Price versus Quantity Competition with Cost Sharing

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Abstract

We inspect the interlink between the endogenous choice of price- and quantity-setting behaviour in an oligopolistic market, and cost sharing among oligopolists. A typical situation of this sort is an oligopoly game where firms invest in product development first, and then play a marketing game later. Only in the initial investment stage, the firms set up a joint venture in order to share the costs. We discover that, in the presence of shared costs, the well-established result by Singh and Vives (1984) that firms endogenously choose quantity (resp., price) as a dominant strategy when their products are substitutes (resp., complements) may not be the only equilibrium outcome. In particular, the *procedural order* between firms' cost sharing decisions and their marketing decisions makes a key difference in the resulting equilibrium profiles.

Keywords : joint venture, repayment, subgame perfection, Nash bargaining.

JEL classification : L13, G31, O32.

1 Introduction

COURNOT VERSUS BERTRAND, whether firms are quantity setters or price setters, has been a long standing dispute. A possible theoretical answer to this dispute is to construct and solve a game in which each firm's choice between price setting and quantity setting is endogenous. The seminal paper by Singh and Vives (1984) establishes that, in a simple two-stage game where the first stage is for firms to choose between price setting and quantity setting, and the second stage is for them to realise their actions which are either prices or quantities, quantity (resp., price) setting behaviour is a dominant strategy if firms supply substitute (resp., complement) products. As a consequence, firms play a Cournot (resp., Bertrand) game.[1] This result is reinforced by Boyer and Moreaux (1987a, b). Maintaining the same demand structure as Singh and Vives, they consider the possibility for firms to move either simultaneously or sequentially in the marketing stage, and prove that the initial choice of the strategic variable dominates the ensuing choice of action timing, in that quantity setting behaviour ensures higher profits than price setting behaviour if firms' products are demand substitutes and *vice versa* if demand complements, irrespectively of whether firms are to play a static Nash or a Stackelberg equilibrium in marketing.

At the same time, it is apparent from our anecdotal observations that the choice between price setting and quantity setting depends largely upon those processes through which the products and their distribution systems are developed, run, and/or maintained. When the marketing game preceded by a sizeable initial investment, it is often observed that part of such an investment is shared among the "competing" firms. A typical economic example of such a multi-firm joint initial investment is an R&D joint venture (often abbreviated as an RJV).[2] Such cost sharing, when interrelated closely to firms' ensuing marketing behaviour, can affect their strategic variable choices. Our attempt in this paper is to inspect the robustness

[1] The issue whether the same holds in an infinitely repeated game is tackled by Lambertini (1997) and Albk and Lambertini (1997).

[2] Such investment pooling is in fact *encouraged* through various industrial *policy measures* in effectively every developed economy. The rationale is that it curtails *effort duplication* among firms and thereby enhances economic efficiency. This is contrasted sharply against the antitrust regulations on firms' *marketing* behaviour, where firms are strictly forced to stay apart.

of the preceding seminal results by embedding firms' strategic choice between price setting and quantity setting in the context of a three-stage oligopoly game where a shared initial investment precedes a two-stage marketing game *à la* Singh and Vives.

More precisely, the gist of the difference between preceding studies and our model is not the mere fact that firms share initial investment costs, but that there can be room for *bargaining among firms* to determine their cost shares. In certain situations, the cost of initial investment can be precisely calculated at the time of, or even before, the commencement of the investment. In such a situation, if all payments are sorted out prior to the marketing stage, then any shared investment costs become entirely sunk by the time the firms start acting as competitors in marketing and choose their respective strategic variables, either prices or quantities. In such a circumstance, the cost sharing rule cannot be made contingent upon the firms' ensuing marketing behaviour. Therefore, all the seminal results continue to stay valid.

Realistically, however, the costs of the initial investment often require continual reassessment and readjustment as the invested activity proceeds. The payment of the investment costs can also take a substantial amount of time to settle. For, it seems plausible for the group of firms to go in debt to commence their joint investment and then repay the costs later, obviously the fund for repayment being raised from the participant firms' marketing profits. Namely, by the time when the investment costs are actually (re)paid, the marketing stage has already begun. Game theoretically, this can imply that there is room for the cost sharing rule to be made contingent upon part of the marketing behaviour of the firms.

Viewed differently, if a joint venture or a multi-firm cooperative aiming any sort of cost pooling is officially recognised as a debt-creditable economic body, the resulting form of market competition may differ quite substantially from otherwise. For, the official institutionalisation of multiple firms' "joint debt" can serve as a device for these firms to borrow costs externally, leaving each firm's cost share up to the firms' internal negotiation. Although in this paper we represent such cost sharing situations by a simple model where the pooled costs are incurred at the beginning of the game in the form of initial investment, the essence of our analysis could carry over to other forms of cost pooling which do not necessarily precede the marketing stage. It is by no means impossible for firms to pool their *operative costs* which are to be paid as their production activities go on. Some kinds of infrastructural maintainance costs or utility costs may offer prime examples.

In section 2 we construct and analyse a pair of simple duopoly games modelling these two different procedures concerning the firms' decisions on cost sharing in their joint initial investment. We discover that the set of equilibrium outcomes can be sensitive to the procedural structure of the game. Our main focus is on the "negotiable cost sharing" game in which the cost sharing rule among firms can be interrelated to their strategic variable choices, as is laid out in subsection 2.2. In section 3, we locate our analysis in the context of the existing literature, by applying our general abstract observations to the popularly studied demand function *à la* Singh and Vives. In fact, we prove that one firm's quantity setting and the other's price setting can be sustained in a pure strategy subgame perfect equilibrium when (and only when) the timing of the firms' actions in the marketing game is endogenously choosable. Then in section 4, we extend our general results to encompass [1] n-firm oligopoly and also [2] structural asymmetry across firms. Section 5 discusses briefly whether/why [i] cost sharing is socially preferable, and [ii] joint ventures needs to be encouraged by policy measures. Section 6 concludes the paper with a brief summary of our main qualitative findings.

