N° 27 PROGETTO D'ATENEO
L'economia Italiana e la sua collocazione internazi
una ridefinizione delle politiche di welfare e
dell'occupazione per una più effciente crescita econ

ON ENDOGENOUS GROWTH AND INCREASING RETURNS: MODELLING LEARNING-BY-DOING AND THE DIVISION OF LABOR

NICOLA DE LISO
GIOVANNI FILATRELLA
NICK WEAVER

N° 336

GRUPPO DI LAVORO 3:
Struttura dell'offerta e sistemi di produzione

On endogenous growth and increasing returns: modelling learning-by-doing and the division of labor[1]

by

Nicola De Liso[2] (IDSE-CNR, Milan, Italy)

Giovanni Filatrella[3] (Department of Physics and INFM, University of Salerno, Italy)

Nick Weaver[4] (School of Economic Studies, University of Manchester, UK)

Abstract

This paper discusses those sources of endogenous growth arising from *labor as labor*. It uses a production function which models the returns to scale as a function of the division of labor and learning. Smithian analysis of the labor process constitutes the basis upon which we build our own approach. The dynamics, therefore, drawn upon are more classical in style than those suggested by Arrow's (1962) article.

Keywords: endogenous growth, returns to scale, division of labor, learning-by-doing

J.E.L. classification: D24, J24, O41

---

[1] The authors wish to thank Bernhard Böhm, Riccardo Leoncini, Stan Metcalfe and Bernard Walters for their comments and suggestions, the usual caveats applying.
The present paper was partly developed within the Progetto di Ateneo of the University of Bologna, Department of Economics, "The Italian economy and its international placing", research line on "Economic growth, labour markets and human resources". Support from the CNR Progetto Speciale on "Technological systems, research evaluation and innovation policies" is also acknowledged.

[2] Corresponding author: IDSE-CNR, via A. M. Ampère 56, 20131 Milan, Italy, e-mail: deliso@idse.mi.cnr.it

[3] Permanent address: Facoltà di Scienze, Università del Sannio, Benevento, Italy.

[4] Permanent address: School of Economic Studies, University of Manchester, M13 9PL Manchester, UK.

On endogenous growth and increasing returns: modelling learning-by-doing and the division of labor

I. Introduction

Endogenous growth is a term that has been widely used. Different authors have used it to mean different things. Sometimes it has been used synonymously with increasing returns to scale, at other times it has been used to describe endogenously created technological change, which in turn leads to increasing returns. The processes involved range from various explanations of the roles of human capital and R&D. All of them basically concentrate on a factor that can be accumulated; that is capital in a broad sense. As Mankiw has put it:

"... capital is a much broader concept than is suggested by the national income accounts. In the national income accounts, capital income includes only the return to *physical capital*, such as plant and equipment. More generally, however, we accumulate capital whenever we forgo consumption today in order to produce more income tomorrow. In this sense, one of the most important forms of capital accumulation is the acquisition of skills. Such human capital includes both schooling and on-the-job-training." (Mankiw, 1995, p. 293)

The role of capital, broadly defined, has been carefully examined in the context of endogenous growth. Indeed it probably explains a good deal of endogenous growth. In the present work, however, we intend to look at the sources of endogenous growth in labor rather than capital. That is we will highlight the role of labor in a narrow sense by deliberately avoiding issues related to human capital. In this context we can identify two sources of increasing returns to scale: the division of labor and learning-by-doing. Both these phenomena have been recognised as important since Adam Smith. He pointed out that a division of labor, since it increases repetition, might create the best conditions for improving *dexterity*.

II. Arrow's analysis

In 1962 Arrow published his famous article on 'The economic implications of learning-by-doing'. His explicitly declared aim was to suggest an "*endogenous* theory of the changes in knowledge which underlie intertemporal and international shifts in production functions" (Arrow, 1962, p. 155, emphasis added). He made use of an aggregate production function in which both capital and labor were used, taking as a starting point many examples found in the economic and technical literature.

