Influencing the Misinformed Misbehaver: An Analysis of Public Policy towards Uncertainty and External Effects*

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Abstract

We study a situation where the government influences consumers' behaviors by providing both information and incentives. More generally, we propose a methodology for solving models of signal cum cheap talk.

We develop the case of consumption choice in the presence of uncertainty and external effects. The instruments used by the government are information campaigns and taxes. A difficulty arises because the government would like to improve its imperfect coercive instruments by delivering biased information to the misbehaver. We study the equilibrium trade-off between informing and giving incentives. Environmental tax policy, anti-smoking campaigns and policy against antibiotics over-consumption serve as illustrations.

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1 Introduction

As for the "mad cow" disease or other hotly debated issues concerning public health, food safety and the environment, risk controversies have grown dramatically. Since policy makers must often assess and communicate such risks, the confidence individuals feel towards the government or other authorities is a decisive component of policy making. Our work focuses on communication as a discloser of the conflicts between a benevolent authority and consumers. Two ingredients are indispensable: the first is the public's lack of knowledge concerning the risk to be regulated and the second is the impossibility for individuals alone to rightly internalize certain negative consequences of their actions. Each problem taken in isolation is relatively easy to solve: the former needs information provided to the public, whereas the latter needs optimized incentives. But put together, these remedies interfere and result in political confusion when incentives and coercive instruments are defective. Indeed, the maximization of social welfare doesn't guarantee truthful policies, and consumers are aware of that. They are even more and more skeptical in front of promotional strategies using the outcome of the scientific literature or citing expert advisory committees.

In the present paper, the policy making process is analyzed as a game where the government wants to influence consumers' behaviors by using both a tax policy and information campaigns, and where rational consumers react in a Bayesian manner. Instruments being umperfect, the government is often tempted to "improve" behaviors by sending biased information. As our work shows, confidence is not easily controlled. Depending on the coordination between the government and the consumers, the same background data can produce a variety of policies and real effects. We determine the structure of implementable policies and we discuss the trade-off between vagueness in communication and distortion in incentives.

Influence Games We introduce a general methodology for tackling influence games, i.e. games in which the principal is the informed party and combines different instruments for transmitting information and providing incentives to the agent. The literature which is influential for this work was initiated by Crawford and Sobel (1982) and Milgrom and Roberts (1986). Crawford and Sobel (1982) show that the precision of the information transmitted depends on the intensity of the conflict existing between the two parties' objectives. Quite recently, these ideas have been applied to political games in which a lobby tries to influence policy makers.[1] In a major contribution to the rational foundations of advertising, Milgrom and Roberts (1986) model a firm signalling its product quality through price and dissipative advertising (burned money) to enhance the

[1]See Helpman (2000), Presidential Address at the Econometric Society World Congress, Seattle and Grossman and Helpman (2001), for surveys.

consumers willingness-to-pay for the product. In line with these articles, and contrary to Maskin and Tirole (1992), characterizing optimal mechanisms is not our primary objective; rather, we study the combination of imperfect mechanisms, giving the priority to their practical structure and consequences.

In a recent development along these lines, Austen-Smith and Banks (2000) show, starting from the classical model of Crawford and Sobel, how burning money can improve cheap talk. In particular, they clearly show why the information transmitted can be perfect, and why the most informative equilibrium need not be the most efficient.

We retrieve these results in our context, but our neatly different methodology, which is conceived to be applicable to a variety of models, enables us to prove other useful findings. First we change the perspective: we show that cheap talk is almost useless when costly signals are available,[2] and why more precise equilibria are typically more distorted. Second, rather than searching for the equilibria of a given economy, we define the minimal set of data (a skeleton) useful for describing an equilibrium, and we determine the full set of economies which admit a given skeleton as an equilibrium. We prove the uniqueness of the fully revealing equilibrium, and we characterize such an equilibrium in detail. More generally, our approach in terms of skeletons facilitates the understanding of the interaction between costly and free signal, and opens the way to interesting comparative statics.

The Analysis of Health and Environmental Policy Our model deals with policies affecting the consumption of commodities which are detrimental to consumers' welfare both at a private and at a public level. Typically, *side effects* are due to *individual* consumption, whereas *external effects* are due to *overall* consumption in the economy. Broad spectrum antibiotics exhibit this double negative impact (besides the obvious beneficial effects): at the individual level, they clear the way to opportunistic infections by more resistant germs,[3] while, at the social level, they enhance the very resistance of the germs involved in contagious diseases. Analogously, in the case of tobacco and alcohol, one clearly distinguishes diseases related to individual consumption from the passive smoking or the cost to the health care system (not to speak of psychosocial issues like drunk driving or addiction).

These two types of negative effects explain why, without governmental intervention in the form of information or incentives, consumers may not consume

[2] Our proof is direct; an indirect proof was available in Manelli (1996).

[3] Some broad-spectrum antibiotics decrease the individual's immunologic reaction and, as a consequence, new diseases can arise. For example, many antibiotics based on penicillin are used to treat diseases like bronchitis, otitis and tonsillitis caused by different bacteria (staphilococcus aureus, haemofilus influenzae, streptococcus pneumoniae). Possible side effects of penicillin consumption are candida albicans and herpes. See Levy (1992) for the medical viewpoint, and Brown and Layton (1996) for an excellent economic analysis of the external effects involved.

efficiently. Firstly, side effects are not necessarily well perceived by consumers. For example, the real strength of side effects remains only a vague notion for the majority of people as far as antibiotics are concerned; likewise, the risk smokers perceive can be under- or over-estimated (Viscusi 1990). Secondly, external effects (e.g. resistance acquisition or passive smoke) are vastly ignored by consumers in the absence of incentives such as taxes, norms, controls.

Political attitudes towards tobacco is typical of the schizophrenia we analyze in the present work. Efficient taxation is, in general, difficult to establish but compared to others, tobacco taxes are an easy source of funds. The government may try to optimize health and budgetary objectives by manipulating consumers beliefs on the individual consequences of smoking. Obviously, rational consumers form their opinion with this threat in mind, and the success of these attempts is uncertain. In the same vein, remark that the opinion that consuming a lot of antibiotics may cause *individual* resistance to the treatment is pervasive, but not founded. (In fact the problem is rather due to the resistance acquired by the germs, which concerns the society rather than the individual *sensu stricto*). Authorities (which may not be directly responsible for this belief) are clearly tempted not to correct it, since it serves (at a low cost) its practical goal: curbing consumption.

We assume that the government is better informed and benevolent, and that it maximizes the utility of the representative consumer.

To support our assumption that the government is better informed, remark that the government may appeal to experts (civil servants, professionals, academics) who are able to transform dispersed data and results in operational knowledge. We do not need to believe that this operation is perfect, only that it is better done by the experts than by the majority of the public. Moreover, informing the public is a never-ending task. We know that attempts to discourage teenagers from smoking require renewed strategies year after year. Though one may think that the "society" is already saturated with information on the relationship between tobacco and cancer, each new cohort of consumers has to be educated.[4]

Nevertheless, the government confronts the following dilemma: taxes are imperfect instruments, and improving their performance with biased information is tempting. This motivates a sort of paternalism: the government wants consumers to consume efficiently, but, being unable to commit to neutral and truthful information transmission, it may send interested messages. In our view, taxes must be understood as a metaphor for more comprehensive policy, like contracts, restrictions, standards or norms, in all circumstances where there is some imperfection which impedes economic efficiency.[5]

[4] Publication of rough scientific findings in the mass media, though in principle a contribution to the formation of public opinion, may result in confusion; sensationalism and caricatures are common. The best examples are found in extreme dietary recommendations.

[5] Causes of imperfection are often related to asymmetric information. In moral hazard

One crucial aspect of the model concerns the analysis of the tax as a signal transmitting information on the value of side effects. The tax has two consequences: first, modifying the consumption price, it provides incentives to internalize the external effects; second, the tax is a signal which informs on the value of side effects.

Information campaigns are analyzed as messages, i.e. short statements aiming at informing individuals about the effects of certain goods. Warning labels on cigarette packs or on hazards related to drinking, are the best examples. A fundamental characteristic of these information campaigns is that they have no direct consequence on the government's and on the consumers' utility: they are inexpensive, and to simplify, enter into the category of cheap talk messages.[6,7] This implies that the literal meaning of the information provided is sufficiently vague not to be falsifiable. Take the warning label "Seriously Harmful to Health" on cigarette packs. This is not false, but the exact nuance it bears is a matter of social convention (specifically the way cheap talk is interpreted). Like taxes, information campaigns are imperfect policy instruments.

Another crucial ingredient concerns the analysis of tax distortions. In the model, the sign of the marginal cost of public funds is not restricted a priori. For this reason, the paper draws practical implications of the literature on the "double dividend" which concerns environmental levies. According to this literature a tax on a polluting good is welfare improving for two reasons: first because it makes pollution decrease, and, second, because it reduces distortions caused by preexisting taxes. Quite recently, it has been shown that the double dividend exists only under certain conditions, which means in particular that the sign of the marginal cost of public funds is not determined a priori (see Goulder 1995 for a survey).[8]

Finally we show, in a framework where authorities are benevolent and consumers are rational, that the lack of credibility of the government is the major cause of trouble. A government able to commit *ex ante* to inform truthfully would not, by definition, encounter the difficulties we discuss. If such a government is in place, fine. We remark however that distrustful attitude on the part of the public towards informed authorities is frequent: people often feel that

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models, for example, outcomes are observable, but the contribution of effort cannot be perfectly separated from random effects. As a consequence, contracts are rarely able to implement first-best efficiency.

[6]See Crawford and Sobel (1982).