2 The basic duopoly model

We start our analysis from a simple symmetric duopoly model, in which an initial investment is followed by a marketing game. The initial investment is made jointly between the two firms, which shall be referred to as a *joint venture*, or a JV for shorthand. One of the most common situations of this sort is an oligopolistic industry where a new product is innovated in an R&D joint venture formed by multiple firms, and then, once the product has been developed, the individual firms *noncooperatively* compete in marketing.

The total cost required in the initial investment is $k > 0$, which is to be split between the two participating firms. In the ensuing marketing stage, each of the two firms simultaneously and independently chooses whether to become a price setter or a quantity setter. Then finally, these firms are to choose their actual actions, i.e., either price or quantity *levels*.

We do not specify any concrete form of market interactions, thereby our results hereinafter stay valid whether the marketing stage is static or multi-period, whether the timing of firms' marketing actions is exogenous or endogenous, whether their products are mutually substitutes or complements, and whether time discounting is adopted or not. We abstract their market interaction into the following profit matrix which, obviously, is to be interpreted as *discounted to the time of R&D investment* if the market interaction is dynamic and firms' time preferences are considered accordingly.

The whole game is symmetric in that the two firms are *a priori identical*. The payoff matrix is therefore symmetric between the two firms.

Firm	Opponent	
	price	quantity
price	Y_{PP}	Y_{PQ}
quantity	Y_{QP}	Y_{QQ}

This matrix defines the notation for equilibrium profits from the marketing stage. Rows and columns are labelled according to each firm's strategic variable, and the cells indicate the row player's net profits from marketing.

In the following, we compare the effects of two different mechanisms of cost sharing. One is a mechanism where each firm's cost share is determined independently of the form of the ensuing market competition. The results in this setting are directly analogous to Singh and Vives (1984) and Boyer and Moreaux (1987a, b), as described briefly in 2.1. The other is a situation where cost sharing can depend upon the firms' strategic variable decisions. In this setting, the equilibrium results differ quite substantially from the preceding seminal results, as explained closely in 2.2.

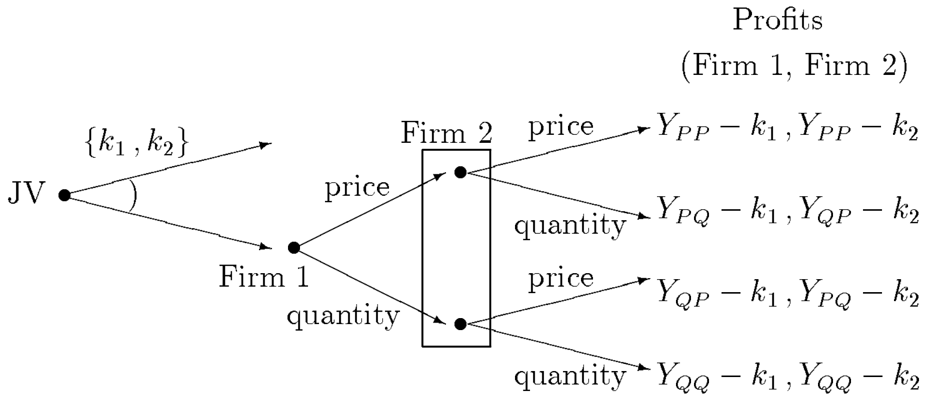
2.1 Non-negotiable cost sharing

This is a situation where decisions on how the investment cost k should be split between the two firms is made *independently of the choice between price setting and quantity setting* in the ensuing marketing stage. In other words, the game is *separable* between the initial investment stage and the ensuing marketing stage in the sense that all the investment costs are completely sunk by the time the marketing stage arrives. This model is appropriate if (and only if) the initial investment and the form of the ensuing marketing behaviour are not interconnected at all, either technologically or institutionally.

This situation also arises when the two firms *prepay* their respective shares of investment costs k_1 and k_2, where $k_1 + k_2 = k$, when they commence the JV. Since it is each individual firm that brings the fund in to start up the JV, the cost shares cannot be changed later.

The procedural structure of the game can be summarised by the tree in Figure 1.

Figure 1 : non-negotiable cost sharing.



In this game, it is straightforward to see that the seminal results in preceding studies would directly carry on. Essentially, if the investment is prepaid and sunk, so that the decision on it cannot hinge upon the ensuing revenues, then the firms end up playing the normal form game described by the profit matrix (see section 2) directly.