The examples he uses, in which learning has occurred, and on which firms' managers can rely, include the production of airframes of a given type and the so-called "Horndal effect" observed at first by Lundberg in the Horndal iron works in Sweden. In the former case Arrow recalls T.P. Wright's study, whose main result was that "the number of labor-hours expended in the production of an airframe ... is a decreasing function of the total number of airframes of the same type previously produced. Indeed, the relation is remarkably precise; to produce the Nth airframe of a given type ... the amount of labor required is proportional to $N^{-1/3}$" (Arrow, 1962, p. 156). In the latter case productivity rose by 2% per annum despite the fact that no investment had occurred in the fifteen years considered.

However Arrow uses as his index of experience gross cumulative investment. This is problematic. Firstly embodied and disembodied forms of technical change are subsumed under a single heading — embodied technical change implies a change in physical capital while disembodied technical change does not. Secondly, Arrow's concept of "learning" is thus a peculiar one since he argues that the choice of cumulative production of capital goods is motivated by the fact that "each new machine produced and put into use is capable of changing the environment in which production takes place, so that learning is taking place with continually new stimuli [and that ... at] any moment of new time, the new capital goods incorporate all the knowledge then available, but once built their productive efficiency cannot be altered by subsequent learning." (Arrow, 1962, p. 157)

Thus, he is referring to two different forms of technological change, rather than learning *per se*: the one developed in capital goods industry and the one developed in the firms using new vintages of capital goods; neither form, however, can be properly classified as "learning-by-doing".

Learning has also been tackled by Rosenberg (1982), who distinguishes between learning-by-doing, which is a characteristic of human beings, and learning-by-using, which is a form of disembodied technical change, concerning the use of machines produced by others. Learning-by-using involves the same capital goods performing better and better as their properties become more known; typical examples are the better exploitation of machine tools and a reduction in the amount of maintenance required by machinery. Rosenberg's point confutes Arrow's statement according to which once built the new capital goods' efficiency cannot be altered — which, by the way, is also contradicted by the Horndal effect.

Learning occurs, often unintentionally, not only in the production of physical goods, but also in the production of services. For instance, as early as 1832 Charles Babbage stressed the role of learning-by-doing in the calculation of logarithms from 1 to 200,000. The calculation process was organized by dividing the job into three phases; in the first a handful of pure mathematicians elaborated formulae as general as possible; in the second a few persons having some knowledge of mathematics produced some examples; in the third persons whose knowledge of mathematics did not go beyond the four basic mathematical operations performed the actual calculations. Babbage noted how this third group of workers quickly improved the speed and accuracy of their calculations.

Nilsson (1995) added to the concept of learning-by-doing the role of workers' cumulative experience in promoting innovation rather than merely speeding up existing processes. He calls this *innovating-by-doing*, and gives three examples. The first related to the standardization of components in the US armouries during the early 1800s, the second to the aircraft industry (the importance of pilots in suggesting procedures), and the third to the steel industry (the introduction of 'thin slab technology' in the late 1980s). Nilsson thus concludes that students of technological change continue to underestimate the importance of 'doing' as a source of innovation.

III. Labor and endogenous growth

The aim of our analysis is to complement the, by now, traditional concern with explanations of endogenously created increasing returns due to human capital accumulation, or R&D activity, or returns to physical capital, with a discussion of the role of labor *as* labor. We use a "new" production function, in which labor is the only input. Physical capital is either non-existent or can be subsumed under the constant A. We use this function as purely a micro-theoretical tool. However we think that this is a useful abstraction because it allows us to examine some of the "origins of endogenous growth"[5] which have not hitherto been formally highlighted.

[5] This is the title of Romer's 1994 article.

Our starting point is a sort of Cobb-Douglas production function in which only one commodity is produced and labor is the only input:

(1) $$Y = AL^{Z}$$

where Y is total production, A is a positive constant, L the labor input and Z represents the returns to scale. Labor is, at the beginning, homogeneous and undifferentiated. If Z is greater, equal to, or smaller than *1* we will have, increasing, constant, and decreasing returns to scale, respectively. We assume that the labor supply is unlimited.

Our principal aim is to explain *why* Z can have different values at different times, and, in particular why it might become greater than *1* over time. The first step is to write a dynamic function:

(2) $$Y_t = AL_t^{Z_t}$$

and to recognise that the exponent itself is some function of the labor input and the way in which labor is divided and learning-by-doing. Before taking into account an explicit function let us remind ourselves that the returns can also be written as the labor/output elasticity:

(3) $$Z_t = \frac{L_t}{Y_t}\frac{\partial Y_t}{\partial L_t}$$

which highlights the link between the returns and output.