[7]Note that our approach is valid if information campaigns are costly, but their cost is independent of the message the authority decides to send. This means that the diffusion cost of "smoke is detrimental to one's health" is the same as that of "smoke is *very* detrimental to one's health". As a consequence, if no-message is not a choice, we can, without generality loss, normalize this cost to zero.

[8]Actually the marginal cost of public funds can be either positive or negative, depending on the relationship between preexisting taxes and the tax on the good harmful to health. Our model covers all the cases.

the government's actions are motivated by *economic* interest more that by the *public* interest (think about information diffusion concerning the HIV contaminated blood in the eighties in France and Germany, or concerning the "mad cow" disease in Europe).

To solve the social game sketched here, we use an approach based on Bayesian equilibrium notions: people are not systematically fooled and the government tries to get the best of its available instruments. The main practical implications of our model are the following. We analyze one of the possible causes of consumers' distrustful behavior against the public authority. We establish the trade-off between the precision of the information transmitted and the optimality of the policy implemented: precision is higher with less efficient political programs, and *vice versa*. We prove that the equilibrium is never efficient *ex ante*, and that there exists a unique fully revealing equilibrium which is almost surely inefficient *ex post*.

Plan Section 2 presents the terms of the policy dilemma in the case of commodities affecting health and the environment. We define the equilibrium in Section 3. Sections 4–7 develop our methodology. After a few results on the structure of the government's preferences (Section 4), the analysis is developed in three steps. Firstly we show that an equilibrium can be summed up by its "skeleton", i.e. a relatively small set of policies satisfying incentive compatibility for the sender (Section 5). Secondly, we show under which circumstances a given "skeleton" can be implemented in an equilibrium. This is crucial to have insights on the structure of partially revealing equilibria (Section 6). Finally we characterize the unique fully revealing equilibrium of the game (Section 7). Implications concerning tax policy for fuels, SO_2 emissions and drugs are discussed in the conclusion.

2 The Model

2.1 The Consumers

Consumers live two periods and the value of their period-two consumption x_2 is negatively affected by period-one consumption x_1. Preferences can be written as:

$$U[x_1] + x_2 - \theta x_1 - \eta \overline{x}_1 \tag{1}$$

where U is the logarithmic utility function.[9] The consequences of x_1 on period-two utility pass through two distinct channels:

[9] Most propositions in Section 4 (characterization of the equilibria) do not rely on this restriction on utility, as can be seen in the proofs. The explicit calculations of the first- and second-best that we can perform with the logarithm are nevertheless more legible.

- The term $-\theta x_1$ measures **side effects** due to the consumer's own consumption in period 1. The intensity θ is not precisely known to consumers. The cumulative distribution function $F(\theta)$ and its density $f(\theta)$, both supported in $[\underline{\theta}, \overline{\theta}]$, represent consumers' priors on θ. In general, f is continuous and non negative on the support.

- The term $-\eta\overline{x}_1$ indicates the negative **externality** which depends on $\overline{x}_1$, that is on *average* period-one consumption in the economy. The intensity η is supposed to be known to all the agents.[10] The consumer does not internalize the social consequence of x_1. This happens because there is a large number of atomistic consumers in the economy: each consumer knows that he would affect the externality only marginally.

Let t be the tax rate used by the government. The representative consumer, not internalizing the externality η, solves:

$$\begin{cases} \max_{x_1,x_2} E\left[U[x_1] + x_2 - \theta x_1\right] \\ \text{s.t.} : (p_1 + t)x_1 + p_2 x_2 = W \end{cases} \tag{2}$$

where the expected value of utility is conditional on the consumer's information; p_1 and p_2 are the prices for, respectively, period-one and -two consumptions, and W is the consumer's endowment.

To further simplify the program we normalize p_2 to 1 and p_1 to 0 without loss of generality since the support of θ can be translated to account for the price, which is exogenous. Then we substitute the budget constraint in the objective function and we drop the subscripts to write the first period consumption as x. The simplified consumer's program is:[11]

$$\max_x E\left[U[x] - (\theta + t)x\right] \tag{3}$$

As a consequence, consumption choice x^* depends on the consumer's information and on the tax rate t:

$$x^*[t, E\theta] \text{ solves } U'[x] = E\theta + t \tag{4}$$

that is

$$x^*[t, E\theta] = \frac{1}{E\theta + t} \tag{5}$$

[10]An alternative model could put uncertainty on η. In general, though, this uncertainty alone would not exhibit the sort of conflict we are pointing at since the consumer's behavior is not affected by the intensity of the externality. In our specification, consumption does not even depend on η.

[11]Notice that the linear part in the preferences also represent the utility from goods other than x.

2.2 Social Welfare and the Marginal Cost of Public Funds

The social welfare function that is maximized by the government corresponds to the consumers' utility once the externality and the exact value of side effects are taken into account. The fiscal revenue from income, capital, or other commodities is exogenous and taxation is distortionary. Within this public finance perspective, we calculate how x should be taxed. In addition to the fact that taxes are imperfect, the government is hampered by its inability to commit to a policy that informs truthfully on the value of θ. In other terms, the government is benevolent since it evaluates consumption in the consumers' best interest, but opportunistic since it doesn't value truthful information *per se*, and would deceive consumers provided this induces "better" behavior (and reduces distortions caused by the tax). This attitude should be seen as a variety of paternalism.

All consumers being identical, in equilibrium $\bar{x} = x$, and the government's objective function can be represented as:

$$U[x] - (\theta + \eta)x + S - (1 + \lambda) R \tag{6}$$

where S is the consumer's surplus from public expenditures R. The government raises R with general taxation at the welfare cost $(1 + \lambda) R$, with $\lambda > -1$. Generally, in partial equilibrium models, the parameter λ is called the shadow cost of public funds and it represents the distortion due to the raising of fiscal revenue.[12]

In our model, R and S are constant, and a new tax on the good x is added to preexisting taxes. As the modern public finance theory has shown, no general conclusion can be drawn about the sign of the shadow cost of taxation when a revenue-neutral substitution between different taxes is implemented. The sign of λ is not restricted *a priori* and depends on the structure of preexisting taxes (in particular their level of efficiency) and on the interaction between them and the new tax.[13]

When the government introduces a tax t on good x and the tax revenue tx is devoted to reducing preexisting taxes, (6) becomes, after simplification:

$$U[x] - (\theta + \eta - \lambda t)x + S - (1 + \lambda)R \tag{7}$$

Comparing (3) and (7), we see that there are three main differences between the government's and the consumer's programs. The first is the superior information on θ; the second is the internalization of η by the government, and the third is

[12]See, for example, the shadow cost of public funds used in the theory of regulation (Laffont and Tirole 1993). In *general* equilibrium models of taxation (e.g. Ramsey), λ would be the (endogenous) Lagrange multiplier associated to the government's budget constraint. Under some regularities conditions, the Lagrange multiplier is equivalent to what the theory of cost-benefit analysis calls the shadow cost of a marginal change in a public project. See Drèze and Stern (1987).

[13]For a synthetic discussion on this issue, the reader can see Ballard and Fullerton (1992) and Goulder (1995).

the presence of λ in the government's objective function. As for the externality, the consumer does not internalize the effect of *his* contribution tx on the total distortion caused by taxation.

The case $\lambda > 0$ (preexisting taxes inflict a welfare cost larger than R) relates to the recently debated "double dividend" effect. According to this literature, a revenue-neutral substitution of environmental taxes for ordinary income taxes might offer a double dividend: not only (1) it improves the environment but also (2) it reduces the costs of the tax system through cuts in distortionary taxes (see Goulder 1995). To grasp an intuition of this result, assume that x and the other taxed goods (labor included) are gross substitutes. In that case, typically, the tax t reduces the consumption of x and increases the consumption of the other taxed goods. Thus, the total fiscal revenue increases as well, and taxes on the other goods can be reduced, which attenuates distortions.[14,15] Notice that we could also reason in terms of relative efficiency: when $\lambda > 0$ a tax on good x is relatively less distortionary than preexisting taxes. When $\lambda < 0$, a tax on good x is relatively more distortionary than preexisting taxes.

Dropping constant terms which are not influent for policy decisions, a reduced form of the government's objective function is:

$$SW[x,t,\theta] \equiv U[x] - (\theta + \eta - \lambda t)x \tag{8}$$

2.3 Constrained Efficient Allocations

The *first-best allocation* is defined as the welfare maximizing allocation the government would choose if it could impose consumption. This implies $x_{\text{FB}}(\theta) = \frac{1}{\theta+\eta}$. This allocation can be implemented even without full control over x with the standard Pigovian tax $t = \eta$ only if $\lambda = 0$ (no distortionary taxes on the other goods).

The *second-best allocation* is defined as the best the government can reach with consumers perfectly informed on θ when (1) it is constrained to a linear tax on x, and (2) the marginal cost of public funds is not zero. This can be written as follows:

$$\begin{cases} \max\limits_{t} U[x] - (\eta + \theta - \lambda t)x \\ \text{s.t.} : x = \frac{1}{\theta+t} \end{cases} \tag{9}$$

Hence the second-best consumption and tax rate

$$\begin{cases} x_{\text{SB}}(\theta) = \frac{1}{\eta+(1+\lambda)\theta} \\ t_{\text{SB}}(\theta) = \eta + \lambda\theta \end{cases} \tag{10}$$

[14] Complementarity between x and the other taxed goods allows the same reasoning to hold when $\lambda < 0$.

[15] The same reasoning in terms of substituability and complementarity between x, the other goods, and the public project (financed by R) may be done. In other words, the cost of public funds also depends on the interaction between the public expenditures and the taxed activities.

Because of the "double dividend", the second-best tax is larger than the first-best tax when $\lambda > 0$. The opposite holds for $\lambda < 0$. For a given λ, the tax rate is strictly increasing (decreasing) with respect to θ when $\lambda > 0$ ($\lambda < 0$). In any case, it is important to remark at this step that the tax rate is potentially informative on the value of side effects.