2.2 Negotiable cost sharing

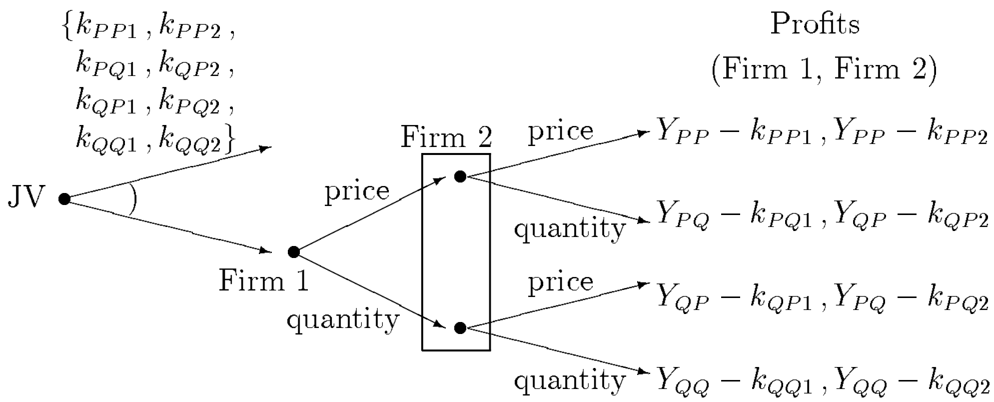
We now consider a slightly different situation where decisions on investment cost sharing between the two firms is made in conjunction with the choice between price setting and quantity setting. This should occur when the initial investment and the form of the ensuing marketing behaviour are indivisibly interconnected.

A similar situation arises when a JV *borrows* the cost of the initial investment first, and then the two participant firms *repay the debt* as they raise profits in marketing. Since it is the *joint* venture that borrows the fund, the two firms' respective cost shares k_{X1} , k_{X2} can remain negotiable between the two firms, thus can be made contingent upon the profile of strategic variables $X \in \{PP, PQ, QP, QQ\}$ the firms choose later in marketing, obviously subject to $k_{X1} + k_{X2} = k$.

As in many other game-theoretic models, there remains a certain degree of ambiguity in the process of collective decision making. In our specific model, we need to impose an assumption on the "equilibrium" cost sharing rule the JV should choose. In this paper we assume that the cost sharing rule be derived from *Nash bargaining*.

The procedural structure of this version of the game can be summarised by the tree in Figure 2.

Figure 2 : negotiable cost sharing.



Nash bargaining stipulates that each firm's respective gains (or losses) as compared to its "outside alternative" be equalised, i.e.

- when both firms set prices, firm 1's incentive (or disincentive ; likewise henceforth) to switch to quantity setting should equal that of firm 2 :

$$(Y_{PP} - k_{PP1}) - (Y_{QP} - k_{QP1}) = (Y_{PP} - k_{PP2}) - (Y_{QP} - k_{QP2}) \,; \qquad (1)$$

- when one firm sets a price and the other a quantity, the price setter's incentive to switch to quantity setting should equal the quantity setter's incentive to switch to price setting :

$$(Y_{PQ} - k_{PQi}) - (Y_{QQ} - k_{QQi}) = (Y_{QP} - k_{QPj}) - (Y_{PP} - k_{PPj}) \qquad (2)$$

where $\{i, j\} = \{1, 2\}$;

- when both firms set quantities, firm 1's incentive to switch to price setting should be equal to firm 2's incentive to do so:

$$(Y_{QQ} - k_{QQ1}) - (Y_{PQ} - k_{PQ1}) = (Y_{QQ} - k_{QQ2}) - (Y_{PQ} - k_{PQ2}) \,. \qquad (3)$$

In conjunction with the feasibility condition

$$k_{X1} + k_{X2} = k \qquad X \in \{PP, PQ, QP, QQ\} \,, \qquad (4)$$

there are eight equations for eight unknowns. However, since the four equations (1), (2), (3) are not mutually independent, there still remains one degree of freedom. Namely, we obtain

$$k_{PPi} = k_{QQi} = \widehat{k}_i \qquad \widehat{k}_1 + \widehat{k}_2 = k \,, \qquad (5)$$

$$k_{PQi} = \widehat{k}_i + \frac{Y_{PP} + Y_{PQ} - Y_{QP} - Y_{QQ}}{2} \,, \qquad (6)$$

$$k_{QPi} = \widehat{k}_i + \frac{Y_{QQ} - Y_{QP} - Y_{PQ} - Y_{PP}}{2} \qquad (7)$$

where $i = 1, 2$.

Hence, the following outcomes can be sustained in pure strategy subgame perfect equilibria (simply "equilibria" hereinafter unless otherwise specified).

Proposition I : In equilibrium,

- either both firms become price setters (Bertrand) or both become quantity setters (Cournot) if

$$Y_{PP} + Y_{QQ} \geq Y_{PQ} + Y_{QP} \,; \qquad (8)$$

- one firm becomes a price setter and the other a quantity setter if

$$Y_{PP} + Y_{QQ} \leq Y_{PQ} + Y_{QP} \,. \qquad (9)$$

In words, {price, price} and {quantity, quantity} are equilibrium outcomes if the choices between price setting and quantity setting entail *supermodular* profits[3] from

[3] Here, we refer to the discrete action version of supermodularity $Y_{PP} - Y_{QP} \geq Y_{PQ} - Y_{QQ}$ and submodularity $Y_{PP} - Y_{QP} \leq Y_{PQ} - Y_{QQ}$. In the former, the incentives to switch from action Q to action P is *superadditive* between the two firms (i.e., one firm's incentive to do so is higher when the other firm does likewise than when the other firm does elsewise), so are the incentives to switch from P to Q. In the latter, these incentives are *subadditive*.

the market (as in inequality (8)). Otherwise, if the strategic variable choices are to yield *submodular* profits in the marketing stage (indicated by inequality (9)), then {price, quantity} and {quantity, price} are equilibrium outcomes.

