# Price vs Quantity in a Duopoly with Technological Spillovers: A Welfare Re-Appraisal


Luca Lambertini[#] and Andrea Mantovani[#,§]
#Department of Economics
University of Bologna
Strada Maggiore 45
I-40125 Bologna, Italy
fax: +39-051-2092664
e-mail: lamberti@spbo.unibo.it
e-mail: mantovan@spbo.unibo.it
§CORE, Université Catholique de Louvain
34, voie du Roman Pays
B-1348 Louvain-la-Neuve, Belgium


November 3, 2000

## Abstract

We analyse the problem of the choice of the market variable in a model where firms activate R&D investments for process innovation. We establish that (i) firms always choose the Cournot behaviour; and (ii) there exists a set of the relevant parameters where a benevolent social planner prefers quantity setting to price setting. This happens when the marginal cost of R&D activities is relatively low while technological externalities are relatively high. In this situation, the conflict between social and private preferences over the type of market behaviour disappears.



J.E.L. classification: L13, O31

Keywords: price, quantity, R&D, spillovers

# 1 Introduction

The interplay between technological choices and market behaviour in oligopoly models has been studied along two main routes. The first has emphasized the link between the kind of competition prevailing on the market and firms' incentives to invest either in process or in product innovation. The second concerns the influence of capacity constraints on market equilibrium.

Most literature on R&D races in oligopoly deals with the evaluation of incentives to undertake cost reducing investments as the number of firms changes. This Schumpeterian approach holds that a major factor determining the pace of technological progress is market structure (amongst the countless contributions in this vein, see Arrow, 1962; Loury; 1979; Lee and Wilde, 1980; Dasgupta and Stiglitz, 1980; Delbono and Denicolò, 1991; for an overview see Reinganum, 1989).

An established result on cost reducing investment in oligopolistic markets under perfect certainty states that there is excess expenditure in R&D under Cournot competition, and conversely under Bertrand competition, due to the opposite slopes of reaction functions at the market stage (Brander and Spencer, 1983; Dixon, 1985). With differentiated products, Bester and Petrakis (1993) maintain that the incentive to invest in cost reducing innovation depends upon the degree of product substitutability. Under both Cournot and Bertrand competition, underinvestment, as compared to the social optimum, obtains when products are fairly imperfect substitutes, while the opposite may occur when products are sufficiently similar. Cournot competition provides a lower (respectively, higher) incentive to innovate than Bertrand competition if substitutability is high (respectively, low). Social welfare may then be higher under Cournot than under Bertrand competition (Delbono and Denicolò, 1990; Qiu, 1997).

Singh and Vives (1984) investigate the choice of the market variable in a duopoly where firms operate costlessly. They find that, independently of the degree of product substitutability, firms choose to be quantity setters at the subgame perfect equilibrium, while social welfare would be higher under price setting behaviour. The opposite holds if products are demand complements.

In this paper, we extend Qiu's analysis to account for the asymmetric case where one firm is a quantity setter while the other is a price setter, and we derive the subgame perfect equilibrium of a two-stage game where firms operate R&D activities aimed at reducing marginal production costs and then compete at the marketing stage. Then, we recast Singh and Vives's analysis in a three-stage game where firms choose whether to be price or quantity setters at the first stage, then invest in cost-reducing R&D, and finally compete on the market. We establish that, at the subgame perfect equilibrium, firms always choose to set quantities. However, we also find that there exists a parameter region where quantity setting behaviour is socially preferable, if marginal R&D costs are sufficiently low and spillover are sufficiently high. In such a situation, we have a second best equilibrium where the usual conflict over the choice of the market variable disappears.

The remainder of the paper is organised as follows. The setup is laid out in section 2. Section 3 describes R&D and market behaviour. Private and social preferences concerning the choice of the market variable are then investigated in section 4. Section 5 concludes.

