Tari¤s vs Quotas in a Model of Trade with Capital Accumulation[1]

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Abstract

This paper examines the equivalence among price-modifying and quantity …xing international trade policies in a di¤erential game. We employ two well known capital accumulation dynamics for …rms, due to Nerlove and Arrow and to Ramsey, respectively. We show that, in both cases, open-loop and closed-loop Nash equilibria coincide. Under the former accumulation the tari¤-quota equivalence holds, but it does not under the latter. Moreover, in the Ramsey model, the country setting the trade policy prefers a quantity-equivalent import quota to the adoption of the tari¤.

JEL Classi…cation: D43, D92, F12, F13, L13

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1 Introduction

An important question in international trade theory and policy has been, since a long time, the comparative evaluation of di¤erent trade policies. In particular, comparing quantity restrictions (such as quotas and voluntary export restraints) versus price-modifying policies (such as tari¤ and subsidy) has taken a prominent position in this debate. These two sets of instruments prove to be equivalent in perfectly competitive markets in the sense that any e¤ect of a price instrument can be replicated by an appropriately chosen quantity policy and vice versa. Bhagwati (1965, 1968) noted, however, that this need not be true when international markets are imperfectly competitive. Since then, a number of papers have dwelled upon this question, showing that either the equivalence holds (as in Eaton and Grossman, 1986; and in Hwang and Mai, 1988, for the Cournot case), or, if it does not, quantity restrictions tend to rise equilibrium prices (Itoh and Ono, 1982, 1984; Harris, 1985; Krishna, 1989).[1]

As Brander (1995) well pointed out, the main limit of the existing literature on strategic trade policy is, with few exceptions, its essentially static nature. One-shot static games are clearly not well suited to analyse long-term interactions characterizing international oligopolistic markets. One may well expect that introducing real time in these models substantially a¤ects …rms' behavior.

Cheng (1987), Driskill and McCa¤erty (1989a, 1996) and Dockner and Haug (1990, 1991) are the few exceptions as they examine trade policies with oligopolistic …rms interacting in a di¤erential game fashion. In this paper, we take this avenue and study the equivalence among price-modifying and quantity-…xing trade policies in a continuous time di¤erential game.

An important di¤erence with the quoted papers is that, following the literature initiated by Spence (1979), we explicitly model …rms' dynamic capital accumulation game. To this end, we will consider both the Nerlove-Arrow (1962) model of reversible investment (i.e. accumulation with capital depreciation) and the Ramsey (1928) model (i.e., the well known "corn-corn" growth model).

Di¤erent strategies and solution concepts may prevail in a di¤erential game and the existing literature mainly concentrated on two kind of strategies:[2] the-open loop and the closed-loop. In the former case, …rms precommit to an investment path over time and the relevant equilibrium concept is the open-loop Nash equilibrium. In the latter, …rms do not precommit on invest-

[1] A ranking of tari¤ and quota policies can be found in Sweeney, Tower and Willett (1977), for the case where domestic production is monopolised.

[2] See Kamien and Schwartz (1981); Başar and Olsder (1982); Mehlmann (1988).

ment path and their strategies at any instant may depend on all the preceding history. In this situation, the information set used by …rms in setting their strategies at any given time is often simpli…ed to be only the current value of the capital stocks at that time. The relevant equilibrium concept is in this (sub-)case the closed-loop no-memory (or Markov Perfect) Nash equilibrium.

In order to further simplify the analysis, the above mentioned papers on international trade di¤erential games have adopted a re…nement of the closed-loop Nash equilibrium, which is known as the feedback Nash equilibrium.[3] In what follows, we will not restrict to this re…nement and deal with the open-loop and closed loop no-memory solutions. We will study how these two solution concepts a¤ect the tari¤-quote equivalence of the trade game.