Straightforward calculations lead to a sort of Ramsey-Boiteux pricing rule:[16]

$$\frac{t_{\mathrm{SB}}(\theta) - \frac{\eta}{1+\lambda}}{t_{\mathrm{SB}}(\theta)} = \frac{\lambda}{1+\lambda}\frac{1}{\varepsilon} \tag{11}$$

where $\varepsilon \equiv -\frac{\partial x^*}{\partial t}\frac{t}{x^*} = \frac{t}{\theta+t}$ is the tax elasticity of demand. Tax elasticity being decreasing in θ for any t, (11) shows that, for positive λ, the stronger the side effects, the higher the tax. The opposite holds for negative λ.

From (8), and for a given tax t, the marginal net external effect of consumption x is $\eta - \lambda t$, where η is for external effects *sensu stricto*, and $-\lambda t$ for the effects of the tax on public finance that the consumers do not internalize. Assume that $\lambda > 0$. Firstly, the social welfare function (8) shows that, others things (in particular x) equal, the government would like to impose a "large" (in fact infinite) tax, in order to "create" large positive externalities. Secondly, given such positive externalities, the government has to choose how to tax x in order to make consumers internalize these huge positive effects; according to the Pigou rule, $|t - (\eta - \lambda t)|$ should be minimized and this is attained at $t = \bar{t} \equiv \frac{\eta}{1+\lambda}$. Unfortunately, these two arguments draw in opposite directions and the two goals rely on the same instrument t. This explains why the optimal trade-off is $t_{\mathrm{SB}}(\theta)$, a value between $\bar{t}$ and $+\infty$.[17]

3 The Influence Game

The timing of the model is as follows: firstly the government observes θ, secondly it chooses its policy and finally the consumer, observing the policy, updates his beliefs on θ and chooses his consumption level.

A policy $P = (t, m) \in I\!R \times M$ is composed of the tax rate t and a (cheap talk) "message" m taken in a certain large set M. Through the choice of a policy P, the government wants the consumer to approach the efficient consumption. The tax has the double role of providing incentives and signaling information, while cheap talk can only transmit information. We can think of m as composed of a "sentence". We assume that M is rich enough to say what needs to be said; for example it can be composed by all reasonably short utterances in English (see, e.g., Farrell and Rabin 1996 on what cheap talk is and is not). It is useful,

[16]A similar expression can be found in Sandmo (1975). See also Bovenberg and van der Ploeg (1994).

[17]When $\lambda < 0$, with a fixed x, the government would like large subsidies ($t \to -\infty$); $t_{\mathrm{SB}}(\theta)$ is between $-\infty$ and $\bar{t}$.

at this point, to make a distinction between the message the government sends and the interpretation that the consumer gives to such a message at the equilibrium. What really matters is not the message itself, but the way the consumer understands the policy. To be clearer, whatever the language that is used to communicate, we will concentrate on the meaning (the revised $E\theta$) the consumer assigns to every policy.[18]

After observing the policy, the consumer updates his priors which are then denoted by $\mu(P)$ (with $\mu(P) \in \Delta([\underline{\theta}, \overline{\theta}])$, the set of probability distributions over $[\underline{\theta}, \overline{\theta}]$). We denote $E(\theta|P)$ by $\widehat{\theta}(P)$.

Definition 1 *A Perfect Bayesian Equilibrium (PBE) of the game is a pure strategy $\mathcal{P}$ mapping $[\underline{\theta}, \overline{\theta}]$ into $\mathbb{R}_+ \times M$ and a belief μ mapping $\mathbb{R} \times M$ into $\Delta([\underline{\theta}, \overline{\theta}])$ such that:*

1. *Policies are optimal given beliefs: for each $\theta \in [\underline{\theta}, \overline{\theta}]$, $\mathcal{P}(\theta)$ solves*

$$\max_P SW[x^*[t, \widehat{\theta}(P)], t, \theta]. \tag{12}$$

2. *Beliefs are rational given equilibrium policy: for each P, $x^*[t, \widehat{\theta}(P)]$ solves*

$$\max_x \int_{\underline{\theta}}^{\overline{\theta}} [U[x] - (\theta + t)x]\mu(\theta|P)d\theta, \tag{13}$$

where $\mu(\theta|P) \equiv \frac{\mathbb{I}_{\{\mathcal{P}(\theta)=P\}} \cdot f(\theta)}{\int_{\underline{\theta}}^{\overline{\theta}} \mathbb{I}_{\{\mathcal{P}(\theta)=P\}} \cdot f(s)ds}$, $\mathbb{I}$ being the indicator function.

The reader worried that revised beliefs may not always be well-defined can see our Proposition 3.

4 Policy Preferences of the Government

Bad news for communication gurus: the consumer's rationality prevents the government to turn lead into gold with nice communication strategies. In other words there is no perverse reason why propaganda would make less desirable states of the world (larger side effects) preferable.

Proposition 1 *In any equilibrium, the larger the side effects, the lower the social welfare.*

[18] As an example, let m_1 and m_2 denote two messages sent in a perfectly revealing equilibrium. Assume that m_1 corresponds to the word "dog" and m_2 corresponds to the word "cat". This is an equilibrium as long as the receiver understands this language and assigns to the message "dog" the meaning, say "$\theta = \theta_1$", and to the message "cat" the meaning, say "$\theta = \theta_2$", where θ_1 and $\theta_2 \in [\underline{\theta}, \overline{\theta}]$.

Proof. Let θ_1 and θ_2 be two possible states of the world, and let $P_1 = (t_1, m_1)$ and $P_2 = (t_2, m_2)$ be two equilibrium policies. The consumption levels induced by the two equilibrium policies respectively are x_1 and x_2. If $\theta_1 < \theta_2$, then $U[x_2]-(\eta+\theta_1-\lambda t_2)x_2 \geq U[x_2]-(\eta+\theta_2-\lambda t_2)x_2$. On the other hand, the incentive constraint of the type-θ_1 social planner reads: $U[x_1]-(\eta+\theta_1-\lambda t_1)x_1 \geq U[x_2]-(\eta+\theta_1-\lambda t_2)x_2$. By transitivity, we get: $U[x_1]-(\eta+\theta_1-\lambda t_1)x_1 \geq U[x_2]-(\eta+\theta_2-\lambda t_2)x_2$. Thus the social planner's pay-off decreases with respect to the side effects.■

In the rest of this section, we analyze the government's incentives to manipulate information, i.e. the reasons why, and to what extent, the government's actions and allegations are likely to be suspicious for the consumer.

Remark 1 *In equilibrium, any policy P can be analyzed without loss of insight as a pair $(t, \widehat{\theta})$, where t is the tax rate, and $\widehat{\theta}$ the belief associated to the policy.*

We define $\overline{SW}[t, \widehat{\theta}, \theta] \equiv SW[x^*[t, \widehat{\theta}], t, \theta]$ as the value of a policy characterized by the tax-beliefs pair $(t, \widehat{\theta})$ for a government of type θ. Reasoning directly on tax-beliefs pairs allows a simpler analysis of incentive constraints, independently of the cheap talk message sent by the government. Indeed, incentive compatibility for $P(\theta) = (t, \widehat{\theta})$ and $P(\theta') = (t', \widehat{\theta}')$ can clearly be checked by comparing $\overline{SW}[t, \widehat{\theta}, \theta]$ with $\overline{SW}[t', \widehat{\theta}', \theta]$, and $\overline{SW}[t, \widehat{\theta}, \theta']$ with $\overline{SW}[t', \widehat{\theta}', \theta']$.

The consumer solves $U'[x] = \widehat{\theta}(P) + t$. Therefore from (8), we can see that the consumer's choice equals the socially optimal consumption when:

$$\widehat{\theta}(P) + (1 + \lambda)\, t = \theta + \eta \tag{14}$$

Suppose that the consumer is naive and believes whatever announcement of the government. The government would exactly set the tax rate and induce beliefs so that (14) is verified. Notice that the right hand side of (14) is a constant. When $\lambda > 0$ (t rises social welfare), the government relatively prefers to set high taxes and to induce low beliefs. In other words it prefers to make the consumer *optimistic* about side effects, and mostly relies on taxation. The opposite is true when $\lambda < 0$ (t yields deadweight losses): the government prefers to make the consumer *pessimistic* about side effects, and to drive taxation to its lowest level.

In front of a rational consumer, this form of policy is obviously never consistent; nevertheless, it gives useful indications on the incentives perceived by the government. For instance, when cheap talk only is available, equation (14) becomes $\widehat{\theta}(P) = \theta + \eta$. In this case, the government has always incentives to overstate the value of θ such as to make the consumer internalize the externality. The setting is then similar to Crawford and Sobel (1982). This explains why the health authority is better off when consumers have an overly high perception of the side effects of antibiotics, as mentioned in the introduction.

We are more formal now. Policies are restricted to induce finite consumption, i.e. interior solutions for the consumer's program. Thus, feasible policies are such that $t + \widehat{\theta} > 0$. The difficulty here is that indifference curves are not monotonic:

there is an optimal policy (unfortunately inconsistent with Bayesian consumers as we will make clear), and utility decreases as the tax and the belief gets farther from the optimum. Nevertheless, the following proposition gives useful properties to go on with the analysis of incentive compatibility.

Proposition 2 *1. For all θ, the upper contours of $\overline{SW}$ with respect to t and $\widehat{\theta}$ are convex.*

2. For all θ, tangents to indifference curves are horizontal along the straight line $(1+\lambda)t+\widehat{\theta} = \eta+\theta$, and vertical along the straight line $t+(1-\lambda)\widehat{\theta} = \eta+\theta$. The overall optimum is the intersection of these lines ($t = \frac{\eta+\theta}{\lambda}, \widehat{\theta} = -\frac{\eta+\theta}{\lambda}$); the optimum with $t = 0$ is $\widehat{\theta} = \theta + \eta$.