# 2 The setup

The demand side is a simplified version of Bowley (1924), subsequently adopted by Spence (1976), Dixit (1979) and Singh and Vives (1984), inter alia. Assume the representative consumer is characterised by the following utility function:

$$U(q_i q_j) = q_i + q_j - \frac{1}{2}\left(q_i^2 + q_j^2 + 2\gamma q_i q_j\right) \tag{1}$$

where $q_i$ and $q_j$ are the quantities of goods $i$ and $j$, respectively. The resulting (symmetric) demand functions under Cournot and Bertrand competition are, respectively

$$p_i = 1 - q_i - \gamma q_j \tag{2}$$

$$q_i = \frac{1}{1+\gamma} - \frac{p_i}{1-\gamma^2} + \frac{\gamma p_j}{1-\gamma^2} \tag{3}$$

In the asymmetric case, where firm $i$ is a quantity setter, while firm $j$ is a price setter, demand functions are:

$$p_i = 1 - q_i + \gamma(p_j + \gamma q_i - 1) \tag{4}$$

$$q_j = 1 - p_j - \gamma q_i \tag{5}$$

Parameter $\gamma \in (0, 1]$ represents product substitutability as perceived by consumers, depending upon the products firms supply. If one supposes that marginal costs are constant and equal to $c$ across firms, then individual profits are $\pi_i^{IJ} = (p_i - c)\, q_i$, where $IJ \in \{PP, QQ, PQ, QP\}$ indicates that, at the market stage, firm $i$ sets variable $I$ while firm $j$ sets variable $J$. In this case, the choice between price and quantity behaviour is summarised by the reduced form represented in Matrix 1.

|  |  | $j$ |  |
|---|---|---|---|
|  |  | $P$ | $Q$ |
| $i$ | $P$ | $\pi_i^{PP};\pi_j^{PP}$ | $\pi_i^{PQ};\pi_j^{QP}$ |
|  | $Q$ | $\pi_i^{QP};\pi_j^{PQ}$ | $\pi_i^{QQ};\pi_j^{QQ}$ |

Matrix 1

On the basis of Matrix 1, Singh and Vives (1984) conclude that quantity setting (resp., price setting) is the dominant strategy (at least weakly) if goods are demand substitutes (resp., complements). As a consequence, firms play a Cournot (resp., Bertrand) equilibrium for all $\gamma \in (0;1]$ (resp., $\gamma \in [-1;0)$).

In the remainder, given the symmetry of the model w.r.t. parameter $\gamma$; we will focus on the case of substitutes. Singh and Vives's result entails that there exists a conflict between firms' profit incentives and the social incentive towards welfare maximization, which requires price setting behaviour when goods are substitutes.

If we abandon the assumption of homogeneous costs, the choice between price and quantity is driven by the sign of the following expressions:

$$\pi_i^{QP}(c_i;c_j) - \pi_i^{PP}(c_i;c_j) \tag{6}$$

$$\pi_i^{QQ}(c_i;c_j) - \pi_i^{PQ}(c_i;c_j) \tag{7}$$

Now observe that, relabelling $1 - c_i \equiv \alpha_i$; the difference in productive efficiency across firms is formally equivalent to a difference in reservation prices for goods $i$ and $j$ in the representative consumer's preferences (see Häckner, 2000), which would now write as follows:

$$U(q_i q_j) = \alpha_i q_i + \alpha_j q_j - \frac{1}{2}\left(q_i^2 + q_j^2 + 2\gamma q_i q_j\right)$$

Hence, as proved by Singh and Vives (1984), $\{Q;Q\}$ is the unique equilibrium outcome.

Given that firms are ex ante symmetric, we need an explanation to justify any asymmetry in marginal costs. The reason for such an asymmetry can be found in the different incentives towards R&D investment that firms have in the four subgames $\{PP;QQ;PQ;QP\}$:

Suppose firms play a non-cooperative two-stage game, where the first stage involves choosing the individually optimal amount of R&D for process innovation, while the second is for marketing. Define as $x_i$ the R&D effort produced by firm $i$: The R&D technology is the same across firms. The cost of R&D activity is $K_i = f(x_i)$, with $f'(x_i) \equiv \partial f(x_i)/\partial x_i > 0$ and $f''(x_i) \equiv \partial^2 f(x_i)/\partial x_i^2 > 0$: That is, we suppose that R&D activity is characterised by decreasing returns to scale. The resulting marginal cost is $c_i \equiv c_i(x_i, \theta x_j)$, where $\theta \in [0,1]$ denotes the spillover received from firm $j$'s R&D investment. The net profits accruing to firm $i$, when the market subgame is $IJ$, are

$$a_i^{IJ} = \pi_i^{IJ}\left[c_i\left(x_i^{IJ}, \theta x_j^{JI}\right), c_j\left(x_j^{JI}, \theta x_i^{IJ}\right)\right] - f\left(x_i^{IJ}\right) \tag{8}$$

In this situation, the reduced form of the game is as in Matrix 2.

