In particular, consumers search for the firm charging the lowest price, and this generates the discontinuities in demands that we are well accustomed with since Edgeworth (Daughety, 1989, pp. 22-23). The extreme evolution of

[12] The above list of quotations gathers but a few instances. Also Sutton (1991, p. 32) briefly presents the price game as the Bertrand game.

this way of thinking about Bertrand is Klemperer's reinterpretation of price oligopoly games as auctions where firms produce the perfectly competitive outcome while striving to conquer the whole market (Klemperer, 2000). That is, the substance of what we now label as the Bertrand paradox is not the fact that a limited number of firms suffices to yield perfect competition (as in the Chicago vein), but rather that they do so by trying to acquire monopoly power.

The foregoing discussion should have made clear that our position concerning the paternity of formal models of price competition in oligopoly markets is the following. The first duopoly model of price setting behaviour is in Edgeworth (1881), where product homogeneity and increasing marginal costs prevent the author from producing a model with continuous demand and reaction functions, and consequently that model cannot yield an equilibrium at marginal cost pricing. The same holds for his subsequent work (1897), due to the assumption of capacity constraints. In Launhardt (1885), instead, one finds a duopoly model with product differentiation and continuous demand and best reply functions, whose price equilibrium encompasses the alleged *pure Bertrand equilibrium* as a special case. Therefore, it is our opinion that Launhardt should be considered the father of price competition.

3 What exactly are firms supposed to know when they play an oligopoly game?

Here, we aim at outlining the (implicit) background to the discussion on duopoly carried out by Cournot and Bertrand through to Shubik (1959). In particular, oligopoly theory ever since its earliest days triggers the doubt, hunting the dreams of generations of industrial economists, that the Cournot model needs an auctioneer to set equilibrium prices and clear the market. This is usually attributed to the idea that, if firms set quantities, then someone else has to find the price(s) at which demand equals supply. This someone is an auctioneer like Walras's. In many undergraduate and postgraduate courses in microeconomics and industrial organization theory, this is actually presented as the main basis for Bertrand's critique to Cournot:

"There is an important objection to quantities as the decision variables of oligopolists, with a market price being determined by the total quantity produced. By what mechanism is the market price established? It is no

problem imagining firms choosing prices and either producing output to order or producing for inventory. But what institutional arrangements accomplish price determination if the firms do not choose prices?" (Friedman, 1977, p. 39)

These considerations are commonly accepted among industrial economists, as it would be most likely confirmed by interviewing a representative sample of them. In the remainder of this section, we will argue that there exist no sound foundations justifying this view.

As a starting point, consider the assumptions adopted by Cournot. He supposes that firms know the market demand function, but refuses to consider the formalisation of consumer utility function, as unrealistic (see the introduction of *Principles*). This situation also characterises the works of Edgeworth (1881, 1897) and Hotelling (1929). Our current way of representing the derivation of market demands in the Hotelling framework is a modern interpretation, but it is completely absent from Hotelling's original formulation, where the partition of customers between the two firms is based upon the vector of 'delivered prices':

$$p_1 + cx = p_2 + cy \tag{2}$$

where $\{p_1, p_2\}$ is the vector of mill prices, $c > 0$ is the transportation cost rate and x and y, $x + y = 1$, are the segments measuring the distance between the indifferent consumer and firms 1 and 2, respectively (Hotelling, 1929, p. 46). The standard formulation that is currently taught to students all over the world is the following (see d'Aspremont, Gabszewicz and Thisse, 1979). A population of consumers is uniformly distributed over a unit segment; a consumer located at $m \in [0, 1]$ enjoys the following net surplus from consumption:

$$U = s - p_i - cd_i \tag{3}$$

where s is gross surplus and d_i is the distance between the consumer and firm $i = 1, 2$, with $d_1 = x$ and $d_2 = y$. The consumer at m is indifferent between the two products if

$$s - p_1 - cx = s - p_2 - cy\,. \tag{4}$$

Of course (2) and (4) are equivalent, with the *caveat* that starting from (2) makes it less than immediate that being able to write the indifference condition and then the demand functions amounts to assuming that a specific

structure of consumer preferences is known to firms (especially if one looks at the problem with the eyes of the XIX or early XX century).

Once again, the correct formulation has been there since 1885. In setting out to treat the problem of transportation and international trade, Launhardt (p. 141 of the English edition) defines as α the marginal utility from unit consumption at a 'priceworthyness' of w, and $p + fz$ the delivery price. Then, the maximum distance, z', at which the marginal consumer locates is determined by solving:

$$\alpha - w\left(p + fz'\right) = 0 \tag{5}$$

that is, by imposing that the net utility of the consumer who is indifferent between buying or not be zero.