3. Let $\mathcal{V}(\theta)$ be an indifference curve for type θ passing through $(t, \widehat{\theta})$. $\mathcal{V}(\theta)$ turns continuously clockwise if $\lambda > 0$ (anti-clockwise if $\lambda < 0$) locally at $(\widehat{\theta}, t)$ as θ increases and indifference curves related to two different types cross once at most.

Proof. 1. It suffices to verify that the utility is quasi-concave. To do this, we check that the successive principal minors of the bordered Hessian matrix alternate signs (odd principal minors have to be positive). The bordered Hessian matrix is:

$$\begin{bmatrix} 0 & \frac{\eta+\theta-t-(1-\lambda)\widehat{\theta}}{(t+\widehat{\theta})^2} & \frac{\eta+\theta-(1+\lambda)t-\widehat{\theta}}{(t+\widehat{\theta})^2} \\ \frac{\eta+\theta-t-(1-\lambda)\widehat{\theta}}{(t+\widehat{\theta})^2} & -\frac{2\eta+2\theta-t-(1-2\lambda)\widehat{\theta}}{(t+\widehat{\theta})^3} & -\frac{2\eta+2\theta-(1+\lambda)t-(1-\lambda)\widehat{\theta}}{(t+\widehat{\theta})^3} \\ \frac{\eta+\theta-(1+\lambda)t-\widehat{\theta}}{(t+\widehat{\theta})^2} & -\frac{2\eta+2\theta-(1+\lambda)t-(1-\lambda)\widehat{\theta}}{(t+\widehat{\theta})^3} & -\frac{2\eta+2\theta-(1+2\lambda)t-\widehat{\theta}}{(t+\widehat{\theta})^3} \end{bmatrix} \tag{15}$$

The first principal minor is equal to zero, the second is negative and we find $\frac{\lambda^2}{(t+\widehat{\theta})^4}$ for the third. This gives the result.

2. and 3. The MRS between t and $\widehat{\theta}$ is

$$\left.\frac{dt}{d\widehat{\theta}}\right|_{\overline{SW}=\text{constant}} = -\frac{\eta+\theta-(1+\lambda)t-\widehat{\theta}}{\eta+\theta-t-(1-\lambda)\widehat{\theta}} \tag{16}$$

Its derivative with respect to θ is

$$-\frac{\lambda(t+\widehat{\theta})}{(\eta+\theta-t-(1-\lambda)\widehat{\theta})^2} \tag{17}$$

which is negative (positive) for $\lambda > 0$ ($\lambda < 0$) for $t+\widehat{\theta} > 0$. Tangents to indifference curves being vertical whenever $\eta+\theta-t-(1-\lambda)\widehat{\theta} = 0$, the claim is correct. The optimum is the singular point where both the numerator and the denominator of (16) equal zero.

Notice that, if one puts aside domain restrictions, upper contours are closed, meaning that two indifference curves related to two different types, if ever they cross, cross twice at least. We proved here that in the relevant range $(t + \widehat{\theta} > 0)$ crossing occurs once at most, which is sufficient to retrieve the standard argument based on single crossing. ■

Figure 1 shows the government's indifference curves for $\eta = 1$, $\underline{\theta} = 0$, $\bar{\theta} = 1$, $\lambda = .7$ and $\theta = 1$.

Insert figure 1 here.

We prove now that the second-best policy is not implementable in a Bayesian equilibrium. Suppose the consumer *thinks* that the government plays the second-best strategy. The tax schedule t_{SB} being invertible, the individual can infer unambiguously θ if $t_{\text{SB}}(\theta)$ is imposed. Nevertheless, it is not possible to implement this allocation in a Bayesian equilibrium. In fact, the fiscal revenue $t_{\text{SB}}(\theta)x_{\text{SB}}(\theta)$ increases as θ decreases. When $\lambda > 0$ and θ is high, the government may have interest in reporting a lower θ, in other terms in making the consumer optimistic. On the contrary, when $\lambda < 0$, the government may have interest in making the consumer pessimistic.

Corollary 1 *The second-best allocation is never an equilibrium if $\lambda \neq 0$.*

Indeed, at $(t_{\text{SB}}(\theta), \theta) = (\eta + \lambda\theta, \theta)$, for all θ, the tangent of the indifference curve of the government is vertical (see point 2 in the Proposition): small changes in the tax have first-order effects, whereas small changes in the beliefs have only second-order effects on the government's objective. In consequence, if $\lambda > 0$, any policy close to the second-best $(\eta + \lambda\theta, \theta)$ but with $t < \eta + \lambda\theta$ is preferred; this is the case for a second-best policy associated to close but smaller type. If $\lambda < 0$, second-best policies associated to close but larger types are preferred. In any case, the second-best allocation is not incentive compatible, which confirms that the government faces strong incentives to provide biased information.

Notice in contrast that, when $\lambda = 0$, the first-best allocation is implementable in a PBE. Indeed, given that the government has no incentive to lie (see (14)), the tax is specifically used to internalize the externality $(t = \eta)$, but the tax rate being uninformative on θ, cheap talk *has to* be used to eliminate asymmetric information. With a slight abuse, equilibrium policies can be written as $P = (t = \eta, \widehat{\theta} = \theta)$, where information is fully transmitted. This is, of course, very particular.

5 Skeletons

A description of all the equilibria given the prior type distribution is difficult to perform. Hence, instead of looking for equilibria in the traditional way for signalling games, we introduce a different technique. We solve the inverse problem: we find the set of types and the distributions of types which are consistent with a certain equilibrium allocation. This new approach to equilibria has some relationship with mathematical tools mostly used in imagery (namely Voronoi diagrams, and its dual, Delaunay triangulation) from which we borrow our vocabulary (the skeleton).[19] The analogy is the following: given a partition of the types, types in each subset applying the same policy, and two different subsets applying different policies, one may be interested in the underlying policies. Conversely, given a certain set of policies, and given that the government responds to its incentives, one may be interested in the types which have to be associated with each policy. In all these problems, preferences can be seen as a measure of distance.

The following proposition generalizes the well-known result of Crawford and Sobel (1982, henceforth CS) that all equilibria are "partition equilibria". See also Austen-Smith and Banks (2000).

Proposition 3 *Any equilibrium allocation can be implemented in an equilibrium in which there exists a partition of* $[\underline{\theta}, \overline{\theta}]$ *into a set of intervals* $\{I_k\}_{i \in K}$ *(*K *is a minimal set of indices) and a set of policies* $\{P_k\}_{k \in K}$ *such that (i) the policy chosen in* I_k *is* P_k*, and (ii)* $k \neq k'$ *implies* $I_k \neq I_{k'}$ *and* $P_k \neq P_{k'}$*. Moreover, the effects of policy* P_k *are entirely characterized by the pair* $(t_k, \widehat{\theta}_k)$*, where* $\widehat{\theta}_k \equiv E(\theta | P_k) = E(\theta | I_k)), \forall k$*.*

Proof. In this proof, optimal is used in the weak sense. In any PBE, for all P being an equilibrium action, the set of types for which P is optimal is a convex subset of $[\underline{\theta}, \overline{\theta}]$. To see this, let's consider the sender's incentive constraint in a given equilibrium. Type θ will prefer policy $P_1 = (t_1, m_1)$ to any $P_2 = (t_2, m_2)$, implying, respectively, consumptions x_1 and x_2, if and only if:

$$
\begin{aligned}
(18) U[x_1] - (\eta + \theta - \lambda t_1) x_1 &\geq U[x_2] - (\eta + \theta - \lambda t_2) x_2 \Leftrightarrow \\
\theta(x_2 - x_1) &\geq U[x_2] - U[x_1] + \eta(x_1 - x_2) + \lambda t_2 x_2 - \lambda t_1 x_1
\end{aligned}
$$

The latter equation defines either a half straight-line in the space of types ($x_1 - x_2 \neq 0$) or the whole real line ($x_1 = x_2$). From this, it follows that if policy P is optimal for two values of θ, then P is optimal for any type between these two values.

[19]The idea is basically the following: the Voronoi diagram of a point set $\mathcal{P}$ is a subdivision of the plane with the property that the Voronoi cell of point p contains all locations that are closer to p than to every other point of $\mathcal{P}$. The points of $\mathcal{P}$ are also called Voronoi generators. Each edge of a Voronoi cell is the bisector of the connection of p to the corresponding neighbour cell. See http://www.voronoi.com/ for theory, algorithms, and examples of applications.

Let's denote by (θ_1, θ_2), with $\theta_1 \neq \theta_2$, an interval in which P_1 is optimal. We check now that there is only one optimal policy in the interval. Suppose it is not the case, e.g. $\exists \theta \in (\theta_1, \theta_2)$ for which both P_1 and $P_2(\neq P_1)$ are optimal. Equation (18) becomes

$$(19) \qquad \theta(x_2 - x_1) = U[x_2] - U[x_1] + \eta(x_1 - x_2) + \lambda t_2 x_2 - \lambda t_1 x_1$$

A consequence is that either $x_1 \neq x_2$, and, according to (18), P_1 is strictly preferred to P_2 on one side of θ, and P_2 is strictly preferred to P_1 on the other side, which is in contradiction with our assumption that P_1 is optimal on (θ_1, θ_2); or $x_1 = x_2$, which implies in turn that $t_1 = t_2$, and, given that consumptions are only a function of the tax and the beliefs, that P_1 and P_2 imply the same beliefs. In this case, P_1 and P_2 are the same in terms of tax and beliefs. Though they may differ in their cheap talk dimension, Remark 1 justifies why they can be seen as identical.