| | | $j$ P | $j$ Q |
|---|---|---|---|
| $i$ | P | $a_i^{PP}, a_j^{PP}$ | $a_i^{PQ}, a_j^{QP}$ |
| | Q | $a_i^{QP}, a_j^{PQ}$ | $a_i^{QQ}, a_j^{QQ}$ |

Matrix 2

Hence, the choice between P and Q is made according to the sign of:

$$\pi_i^{QP}\left[c_i(\cdot,\cdot), c_j(\cdot,\cdot)\right] - \pi_i^{PP}\left[c_i(\cdot,\cdot), c_j(\cdot,\cdot)\right] - f\left(x_i^{QP}\right) + f\left(x_i^{PP}\right) \tag{9}$$

$$\pi_i^{QQ}\left[c_i(\cdot,\cdot), c_j(\cdot,\cdot)\right] - \pi_i^{PQ}\left[c_i(\cdot,\cdot), c_j(\cdot,\cdot)\right] - f\left(x_i^{QQ}\right) + f\left(x_i^{PQ}\right) \tag{10}$$

The additional task consists in reassessing social preferences over the choice of market variables in this new setting. A priori, one cannot presume that the conflict between social and private incentives that characterises the previous setting extends to this case. Indeed, we know from Qiu (1997) that there are situations where social welfare is higher at the Cournot equilibrium than at the Bertrand equilibrium.

In order to carry out the analysis of this problem, we model the R&D stage as in Qiu (1997), by assuming that

$$c_i = c - x_i - \theta x_j\,;\; c \in (0,1) \tag{11}$$

$$K_i = \frac{\gamma x_i^2}{2} \tag{12}$$

Firms play a non-cooperative three-stage game. At the first stage, they choose between price and quantity. The following two stages describe (i) the choice of R&D efforts and (ii) marketing. As usual, we proceed by backward induction, using subgame perfection as the solution concept.

# 3 R&D and market subgames

We borrow from Qiu (1997) the characterisation of subgames $\{PP; QQ\}$; i.e., Bertrand and Cournot.

## 3.1 Symmetric subgames

Bertrand equilibrium prices are:

$$p_i^{PP} = \frac{(2+\gamma)(1-\gamma+c) - (2+\theta\gamma)x_i - (2\theta+\gamma)x_j}{4-\gamma^2} \tag{13}$$

The associated profits are:

$$\pi_i^{PP} = \frac{\left[(1-c)(\gamma^2+\gamma-2) - (2-\theta\gamma-\gamma^2)x_i + (\gamma-2\theta+\theta\gamma^2)x_j\right]^2}{(4-3\gamma^2)^2-\gamma^6} - \frac{\nu x_i^2}{2} \tag{14}$$

Solving the R&D stage, one obtains:

$$x^{PP} = \frac{2(1-c)(2-\theta\gamma-\gamma^2)}{\Delta^{PP}} \tag{15}$$

$$\Delta^{PP} = \nu(1+\gamma)(2-\gamma)\left(4-\gamma^2\right) - 2(1+\theta)\left(2-\theta\gamma-\gamma^2\right) \tag{16}$$

The resulting per-firm equilibrium quantity is

$$q^{PP} = \frac{\nu(1-c)(4-\gamma^2)}{\Delta^{PP}} \tag{17}$$

Consumer surplus is

$$CS^{PP} = (1+\gamma)\left[\frac{\nu(1-c)(4-\gamma^2)}{\Delta^{PP}}\right]^2 \tag{18}$$

so that social welfare is

$$SW^{PP} = \sum_i \pi_i^{PP} + CS^{PP} = \tag{19}$$

$$= \frac{\nu(1-c)^2\left[\nu(3+\gamma-2\gamma^2)(4-\gamma^2)^2 - 4(2-\theta\gamma-\gamma^2)^2\right]}{(\Delta^{PP})^2}$$

The following holds:

**Lemma 1** (Qiu, 1997, p. 217) The condition $\nu > 1/c$ is i] sufficient but not necessary for $\Delta^{PP} > 0$; for all $\theta$ and $\gamma$ (stability)

ii] necessary and sufficient for post-innovation costs to be positive, i.e., $c^{PP} = c - (1+\theta)\,x^{PP} > 0$

iii] sufficient to ensure $\partial^2 \pi_i^{PP}/\partial x_i^2 < 0$ iff $\theta = 1$