Whenever we add units of labor to a function such as (2) organizational adjustments, which in this case are synonymous with an increased division of labor, are *necessary*. We will emphasise the role of a more detailed division of labor and of learning-by-doing.

Such a function is dynamic according to two different criteria: because it takes time into account, and because it shows irreversibilities. Once certain phenomena have occurred, they become a permanent feature of the productive system. That is, once we have learnt how to produce more efficiently, we cannot forget it — though this does not mean that output could not be restrained voluntarily, but this would be an intentional action.

We should be able to observe, at least conceptually, three distinct effects which individually and jointly affect total production and the productivity of the individual worker; that is total production can be superadditive because:

(i) of a more detailed division of labor in the work process;

(ii) there occurs learning-by-doing at the individual level, so that each component becomes more productive as time goes by;

(iii) there occurs learning-by-doing at the level of the firm as a whole; an improvement in collective competence.

The three effects can be observed in the dynamics of Z_t. It is important to emphasise the fact that we will concentrate on Z_t as our 'index of experience', and not on cumulative output directly, as more 'traditional' works (e.g. Fellner, 1969) do. The reasons for doing this are threefold: first of all, we are specifically interested in the evolution of Z as time goes by; secondly, as equation (3) shows, Z_t and Y_t are linked, so that there is always a relationship between the two, i.e. Z can capture the 'cumulative output effect', while, thirdly, at the same time, by giving us the level of the returns Z_t, takes into account the effect of time on learning. In other words, the level of the returns today depend on their level of the previous period(s), while capturing the contribution of learning-by-doing which grows, up to a certain maximum, as time passes; learning-by-doing affects the returns, depending on how long one has been performing the same task. We do not put any time index on A, as we are not interested in exogenous variations in total output.

We want to write explicitly a function for Z in order to take into account both the increased division of labor and learning, and one way of obtaining this is to use the following expression:

$$(4) \qquad Z_{t+1} = \frac{\lambda}{1+\beta e^{-\alpha L_{t+1}}} + \gamma \sum_{j=0}^{t} \omega^{j} Z_{t-j}$$

Thus, the exponent can be written as the sum of two components, the first, represented by a logistic equation, which shows the pattern along which labor can be 'divided', while the second describes the way in which past returns, via past production, affect present production.

We can write our function in a slightly different way, so that we can make use of the mapping technique. The first step consists of the assumption that the labor force employed in the work process is increased by ΔL_0 each period:

$$(5) \qquad Z_{t+1} = \frac{\lambda}{1+\beta e^{-\alpha \Delta L_0 (t+1)}} + \gamma \sum_{j=0}^{t} \omega^{j} Z_{t-j}$$

The second step consists in introducing a new variable which we call W, and we will write it as:

$$(6) \qquad W_{t+1} = \sum_{j=0}^{t} \omega^{j} Z_{t-j} = \sum_{j=1}^{t} \omega^{j} Z_{t-j} + Z_t$$

which can be rewritten as:

(6') $$W_{t+1} = \omega \sum_{j=1}^{t} \omega^{j-1} Z_{t-j} + Z_t$$ and, by setting $i = j-1$

(6'') $$W_{t+1} = \omega \sum_{i=0}^{t-1} \omega^{i} Z_{t-i-1} + Z_t = \omega W_t + Z_t$$

Thus, substituting (6'') into (5) we get:

(7) $$\begin{cases} Z_{t+1} = \dfrac{\lambda}{1+\beta e^{-\alpha \Delta L_0 (t+1)}} + \gamma (Z_t + \omega W_t) \\ W_{t+1} = \omega W_t + Z_t \end{cases}$$

This is a linear non-autonomous two-dimensional map[6] capable of describing the value of the returns at time $t+1$ as a function of t. The returns of each period are weighted by a parameter ω, while γ 'translates' the learning effects into production, so that the higher is γ, the more the learning affects Z and the function as a whole.