The main results are as follows. Interestingly enough, as it is also shown in Cellini and Lambertini (2000b), both under the Nerlove-Arrow and the Ramsey capital accumulation dynamics, the open-loop Nash equilibrium coincides with the closed-loop (no-memory) one (and hence it is subgame perfect). Moreover, under the Nerlove-Arrow accumulation, with quantity-equivalent import tari¤ and quota, the steady state equilibrium price in the domestic market is the same under both trade policy regimes. Hence, the tari¤-quota equivalence holds.

On the contrary, with the Ramsey accumulation, the adoption of any import quota drives the domestic …rm to the Ramsey equilibrium. This does not always happen when imposing a tari¤ on imports and the equivalence of tari¤s and quotas does not hold. Moreover, we show that if the government setting the trade policy aims at favouring the domestic …rm, and/or lowering the domestic price, the adoption of a quantity-equivalent import quota is preferable to the adoption of the tari¤, in that total output is larger under the former policy than under the latter.

The paper is organized as follows. The general setting is laid out in section 2. Section 3 is devoted to the analysis of the Nerlove-Arrow capital accumulation, while the Ramsey model is investigated in section 4. Concluding remarks are in section 5.

2 The setup

As in the previous literature on this topic, we consider a duopoly market supplied by a domestic producer (…rm D) and a foreign rival (…rm F). For the sake of simplicity, we assume that …rms sell homogeneous goods, although the ensuing analysis could be easily extended to account for product

[3] For a clear exposition of the di¤erence among these equilibrium solutions see Başar and Olsder (1982, pp. 318-327, and chapter 6, in particular Proposition 6.1).

differentiation.

The model is built in continuous time. The market exists over $t \in [0; \infty)$. Let $q_i(t)$ define the quantity sold by firm i, $i = D; F$; at time t. The marginal production cost is constant and equal to c for both firms. Firms compete à la Cournot, the demand function at time t being:

$$p(t) = a - q_D(t) - q_F(t). \tag{1}$$

In order to produce, firms must accumulate capacity or physical capital $k_i(t)$ over time. In the remainder of the paper, we will investigate two alternative models of capital accumulation:

A] The Nerlove-Arrow (1962) model, where the relevant dynamic equation is:

$$\frac{\partial k_i(t)}{\partial t} = I_i(t) - \delta k_i(t); \tag{2}$$

where $I_i(t)$ is the investment carried out by firm i at time t, and δ is the constant depreciation rate. The instantaneous cost of investment is $C_i[I_i(t)] = b[I_i(t)]^2$; with $b > 0$. To solve this model explicitly, we also assume that firms operate with a constant returns technology $q_i(t) = k_i(t)$; so that the demand function rewrites as:[4]

$$p(t) = a - k_D(t) - k_F(t). \tag{3}$$

Here, the control variable is the instantaneous investment $I_i(t)$, while the state variable is obviously $k_i(t)$.

B] The Ramsey (1928) model, whit the following dynamic equation:

$$\frac{\partial k_i(t)}{\partial t} = f(k_i(t)) - q_i(t) - \delta k_i(t); \tag{4}$$

where $f(k_i(t)) = y_i(t)$ denotes the output produced by firm i at time t. In this case, capital accumulates as a result of intertemporal relocation of unsold output $y_i(t) - q_i(t)$. This can be interpreted in two ways. The first consists in viewing this setup as a corn-corn model, where unsold output is reintroduced in the production process. The second consists in thinking of a two-sector economy where there exists an industry

[4] Notice that this assumption entails that firms always operate at full capacity. This, in turn, amounts to saying that this model encompasses the case of Bertrand behaviour under capacity constraints, as in Kreps and Scheinkman (1983). The open-loop solution of the Nerlove-Arrow differential duopoly game in a model without trade is in Fershtman and Muller (1984) and Reynolds (1987).

producing the capital input which can be traded against the final good at a price equal to one (see Cellini and Lambertini, 1998, 2000a).

In this model, the control variable is $q_i(t)$, while the state variable remains $k_i(t)$.