If an equilibrium allocation were not implementable by a strategy based on a partition in intervals, then the latter result would be false. This proves the claim. ■

One substantial implication of this result is that Condition 1 in the definition of the equilibrium (Section 3) implies the non evident property that beliefs in Condition 2 are well-defined (indeed, strategies inherit the measurability of the space of types).

Proposition 3 suggests that only "few" tax-beliefs pairs are interesting. We can go further and show that only "few" incentive compatibility constraints have to be checked to ensure that an allocation is an equilibrium.

Definition 2 (Skeleton) *Let $\{\theta_k\}_{k\in K}$ be a close subset of $[\underline{\theta}, \overline{\theta}]$ in which $k \neq k'$ implies $\theta_k \neq \theta_{k'}$ (K is a minimal set of indices), and let $\{t_k\}_{k\in K}$ be a set of real numbers. $\{(t_k, \theta_k)\}_{k\in K}$ is said to be a* skeleton *if and only if $\forall k, k' \in K, \overline{SW}[t_k, \theta_k, \theta_k] \geq \overline{SW}[t_{k'}, \theta_{k'}, \theta_k]$ (incentive compatibility).*

One particularity of the skeleton is that any equilibrium to which it is connected is revealing for the types of the skeleton, and for these types only (this is represented by the incentive constraints in the definition of skeleton).

Proposition 4 below is the reciprocal of Proposition 3. We exploit the idea that the skeleton represents the essential data that characterize an equilibrium. The type support can be divided into intervals in which the strategy is pooling, and we specify the restrictions on the "flesh" (the distribution F) that can be put on the "bones" (the skeleton) to have an equilibrium.

Proposition 4 *Let $\mathcal{F}$ be the set of type distribution F such that the skeleton $\{(t_k, \theta_k)\}_{k\in K}$ is an equilibrium set of policies. There exists a partition of $[\underline{\theta}, \overline{\theta}]$ into a set of intervals $\{I_k\}_{k\in K}$ with $\theta_k \in I_k$ such that: $\forall F \in \mathcal{F}, \forall k$, $t(\cdot) = t_k$ over I_k and $E(\theta|I_k) = \theta_k$.*

Proof. By convention, we denote the lowest element of $\{\theta_k\}_{k\in K}$ as θ_1, and the largest as θ_∞. Given θ_k, we define its successor in $\{\theta_k\}_{k\in K}$ as $\theta_{k+1} \equiv \min_{k'\in K}\{\theta_{k'} > \theta_k\}$ (this "+1" is just a convention, inspired by the fact that when K is finite, it can be reformulated as a set of successive integers). The type θ_{k+1} is well defined since a skeleton is close.[20] We reason on incentive compatibility.

If $\theta_{k+1} \neq \theta_k$, we denote by τ_k a type which is indifferent between $P(\theta_k)$ and $P(\theta_{k+1})$, i.e. $\overline{SW}[t_k, \theta_k, \tau_k] = \overline{SW}[t_{k+1}, \theta_{k+1}, \tau_k]$. Given the single crossing property, and given the continuity of the government's welfare function with respect to the true type, τ_k is unique and belongs to $[\theta_k, \theta_{k+1}]$. We define $I_k = (\tau_{k-1}, \tau_k]$. If the successor of θ_k is θ_k itself (this happens if θ_k is, on the right, an accumulation point in $\{\theta_k\}_{k\in K}$), then $I_k = \{\theta_k\}$. The lower bound of the lowest interval (i.e. containing θ_1) is $\underline{\theta}$, and the upper bound of the upper interval (containing θ_∞) is $\overline{\theta}$. Given Proposition 3, $t(\cdot) = t_k$ over I_k for all k is incentive compatible. Finally, to ensure that the equilibrium beliefs of the consumer are Bayesian, it is necessary and sufficient that $F(\cdot)$ be such that $E(\theta|I_k) = \theta_k$. ■

Conditional expectations (with respect to the policy, or to the interval) are independent from each other. The probability associated to the *interval* I_k not being constrained, $F \in \mathcal{F}$ can be chosen as smooth as wanted.

Corollary 2 *If two different intervals are associated with two different tax rates, then the tax rate is sufficiently informative for the consumer, and the message can be ignored. If there exists $k \neq k'$ such that $t_k = t_{k'}$, then messages are indispensable to signal the right interval and ensure the right beliefs.*

Indeed, when the tax rate is the same for two or more intervals, cheap talk serves to transmit some information. In the terminology of Austen-Smith and Banks (2000), costless signalling is *influential* if two different cheap talk messages associated with the same tax rate have to be used to distinguish two different intervals.

Similarities with CS are obvious: Propositions 3 and 4 show that the government can use meaningful yet imprecise policies to communicate on the side effects to consumers. The government having interest in lying on the value of θ to make consumers internalize the externality, powerful communication campaigns would give the government the means of manipulating consumers' beliefs. As a consequence, the government is restricted in equilibrium to vague statements that only specify broad ranges within which θ may lie.

This trade-off is classical for readers accustomed to cheap talk: the partition $\{I_k\}_{k\in K}$ entails a loss in precision, but now, if the government wants to lie, it

[20] Notice that we assume that $\{\theta_k\}_{k\in K}$ is close only to simplify our reasoning. This assumption is indeed without generality loss: if an accumulation point of $\{\theta_k\}_{k\in K}$ were missing (i.e. if $\{\theta_k\}_{k\in K}$ were not complete), we could add it to $\{\theta_k\}_{k\in K}$, with a corresponding accumulation point in $\{t_k\}_{k\in K}$. Due to the continuity of the incentive constraints, incentives are not reversed, and the skeleton is completed.

has to pretend that side effects are in a different subinterval, which changes consumers' consumption by a discrete amount. Such "big lies" are less attractive than telling the truth.

A less evident conclusion is that there are also considerable differences with CS. Our emphasis on skeletons enables us to show that partitions need not be finite, meaning that the precision of the message may be arbitrarily high locally. In this sense a new trade-off arises: as precision increases, tax policies are more severely constrained by incentive compatibility, and distortions away from the second-best become large.

6 Partially Revealing Equilibria

We can start to build an equilibrium by choosing a skeleton, and fill the distribution while preserving conditional expectations. Proceeding in this way, we can easily give examples in which the tax rate is not monotonic, where it is revealing on certain subsets of $[\underline{\theta}, \overline{\theta}]$ with bundles elsewhere, etc. As a consequence a multiplicity of partially revealing and pooling equilibria are conceivable.

Proposition 4 does not claim that some distribution F always exists. Indeed, even off-equilibrium beliefs are constrained to be in $[\underline{\theta}, \overline{\theta}]$, and we may be short of sufficiently dissuasive off-equilibrium beliefs to support a skeleton. We are nevertheless able to give a simple way of extending a skeleton to make $\mathcal{F}$ non empty, in other words, to implement the skeleton in a PBE.

Proposition 5 *Any skeleton is either directly implementable or can be made implementable by adding* one *policy (one belief and its associated tax).*

Proof. Let us take a non implementable skeleton. If $\lambda > 0$, the simplest way to complete it is to add a sufficiently low type, say $\theta_{\min} < \theta_1$, coupled with $t_{\mathrm{SB}}(\theta_{\min})$. This may entail enlarging $[\underline{\theta}, \overline{\theta}]$ by replacing $\underline{\theta}$ by $\theta_{\min}$. If we associate belief $\theta_{\min}$ to any tax outside $\{t_k\}_{k \in K}$, we still have a skeleton. To check incentive compatibility, remark that if the belief $\theta_{\min}$ is sufficiently small compared to θ_0, such a belief is necessarily too small compared to any type of government drawn in $\{\theta_k\}_{k \in K}$. Moreover $t = t_{\mathrm{SB}}(\theta_{\min})$ is better than any other value of the tax for a government of type $\theta_{\min}$. The new skeleton is now implementable. If $\lambda < 0$, the same reasoning with a large $\theta_{\max} > \theta_{\infty}$ (coupled with $t_{\mathrm{SB}}(\theta_{\max})$) is applicable. ■

This suggests that, provided priors are defined over a sufficiently large set, and even if extreme types are extremely unlikely, one may take advantage of the presence of "scarecrow" types to build equilibria.

An important difference exists between pure cheap talk models and ours. Indeed, with finite skeletons, cheap talk doesn't really need to be influential (i.e. useful) since, either all taxes are different, or some are identical and we can use the continuity of the incentive constraints to modify the skeleton slightly and make all tax rates different, in which case cheap talk is useless. In other words, suppose that (t_k, m_k) and $(t_{k'}, m_{k'})$ are two equilibrium policies with $t_k = t_{k'} = t$ and $m_k \neq m_{k'}$ (cheap talk is influential). If we change one of the two taxes, the partition in the skeleton has to be modified, but changes remain small because there is only a finite number of bones (hence a finite number of continuous incentive constraints) and the welfare cost of doing so is arbitrarily low. The extension of this intuition to a large set of signals, is not developed here. See Manelli (1996) for another approach to the same sort of result (i.e. cheap talk closes but does not substantially extend the set of equilibrium allocations). The previous reasoning shows that the role of cheap talk as stated in Proposition 4 is neatly diminished, since in a quite strong sense, cheap talk is almost useless when a costly message (here the tax) is available.

7 Fully Informative Equilibria

By definition, fully informative equilibria have exhaustive skeletons in which all types are represented. Moreover, notice that Proposition 4 implies that the corresponding allocation is a *universal skeleton*, that is an equilibrium for *any* distribution F in $[\underline{\theta}, \overline{\theta}]$. The following proposition establishes that for a given $[\underline{\theta}, \overline{\theta}]$, there is a unique fully revealing equilibrium (or a unique universal skeleton) which we characterize in detail.