It is convenient to introduce a map because while eq.(5) retains memory of *all* the previous steps (namely, the retuns to scale at each time step, t), eq. (7) gives the dynamics in terms of the configuration one step before. The 'price' that must be paid is the introduction of a new variable, W_t, and the map becomes two-dimensional — and yet it is much more convenient than a one-dimensional map with memory. As for the values of the parameters we will see in the next section the values that they can take.

IV Analytical properties of the map

First of all we consider the system's asymptotic behaviour, so that given t sufficiently large one gets:

[6] A full analysis of the mapping techniques, including nonlinear cases, can be found in Thompson and Stewart (1986).

(8) $$\begin{cases} \overline{Z} = \lambda + \gamma \overline{Z} + \omega\gamma \overline{W} \\ \overline{W} = \overline{Z} + \omega \overline{W} \end{cases}$$

and after solving:

(9) $$\begin{cases} \overline{Z} = \dfrac{\lambda(1-\omega)}{1-\omega-\gamma} \\ \overline{W} = \dfrac{\lambda}{1-\omega-\gamma} \end{cases}$$

We need to check that the stability conditions are fulfilled, that is we have to calculate the eigenvalues of the Jacobian (cf. the appendix) and we have to impose the condition that their modulus be smaller than 1. Thus, we obtain:

(10) $\omega + \gamma < 1$

We can now run a simulation with the following values:

$\lambda = 0.6; \beta = 0.8; \alpha = 0.1; \omega = 0.8; \gamma = 0.1; \Delta L_0 = 0.1$

[See figure 1 – returns /time]

Figure 1 shows how returns change as we add labor units within the work process. The solid line corresponds to the case in which there also occurs learning ($\gamma > 0$, i.e. $\gamma = 0.1$), while the dashed line shows what happens if learning is zero. It is very interesting to compare the two lines, as returns in one case ($\gamma > 0$) turn to increasing, while in the other ($\gamma = 0$) do not. Of course the higher is γ the greater will be the difference between the two paths. It is important to underline that in either case there exists an asymptotic value for Z, which means that neither the division of labor, nor learning-by-doing can have positive effects for ever. There exists an upper limit for both of them — and we will comment on this later.

In figure 2, instead, the relationship between labor input and total output is shown. As one can see the presence of positive learning is quite important.

[See figure 2, production/time]

The difference between the solid line (with learning-by-doing) and the dashed line (without learning-by-doing) grows continuously, as it is magnified by the exponent of the production function. Thus, the solid line shows increasing returns to scale, while the dashed line shows decreasing returns.

A specific comment must be made about β. In the previous simulations we have shown the results and we have considered a positive β, which is the parameter affecting the 'division of labor effect', as it is part of the denominator of the logistic, i.e. the first component of Z. A positive sign means that if we add labor units, there always exists a positive, though decreasing, effect. Should we make use of a negative β, as we add labor units the effect on the returns would be immediately negative. Such a phenomenon can be partially compensated by learning-by-doing, so that the net effect on the returns could be compensated: this is a good example of a dynamics in which two forces work in opposite directions.

We can conclude this section by pointing out the links between the parameters and the 'real' production system, so that we can observe the time pattern along with the higher level of the returns, due respectively to 'dimension' (i.e. we abserve what happens when we add labor ΔL_0 units) and 'learning', is reached. First of all let us define:

(10) $\tau_1 = \frac{1}{\alpha \Delta L_0}$ which depicts the pattern along which the asymptotic value concerning organization and dimension Z^D, $\lambda, \frac{\lambda}{1+\beta}$ is reached (see figure 3); similarly, we can also define:

(11) $\tau_2 = -\frac{1}{\log(\omega)}$ which depicts the transitional time between the two levels of productivity due to learning-by-doing Z^L; τ_2 describes the way in which workers experience a change in productivity, the two levels of which depend on the parameters γ and ω, but also on the value of the initial conditions; the two productivity values thus are $\lambda, \frac{\lambda(1-\omega)}{1-\omega-\gamma}$ (see figure 4).

V Division of labor, organization and learning-by-doing

In the previous section we have seen how returns behave given an explicit production function and some values of the parameters. As we have also pointed out, the forces operating behind the function and the parameters are the more detailed division of labor, which necessarily emerges as more workers are introduced within production, and learning-by-doing.