Both in model [A] and in model [B], we address the issue whether the equivalence of import tariff and quota holds. Following Dockner and Haug (1990), one should check the existence or non-existence of such equivalence under both open-loop and closed-loop solutions. As we show below, the present games [A-B] are such that open- and closed-loop equilibria coincide.

3 The Nerlove-Arrow model

In the Nerlove-Arrow model, the Hamiltonian of the domestic firm writes as follows:

$$\begin{aligned} H_D(t) &= e^{-\rho t} \cdot \Big\{ [a - k_D(t) - k_F(t) - c]\, k_D(t) - b[I_D(t)]^2 + \\ &\quad + \lambda_{DD}(t)[I_D(t) - \delta k_D(t)] + \lambda_{DF}(t)[I_F(t) - \delta k_F(t)] \Big\} \end{aligned} \tag{5}$$

where $\lambda_{Di}(t) = \mu_{Di}(t)e^{\rho t}$, and $\mu_{Di}(t)$ is the co-state variable associated to $k_i(t)$, $i = D, F$.

If the government adopts an import tariff τ, the Hamiltonian of the foreign firm is:

$$\begin{aligned} H_F(t) &= e^{-\rho t} \cdot \Big\{ [a - k_D(t) - k_F(t) - c - \tau]\, k_F(t) - b[I_F(t)]^2 + \\ &\quad + \lambda_{FF}(t)[I_F(t) - \delta k_F(t)] + \lambda_{FD}(t)[I_D(t) - \delta k_D(t)] \Big\} \end{aligned} \tag{6}$$

First note that, as the tariff (directly) affects only the foreign firm's profit one cannot rely on symmetry to solve the game.

Necessary conditions for the domestic firm require

$$\begin{aligned} &(i)\ \frac{\partial H_D(t)}{\partial I_D(t)} = 0 \Rightarrow -2bI_D(t) + \lambda_{DD}(t) = 0 \\ &(ii)\ -\frac{\partial H_D(t)}{\partial k_D(t)} - \frac{\partial H_D(t)}{\partial I_F(t)}\frac{\partial I_F(t)}{\partial k_D(t)} = \frac{\partial \lambda_{DD}(t)}{\partial t} - \rho\lambda_{DD}(t) \Rightarrow \\ &\Rightarrow -\frac{\partial \lambda_{DD}(t)}{\partial t} + \rho\lambda_{DD}(t) = a - c - 2k_D(t) - k_F(t) - \delta\lambda_{DD}(t) \\ &(iii)\ -\frac{\partial H_D(t)}{\partial k_F(t)} - \frac{\partial H_D(t)}{\partial I_F(t)}\frac{\partial I_F(t)}{\partial k_F(t)} = \frac{\partial \lambda_{DF}(t)}{\partial t} - \rho\lambda_{DF}(t) \\ &(iv)\ \lim_{t\to\infty} \mu_{DD}(t) \cdot k_D(t) = 0\,;\ \lim_{t\to\infty} \mu_{DF}(t) \cdot k_F(t) = 0\,; \end{aligned} \tag{7}$$

where (iv) is the transversality condition.

Similarly for the foreign firm

$$
\begin{aligned}
&\text{(i)}\ \frac{\partial H_F(t)}{\partial I_F(t)} = 0 \Rightarrow -2bI_F(t) + \lambda_{FF}(t) = 0\\
&\text{(ii)}\ -\frac{\partial H_F(t)}{\partial k_F(t)} - \frac{\partial H_F(t)}{\partial I_D(t)}\frac{\partial I_D(t)}{\partial k_F(t)} = \frac{\partial \lambda_{FF}(t)}{\partial t} - \rho\lambda_{FF}(t) \Rightarrow\\
&\Rightarrow -\frac{\partial \lambda_{FF}(t)}{\partial t} + \rho\lambda_{FF}(t) = a - c - 2k_F(t) - k_D(t) - \delta\lambda_{FF}(t) - \tau\\
&\text{(iii)}\ -\frac{\partial H_F(t)}{\partial k_D(t)} - \frac{\partial H_F(t)}{\partial I_D(t)}\frac{\partial I_D(t)}{\partial k_D(t)} = \frac{\partial \lambda_{FD}(t)}{\partial t} - \rho\lambda_{FD}(t)\\
&\text{(iv)}\ \lim_{t\to\infty} \mu_{FF}(t)\cdot k_F(t) = 0\,;\ \lim_{t\to\infty} \mu_{FD}(t)\cdot k_D(t) = 0\,;
\end{aligned}
\tag{8}
$$