As far as the market demand for homogeneous products (assumed by Cournot and inherited by Edgeworth) is concerned, what we know now is that it obtains from the solution of the consumer choice problem consisting in

$$\begin{aligned} \max_{q_1,q_2} U &\equiv a\left(q_1 + q_2\right) - \frac{1}{2}b\left(q_1^2 + q_2^2 + 2q_1q_2\right) \\ s.t. &: R \geq p_1q_1 + p_2q_2 \end{aligned} \tag{6}$$

where R is income and $\{a, b\}$ are positive parameters. This deserves a few remarks. First, the model postulates the existence of a representative consumer, and therefore what (following Cournot) is usually referred to as the market demand is actually the result of the solution to and individual optimum problem. Second, the consumer carries out an activity of quasi-production, in that his preferences are defined over the set of available products,[13] and, unlike what happens in Hotelling-like models, he patronises both firms. Third, the utility function in (6) appears first in Bowley (1924) in its more general form apt to account for price competition with product differentiation:

$$U \equiv a\left(q_1 + q_2\right) - \frac{1}{2}\left[b\left(q_1^2 + q_2^2\right) + 2\sigma q_1q_2\right] \tag{7}$$

where $\sigma \in [0, b]$ is the degree of product substitutability, i.e., it is an inverse measure of product differentiation. When $\sigma = b$, (7) reproduces the case of perfect substitutability, i.e., product homogeneity. When $\sigma = 0$, the two

[13]For simplicity we keep on discussing the two-product setting, but the extension to n products is straightforward.

firms are independent monopolists.[14] The direct demand functions for the price duopoly model obtaining from (7) write as follows:

$$q_i = \frac{a}{b+\sigma} - \frac{p_i}{b^2-\sigma^2} + \frac{\sigma p_j}{b^2-\sigma^2} \ , \ i \neq j \ , \ i,j = 1,2 \, . \tag{8}$$

Using (8), a price equilibrium can be obtained such that, as σ tends to b *in the limit*, the Bertrand paradox arises with market demand being equally split across firms.

However, the early theory of markets before Bowley (1924) disregards the problem of modelling consumer preferences and therefore cannot produce an appropriate setup for analysing price behaviour. Interestingly enough, this specific aspect of Bowley's contribution to mathematical economics in general and to the theory of markets in particular remains disregarded for several decades, until Spence (1976) and Dixit (1979). For a long time, his main contribution is considered to be the formalisation of conjectural variations (see Friedman, 1977, pp. 77-78, *inter alia*).

Shubik (1980) independently introduces another formulation of consumer utility defined over a set of differentiated products:

$$U \equiv \frac{1}{\beta} \left[\alpha \left(q_1 + q_2 \right) - \frac{1}{2} \left(q_1^2 + q_2^2 \right) - \frac{\left(q_1 - q_2 \right)^2}{2 \left(1 + \gamma \right)} \right] \tag{9}$$

where $\gamma \in [0, \infty)$ measures product substitutability. Maximising (9) under the usual budget constraint yields:

$$q_i = \frac{1}{2} \left[\alpha - \frac{\beta \left(2 + \gamma \right) p_i - \beta \gamma p_j}{2} \right] \ , \ i \neq j \ , \ i,j = 1,2 \, . \tag{10}$$

The two models (Bowley's and Shubik's) have been used independently by many industrial economists.[15] To the best of our knowledge, the first proof of the isomorphism between (7) and (9), and therefore also (8) and (10), is due to Albæk and Lambertini (1998).

This relatively long digression into the contemporary developments of the theory of industrial organization should contribute to make it clear that

[14] Moreover, in the range $\sigma \in (0, b]$ products are complements. Duality theorems can be found in Bowley (1924), Shubik (1980) and Singh and Vives (1984), *inter alia*.

[15] Spence (1976), Dixit (1979), Singh and Vives (1984), Vives (1985), Klemperer and Meyer (1986, 1989), Okuguchi (1987) and Lambertini (1997) are among those who use Bowley's formulation. Shubik's model is adopted by Rothschild (1992, 1995). A general approach is in Tanaka (2001).

choosing not to formalise the structure of consumer preferences leaves concealed for a long time the main issue related to the early studies of duopoly behaviour. That is, what is now called the assumption of full or complete information concerning the market game involves necessarily that not only the features of market demand but also, and more crucially, consumer preferences are known to firms independently of whether (i) they optimise w.r.t. prices or quantities, and (ii) products are differentiated or not.