The importance of both forces was clearly indicated in Adam Smith's *Inquiry into the Nature and Causes of the Wealth of Nations* (1776) and the fact that division of labor and learning are connected has been widely recognised. Becker and Murphy (1992) devote an article to the division of labor, coordination costs and knowledge, while as

Loasby (1996, p. 301) has pointed out, the division of labor should be thought of as a method of fostering the development of skills, and indeed generating other kinds of knowledge. The importance of learning is increasingly being recognised in economics; from endogenous growth literature to the theories of organization. The nature of *how* one (person, organization, etc.) learns and *what* is learnt is quite important. Hilgard, a psychologist, defines learning as:

"the process by which an activity originates or is changed through reacting to an encountered situation, provided that the characteristics of the change in activity cannot be explained on the basis of native response tendencies, maturation, or temporary states of the organism" (Hilgard, 1956, p. 3)[7].

Whilst he highlights two answers to the question of 'what is learnt':

"The stimulus-response theorist and the cognitive theorist come up with different answers to the question, What is learned? The answer of the former is 'habits'; the answer of the latter is 'cognitive structures'. (Hilgard, 1956, p. 10)

Within production both forms of learning are at work. How they are at work depends, in part, upon the prevailing 'production era'. Whichever view we take of the learning mechanisms, human learning capabilities are limited so that when we specialise in one particular task (be it more or less complex), there exist an upper limit beyond which it will not be possible to go — as we can see in figure 4 —, unless some discontinuity occurs. It must be said that these discontinuities can be endogenously generated by either the division of labor, or learning itself, or both.

It is important to emphasise the existing link between the division of labor and learning, both in terms of effectiveness and type. It seems reasonable that the more a work process is subdivided the more effectively learning will occur within a specific

[7] This reference was originally found in Arrow 's 1962 bibliography.

phase or task, though at the cost of loosing the general knowledge of the process as a whole. The second point is that different kinds of division of labor will lead to different types and patterns of specialisation.

Also, it is important to emphasise that even if no further division of labor is possible, learning effects might still increase returns, or if there is no further learning possible an increased division of labor might help.

A further effect that must be taken into account concerns the changes which affect the labor force as time passes and cumulated production grows. If at the beginning of our history we have simple homogeneous workers, as time passes each individual becomes more and more specialised in the specific task that he or she is engaged in. Starting from a 'general knowledge', which allows the individuals to perform the task that they are supposed to work on, the specialisation process leads to a decreasing possibility of substitution between workers. Substitution implies the loss of all the learning embodied in that worker. If the work process is organized rigidly, that is, if the phases and tasks follow each other as in a Fordist organization, the whole production will be affected by the bottleneck engendered by the new worker.

A final point worth stressing is that different tasks are likely to allow for different learning speeds, so that the process as a whole will be affected by the 'laggard' components.

VI Some implications of the changes affecting returns to scale

As already touched upon in the previous section, whenever we add labor units to such a production function we are re-organizing the work process, that is, in general, we may be applying a more detailed division of labor. The way in which we write the function, however, is such that we experience growing returns to scale which, when there occurs learning, switch to increasing, however converging to an asymptotic value at some point in time. There exist technological and economic reasons for such a phenomenon.

Technologically, the division of labor is limited by the nature of the work process, and, economically, by the extent of the market. With regards to the technological component, the work process can be more or less suitable to further and further subdivision. As Ford taught us, the limits to subdivision of labor do not finish at the ratio 1/1, that is one person per task, as what we see today as the simplest task might be further split into sub-operations at some future time. Learning itself is limited, and increased working speed through repetition has both a physical and an intellectual limit. If we want to stick to the Smithian example of pin production, the worker drawing the wire will approach a maximum speed beyond which he/she could not go. Furthermore, different phases and tasks may experience different rates of learning, because of different abilities or because of different intrinsic difficulties.

As far as economic forces are concerned, the fact that the division of labor is limited by the extent of the market was emphasised by Adam Smith; others following him include Verdoorn (1949), Stigler (1951) and, much more recently, Kelly (1997) and Becker and Murphy (1992). Verdoorn in his studies on the factors which govern the development of labor productivity, emphasised that:

"a more detailed division of labour occurs only together with an increase in the volume of production; thus the expansion of production creates the possibility of a further rationalisation with the same effects of mechanisation." (Verdoorn, 1949, p. 46)

Technological and market-extension considerations thus define two endogenous forces which determine how returns to scale evolve. At this point it is necessary to point out that there exist feedbacks between division of labor and extent of the market, so that 'pure' technological forces merge with economic ones.