Notice that by (7.i) we have $\frac{\partial I_i(t)}{\partial k_j(t)} = 0$ for i different from j. Moreover, condition (7.iii), which yields $\partial\lambda_{DF}(t)/\partial t$, is redundant in that $\lambda_{DF}(t)$ does not appear in the first order conditions (7.i) and (7.ii). Therefore, the open-loop solution is indeed a degenerate closed-loop solution.[5]

Replace (7.i) into (7.ii) obtaining

$$\frac{\partial \lambda_{DD}(t)}{\partial t} = bI_D(t)(\rho + \delta) - [a - c - 2k_D(t) - k_F(t)]\,.$$

Then, differentiating (7.i) w.r.t. time and substituting the previous condition we obtain

$$\frac{\partial I_D(t)}{\partial t} = \frac{I_D(t)(\rho + \delta)}{2} - \frac{a - c - 2k_D(t) - k_F(t)}{2b}\,. \tag{9}$$

Similarly, condition (8.iii) yields $\partial\lambda_{FD}(t)/\partial t$, is redundant.

The discussion carried out so far establishes:

Proposition 1 Under the Nerlove-Arrow capital accumulation dynamics, the open-loop Nash equilibrium is subgame perfect.

Now we can explicitly look for steady state points. We obtain

$$\frac{\partial I_F(t)}{\partial t} = \frac{I_F(t)(\rho + \delta)}{2} - \frac{a - c - k_D(t) - 2k_F(t) + \tau}{2b}\,. \tag{10}$$

[5] Note that, however, the open-loop solution does not coincide with the feedback solution (see Reynolds, 1987). For further details, see Cellini and Lambertini (2000b), as well as the discussion in Driskill and McCafferty (1989b, pp. 326-8). Classes of games where this coincidence arises are illustrated in Clemhout and Wan (1974); Reinganum (1982); Mehlmann and Willing (1983); Dockner, Feichtinger and Jørgensen (1985); Fershtman (1987). For an overview, see Mehlmann (1988); Fershtman, Kamien and Muller (1992).

Now, solving the system:

$$\frac{\partial I_i(t)}{\partial t} = 0\,; \quad \frac{\partial k_i(t)}{\partial t} = 0\,; \quad i = D; F; \tag{11}$$

we calculate the steady state levels of states and controls:

$$\begin{aligned}
I_D^{ss} &= \frac{\delta\left[(a-c)\left(1+2b\delta(\rho+\delta)\right)+\tau\right]}{3+4b\delta\left[\delta\left(2+b\delta^2\right)+\rho\left(2+2b\delta^2+b\delta\rho\right)\right]}\,; \\
I_F^{ss} &= \frac{\delta\left[a-c-2(a-c-\tau)\left(1+b\delta(\rho+\delta)\right)\right]}{1-4\left[1+b\delta(\rho+\delta)\right]^2}\,; \\
k_D^{ss} &= \frac{I_D^{ss}}{\delta}\,; \; k_F^{ss} = \frac{I_F^{ss}}{\delta}\,:
\end{aligned} \tag{12}$$