This point receives a brilliant illustration by Singh and Vives (1984), who, using the Bowley model, show that quantity is a dominant strategy for all degrees of substitutability, so one should expect firms to play a Cournot equilibrium. Concerning the problem of who announces prices, one has to keep in mind the assumption of full information, implying that firms know the demand structure - that is, as we are now aware of, they know the functional form of consumer preferences from which the demand curves derive. Hence, in choosing outputs, firms also know what the prices are going to be in equilibrium, and there is no need of any auctioneer. This is exactly the point that Singh and Vives (1984, p. 546) seem to make when they stipulate that firms offer either a 'price contract' or a 'quantity contract', which means the following. Firms optimise either in prices or in quantities. They then propose a contract which is a price-quantity package along the demand curve, which is known to both consumers and firms. The only difference lies in the derivation of the alternative packages. If firms pick the Cournot outputs, then they check along the (inverse) demand functions for the equilibrium prices and propose such a pair to consumers, whose answer would clearly be positive. the opposite procedure takes place in case of price behaviour.

If assessed under this perspective, the 'auctioneer argument' commonly associated with Bertrand's criticism of the Cournot model appears unreasonable. The delicate point is the assumption of full information, rather than who is to set prices if firms set quantities, because if firms know the demand structure then there is no need of an auctioneer. A relevant implication of this argument is: why should one worry about Bertrand competition (with or without capacity constraints) at all? Within the limits of available capacity, firms should in any case play *à la* Cournot, at least from the standpoint of pure profit incentives.

Another, and possibly even more important implication of the above discussion is that full information also eliminates the auctioneer from Walras's model of general equilibrium (Walras, 1874, see also 1883), where full (or complete) information is assumed, and nonetheless the auctioneer is intro-

duced as the agent whose specific task is to clear the market. But then the question arises, in retrospect: how can there be any excess demand or supply, if all traders are completely informed? One should expect the market to clear instantaneously, without any adjustment towards the equilibrium. Of course, one should take into account that the ultimate implications of the full information hypothesis were much less than evident to all those who write before Nash. It is indeed revealing that this applies also to Shubik, who adopts the intermediary known as the 'marketing board' to match demand and supply. Overall, it seems that Walras's auctioneer and Shubik's board are the result of an attempt on the part of the theory to come to terms with reality, where full information is very seldom, if ever, available.

4 Concluding remarks

We have revisited the Cournot-Bertrand debate in the light of the subsequent contributions of Cournot (1863), Edgeworth (1881, 1897) and Launhardt (1885). This has lead us to identify Launhardt as the source of the price duopoly model. Our discussion of the formalisation of consumer utility function for differentiated products, first appearing in Launhardt (1885) and then in Bowley (1924), has allowed us to point out that assuming that firms know the demand function(s) is equivalent to assuming that they know the structure of consumer preferences. As a consequence, we have argued that there is no role for the auctioneer, either in Cournot or in Walras.

References

[1] Albæk, S. and L. Lambertini (1998), "Collusion in Differentiated Duopolies Revisited", *Economics Letters*, **59**, 305-8.

[2] Anderson, S., A. De Palma and J.-F. Thisse (1992), *Discrete Choice Theory of Product Differentiation*, Cambridge, MA, MIT Press.

[3] Anderson, S.P. and R. Renault (1999), "Product Diversity, and Search Costs: A Bertrand-Chamberlin-Diamond Model", *RAND Journal of Economics*, **30**, 719-35.

[4] Backhaus, U. (2000), "An Engineer's View of Economics: Wilhelm Launhardt's Contributions", *Journal of Economic Studies*, **27**, 424-76.

[5] Baumol, W., J. Panzar and R. Willig (1982), *Contestable Markets and the Theory of Industry Structure*, New York, Harcourt Brace Jovanovich.

[6] Bertrand, J. (1883), Review, *Journal des Savants*, 68, 499-508. Reprinted in English translation by J.W. Friedman in Daughety, A. (ed. 1989), *Cournot Oligopoly. Characterization and Applications*, Cambridge, Cambridge University Press, 73-81.

[7] Blaug, M. (1986), *Great Economists Before Keynes*, Brighton Wheatsheaf.

[8] Bowley, A.L. (1924), *The Mathematical Groundwork of Economics*, Oxford, Oxford University Press.