Proposition 6 *There exists a unique fully revealing equilibrium. The tax rate is the unique solution to the ordinary differential equation* $\frac{t'}{1+\lambda} = -\frac{t-\frac{\eta}{1+\lambda}}{t-t_{SB}(\theta)}$ *with the boundary condition* $t(\overline{\theta}) = t_{SB}(\overline{\theta})$ *if* $\lambda > 0$, *and* $t(\underline{\theta}) = t_{SB}(\underline{\theta})$ *if* $\lambda < 0$. *In particular:*

1. *Cheap talk is ineffective, and the strategy* $t(\cdot)$ *is strictly increasing and differentiable.*

2. *Consumption decreases with respect to* θ.

3. *If* $\lambda > 0$, *the tax rate exhibits no distortion at* $\overline{\theta}$. *For other values, the tax rate is smaller than the second-best tax rate and larger than* $\frac{\eta}{1+\lambda}$.

4. *If* $\lambda < 0$, *the tax rate exhibits no distortion at* $\underline{\theta}$. *For other values the tax rate is larger than the second-best tax rate and lower than* $\frac{\eta}{1+\lambda}$

Proof. See the Appendix. ■

Concerning the role of cheap talk, it is clear from point 1 that in the fully informative equilibrium, all the information is transmitted through the tax rate. When $\lambda > 0$ ($\lambda < 0$), the fully informative equilibrium allocation, compared to the second-best one, is characterized by too low (too large) taxes. Moreover, when $\lambda > 0$ ($\lambda > 0$) taxes are decreasing (increasing) with respect to the type.

The differential equation gives essential roles to the second-best tax and to $\bar{t} = \frac{\eta}{1+\lambda}$. Indeed, the government tries (though under incentive constraints) to maximize welfare, therefore to approach $t_{\text{SB}}(\theta)$ as much as possible. On the one hand, the closer to $t_{\text{SB}}(\theta)$ the tax, the higher the social welfare, but the stronger the incentives to lie and the steeper the slope of the revealing tax schedule. On the other hand, for tax rates approaching the suboptimal $\bar{t}$, incentives to manipulate beliefs vanish, and the revealing tax schedule flattens.[21]

Here is the origin of the distortion: credibility is gained by moving away from the optimal schedule. Remark that if the government could commit *ex ante* to that tax $\bar{t}$ whatever the state of nature θ, then telling the truth by means of cheap talk would be sequentially optimal since no credibility problem would arise. Unfortunately, this easy credibility would be bought at the cost of a severe lack of efficiency!

Figure 2 shows the fully revealing tax rate for $\eta = 1$, $\underline{\theta} = 0$, $\bar{\theta} = 1$ and $\lambda = .3$. Notice the indifference curves passing through the equilibrium value for $\theta = .8$ and $\theta = 1$. Figure 3 corresponds to $\lambda = -.3$ (other parameters are equal to those in figure 2). This illustrates the non negligible size of the distortion.

Insert figure 2 here.
Insert figure 3 here.

Another view of the limited role of cheap talk is the following. As $\lambda \to 0$, one can find a sequence of fully revealing equilibrium allocations converging to the first-best where $t = \eta$ for all θ. However, the first-best is not an equilibrium if we keep restricting the signal to be supported only by the tax, since no precise information on θ can be transmitted: at the limit, cheap talk is indispensable, but very close approximations in which it is not used are available.

In CS, the most informative equilibrium Pareto-dominates, ex ante, the others.[22] Austen-Smith and Banks (2000) find that this is not true when cheap talk and burning money to signal the type are used together. With our approach in terms of skeleton, it is relatively easy to see that the unique fully informative equilibrium allocation need not be efficient. To see this, take an equilibrium and take its skeleton. The substance of Proposition 4 is that any economy which satisfies the restrictions on the conditional expected type in the intervals associated to the skeleton can implement the skeleton in equilibrium. If the mass of an interval where the distortion is substantial is sufficiently large, then the equilibrium is necessarily inefficient *ex ante*. More generally, given two skeletons, one being more informative than the other (a finer partition in intervals), the

[21]On the properties of $\bar{t}$, see Subsection 2.3.

[22]See Theorem 3 and 5 in CS which say that both the sender and the receiver strictly prefer equilibrium partition with more steps.

less informative skeleton can be made more efficient by choosing adequately the distribution.

8 Conclusion

This work studies the conflict between providing incentives and transmitting information which arises when an informed and benevolent government combines linear taxes and information campaigns. Our model suggests that the government may have a hard time gaining credibility for its actions and messages, this even though its objective and the consumers' one are aligned. The problem is that the government cannot commit to reveal information truthfully. Its instruments being imperfect, it faces, in the course of action, strong incentives to improve their impact by providing biased information.

Depending on the kind of distortion prevailing in the fiscal system (i.e. whether the tax generates a "double dividend" or not), the government would like to make consumers either pessimistic or optimistic about the individual effect of consumption. In the likely case of a positive marginal cost of public funds, if the consumers were more optimistic about side effects, the government could set higher taxes without curbing too much consumption, and the distortions created by preexisting taxes could easily be alleviated. Fuel taxes are an example where the government may wish not to stress automobile dangers to preserve this easy source of money. The paradoxical consequence is that, at the fully revealing equilibrium, there is a bias towards exaggeratedly low taxes.

Our example of negative costs of public funds is quite informal. In France, SO_2 emissions are submitted to a "parafiscal" tax, meaning that there is an agency in charge of tax collection which also redistributes the proceeds as subsidies for abatement efforts. The agency is independent of the Treasury. Even if the latter faces a positive marginal cost of public funds, the former may face a negative marginal cost. Exaggerating local effects (represented by our θ in the agents'—here the firms—programs) to economize resources wasted in the costly collection/redistribution process may then be tempting. As a result, at the fully revealing equilibrium, the agency should paradoxically be biased towards exaggeratedly large taxes.[23]

Another policy implication of the model is that information campaigns à la Crawford and Sobel are almost superfluous when the social planner can use costly signals too. The costly tax is simply taken more seriously than cheap propaganda, and the efficacy of information campaigns characterized by short phrases whose contents is too vague to be verifiable ("tobacco is harmful to

[23]As far as health policy is concerned, one can imagine that health autorities are incited to deliver cautious messages on the side effects of antibiotics. In equilibrium, however, they have to limit seriously reimbursements (which is similar to imposing high tax rates) to signal toxic drugs.

health") is seriously limited. This result is in line with the empirical evidence of Bardsley and Olekalns (1999) on the impact of health warnings on cigarette packs. For sure, we have to make here a distinction between information campaigns and the so-called *hard information*. The first takes the form of "free" advertising while the second implies that the government collects and presents detailed scientific evidence corroborating its views, and that it employs other relays (academics, teachers, social workers, newspapers, etc.), with the hope that credibility will cease to be an issue. This process is long, but presumably more effective.

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A Appendix

A.1 Proof of Proposition 6

We establish the result in two steps. The first analyzes *differentiable* fully revealing equilibria; uniqueness in this category is proved. The second step shows that any fully revealing equilibrium is essentially identical to the differentiable one.

A.1.1 Differentiable Equilibrium

The analysis follows this plan: reasoning on local incentive compatibility, we find the ordinary differential equation satisfied by any fully revealing equilibrium tax policy and we eliminate solutions with tax rates which are not between $\frac{\eta}{1+\lambda}$ and the second-best schedule (whichever is the higher); we check global incentive compatibility along the equilibrium policy; we search for off-equilibrium beliefs (i.e. associated with off-equilibrium tax rates) that discourage deviations. This gives a unique equilibrium.

Local Incentive Compatibility The government prefers $t(\theta)$ (and the implied $x(\theta)$) to $t(\theta + d\theta)$ and to $t(\theta - d\theta)$; taking limits we get

$$x'U' + \lambda t'x - (\eta + \theta - \lambda t)x' = 0. \tag{20}$$

Given that the consumer's first-order condition is

$$U' = \theta + t, \tag{21}$$

we can eliminate U' to get (after simplification)

$$tx' = -\lambda t'x + (\eta - \lambda t)x'. \tag{22}$$

t and x being separable, (22) is easily integrated to give

$$\forall \theta, \theta_0 : \frac{\frac{\eta}{1+\lambda} - t(\theta)}{\frac{\eta}{1+\lambda} - t(\theta_0)} = \left(\frac{x(\theta)}{x(\theta_0)}\right)^{\frac{1+\lambda}{\lambda}} \tag{23}$$

where θ_0 and $t_0 = t(\theta_0)$ are initial conditions. Equations (23) and (5) determine implicitly but entirely the solutions $t(\theta)$ and $x(\theta)$. In particular, x solves

$$\left((\theta + \frac{\eta}{1+\lambda})x - 1\right) x^{\frac{1}{\lambda}} = \left((\theta_0 + \frac{\eta}{1+\lambda})x_0 - 1\right) x_0^{\frac{1}{\lambda}} = \text{Constant} \tag{24}$$

By differentiation, we get

$$x' = \frac{\lambda x^2}{1 - (\eta + (1+\lambda)\theta)x} \tag{25}$$

The second-order condition is:

$$0 \geq x'^2 U'' + x''U' + \lambda t''x + 2\lambda t'x' - (\eta + \theta - \lambda t)x'', \tag{26}$$

while the derivative of the first-order condition is:

$$0 = x'^2 U'' + x''U' + \lambda t''x + 2\lambda t'x' - x' - (\eta + \theta - \lambda t)x'', \tag{27}$$

Simplifying (26) with (27) we get:

$$x' \leq 0 \tag{28}$$

Applying (28) to (25) and using (5), we can see that, when $\lambda > 0$, $x' \leq 0$ if and only if $t < \eta + \lambda\theta$ and when $\lambda < 0$, $x' \leq 0$ if and only if $t > \eta + \lambda\theta$.