Steady state capital levels in (12) can be usefully rewritten as:

$$\begin{aligned}
k_D^{ss} &= \frac{(a-c)A+\tau}{A(B+1)} \\
k_F^{ss} &= \frac{(a-c)A-\tau B}{A(B+1)}
\end{aligned} \tag{13}$$

where

$$\begin{aligned}
A &\equiv 2b(\rho+\delta)+1 > 0\,; \\
B &\equiv 2\left[b(\rho+\delta)\delta+1\right] > 0\,:
\end{aligned} \tag{14}$$

Then, from (13) one can easily check that

$$\frac{\partial k_D^{ss}}{\partial \tau} = \frac{1}{A(B+1)} > 0\,; \quad \frac{\partial k_F^{ss}}{\partial \tau} = -\frac{B}{A(B+1)} < 0\,: \tag{15}$$

In the case of an equivalent import quota, the domestic firm's optimization problem is

$$\max_{I_D(t)} H_D(t) = e^{-\rho t}\cdot\left\{\left[a-k_D(t)-\overline{k}_F(t)-c\right]k_D(t)-b\left[I_D(t)\right]^2+\right. \tag{16}$$

$$\left. +\lambda_{DD}(t)\left[I_D(t)-\delta k_D(t)\right]+\lambda_{DF}(t)\left[I_F(t)-\delta k_F(t)\right]\right\}$$

where $\overline{k}_F(t) = k_F^{ss} = \frac{I_F^{ss}}{\delta}$: It is immediate to verify that the first order conditions for the optimum of firm D coincide with (7).

The above discussion proves the following result:

Proposition 2 Under the Nerlove-Arrow capital accumulation dynamics, with quantity-equivalent import tariff and quota, the steady state equilibrium price in the domestic market is the same under both trade policy regimes.

Essentially, the above result is driven by the fact that, in the Nerlove-Arrow model, there is no strategic interaction in the choice of optimal investment on the part of firms, i.e., firm i's first order condition on investment (7.i and 8.i) only contain the own control, and not the rival's. Hence, the behaviour of firm D is the same irrespective of the policy adopted by the home government towards firm F.

4 The Ramsey model

Under the capital accumulation rule (4), the problem of the domestic firm is the following:

$$\begin{aligned} H_D(t) &= e^{-\rho t}\{q_D(t)[a - q_D(t) - q_F(t) - c] + \\ &\quad + \lambda_{DD}(t)[f(k_D(t)) - q_D(t) - \delta k_D(t)] + \\ &\quad + \lambda_{DF}(t)[f(k_F(t)) - q_F(t) - \delta k_F(t)]\}, \qquad (17) \end{aligned}$$

where $\lambda_{Di}(t) = \mu_{Di}(t)e^{\rho t}$, and $\mu_{Di}(t)$ is the co-state variable associated to $k_i(t)$.

If the government of the domestic country imposes an import tariff τ, the Hamiltonian of the foreign firm is:

$$\begin{aligned} H_F(t) &= e^{-\rho t}\cdot\{q_F(t)[a - q_D(t) - q_F(t) - c - \tau] + \\ &\quad + \lambda_{FF}(t)[f(k_F(t)) - q_F(t) - \delta k_F(t)] + \\ &\quad + \lambda_{FD}(t)[f(k_D(t)) - q_D(t) - \delta k_D(t)]\}. \qquad (18) \end{aligned}$$

The first order conditions concerning the control variables are:

$$\begin{aligned} \frac{\partial H_D(t)}{\partial q_D(t)} &= a - 2q_D(t) - q_F(t) - c - \lambda_{DD}(t) = 0, \qquad (19) \\ \frac{\partial H_F(t)}{\partial q_F(t)} &= a - 2q_F(t) - q_D(t) - c - \tau - \lambda_{FF}(t) = 0. \end{aligned}$$