3 Examples

To relate our result to some of the literature, consider the marketing stage arising in a one-shot simultaneous-move linear duopoly game *à la* Singh and Vives (1984), where the inverse demand function for firm i is

$$p_i \;=\; 1 - q_i - \gamma q_j \qquad\qquad i = 1, 2\,. \tag{10}$$

Parameter $\gamma \in (-1, 1)$ measures the degree of differentiation between products,[4] and establishes that the direct effect on p_i of a variation in q_i is always at least as large as the effect of a variation in q_j. If $\gamma \in (0, 1)$, products are substitutes, while if $\gamma \in (-1, 0)$. they are complements in demand.

3.1 Static market with linear demand

Our results from section 2 will carry over as far as firms play a Nash equilibrium, whatever form it may take, in the relevant variables at the market stage. We first consider a setting where firms play a simultaneous-move market equilibrium.

Lemma i : • In the non-negotiable cost sharing game, {quantity, quantity} is the only equilibrium outcome when the two firms produce *demand substitutes* ($\gamma \in (0, 1)$), whilst {price, price} is the unique equilibrium outcome when the two firms produce *demand complements* ($\gamma \in (-1, 0)$).

[4] The supply of imperfect substitutes goods may result from an RJV where firms jointly develop the basic features of the good and then decide to sell differentiated varieties. This behaviour can be observed in the hi-fi industry. E.g., in 1972, Ivor Tiefenbrun at Linn Ltd and John Dunlop at Ariston Audio Ltd (both in Glasgow) started marketing Sondek LP12 and Ariston RD11, respectively. These turntables were internally identical, while their external layouts were slightly different. The project of the mechanics was carried out jointly by Tiefenbrun and Dunlop a few years earlier.

- In the negotiable cost sharing game, both {price, price} and {quantity, quantity} are equilibrium outcomes *irrespective of substitutability or complementarity between the two firms' products.*

Proof : See Appendix 7.1.

3.2 Market with endogenous timing of moves

We now move on to another example where firms may endogenously select the timing of their respective moves, as in Boyer and Moreaux (1987a, b) and in Hamilton and Slutsky (1990).[5] Consider the same inverse demand function (10). We assume that firms choose their strategic variables first, and then choose their respective timing of moves in an *extended game with observable delay* (Hamilton and Slutsky, 1990).

Contrary to our previous example, the following equilibrium result will emerge in this setting.

Lemma ii :

- In the non-negotiable cost sharing game, both firms' quantity setting is the unique equilibrium outcome if the two firms' products are *demand substitutes*, and both firms' price setting is the unique equilibrium outcome if their products are *demand complements.*
- In the negotiable cost sharing game, {price, quantity} and {quantity, price} are equilibrium outcomes *irrespective of demand complementarity or demand substitutability between the two firms' products.*

Proof : See Appendix 7.2.

3.3 Dynamic market with time discounting

Finally, observe that it is straightforward to apply our results from section 2 to a dynamic marketing game. To see this, consider another example where the marketing stage is a *supergame.* Namely, suppose that the development stage is at time $t = 0$, and that the marketing supergame unravels over $t = 1, 2, \cdots, \infty$. Given

[5] Earlier investigations on firms' preferences over their respective roles in market games can be found in Gal-Or (1985) and Dowrick (1986).

the two firms' strategic variable choices $X \in \{PP, PQ, QP, QQ\}$, if the marketing supergame has respective subgame perfect equilibria which yield stage profits W_X, then the operative payoff matrix in section 2 shall be derived as $Y_X = \frac{\delta}{1-\delta} W_X$, where $\delta \in (0, 1)$ is the time discount factor common to both firms.[6] Note in particular that all payoffs must be discounted *to the beginning of the game*. The remainder of the analysis follows unmodified.

In general, whatever the structure of the marketing game is, as long as the payoff matrix is written in terms of payoffs discounted to the beginning of the game, our observations in section 2 will continue to hold.

4 General oligopoly

It is straightforward to extend our basic duopoly model presented in the previous section into more general n-firm oligopoly. When cost sharing is non-negotiable, the equilibrium results depend solely upon the market structure, as there is no strategic interlink between firms' shared initial investment and their ensuing marketing behaviour. Henceforth we focus on the negotiable cost sharing case.

The generalisation contemplated in this section encompasses two directions. First, we allow profit asymmetry across firms. Namely, the profit matrix in the marketing stage (once again, discounted to the beginning of the game if necessary) need no longer be symmetric. Second, we extend our analysis into a general n-firm oligopoly game with shared initial investment.

Now that firms are not necessarily *a priori* identical, the marketing profit of firm i $(i = 1, \cdots, n)$ is denoted generally by $Y_i[X]$, and likewise the cost share of firm i is $k_i[X]$, where X is the profile of strategic variables. Following the conventional notation in game theory, subscripts attached to strategic variables indicate the player. Namely, $X_i \in \{P_i, Q_i\}$ indicates the strategic variable adopted by firm

[6] To adhere to our assumption of *a priori* symmetry between the two firms, we assume for the time being that they have a common discount factor. This assumption can easily be relaxed without substantial complication, as shall be seen in section 4. On the other hand, if the game is infinitely repeated and nevertheless no time discounting is introduced, then we would need to invoke other criteria than those well-defined payoffs Y_X, such as the overtaking criterion.