Starting from (25) and (22), straightforward calculations prove that the differential equation satisfied by t is

$$t' = \frac{\eta - (1+\lambda)t}{t - \eta - \lambda\theta}. \tag{29}$$

Global Incentive Compatibility We want to exclude cases where infinitesimal deviations are rejected (of this we are sure because of the first- and second-order conditions) whereas discrete deviations are possible.

Let θ be the true value of the side effects parameter. Using (5) we calculate the derivative of the government's utility with respect to $\widehat{\theta}$ assuming that the government offers $t(\widehat{\theta})$, thereby inducing $x(\widehat{\theta})$:

$$(30)\qquad \begin{aligned} & x'(\widehat{\theta})U'[x(\widehat{\theta})] + (\lambda t'(\widehat{\theta})x(\widehat{\theta}) - (\eta + \theta - \lambda t(\widehat{\theta}))x'(\widehat{\theta})) \\ = \; & \frac{x'(\widehat{\theta})}{x(\widehat{\theta})} + \lambda t'(\widehat{\theta})x(\widehat{\theta}) - (\eta + \theta - \lambda t(\widehat{\theta}))x'(\widehat{\theta}) \end{aligned}$$

From (22), we find that the following expression has the same sign of (30)

$$(31)\qquad x(\widehat{\theta})(t(\widehat{\theta}) + \theta) - 1$$

Using (5), it follows that (30) is positive for $\widehat{\theta} < \theta$ and negative for $\widehat{\theta} > \theta$. This means that incentive compatibility is satisfied everywhere for equilibrium actions.

Uniqueness of the Differentiable Equilibrium We give the full reasoning for $\lambda > 0$. A symmetric argument proves the proposition for $\lambda < 0$. We show now that the boundary condition $t(\overline{\theta}) = \eta + \lambda\overline{\theta}$ is necessary.

Reasoning by contradiction, we show that there exist beliefs compatible with the equilibrium for off-equilibrium actions if and only if the condition is satisfied. Let $t(\cdot)$ be a solution to (29) such that $t(\overline{\theta}) < \eta + \lambda\overline{\theta}$.[24] Given (29), either $t(\theta)$ is systematically below $\frac{\eta}{1+\lambda}$ or $t(\theta)$ is strictly increasing. In any case $\max_\theta t(\theta) < \eta + \lambda\overline{\theta}$; we choose an arbitrary (off-equilibrium) t in the interval $(\max_\theta t(\theta), \eta + \lambda\overline{\theta})$ and we denote by $\widehat{\theta}$ the associated belief. Now we prove that there always exists a type θ such that the government prefers policy $(t,\widehat{\theta})$ to policy $(t(\theta), \theta)$.

From Proposition 2, we know that for each θ, the absolute best policy is $\frac{\eta+\theta}{\lambda}$ for the tax rate and $-\frac{\eta+\theta}{\lambda}$ for the belief; moreover, the second-best $(\eta + \lambda\theta, \theta)$ is preferred to $(t(\theta), \theta)$. The convexity of the upper contours of the government's objective function imply that any policy in the triangle $\triangle(\theta) = ((t(\theta), \theta), (\eta + \lambda\theta, \theta), (\frac{\eta+\theta}{\lambda}, -\frac{\eta+\theta}{\lambda}))$, except $(t(\theta), \theta)$, is strictly better than $(t(\theta), \theta)$ when θ is the type. It suffices now to check that $(t,\widehat{\theta})$ is necessarily in $\triangle(\theta)$ for a certain $\theta \in [\underline{\theta}, \overline{\theta}]$. Indeed, $\cup_{\theta \in [\underline{\theta},\overline{\theta}]} \triangle(\theta)$ contains (a) the triangle $((\eta + \lambda\underline{\theta}, \underline{\theta}), (\eta + \lambda\overline{\theta}, \overline{\theta}), (\frac{\eta+\overline{\theta}}{\lambda}, -\frac{\eta+\overline{\theta}}{\lambda}))$, and (b) the policies between $(t(\theta), \theta)$ and $(t_{\text{SB}}(\theta), \theta)$ for $\theta \in [\underline{\theta}, \overline{\theta}]$. Provided $\frac{\eta+\overline{\theta}}{\lambda}$ is larger than $\eta + \lambda\overline{\theta}$, then $(t,\widehat{\theta})$ is either in (a) or (b) in the latter union, hence the existence of a θ for which the deviation is desirable. Given that $\frac{\eta+\overline{\theta}}{\lambda} > \eta + \lambda\overline{\theta}$, we are done. The consequence is that $t(\overline{\theta}) = \eta + \lambda\overline{\theta}$ (no distortion at the top).

[24] For $\lambda > 0$, we already excluded that the tax rate be larger than the second-best tax rate in the preceding subsection.

Now we prove associating belief $\overline{\theta}$ to any tax rate above $t(\overline{\theta})$ does not induce deviations. The value to the government of type θ of imposing $t > t(\overline{\theta})$, thereby inducing belief $\overline{\theta}$, is: $-\log(\overline{\theta}+t) - \frac{\eta+\theta-\lambda t}{\overline{\theta}+t}$. The root of the derivative with respect to t is $\eta+\theta-(1-\lambda)\overline{\theta}$ which is lower than $\eta+\lambda\overline{\theta} = t(\overline{\theta})$. The value being decreasing with respect to t over $[t(\overline{\theta}), +\infty[$, $t(\overline{\theta})$ is a better move than any $t > t(\overline{\theta})$. Given that equilibrium actions are incentive compatible, neither $t(\overline{\theta})$ nor t are desirable, compared to $t(\theta)$. By the same reasoning, we can check that, if for $t < t(\underline{\theta})$, beliefs are $\underline{\theta}$, then t is not attractive: the value to the government of type θ of imposing $t < t(\underline{\theta})$ is smaller than the value of imposing $(t(\underline{\theta}), \underline{\theta})$.

We conclude that the unique revealing allocation found *is* an equilibrium.

A.1.2 Uniqueness in General

Let us take a fully revealing equilibrium. Given that the government's preferences, for constant beliefs, are single-peaked with respect to t (a direct consequence of the convexity in Proposition 2), and given the value of its equilibrium strategy, there exist a maximum of two tax rates per θ, $t_L(\theta)$ and $t_U(\theta)$, both being suboptimal (as compared to the second-best) when different. More precisely, $t_L(\theta) \leq \eta + \lambda\theta \leq t_U(\theta)$). The Theorem of the maximum ensures that the *value* of the government's equilibrium strategy is continuous with respect to θ, therefore functions $t_L(\cdot)$ and $t_U(\cdot)$ are continuous with respect to θ. We denote by Θ_L and Θ_U the subsets of $[\underline{\theta}, \overline{\theta}]$ leading to a move in the lower, respectively in the upper, selection. Notice that $\Theta_L \cup \Theta_U = [\underline{\theta}, \overline{\theta}]$ but $\Theta_L \cap \Theta_U \neq \emptyset$ if mixed strategies are used. For fixing ideas, the following reasoning assumes that $\lambda > 0$.

The first step is to prove that Θ_U is not dense in any interval of $[\underline{\theta}, \overline{\theta}]$. We reason by contradiction: let us take J an interval in $[\underline{\theta}, \overline{\theta}]$ in which Θ_U is dense. Let us take $\theta_0 \in J$, and a strictly monotonic sequence $(\theta_n)_{n\geq 1}$ in Θ_U converging to θ_0. We prove that for all sequence $(\theta_n)_{n\geq 1}$, $\lim_{n\to\infty} \frac{t_n - t_0}{\theta_n - \theta_0} = \frac{\eta-(1+\lambda)t_0}{t_0-\eta+\lambda\theta_0}$, where t_n denotes $t_U(\theta_n)$. Indeed, incentive constraints (θ_n wish not to mimic θ_0, and vice-versa) imply that:

$$(32) \qquad -\log(\theta_n + t_n) - \frac{\eta + \theta_n - \lambda t_n}{\theta_n + t_n} \geq -\log(\theta_0 + t_0) - \frac{\eta + \theta_n - \lambda t_0}{\theta_0 + t_0}$$

$$(33) \qquad -\log(\theta_n + t_n) - \frac{\eta + \theta_0 - \lambda t_n}{\theta_n + t_n} \leq -\log(\theta_0 + t_0) - \frac{\eta + \theta_0 - \lambda t_0}{\theta_0 + t_0}$$

Therefore, taking a first-order approximation, and multiplying by $(\theta_0 + t_0)^2$ yields

(34)
$$0 \geq ((1+\lambda)t - \eta)(\theta_n - \theta_0) + (t - \eta - \lambda\theta)(t_n - t_0) + o(\theta_n - \theta_0) + o(t_n - t_0)$$
(35)
$$0 \leq ((1+\lambda)t - \eta)(\theta_n - \theta_0) + (t - \eta - \lambda\theta)(t_n - t_0) + o(\theta_n - \theta_0) + o(t_n - t_0)$$

The limit of the rate of variations is the same for all sequences, which implies that t_U is differentiable at θ_0, hence differentiable on interval J .

A solution of the differential equation (29) situated above the second-best taxes is incentive compatible at no point because the second-order condition is never satisfied. We can conclude that strategy t_U is not incentive compatible, and that the interval J does not exist.

It is easy now to conclude that Θ_L is dense in $[\underline{\theta}, \overline{\theta}]$: Θ_L, being the complementary set (in an interval) of a set Θ_U which is nowhere dense, is dense. In consequence, t_L satisfies the differential equation (29) in a dense subset of $[\underline{\theta}, \overline{\theta}]$, which implies that it does so everywhere. The lower selection is necessarily equal to the unique differentiable equilibrium strategy, since we can apply to $t_L(\cdot)$ the reasoning suited for differentiable equilibria.