The dynamic process between the division-of-labor and the extent-of-the-market is particularly relevant as it creates an endogenous path of growth and development. Once such a process is embarked upon path-dependency, lock-in phenomena and irreversibilities become part of our production system. In fact, the initial direction undertaken by the process of the division of labor can set particular conditions which affect all the subsequent events, and this explains path dependency and eventual lock-in phenomena. Irreversibility concerns the technique of production and all the related activities. Once the division of labor appears, it becomes part of the permanent memory of the economic system. Thus, the producers not imitating it will find it more and more difficult to stay in the market, and at some point they will be forced out. The actual way in which the work process comes to be subdivided, is a different matter. What matters is the fact that the division of labor becomes the benchmark against which all other forms of organization of production have to confront themselves.

The relationship between the division-of-labor and the extent-of-the-market is bi-directional, that is, a more detailed division of labor favours a more extended market, while a more extended market is likely to lead to a more detailed division of labor. Following this reasoning, as Young (1928, p. 533) observed, we arrive at the conclusion that the division of labor depends on the division of labor, which, however, is something more than a tautology: in fact, we have here an endogenous force capable of generating different forms of structural economic dynamics, characterised at the least by the growth of total output, evolution of labor skills and performance, and decreasing prices.

Exogenous forces contributing to extend the market are of course relevant. Thus, if Smith emphasised the role of new canals and roads in favouring transport, the macro-processes that we observe today — what we call globalisation — are important in shaping the paths of specialisation. Thus, abandoning for a moment the microeconomic perspective, new forms of social and international division of labor are taking place.

The emphasis on increasing-decreasing returns to scale depends on the problems related to the permanence in the economic system of competitive conditions. If increasing returns were always to prevail, there would occur a monopolistic situation. In fact, given a cost structure output would increase more than proportionately, so that one producer would end up with the whole market. In this case the natural monopoly would be engendered by the law of production, rather than from sub-additivity in the cost function. This is not the case with our production function, and anyway, the process leading to monopoly not necessarily could reach the end because of changes engendered by the demand side, from differentiated goods to simple 'death' of the demand for that particular good.

A different route was chosen by Richardson who suggests that firms should be regarded as undertaking *activities* rather than making products. Unless strict inseparability occurs, he says, firms will normally be able to expand or acquire some activities and to reduce or abandon others. Thus, as a rule, increasing returns lead to specialisation and interdependence rather than to straightforward concentration (Richardson, 1975, pp. 354-356).

In a changing technological environment, to which division of labor and learning contribute, the processes leading to monopoly cannot exhaust all their potentialities, so that the same phase of decreasing returns could never be reached. Such a statement would be reinforced in a multi-factor production function, in which many forms of learning and division of labor would be developed. Furthermore, for the best practice technology to take full advantage over other practices would take some time, so that other, maybe Schumpeterian new entrants, could enter the game revolutionising the whole sector. Also, structural change is at work, with a continuous variation of the relative importance of the different sectors.

Returning to our production function, we should note that there exists an upper limit to both the possibilities to subdivide labor on the one hand, and on learning capabilities on the other, so that the existence of a monopoly would depend on how the market size could be 'extended'.

Conclusion

The main aim of the present work has been to analyse some of the determinants of endogenous growth. Rather than focusing, as is conventional, on the roles of both physical and human capital, we have focused our attention on 'simple' labor. The aim has been to show the existence of properly defined endogenous change through the use of labor itself.

Taking as a starting point a production function, we have pointed out three main reasons that can lead to endogenously created increasing returns, namely a more detailed division of labor, learning-by-doing concerning the individual and learning-by-doing concerning the organization as a whole. The principles which pertain to the division of labor and learning are not new in economics; however, it is within this new context that they acquire a formal treatment.

We have then divided the exponent of the production function — using labor as the only input — into two varying entities, the first being related to the process of division of labor, the second being related to learning. Mathematically, the first component can be approximated by a logistic equation, while the second, which subsumes both the individual and the organization as a whole, is approximated by a difference equation relating the stock of knowledge, via the exponent Z, to cumulative output.