i, and $X_{-i} \in \times_{h \neq i}\{P_h, Q_h\}$ the profile of strategic variables adopted by all other players. For example, our previous duopoly notation can be transcribed as

$$\begin{aligned} \mathrm{Y}_{PP} &= Y_i[P_i, P_{-i}], \quad Y_{PQ} = Y_i[P_i, Q_{-i}], \quad Y_{QP} = Y_i[Q_i, P_{-i}], \quad Y_{QQ} = Y_i[Q_i, Q_{-i}], \\ \mathrm{k}_{PPi} &= k_i[P_i, P_{-i}], \quad k_{PQi} = k_i[P_i, Q_{-i}], \quad k_{QPi} = k_i[Q_i, P_{-i}], \quad k_{QQi} = k_i[Q_i, Q_{-i}]. \end{aligned}$$

In an n-firm oligopoly, our previous system of equations (1) through (3) is generalised as follows: for any players i, j $(j \neq i)$ and for any strategic variable profile $X = \{X_i, X_{-i}\} = \{X_j, X_{-j}\}$ (namely, X is decomposed in different ways depending upon which player is in focus),

$$\begin{aligned} &(Y_i[X_i, X_{-i}] - k_i[X_i, X_{-i}]) - (Y_i[\neg X_i, X_{-i}] - k_i[\neg X_i, X_{-i}]) \\ &= (Y_j[X_j, X_{-j}] - k_j[X_j, X_{-j}]) - (Y_j[\neg X_j, X_{-j}] - k_j[\neg X_j, X_{-j}]) \end{aligned} \tag{11}$$

where $\{\neg X_i\} = \{P_i, Q_i\} \setminus \{X_i\}$, and likewise $\{\neg X_j\} = \{P_j, Q_j\} \setminus \{X_j\}$. Note that there are 2^n different choices of strategic variables X, each of which leads to effectively $n-1$ equations (with $\{i,j\} = \{1,2\}, \{2,3\}, \cdots, \{n-1, n\}$). Hereby this system consists of $2^n(n-1)$ equations.

Also, (4) is replaced with

$$\sum_{i=1}^{n} k_i[X] = k \qquad\qquad X \in \times_{h=1}^{n}\{P_h, Q_h\}. \tag{12}$$

This gives additional 2^n equations. Hence in total, we have $2^n n$ equations with $2^n n$ unknowns $k_i[X]$. However, since those $2^n(n-1)$ equations in (11) are not independent, we leave $n-1$ degrees of freedom. The solutions to the system (11)-(12) are

$$\begin{aligned} k_i[X] \;=\; & Y_i[X] + \widehat{k}_i - \frac{1}{n}\sum_{j=1}^{n} \sum_{V_{-i} \in \times_{h \neq i}\{P_h, Q_h\}} \Bigg[\\ & \left(\prod_{m=1}^{|V_{-i}-X_{-i}|} \frac{-m}{n-m} + \frac{(-1)^{|V_{-i}-X_{-i}|}}{2^{n-1}} \sum_{\ell=1}^{|V_{-i}-X_{-i}|} \prod_{m=1}^{\ell} \frac{m}{n-m} \right) Y_j[X_i, V_{-i}] \\ & + \left(\prod_{m=1}^{|V_{-i}-X_{-i}|} \frac{-m}{n-m} + \frac{(-1)^{|V_{-i}-X_{-i}|}}{2^{n-1}} \sum_{\ell=0}^{|V_{-i}-X_{-i}|} \prod_{m=1}^{\ell} \frac{m}{n-m} \right) Y_j[\neg X_i, V_{-i}] \Bigg] \end{aligned}$$

where $|V_{-i} - X_{-i}|$ is the number of players whose strategic variables change between the two partial profiles X_{-i} and V_{-i}, and

$$\sum_{i=1}^{n} \widehat{k}_i = k.$$

Particularly noteworthy about these Nash bargaining solutions is the property that

$$\begin{aligned}
&(Y_i[X] - k_i[X]) - (Y_i[\neg X_i\,, X_{-i}] - k_i[\neg X_i\,, X_{-i}]) \\
&= \frac{1}{2^{n-1}n}\sum_{j=1}^{n}\sum_{V_{-i}\in\times_{h\neq i}\{P_h,Q_h\}}(-1)^{|V_{-i}-X_{-i}|}\left(Y_j[X_i\,, V_{-i}] - Y_j[\neg X_i\,, V_{-i}]\right) \\
&= \frac{1}{2^{n-1}n}\sum_{j=1}^{n}\sum_{V\in\times_{h=1}^{n}\{P_h,Q_h\}}(-1)^{|V-X|}Y_j[V]
\end{aligned}$$

where, as before, $|V-X|$ is the number of firms who switch their strategic variables between the two profiles X and V.

We hereby arrive in the following equilibrium result, which can be viewed as an extension of Proposition I.

Proposition II : X is an equilibrium strategic variable profile if and only if

$$\sum_{j=1}^{n}\sum_{V\in\times_{h=1}^{n}\{P_h,Q_h\}}(-1)^{|V-X|}Y_j[V] \;\geq\; 0\,.$$

It is straightforward to confirm that Proposition I is indeed a special case of proposition II, where $n = 2$ and the profit structure is symmetric between the duopolists.

The literal implication of Proposition II is that it is only the *parity* of the numbers of price setters and quantity setters that is determined by the profit structure of the marketing stage game. Therefore, as n grows large, effectively any arbitrary ratio between the numbers of price setting firms and of quantity setting firms can be consistent with subgame perfection insofar as their numbers maintain the designated parity. On the other hand, if the number of participant firms in the joint venture is relatively small, the same intuition as in Proposition I continues to hold, i.e., the supermodularity or submodularity of strategic variable choices is reflected on the equilibrium profiles.