Now look at the generic co-state equation of firm i, for the closed-loop solution of the game:

$$-\frac{\partial H_i(t)}{\partial k_i(t)} - \frac{\partial H_i(t)}{\partial q_j(t)}\frac{\partial q_j(t)}{\partial k_i(t)} = \frac{\partial \mu_{ii}(t)}{\partial t} \qquad (20)$$

where

$$\frac{\partial q_j(t)}{\partial k_i(t)} = 0 \qquad (21)$$

as it appears from a quick inspection of best replies obtained from (19):

$$q_D^{br}(t) = \frac{a - c - q_F(t) - \lambda_{DD}(t)}{2}, \qquad (22)$$

$$q_F^{br}(t) = \frac{a - c - \tau - q_D(t) - \lambda_{FF}(t)}{2}. \tag{23}$$

Moreover, (22) and (23) suffice to establish that the co-state equation:

$$-\frac{\partial H_i(t)}{\partial k_j(t)} - \frac{\partial H_i(t)}{\partial q_j(t)}\frac{\partial q_j(t)}{\partial k_j(t)} = \frac{\partial \mu_{ij}(t)}{\partial t} \tag{24}$$

is indeed redundant since $\mu_{ij}(t) = \lambda_{ij}(t)e^{-\rho t}$ does not appear in the first order conditions concerning controls. That is, the Ramsey game yields that the open-loop solution is a degenerate closed-loop solution because the best reply function of firm i does not contain the state variable pertaining to the same firm. Therefore, we have proved the analogous to Proposition 1:

Proposition 3 Under the Ramsey capital accumulation dynamics, the open-loop Nash equilibrium is subgame perfect.

Now move on to the solution of the system. The co-state equation of firm i writes as follows:

$$-\frac{\partial H_i(t)}{\partial k_i(t)} = \frac{\partial \mu_{ii}(t)}{\partial t} \Rightarrow \frac{\partial \lambda_{ii}(t)}{\partial t} = [\rho + \delta - f'(k_i(t))]\lambda_{ii}(t). \tag{25}$$

The best reply functions (22-23) can be differentiated w.r.t. time to yield:

$$\frac{dq_i(t)}{dt} = -\frac{dq_j(t)/dt + d\lambda_{ii}(t)/dt}{2}. \tag{26}$$

Then, using

$$\begin{aligned} \lambda_{DD}(t) &= a - c - 2q_D(t) - q_F(t) \\ \lambda_{FF}(t) &= a - c - \tau - q_D(t) - 2q_F(t) \end{aligned} \tag{27}$$

and (25), we obtain:

$$\begin{aligned} \frac{dq_D(t)}{dt} &= \frac{dq_F(t)/dt + [a - c - 2q_D(t) - q_F(t)][\rho + \delta - f'(k_D(t))]}{2} \\ \frac{dq_F(t)}{dt} &= \frac{dq_D(t)/dt + [a - c - \tau - 2q_F(t) - q_D(t)][\rho + \delta - f'(k_F(t))]}{2} \end{aligned} \tag{28}$$

which can be solved to yield:

$$\begin{aligned} \frac{dq_D(t)}{dt} = &\frac{[a - c - \tau - 2q_F(t) - q_D(t)][\rho + \delta - f'(k_F(t))]}{3} + \\ &- 2[a - c - 2q_D(t) - q_F(t)][\rho + \delta - f'(k_D(t))] \end{aligned} \tag{29}$$

$$\frac{dq_F(t)}{dt} = \frac{[a - c - 2q_D(t) - q_F(t)][\rho + \delta - f'(k_D(t))]}{3} + \\ - \frac{2[a - c - \tau - 2q_F(t) - q_D(t)][\rho + \delta - f'(k_F(t))]}{3} \tag{30}$$