[9] Chamberlin, E.H. (1929), "Duopoly: Value when Sellers Are Few", *Quarterly Journal of Economics*, **44**, 63-100.

[10] Cournot A. (1838), *Recherches sur les principes mathématiques de la théorie des richesses*, Paris, Hachette. Translated by N.T. Bacon as Researches into the Mathematical Principles of the Theory of Wealth, New York, Macmillan, 1897. Chapter 7 is reprinted in English in A. Daughety (ed., 1989), *Cournot Oligopoly. Characterization and Applications*, Cambridge, Cambridge University Press, 63-72.

[11] Cournot A. (1863), *Principes de la théorie des richesses*, Paris, Hachette.

[12] Creedy, J. (1993), Introduction to W. Launhardt, *Mathematical Principles of Economics*, Aldershot, Edward Elgar, 1-10.

[13] d'Aspremont, C., J.J. Gabszewicz and J.-F. Thisse (1979), "On Hotelling's 'Stability in Competition"', *Econometrica*, **47**, 1045-50.

[14] Daughety, A.F. (1989), Introduction to *Cournot Oligopoly. Characterization and Applications*, Cambridge, Cambridge University Press.

[15] Davidson, C. and R. Deneckere (1986), "Long-Run Competition in Capacity, Short-Run Competition in Price, and the Cournot Model", *RAND Journal of Economics*, **17**, 404-15.

[16] Deneckere, R. and D. Kovenock (1996), "Bertrand-Edgeworth Duopoly with Unit Cost Asymmetry", *Economic Theory*, **8**, 1-25.

[17] Dimand, R.W. and M.H.I. Dore (1999), "Cournot, Bertrand, and Game Theory: A Further Note", *Atlantic Economic Journal*, **27**, 325-33.

[18] Dixit, A. (1979), "A Model of Duopoly Suggesting the Theory of Entry Barriers", *Bell Journal of Economics*, **10**, 20-32.

[19] Dos Santos Ferreira, R. (1998), "Launhardt and Hotelling", *Recherches Economiques de Louvain*, **64**, 97-109.

[20] Dos Santos Ferreira, R. and J.-F. Thisse (1996), "Horizontal and Vertical Differentiation: The Launhardt Model", *International Journal of Industrial Organization*, **14**, 485-506.

[21] Edgeworth, F. (1881), *Mathematical Psychics*, London, Kegan Paul.

[22] Edgeworth, F. (1897), "La Teoria Pura del Monopolio", *Giornale degli Economisti*, 40, 13-31. Reprinted in English as "The Pure Theory of Monopoly", in F. Edgeworth (1925), *Papers Relating to Political Economy*, Vol. 1, London, Macmillan & Co., Ltd., 111-42.

[23] Fellner, W. (1949), *Competition among the Few*, New York, Knopf.

[24] Fisher, I. (1898), "Cournot and Mathematical Economics", *Quarterly Journal of Economics*, **12**, 119-38.

[25] Friedman, J.W. (1977), *Oligopoly and the Theory of Games*, Amsterdam, North-Holland.

[26] Friedman, J.W. (1983), *Oligopoly Theory*, Cambridge, Cambridge University Press.

[27] Hotelling, H. (1929), "Stability in Competition", *Economic Journal*, **39**, 41-57.

[28] Klemperer, P. (2000), "Why Every Economist Should Learn Some Auction Theory", invited lecture to *VIII World Congress of the Econometric Society*, Seattle; and *XXVII EARIE Conference*, Lausanne.

[29] Klemperer, P. and M. Meyer (1986), "Price Competition vs Quantity Competition: The Role of Uncertainty", *RAND Journal of Economics*, **17**, 618-38.

[30] Klemperer, P. and M. Meyer (1989), "Supply Function Equilibria in Oligopoly under Uncertainty", *Econometrica*, **57**, 1243-77.

[31] Kovenock, D. and S. Roy (1998), "Dynamic Capacity Choice in a Bertrand-Edgeworth Framework", *Journal of Mathematical Economics*, **29**, 133-58.

[32] Kreps, D.M. and J.A. Scheinkman (1983), "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes", *Bell Journal of Economics*, **14**, 326-37.

[33] Lambertini, L. (1997), "Prisoners' Dilemma in Duopoly (Super)Games", *Journal of Economic Theory*, **77**, 181-91.

[34] Launhardt, W. (1885), *Mathematische Begründung der Volkswirtschaftslehre*, Leipzig, B.G. Teubner. Reprinted in English translation as *Mathematical Principles of Economics*, Aldershot, Edward Elgar, 1993.