If instead $\theta \neq 1$;

$$\frac{\partial^2 \pi_i^{PP}}{\partial x_i^2} < 0 \;\Leftrightarrow\; \alpha > \frac{2\,(2-\theta\gamma-\gamma^2)^2}{(1-\gamma^2)\,(4-\gamma^2)^2}$$

Examine now the Cournot case. The equilibrium output is

$$q_i^{QQ} = \frac{(1-c)\,(2-\gamma) + (2-\theta\gamma)\,x_i + (2\theta-\gamma)\,x_j}{4-\gamma^2} \tag{20}$$

The associated profits are:

$$\pi_i^{QQ} = \frac{\left[(1-c)\,(2-\gamma) + (2-\theta\gamma)\,x_i + (2\theta-\gamma)\,x_j\right]^2}{(4-\gamma^2)^2} - \frac{\alpha x_i^2}{2} \tag{21}$$

Proceeding backward to solve the R&D stage, one obtains:

$$x^{QQ} = \frac{2\,(1-c)\,(2-\theta\gamma)}{\Delta^{QQ}}, \tag{22}$$

$$\Delta^{QQ} = \alpha\,(2+\gamma)\left(4-\gamma^2\right) - 2\,(1+\theta)\,(2-\theta\gamma) \tag{23}$$

The resulting per-firm equilibrium quantity is

$$q^{QQ} = \frac{\alpha\,(1-c)\,(4-\gamma^2)}{\Delta^{QQ}} \tag{24}$$

The resulting consumer surplus is

$$CS^{PP} = (1+\gamma)\left[\frac{\alpha\,(1-c)\,(4-\gamma^2)}{\Delta^{QQ}}\right]^2 \tag{25}$$

so that social welfare is

$$SW^{QQ} = \sum_i \pi_i^{QQ} + CS^{QQ} = \tag{26}$$

$$= \frac{\alpha\,(1-c)^2\left[\alpha\,(3+\gamma)\,(4-\gamma^2)^2 - 4\,(2-\theta\gamma)^2\right]}{(\Delta^{QQ})^2}$$

The following holds:

Lemma 2 (Qiu, 1997, p. 216) The condition º > 1=c is

i] su¢cient but not necessary for

$$\begin{aligned} ¢^{QQ} &> 0 \text{ (stability)} \\ \frac{@^2 a_i^{QQ}}{@x_i^2} &< 0 \text{ (concavity)} \end{aligned}$$

for all µ and °

ii] necessary and su¢cient for post-innovation costs to be positive, i.e., $c^{QQ} = c \; ¡ \; (1 + µ)\, x^{QQ} > 0$

## 3.2 The asymmetric subgames

Cases QP and PQ are symmetric up to a permutation of …rms. Therefore, we con…ne our attention to the situation where …rm i is a quantity-setter, while …rm j is a price-setter. The demand functions are (4) and (5). The Nash equilibrium at the market stage is given by:

$$q_i^{QP} = \frac{(2 \; ¡ \; °)(1 \; ¡ \; c) + (2 \; ¡ \; µ°)\, x_i + (2µ \; ¡ \; °)\, x_j}{4 \; ¡ \; 3°^2} \tag{27}$$

$$p_j^{PQ} = \frac{1 \; ¡ \; c \; ¡ \; µx_i \; ¡ \; x_j}{4} + \; ¡ \; \frac{°\, [(2 \; ¡ \; °)(1 \; ¡ \; c) + (2 \; ¡ \; µ°)\, x_i + (2µ \; ¡ \; °)\, x_j]}{4 \; ¡ \; 3°^2} \tag{28}$$

The associated pro…ts are:

$$a_i^{QP} = \frac{(1 \; ¡ \; °^2)\,[(1 \; ¡ \; c)(2 \; ¡ \; °) + (2 \; ¡ \; µ°)\, x_i + (2µ \; ¡ \; °)\, x_j]^2}{(4 \; ¡ \; 3°^2)^2} \; ¡ \; \frac{ºx_i^2}{2} \tag{29}$$

$$a_j^{PQ} = \frac{[(1 \; ¡ \; c)(°^2 + ° \; ¡ \; 2) \; ¡ \; (µ°^2 + ° \; ¡ \; 2)\, x_i + (°^2 + µ° \; ¡ \; 2)\, x_j]^2}{(4 \; ¡ \; 3°^2)^2} \; ¡ \; \frac{ºx_j^2}{2} \tag{30}$$