An economic interpretation of this result is that, in the presence of shared initial investment, the seminal result *à la* Singh and Vives (1984) that firms supplying substitute (resp., complement) products would choose quantity (resp., price) setting as a dominant strategy would no longer hold as a general principle. Cost sharing in the initial investment, in conjunction with Nash bargaining, entails vast multiplicity of pure strategy subgame perfect equilibria as to the participant firms' strategic variable choices, thereby any game-theoretic prediction will be a matter of *equilibrium selection* rather than strategic dominance.

5 Cost sharing, or no cost sharing?

One needs to be careful when discussing the welfare implications of cost sharing among oligopolists. For, this can refer to two entirely distinct contents. One is the obvious effect of *aggregate cost saving* by *curtailing effort duplication*. This is often cited as a rationale for encouraging joint ventures by means of various policy measures. Note that this is not what we have been analysing in this paper: we have simply accepted this rationale, assuming that firms are to share their initial investment to minimise the aggregate costs, and let k be this *minimised* amount.

The other content, which we have been focusing upon throughout this paper, is the effects of the institution that allows the oligopolists to "open a joint account" and run a joint debt in financing their costs, whether for initial investment or for operative expenses. Namely, firms can cross-subsidise one another by reshuffling their cost-shares when, and only when, their costs are expended through a joint venture, or a cooperative, which is *externally recognised as an official economic body*. Whether or not firms are allowed to joint their account does not, presumably, affect directly the amount of aggregate costs (denoted by k in our model). Instead, it can affect indirectly by changing the firms' *marketing behaviour*. As we have seen, the set of equilibrium profiles can change, or enlarge, when firms can joint their cost accounts.

Now, perhaps the most natural questions are [i] whether such jointing is *socially desirable*, and [ii] whether jointing needs to be *artificially encouraged*. We shall discuss these issues respectively in the following two subsections.

5.1 Welfare comparison

It is unfortunately not straightforward to make general statements about firms' strategic variable choices *with general demand and cost structures*. Instead, we glance at those marketing games used in the seminal studies, and intuitively presume that welfare results we find in these specific market structures can possibly be applied to a large class of, if not all, marketing games.

- When the marketing stage game is a simultaneous move duopoly as in Singh and Vives, *socially* the most desirable outcome is for both firms to set prices (resp., quantities) if they produce demand substitutes (resp., complements). In the absence of negotiable cost sharing, however, firms supplying demand substitutes (resp., complements) would choose to set quantities (resp., prices), which is socially less desirable.

  As we have seen in Lemma i (subsection 3.1), the presence of negotiable cost sharing enables the socially desirable outcome to be one of the equilibrium profiles. Moreover, once cost shares are agreed upon according to (5) through (7) by Nash bargaining, the two equilibrium profiles {price, price} and {quantity, quantity} are *equally risk dominant* despite that one profit-dominates the other.

- When the marketing stage game is an endogenous timing duopoly as in Boyer and Moreaux, once again, without negotiable cost sharing, firms supplying demand substitutes (resp., complements) would choose to set quantities (resp., prices), which is socially suboptimal.

  As we have seen in Lemma ii (subsection 3.2), negotiable cost sharing encourages one of the firms to set a price and the other a quantity, which enhances social welfare.

In both of these examples, the presence of negotiable cost sharing, which is institutionally made possible by allowing firms to set up a joint venture, can improve welfare.

5.2 Incentive comparison

One might spontaneously suspect that, if a joint venture can curtail effort duplication and thereby enhance cost efficiency, then firms would opt for it *voluntarily*, so that there is little need for economic *policy* to "encourage" such jointing. On the contrary, evidence abounds that social planners regularly intervene in attempt to encourage JVs beyond individual firms' private incentives,[7] in doing which the establishment of a multi-firm joint account is frequently the preferred institution.

A possible way to shed light on this seeming contradiction is to hypothesise what firms would do if they were free to choose between jointing and not jointing. If firms decide not to joint, then the seminal results from Singh and Vives or from Boyer and Moreaux shall be invoked. Otherwise, if firms joint, then as we have already seen, Lemma i (simultaneous moves) or Lemma ii (endogenous timing) shall be invoked, and some of these outcomes are welfare superior to those without jointing. Let us assume that these welfare superior outcomes are to be chosen in the case of jointing.

Then, in either of these two types of marketing games (i.e, the simultaneous move duopoly *à la* Singh and Vives, and the endogenous timing game *à la* Boyer and Moreaux), the equilibrium outcome resulting from *not jointing* is more profitable from the firms' point of view. Hence, if this profit enhancement by not jointing outweighs the cost saving benefit from curtailing effort duplication, then firms would choose against jointing, which would damage social welfare.

Hereby somewhat paradoxically, joint ventures need to be promoted by means of policy measures when, and only when, the cost saving effect from curtailing effort duplication is *not* overwhelmingly large. The intuition is, precisely because the cost saving is not overwhelming, firms need to be atrificially encouraged to choose joint ventures instead of the capability to exercise market power, the latter being potentially more profitable than the former.

6 Concluding remarks

We have shown that, in the presence of cost sharing among oligopolists, quantity (resp., price) setting may not always be dominant even when firms' products are demand substitutes (resp., complements). The dominance relation stands intact insofar as the cost sharing rule is independent of the choice of strategic variables. Otherwise, the bargaining between firms can upset the dominance relation even

[7] See the National Cooperative Research Act in the US; EC Commission (1990); and Goto and Wakasugi (1988).

though all firms' quantity (resp., price) setting is still the unique optimal profile in terms of *aggregate* profits. This observation can straightforwardly extend to general n-firm oligopoly.