It remains to prove now that Θ_U contains a finite number of points. Let us take θ_1 and $\theta_2 \in \Theta_U$ (where $\theta_1 \neq \theta_2$) with corresponding tax rates t_1 and t_2. Let us denote by $t_i(\cdot)$ $(i = 1, 2)$ the solution to (29) with maximal definition domain passing through t_i at θ_i. Note that either $t_1(\cdot)$ and $t_2(\cdot)$ are the same, or one is systematically above the other, because, according to the Cauchy-Lipschitz Theorem, two different solutions to differential equation (29) never cross.

Assume for fixing ideas that $t_2(\cdot)$ is above $t_1(\cdot)$. (a) If the two curves are sufficiently close to each other, $t_2(\theta_1)$ is defined and is larger than t_1. Notice that $t_2(\theta_1)$ is closer to the second-best than t_1. Our study of the incentives when taxes are above the second-best shows that solutions to the differential equations are minimizing welfare (the second-order conditions is violated everywhere): when the type is θ_1, t_2 with belief θ_2 is preferred to $t_2(\theta_1)$ with belief θ_1. By transitivity, t_2 is preferable to t_1 when the true type is θ_2. This is in contradiction with incentives. (b) If there is an infinite number of types in Θ_U, we can always exhibit θ_1 and θ_2 which are close enough to each other to apply the reasoning (a). We conclude that Θ_U contains a finite number of points.

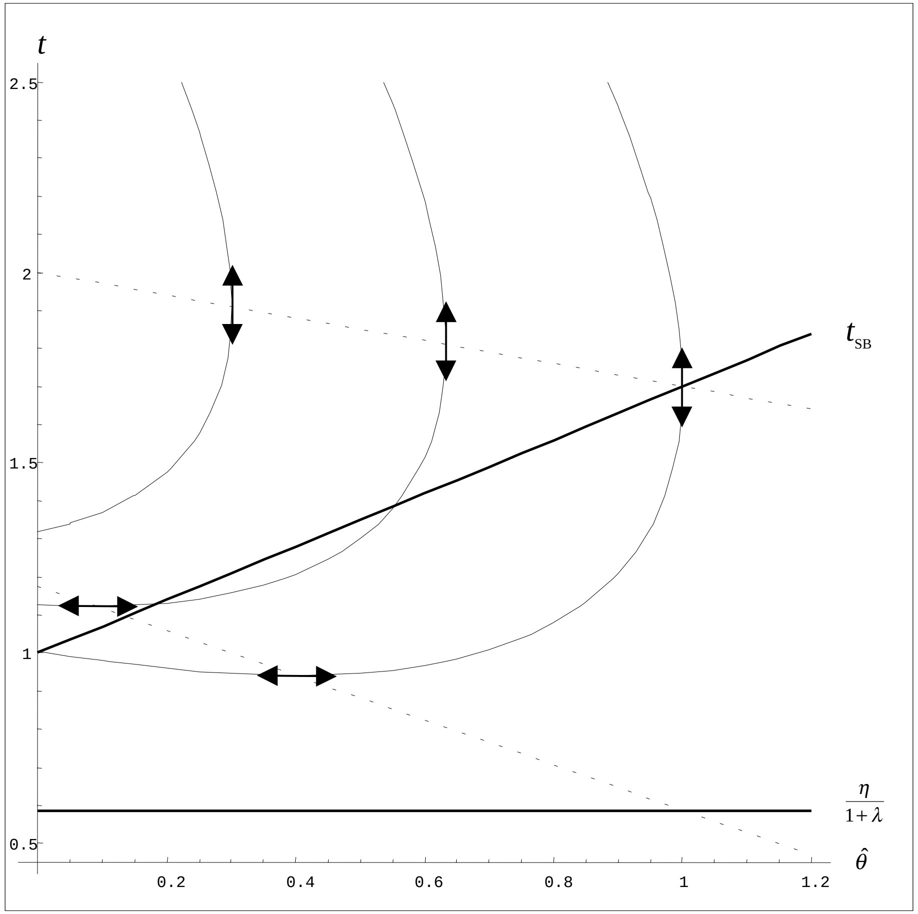

Figure 1: the government indifference curves with λ>0.

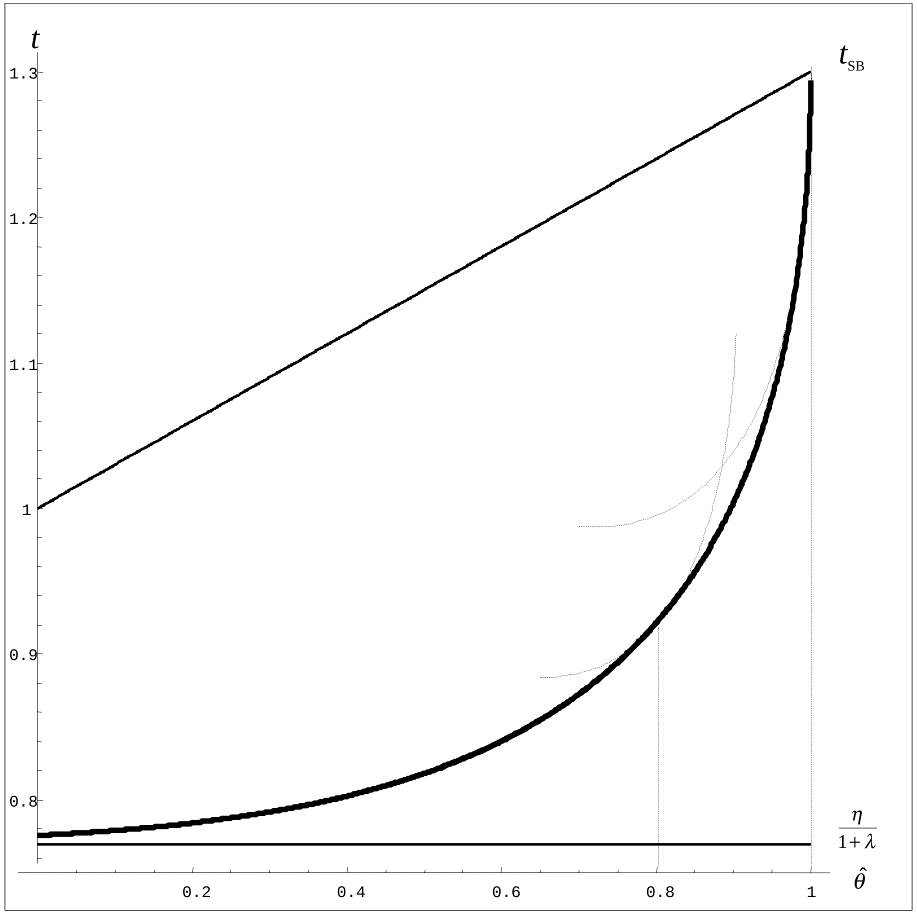

Figure 2: the fully revealing equilibrium with λ>0.

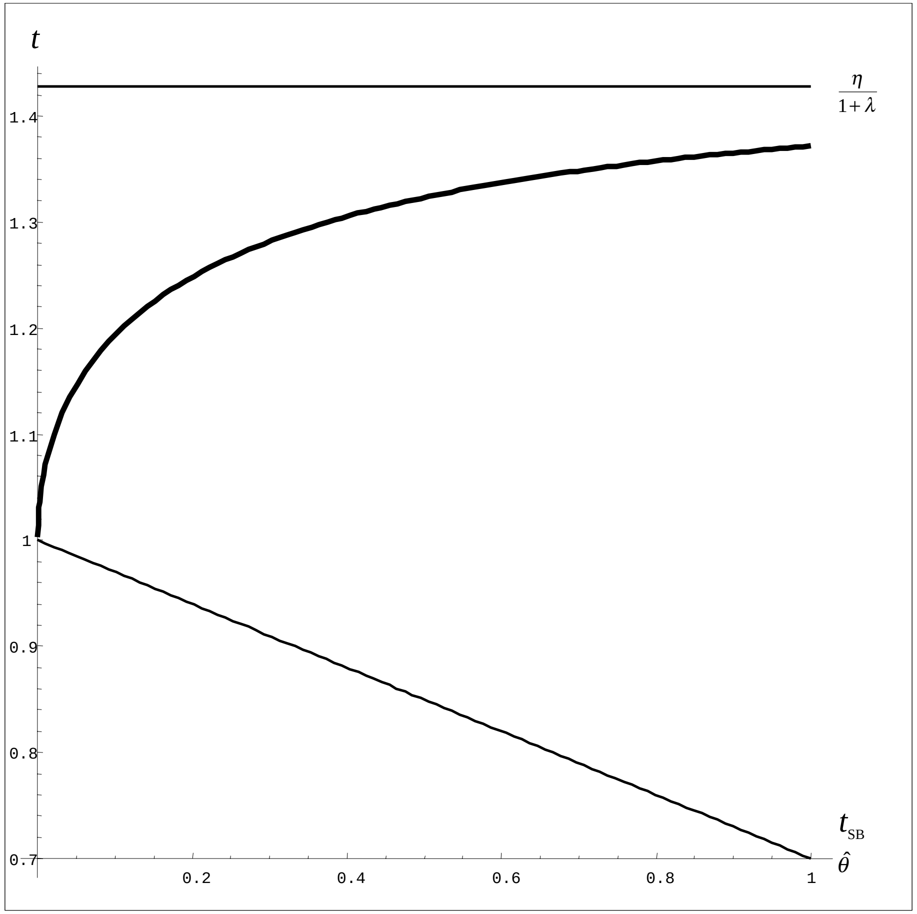

Figure 3: the fully revealing equilibrium with λ<0.