At the …rst stage, …rms non-cooperatively maximise their respective pro…ts w.r.t. R&D e¤ort levels, solving:

$$\frac{@a_i^{QP}}{@x_i} = \frac{2(1 \; ¡ \; °^2)(2 \; ¡ \; µ°)(1 \; ¡ \; c)(2 \; ¡ \; °)}{(4 \; ¡ \; 3°^2)^2} + \tag{31}$$

$$+ \frac{2(1 \; ¡ \; °^2)(2 \; ¡ \; µ°)\,[(2 \; ¡ \; µ°)\, x_i + (2µ \; ¡ \; °)\, x_j]}{(4 \; ¡ \; 3°^2)^2} \; ¡ \; ºx_i = 0$$

$$\frac{\partial \Psi_j^{PQ}}{\partial x_j} = \frac{2(\theta\gamma^2+\gamma-2)(1-c)(\gamma^2+\gamma-2)}{(4-3\gamma^2)^2} + \tag{32}$$
$$-\frac{2(\theta\gamma^2+\gamma-2)\left[(\theta\gamma^2+\gamma-2)x_i + (\gamma^2+\theta\gamma-2)x_j^2\right]}{(4-3\gamma^2)^2} - \nu x_j = 0$$

whose solution yields the equilibrium R&D investments:

$$x_i^{QP} = \frac{2(1-c)(1-\gamma^2)(\theta\gamma-2)\alpha_i}{\Delta_{xQP}} \tag{33}$$

$$x_j^{PQ} = \frac{2(1-c)(1-\gamma)(4-3\gamma^2)(\gamma^2+\theta\gamma-2)\alpha_j}{\Delta_{xPQ}} \tag{34}$$

where:

$$\alpha_i = 2(1-\theta)\left(\gamma^2+\theta\gamma-2\right) + \nu(2-\gamma)\left(4-3\gamma^2\right)$$

$$\alpha_j = 2(1-\theta)\left(2-\theta\gamma^2\right) + 2\left(2-3\theta+\theta^2\right)\gamma - \nu(2+\gamma)\left(4-3\gamma^2\right)$$

$$\Delta_{xQP} = \left(\theta^2-1\right)\left[5\left(4-\gamma^2\right)\theta\gamma^3 + 16(1-\theta\gamma)\right] + 4\left[2\left(3-\gamma^2\right) - (7-3\gamma)\theta^2\right]\gamma^2 +$$
$$+\nu\left[16\left(4-7\gamma^2\right) + 2\left(28-3\gamma^2\right)\gamma^4 - 32\left(2-3\gamma^2\right)\theta\gamma - 6(6-\gamma)\theta\gamma^5 +\right.$$
$$\left. +4\left(4-5\gamma^2\right)\theta^2\gamma^2 - 16\left(4-9\gamma^2\right)\nu - 27\left(4-\gamma^2\right)\gamma^2\right]\gamma^2$$

$$\Delta_{xPQ} = 4\left(\gamma^2-1\right)(\gamma-2\theta)(2-\theta\gamma)\left(\gamma^2+\theta\gamma-2\right)\left(\gamma-2\theta+\theta\gamma^2\right) +$$
$$+\left\{2\left(1-\gamma^2\right)(4-\theta\gamma)^2 - \nu\left(4-3\gamma^2\right)^2\right\}\left[2\left(4\left(1-\gamma^2\right)-\theta\gamma\right) + (\gamma+\theta)^2\gamma^2\right] +$$
$$\left. -\nu\left(4-3\gamma^2\right)^2\right\}$$

The equilibrium output levels are:[1]

$$q_i^{QP} = \frac{(1-c)(3\gamma^2-4)\left[2(1-\theta)(\gamma^2+\theta\gamma-2) + \nu(2-\gamma)(4-3\gamma^2)\right]}{\Delta_{xQP}} \tag{35}$$

$$q_j^{PQ} = \frac{(1-c)(1-\gamma)(4-3\gamma^2)\nu\left[2(1-\theta)(2-\theta\gamma^2) + \right.}{\Delta_{xPQ}} \tag{36}$$
$$\frac{\left. +2\left(2-3\theta+\theta^2\right)\gamma - \nu(2+\gamma)(4-3\gamma^2)\right]}{\Delta_{xPQ}}$$

[1] For the sake of brevity, we omit the expressions of equilibrium prices, which are available upon request.