In addition, we have applied these results to a widely accepted linear demand function, and discovered that the endogenisation of the timing of moves in the marketing stage can give rise to a pure strategy subgame perfect equilibrium in which one firm sets a price and another firm a quantity.

For simplicity we have assumed Nash bargaining throughout this paper, mainly in order to suppress the need for detailed descriptions of concrete bargaining procedures. Obviously, this assumption can be relaxed or waived without qualitatively altering our findings. For instance, if we explicitly modelled a bargaining process such as an alternate bargaining, the equilibrium results could still retain their qualitative essence albeit algebraically *far* more intricate.

Note finally that economic examples of multi-firm cost sharing include, but by no means confined to, RJVs in product development. Any initial investment which legitimises cost sharing and time-consuming repayment of the costs, can potentially entail those qualitative effects analysed in this paper. The requirements for "negotiable cost sharing" as opposed to "non-negotiable cost sharing" are [1] that the debt is run by the "joint venture" not by individual participant firms, so as to enable the reshuffling of its shares among the participant firms, and [2] that the repayment takes long enough to continue into the time period when the strategic variables have already been chosen by the firms. These conditions [1] and [2] can also be met when firms are to pool their *operative costs* in lieu of initial investment.

7 Appendix

7.1 Proof of Lemma i

Each firm's marketing payoffs are computed as

$$Y^N_{PP} = \frac{1-\gamma}{(2-\gamma)^2(1+\gamma)}\,, \qquad Y^N_{PQ} = \frac{(1-\gamma)^2(2+\gamma)^2}{(4-3\gamma^2)^2}\,,$$

$$Y^N_{QP} = \frac{(2-\gamma)^2(1-\gamma^2)}{(4-3\gamma^2)^2}\,, \qquad Y^N_{QQ} = \frac{1}{(2+\gamma)^2}$$

where the superscript N, standing for simultaneous-move market equilibria, is introduced for later reference (see Proof of Lemma ii, subsection 6.2).

When the two firms' products are demand substitutes, i.e., $\gamma \in (0,1)$, the dominance relation is

$$Y^N_{PQ} < Y^N_{PP} < Y^N_{QP} < Y^N_{QQ}\,. \tag{13}$$

Therefore, in the non-negotiable cost sharing game, the relations $Y^N_{PP} < Y^N_{QP}$ and $Y^N_{PQ} < Y^N_{QQ}$ make {quantity, quantity} the dominant strategy equilibrium profile.

When the products are demand complements, i.e., $\gamma \in (-1,0)$, the dominance relation becomes

$$Y^N_{QP} < Y^N_{QQ} < Y^N_{PQ} < Y^N_{PP}\,. \tag{14}$$

In the non-negotiable cost sharing game, the relations $Y^N_{PP} > Y^N_{QP}$ and $Y^N_{PQ} > Y^N_{QQ}$ entail the dominant strategy equilibrium profile {price, price}. These observations are identical to the seminal results by Singh and Vives (1984).

On the other hand, either of these two dominance chains (13) and (14) implies $Y^N_{PP} + Y^N_{QQ} > Y^N_{PQ} + Y^N_{QP}$. Hence, by Proposition I, in the negotiable cost sharing game, both {price, price} and {quantity, quantity} are equilibrium outcomes *irrespective of the sign of* γ.

7.2 Proof of Lemma ii

To begin with, when both firms act as quantity-setters, each firm attains the following equilibrium profits:

$$Y^N_{QQ} = \frac{1}{(2+\gamma)^2}\,; \quad Y^L_{QQ} = \frac{(2-\gamma)^2}{8(2-\gamma^2)}; \quad Y^F_{QQ} = \frac{(4-2\gamma-\gamma^2)^2}{16(2-\gamma^2)^2}, \tag{15}$$

where superscripts L and F stand for leader and follower, respectively.

The Bertrand game yields the following profits:

$$Y_{PP}^{N} = \frac{(1-\gamma)}{(2-\gamma)^2(1+\gamma)}\,; \quad Y_{PP}^{L} = \frac{(1-\gamma)(2+\gamma)^2}{8(1+\gamma)(2-\gamma^2)}\,; \quad Y_{PP}^{F} = \frac{(1-\gamma)(4+2\gamma-\gamma^2)^2}{16(1+\gamma)(2-\gamma^2)^2}\,. \tag{16}$$

Finally, in the price-versus-quantity game, the resulting profits are:

$$Y_{QP}^{N} = \frac{(2-\gamma)^2(1-\gamma^2)}{(4-3\gamma^2)^2}\,; \quad Y_{QP}^{L} = \frac{(2-\gamma)^2}{8(2-\gamma^2)}\,; \quad Y_{QP}^{F} = \frac{(1-\gamma)(4+2\gamma-\gamma^2)^2}{16(1+\gamma)(2-\gamma^2)^2}\,; \tag{17}$$

$$Y_{PQ}^{N} = \frac{(1-\gamma)^2(2+\gamma)^2}{(4-3\gamma^2)^2}\,; \quad Y_{PQ}^{L} = \frac{(1-\gamma)(2+\gamma)^2}{8(1+\gamma)(2-\gamma^2)}\,; \quad Y_{PQ}^{F} = \frac{(4-2\gamma-\gamma^2)^2}{16(2-\gamma^2)^2}\,. \tag{18}$$

Obviously, $Y_{QQ}^{L} = Y_{QP}^{L}$, $Y_{QQ}^{F} = Y_{PQ}^{F}$, $Y_{PP}^{L} = Y_{PQ}^{L}$ and finally $Y_{PP}^{F} = Y_{QP}^{F}$. These equalities imply that in any sequential play, both firms are just indifferent as to whether the follower acts as a price- or a quantity-setter.