[35] Leonard, R.J. (1994), "Reading Cournot, Reading Nash: The Creation and Stabilisation of the Nash Equilibrium", *Economic Journal*, **104**, 492-511.

[36] Levitan, R. and M. Shubik (1972), "Price Duopoly and Capacity Constraints", *International Economic Review*, **13**, 111-23.

[37] Magnan de Bornier, J. (1992), "The 'Cournot-Bertrand Debate': A Historical Perspective", *History of Political Economy*, **24**, 623-56.

[38] Marshall, A. (1890), *Principles of Economics*, London, Macmillan.

[39] Martin, S. (2001), *Advanced Industrial Economics, Second Edition*, Oxford, Blackwell, in print.

[40] Mayberry, J.P., J.F. Nash, Jr., and M. Shubik (1953), "A Comparison of Treatments of a Duopoly Situation", *Econometrica*, **21**, 141-54.

[41] Morrison, C.C. (1998), "Cournot, Bertrand, and Modern Game Theory", *Atlantic Economic Journal*, **26**, 172-4.

[42] Morrison, C.C. (1999), "Dimand-Dore on Cournot-Bertrand: A Reply and More", *Atlantic Economic Journal*, **27**, 334-9.

[43] Nash, J.F., Jr. (1950), "Equilibrium Points in N-Person Games", *Proceedings of the National Academy of Sciences*, **36**, 48-9.

[44] Okuguchi, K. (1987), "Equilibrium Prices in the Bertrand and Cournot Oligopolies", *Journal of Economic Theory*, **42**, 128-39.

[45] Osborne, M. and C. Pitchik (1986), "Price Competition in a Capacity-Constrained Duopoly", *Journal of Economic Theory*, **38**, 238-60.

[46] Perreur, J. (1998), "Industrial Location Theory in German Thought – Launhardt and Weber", *Recherches Economiques de Louvain*, **64**, 75-96.

[47] Pinto, J.V. (1977), "Launhardt and Location Theory: Rediscovery of a Neglected Book", *Journal of Regional Science*, **17**, 17-29.

[48] Rothschild, R. (1992), "On the Sustainability of Collusion in Differentiated Duopolies", *Economics Letters*, **40**, 33-7.

[49] Rothschild, R. (1995), "Sustaining Collusion When the Choice of Strategic Variable Is Endogenous", *Journal of Economic Behavior and Organization*, **28**, 373-85.

[50] Schumpeter, J. (1954), *History of Economic Analysis*, London, Allen and Unwin.

[51] Shubik, M. (1959), *Strategy and Market Structure*, New York, John Wiley & Sons.

[52] Shubik, M. (with R. Levitan, 1980), *Market Structure and Behavior*, Cambridge, MA, Harvard University Press.

[53] Singh, N. and X. Vives (1984), "Price and Quantity Competition in a Differentiated Duopoly", *RAND Journal of Economics*, **15**, 546-54.

[54] Sonnenschein, H. (1968), "The Dual of Duopoly is Complementary Monopoly. Or, Two of Cournot's Theories Are One", *Journal of Political Economy*, **76**, 316-8.

[55] Spence, A.M. (1976), "Product Differentiation and Welfare", *American Economic Review*, **66**, 407-14.

[56] Stackelberg, H. von (1934), *Marktform und Gleichgewicht*, Berlin and Vienna, Springer.

[57] Stigler, G. (1940), "Notes on the Theory of Duopoly", *Journal of Political Economy*, **48**, 521-41.

[58] Sutton, J. (1991), *Sunk Costs and Market Structure*, Cambridge, MA, MIT Press.

[59] Tanaka, Y. (2001), "Profitability of Price and Quantity Strategies in an Oligopoly", *Journal of Mathematical Economics*, **35**, 409-18.

[60] Tirole, J. (1988), *The Theory of Industrial Organization*, Cambridge, MA, MIT Press.

[61] Vives, X. (1985), "On the Efficiency of Bertrand and Cournot Equilibria with Product Differentiation", *Journal of Economic Theory*, **15**, 166-75.

[62] Walras, L. (1874), *Élements déconomie politique*, Lausanne. Reprinted in English translation as *Elements of Pure Economics*, London, Allen and Unwin, 1954.

[63] Walras, L. (1883), *Théorie mathématique de la richesse sociale*, Lausanne.

[64] Weintraub, R. (1992, ed.,), *Toward a History of Game Theory*, *History of Political Economy*, **24** (Supplement).