The corresponding equilibrium profits are:

$$\pi_i^{QP} = \frac{(1-c)^2(\gamma^2-1)\beta\left[2(1-\theta)(\gamma^2+\theta\gamma-2)+\beta(2-\gamma)(4-3\gamma^2)\right]^2\left[2(1-\gamma^2)(2-\theta\gamma)^2-\beta(4-3\gamma^2)\right]}{(\Delta_{xQP})^2} \tag{37}$$

$$\pi_j^{PQ} = \frac{(1-c)^2(1-\gamma)\beta\left[2(1-\theta)(2-\theta\gamma^2)+2\left(2-3\theta+\theta^2\right)\gamma-\beta(2+\gamma)(4-3\gamma^2)\right]^2\left[8(1-\gamma^2-\theta\gamma)+2(\gamma+\theta)^2\gamma^2-\beta(4-3\gamma^2)\right]}{(\Delta_{xPQ})^2} \tag{38}$$

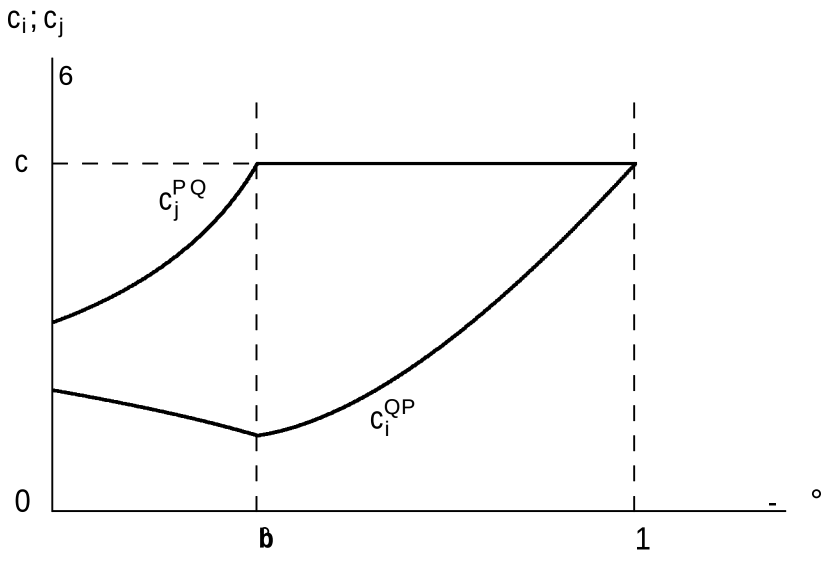
On the basis of the above expressions, we cannot derive analytically the equivalent of Lemmata 1-2 for the asymmetric case. Therefore, we must rely upon numerical calculations to ensure that (i) concavity and stability conditions are satisfied; and (ii) post-innovation marginal costs are non-negative.

However, on the basis of (33-34), the following holds:

**Lemma 3** Given acceptable values of $\{c;\gamma;\theta;\beta\}$; we have that $x_i^{QP} > x_j^{PQ}$: Moreover, given acceptable values of $\{c;\theta;\beta\}$; there exists $\hat{\gamma} \in (0;1)$; such that $\left\{x_j^{PQ}=0;\ c_j^{PQ}=c\right\}$:

That is, the quantity-setter invests more than the price setter, and the latter does not invest at all to reduce her own marginal cost, if product substitutability is larger than a critical threshold. The general behaviour of $c_i^{QP}$ and $c_j^{PQ}$ for $\gamma \in [0;1]$ is described in Figure 1.

Figure 1 : Marginal costs and product substitutability




First of all, notice that, as substitutability increases, the marginal cost of the price-setter becomes increasingly larger than the marginal cost borne by the quantity-setter, for all $\gamma \in [0; \hat{\gamma})$.[2] This reflects the higher incentive towards investment in process innovation for the quantity-setter compared to the price-setter, in line with previous findings by Brander and Spencer (1983), Dixon (1985), Bester and Petrakis (1993). Moreover, at $\gamma = \hat{\gamma}$ we have that $c_j^{PQ} = c$ and, therefore, the price-setter stops investing in R&D. That is, for $\gamma \geq \hat{\gamma}$; we set $x_j^{PQ} = 0$ and recalculate $x_i^{QP}$ from (31). This reveals that the quantity setting firm reduces her investment in R&D in response to the fact that the price setting rival is not investing at all. As a consequence, $c_i^{QP}$ increases in the degree of product substitutability, for all $\gamma \in [\hat{\gamma}; 1]$: When $\gamma = 1$; i.e., products are homogeneous, also the quantity-setter stops investing and both firms operate at $c$:

In general, the solution to firm i's investment problem, when firm j does

[2] For example, when

$$c = \frac{3}{4}; \ \alpha = 1.34; \ \gamma \in [0; 1]; \ \theta = \frac{1}{100};$$

concavity and stability conditions are met and we have $\hat{\gamma} = 0.83622$:

not invest is given by:

$$x_i^{QP}\left(x_j^{PQ}=0\right)=\frac{2(1-\gamma^2)(\gamma\theta-2)(1-c)(2-\gamma)}{2(1-\gamma^2)(2-\gamma\theta)^2-\beta(4-3\gamma^2)^2} \tag{39}$$

Finally, social welfare in the asymmetric case is:

$$SW^{PQ}=SW^{QP}=\sum_i \pi_i^{IJ}+CS^{IJ} \tag{40}$$

where $CS^{IJ}=(1+\gamma)\left(q_i^{IJ}+q_j^{JI}\right)^2/4$.

# 4 The first stage: private vs social preferences

Equilibrium profits $\pi^{PP}$, $\pi^{QQ}$, $\pi^{PQ}$ and $\pi^{QP}$ can be plugged into Matrix 2 to yield the reduced form of the first stage of the game, where firms non-cooperatively choose whether to be price- or quantity-setters.

We obtain the following:

**Claim 1** For all admissible values of parameters $\{c;\gamma;\theta;\beta\}$; we have that $Q \succeq P$: Hence, the Cournot equilibrium is unique and results from (at least weakly) dominant strategies.

Explicit calculations over the relevant inequalities, i.e.:

$$\pi^{QQ}-\pi^{PQ}>0 \tag{41}$$

$$\pi^{QP}-\pi^{PP}>0 \tag{42}$$

are omitted for the sake of brevity. Verifying that (41-42) hold for all positive $\left\{x_i^{IJ}\right\}$ is a matter of simple albeit tedious algebra.[3] Having done that, the extension to the case where $x^{QP}>0$ and $x^{PQ}=0$ is immediate, in that the price-setter's profits $\pi^{PQ}$ are decreasing over $\gamma\in[0;1]$; for two reasons. The first is the increase in product substitutability. As $\gamma$ increases towards one, it becomes increasingly harder for the price setting firm to keep her price above marginal cost. The second reason is that the quantity-setter keeps investing in cost-reducing R&D for all $\gamma\in[0;1)$.

[3] As anticipated in section 3.2, the only complication consists in verifying the concavity and stability conditions, and the positivity of post innovation costs, for the asymmetric case, together with the corresponding conditions for the symmetric cases as from Lemmata 1-2. This can only be done numerically. Calculations are available upon request.

Claim 1 extends Singh and Vives's findings to the case where firms invest in R&D to reduce marginal costs. The interpretation of this result is that the lower incentive to invest that characterise a price-setter as compared to a quantity-setter, (see Brander and Spencer, 1983; Dixon, 1985; Bester and Petrakis, 1993), is insufficient to generate equilibria where at least one firm is a price-setter, over the whole parameter space.

Now we are in a position to compare private and social incentives concerning the choice between $P$ and $Q$: We know that the following holds:

**Proposition 1 (Qiu, 1997, p. 223)** Suppose $\theta \in (0; 1)$; $\nu > 1/c$; and

$$\nu > \frac{2\left(2-\theta\gamma-\gamma^2\right)^2}{\left(1-\gamma^2\right)\left(4-\gamma^2\right)^2} \text{ for all } \gamma \in (0; 1) :$$

Then, given $\gamma$; either

i] $SW^{PP} > SW^{QQ}$ for all $\nu$ and $\theta$; or

ii] there exists a unique $\hat{\nu} > 1$; such that

- for all $\nu \geq \hat{\nu}$ and all $\theta \in (0; 1)$; we have $SW^{PP} > SW^{QQ}$
- for all $\nu < \hat{\nu}$; there exists $\hat{\theta} \in (0; 1)$ such that

$$SW^{PP} - SW^{QQ} \begin{array}{ll} > 0 & \text{for all } \theta < \hat{\theta} \\ = 0 & \text{for } \theta = \hat{\theta} \\ < 0 & \text{for all } \theta > \hat{\theta} \end{array}$$

III] For $\gamma \to 0$; [i] holds; for $\gamma \to 1$; [ii] holds.