In the case of *demand substitutability* between the two products ($\gamma \in (0,1)$), the marketing profits in (15) through (18) can be ranked according to the following sequence of inequalities:

$$Y_{QQ}^{L} = Y_{QP}^{L} > Y_{QQ}^{N} > Y_{QP}^{N} > Y_{QQ}^{F} = Y_{PQ}^{F} > Y_{QP}^{F} = Y_{PP}^{F} > Y_{PP}^{L} = Y_{PQ}^{L} > Y_{PP}^{N} > Y_{PQ}^{N}\,. \tag{19}$$

From the above chain of inequalities, it emerges that

$$Y_{PP}^{F} > Y_{PP}^{L} > Y_{PP}^{N}\,, \qquad Y_{QP}^{L} > Y_{QP}^{N}\,, \qquad Y_{PQ}^{F} > Y_{PQ}^{N}\,.$$

Namely, in the Bertrand game and in the price-versus-quantity game (where one firm sets a quantity and the other sets a price), there exists at least one Stackelberg equilibrium which Pareto-dominates the Nash equilibrium. Hence, these two games are Stackelberg-solvable in the sense of d'Aspremont and Gérard-Varet (1980), and if firms had the possibility of endogenously choosing the timing of moves, they would indeed play a Stackelberg game. In the Cournot game, on the other hand, both firm's simultaneous play is the only sustainable outcome of an extended game with observable delay (Hamilton and Slutsky, 1990).

In the light of these considerations, the games described in section 2 can be recast as follows. The relevant Cournot payoff to each firm remains Y_{QQ}^{N}. In the price-versus-quantity game, the price setter's operative payoff is Y_{PQ}^{F}, and the quantity setter's is Y_{QP}^{L}. In the Bertrand game, there are two Stackelberg equilibria, in each of which one of the firms earns Y_{PP}^{L} whilst the other Y_{PP}^{F}. For notational convenience we define $\widetilde{Y}_{PP} \in \left\{Y_{PP}^{L}, Y_{PP}^{F}\right\}$. Recall that, as long as the two firms' products are substitutes, i.e., $\gamma \in (0,1)$, the relation (19) remains effective. Thereby the relevant dominance relation becomes

$$Y_{QP}^{L} > Y_{QQ}^{N} > Y_{PQ}^{F} > \widetilde{Y}_{PP}\,. \tag{20}$$

Hereby in the non-negotiable cost sharing game, the seminal result by Singh and Vives (1984) robustly holds true, by $\widetilde{Y}_{PP} < Y_{QP}^{L}$ and $Y_{PQ}^{F} < Y_{QQ}^{N}$. At the same time, the relation (20) implies

$$Y_{QP}^{L} + Y_{PQ}^{F} > Y_{QQ}^{N} + \widetilde{Y}_{PP} \tag{21}$$

which, by Proposition I, entails equilibrium outcomes {price, quantity} and {quantity, price} in the negotiable cost sharing game.

When the two firms' products are *demand complements* ($\gamma \in (-1,0)$), marketing profits are ranked as

$$Y_{PP}^{L} = Y_{PQ}^{L} > Y_{PP}^{N} > Y_{PQ}^{N} > Y_{PP}^{F} = Y_{QP}^{F} > Y_{PQ}^{F} = Y_{QQ}^{F} > Y_{QQ}^{L} = Y_{QP}^{L} > Y_{QQ}^{N} > Y_{QP}^{N}\,. \tag{22}$$

This chain of inequalities implies that

$$Y_{QQ}^{F} > Y_{QQ}^{L} > Y_{QQ}^{N}\,, \qquad Y_{PQ}^{L} > Y_{PQ}^{N}\,, \qquad Y_{QP}^{F} > Y_{QP}^{N}\,,$$

namely, the Cournot game and the price-versus-quantity game are Stackelberg solvable. On the other hand, contrary to the previous case of substitute products, the Bertrand game is to be played as a simultaneous-move game.

Hence the profit matrix in the marketing stage game should be filled with Y_{PP}^{N} for the Bertrand case, Y_{PQ}^{L} and Y_{QP}^{F} for the price-versus-quantity case, and either Y_{QQ}^{L} or Y_{QQ}^{F} for the Cournot case. Once again for notational convenience, we define $\widetilde{Y}_{QQ} \in \left\{Y_{QQ}^{L}, Y_{QQ}^{F}\right\}$. The ranking relation (22) implies

$$Y_{PQ}^{L} > Y_{PP}^{N} > Y_{QP}^{F} > \widetilde{Y}_{QQ}\,. \tag{23}$$

Singh and Vives' result is confirmed once again in the non-negotiable cost sharing game, by $Y_{PP}^{N} > Y_{QP}^{F}$ and $Y_{PQ}^{L} > \widetilde{Y}_{QQ}$. The relation (23) also implies

$$Y_{PQ}^{L} + Y_{QP}^{F} > Y_{PP}^{N} + \widetilde{Y}_{QQ}\,. \tag{24}$$

Hence {price, quantity} and {quantity, price} are equilibrium outcomes in the negotiable cost sharing game *irrespective of substitutability* (by (21)) *or complementarity* (by (24)) *of the two firms' products.*

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