Imposing that (29) and (30) be zero and solving, we obtain the following set of solutions:

$$f'(k_D(t)) = f'(k_F(t)) = f'(k(t)) = \rho + \delta \tag{31}$$

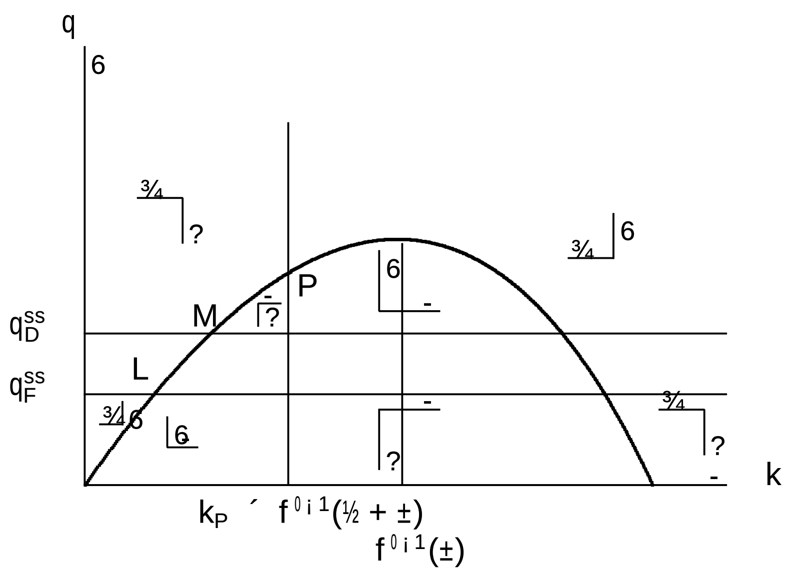
and

$$q_D^{ss} = \frac{a - c + \tau}{3}\,;\; q_F^{ss} = \frac{a - c - 2\tau}{3}\,; \tag{32}$$

where $\{q_D^{ss}; q_F^{ss}\}$ is the solution driven by demand and cost conditions, while $f'(k(t)) = \rho + \delta$ is the Ramsey equilibrium dictated by intertemporal capital accumulation alone. Note that the optimal outputs in (32) exhibit the standard properties $\partial q_D^{ss}/\partial\tau > 0$ and $\partial q_F^{ss}/\partial\tau < 0$.

The phase diagram illustrating the dynamics of the system is in figure 1, where the locus $\partial k/\partial t = 0$ as well as the behaviour of k, depicted by horizontal arrows, derive from (4). Steady states are identified by the intersections between loci.

Figure 1: Steady state equilibrium under a tariff



It is worth noting that the situation illustrated in figure 1 is only one out of several possible configurations, due to the fact that the position of the vertical line $f'(k) = \rho + \delta$ is independent of demand parameters, while the horizontal loci q_D^{ss} and q_F^{ss} shifts upwards (downwards) as a (c) increases. Moreover, $\partial q_D^{ss}/\partial \tau > 0$ and $\partial q_F^{ss}/\partial \tau < 0$. Here, we confine to the case where horizontal loci q_D^{ss} and q_F^{ss} intersect locus $\partial k/\partial t = 0$ in the region where it is increasing in k, to the left of the Ramsey equilibrium $f'(k(t)) = \rho + \delta$. Such steady state points are identified as L for firm D and M for firm F. Intersections to the right of $k = f'^{-1}(\delta)$ are clearly inefficient and therefore can be disregarded. Stability analysis reveals that $\{L, M, P\}$ are saddle points.[6]

The foregoing discussion can be summarised as follows:

Lemma 1 Under the import tariff τ, for all $\{a, c, \tau\}$ such that

$$\frac{a - c + \tau}{3} \leq f(k_P),$$

the system reaches a steady state at

$$q_D^{ss} = \frac{a - c + \tau}{3}; \quad q_F^{ss} = \frac{a - c - 2\tau}{3},$$

which is a saddle.