Now, consider Claim 1 and Proposition 1 jointly. If Proposition 1[ii] holds, and, in particular, $\nu < \hat{\nu}$ and $\theta > \hat{\theta}$; then firms play $\{Q; Q\}$ which is also the socially preferred equilibrium. Therefore, we have our final result:

**Proposition 2** Suppose

- $\theta \in (0; 1)$; $\theta > \hat{\theta}$;
- $\nu \in \left( \max \left\{ \frac{1}{c}; \frac{2\left(2-\theta\gamma-\gamma^2\right)^2}{\left(1-\gamma^2\right)\left(4-\gamma^2\right)^2} \right\}; \hat{\nu} \right)$ for all $\gamma \in (0; 1)$ :

If so, $\{Q; Q\}$ is a second best equilibrium where social and private preferences over the choice of the market variable coincide.

Therefore, the introduction of an additional stage describing cost-reducing R&D activities into Singh and Vives's framework produces a subgame perfect equilibrium where, provided marginal R&D costs are sufficiently low and spillover are sufficiently high, quantity setting behaviour is preferred from both the social and the private standpoint. Obviously, there remains the inefficiency associated with the profit-maximising decisions of firms at the market stage, entailing a distortion in output and price levels as compared to the social optimum.

# 5 Concluding remarks

The foregoing analysis recasts the problem of the choice of the market variable first investigated by Singh and Vives (1984) into a picture where an additional stage describes firms' R&D investments in cost-reducing activities, as in Qiu (1997). This allows us to establish that (i) firms always choose the Cournot behaviour; and (ii) there exists a set of the relevant parameters where a benevolent social planner prefers quantity setting to price setting. This happens when the marginal cost of R&D activities is relatively low while technological externalities are relatively high. In this situation, the overinvestment in R&D associated with Cournot behaviour (Brander and Spencer, 1983) is welcome in that it produces positive welfare effects, to such an extent that the conflict between social and private preferences over the type of market behaviour disappears.

## References


[1] Arrow, K. (1962), "Economic Welfare and the Allocation of Resources for Inventions", in Nelson, R. (ed.), The Rate and Direction of Inventive Activity, Princeton, NJ, Princeton University Press.

[2] Bester, H. and E. Petrakis (1993), "The Incentives for Cost Reduction in a Di¤erentiated Industry", International Journal of Industrial Organization, 11, 519-34.

[3] Bowley, A.L. (1924), The Mathematical Groundwork of Economics, Oxford, Oxford University Press.

[4] Brander, J. and B. Spencer (1983), "Strategic Commitment with R&D: The Symmetric Case", Bell Journal of Economics, 14, 225-35.

[5] Dasgupta, P. and J. Stiglitz (1980), "Uncertainty, Industrial Structure, and the Speed of R&D", Bell Journal of Economics, 11, 1-28.

[6] Delbono, F. and V. Denicolò (1990), "R&D Investment in a Symmetric and Homogeneous Oligopoly: Bertrand vs Cournot", International Journal of Industrial Organization, 8, 297-313.

[7] Delbono, F. and V. Denicolò (1991), "Incentives to Innovate in a Cournot Oligopoly", Quarterly Journal of Economics, 106, 951-61.

[8] Dixit, A. (1979), "A Model of Duopoly Suggesting a Theory of Entry Barriers", Bell Journal of Economics, 10, 20-32.

[9] Dixon, H.D. (1985), "Strategic Investment in a Competitive Industry", Journal of Industrial Economics, 33, 205-12.

[10] Häckner, J. (2000), "A Note on Price and Quantity Competition in Di¤erentiated Oligopolies", Journal of Economic Theory, 93, 233-9.

[11] Lee, T. and L. Wilde (1980), "Market Structure and Innovation: A Reformulation", Quarterly Journal of Economics, 94, 429-36.

[12] Loury, G. (1979), "Market Structure and Innovation", Quarterly Journal of Economics, 93, 395-410.

[13] Qiu, L. (1997), "On the Dynamic E¢ciency of Bertrand and Cournot Equilibria", Journal of Economic Theory, 75, 213-29.

[14] Reinganum, J. (1989), "The Timing of Innovation: Research, Development and Di¤usion", in Schmalensee, R. and R. Willig (eds.), Handbook of Industrial Organization, vol. 1, Amsterdam, North-Holland.

[15] Singh, N. and X. Vives (1984), "Price and Quantity Competition in a Di¤erentiated Duopoly", RAND Journal of Economics, 15, 546-54.

[16] Spence, A.M. (1976), "Product Di¤erentiation and Welfare", American Economic Review, 66, 407-14.