Now we shall take into consideration the alternative setting where the policy maker of country D adopts an equivalent import quota. The issue can be quickly dealt with by observing how the best reply of firm D modifies the quota. Now (22) writes as follows:

$$q_D^{br}(t) = \frac{a - c - \overline{q}_F - \lambda_{DD}(t)}{2}, \tag{33}$$

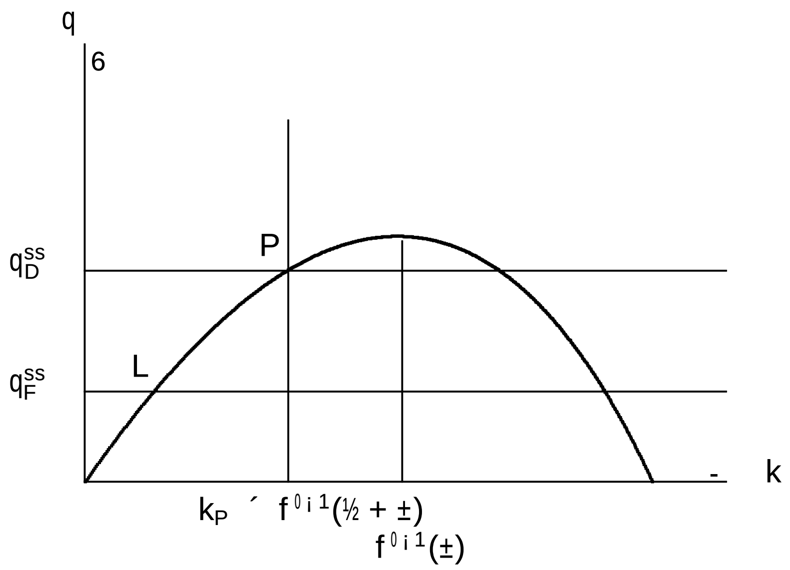
where $\overline{q}_F \equiv \dfrac{a - c - 2\tau}{3}$. It is immediate to verify that

$$\frac{dq_D(t)}{dt} = -\frac{d\lambda_{DD}(t)}{dt} = -[\rho + \delta - f'(k_D(t))]\lambda_{DD}(t). \tag{34}$$

Notice that the above condition holds irrespective of whether the quota is quantity-equivalent to the tariff or not. This situation is illustrated in figure 2 (the horizontal and vertical arrows describing the dynamics of $\{k, q\}$ are omitted).

[6] The stability analysis is omitted for the sake of brevity. See Cellini and Lambertini (1998) for an illustration of the symmmetric case.

Figure 2: Steady state equilibrium under a quota



This proves the following result:

Lemma 2 Under the Ramsey capital accumulation constraint, the adoption of any import quota drives the domestic firm to the Ramsey equilibrium where $f'(k_D(t)) = \rho + \delta$ and $q_D^{ss} = f(k_P)$:

Hence, if the government of the domestic country aims at (i) favouring the domestic firm, and (ii) lowering the domestic price, the adoption of a quantity-equivalent import quota is preferable to the adoption of the tariff, in that total output is larger under the former policy than under the latter.

Lemmata 1-2 produce the main result:

Proposition 4 Under the Ramsey capital accumulation constraint, the domestic price equivalence of tariffs and quotas does not hold.

5 Conclusions

In this paper, we have analyzed the equivalence among price-modifying and quantity fixing trade policies in a continuous time differential game. We have explicitly introduced the firms' accumulation dynamics and showed that, in two well known accumulation models, open-loop and closed-loop (no-memory) Nash equilibria coincide. Under the Nerlove-Arrow (1962) accumulation dynamics, the tari¤-quota equivalence holds, while under the Ramsey (1928) accumulation dynamics it does not. In the latter case, we have shown that the trade policy setting country prefers a quantity-equivalent import quota to the adoption of the tari¤.

The two accumulation schemes used in this paper and a similar analysis can be employed to deal with voluntary export restraints.[7] One could verify if and when export restraints set at a free trade level may increase pro…ts of the exporting …rm. This interesting possibility is left for further research.

[7] See also Dockner and Haug (1991) on this.

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