As we have tried to clarify, there exist technological and economic reasons which justify the fact that the returns experience a change from decreasing to increasing, but the same forces set an upper limit to the growth of returns. The division of labor cannot be pushed beyond a certain point, while the speed that can be reached by each individual is limited by their physical and mental abilities. The organization as a whole, will be thus influenced by each of the members and, if rigidly set, the whole process will evolve according to the slowest member. Each operation, in fact, can lend itself to a different degree of learning not only because of human limitations but also because of the intrinsic difficulties. Economically, we have recalled that the division of labor — and, we may add, learning — is limited by the extent of the market. Conversely, the extent of the market is limited by the degree of division of labor and the learning having taken place. This leads us to a virtuous circle leading to specialisation and interdependence. The other economic force at work would come from the demand side, as consumers can refuse to buy more of the same good even at a falling price.

A final comment concerns the changes that might have occurred in the labor force. If at the beginning of the production process we have undifferentiated labor, after a few periods as a result of specialisation we will have different kinds of labor involved in different tasks. If one of the members of the work process has to be changed, the organization will loose their experience acquired through the simple process of *doing*.

References

Aghion Ph. (1994), Endogenous Growth: A Schumpeterian Approach, paper presented at the Siena Summer School, July.

Arrow K. (1962), The Economic Implications of Learning-by-Doing, *Review of Economic Studies*, vol. 29, pp. 155-173.

Babbage C, (1832), *On the Economy of Machinery and Manufactures*, London.

Barro R. J. and Sala-i-Martin X. (1995), *Economic Growth*, New York, McGraw-Hill.

Becker G. S. and Murphy K. M. (1992), The Division of Labor, Coordination Costs, and Knowledge, *Quarterly Journal of Economics*, Vol. 107, Nov., pp. 1137-1160.

Eltis W. (1984), *The Classical Theory of Economic Growth*, London, Macmillan.

Fellner W. (1969), Specific Interpretations of Learning-by-Doing, *Journal of Economic Theory*, Vol. 1, no. 2, Aug., pp. 119-140.

Helpman E. (1992) Endogenous Macroeconomic Growth Theory, *European Economic Review*, Vol. 36, pp. 237-267.

Hilgard E. R. (1956/2), *Theories of Learning*, New York, Appleton-Century-Crofts.

Kelly M. (1997), The Dynamics of Smithian Growth, *Quarterly Journal of Economics*, Vol. 112, Aug., pp. 939-964.

Loasby B. J. (1996), The Division of Labour, *History of Economic Ideas*, Vol. 4, no. 1-2, pp. 299-323.

Mankiw N. G. (1995), The Growth of Nations, in *Brookings Papers on Economic Activity*, ed. by W. C. Brainard and G. L. Perry, Washington, Brookings Institution, pp. 275-311.

Marshall A. (1920), *Principles of Economics*, London, Macmillan, eight edition, first edition 1890.

Nilsson E. A. (1995), Innovating-by-Doing: Skill Innovation as a Source of Technological Advance, *Journal of Economic Issues*, vol. XXIX, no. 1, March, pp. 33-46.

Ott E. (1981), Strange attractors and chaotic motions of dynamical systems, *Review of Modern Physics*, vol. 53, October, pp. 655-672.

Parker T.S. and Chua L. O. (1987), Chaos: A tutorial for engineers, IEEE, vol. 75, August, pp. 982-1008.

Richardson G. B. (1975), Adam Smith on Competition and Increasing Returns, in Skinner A. S. and Wilson T. (eds), *Essays on Adam Smith*, pp. 350-360.

Romer P. M. (1994), The Origins of Endogenous Growth, *Journal of Economic Perspectives*, vol. 8, no. 1, Winter, pp. 3-22.

Rosenberg N. (1982), *Inside the Black Box: Technology and Economics*, Cambridge University Press, Cambridge.

Smith A. (1776), *An Inquiry into the Nature and Causes of the Wealth of Nations*.

Solow R. M. (1994), Perspectives on Economic Growth, *Journal of Economic Perspectives*, vol. 8, no. 1, Winter, pp. 45-54.

Thompson J.M.T. and Stewart H.H. (1986), *Nonlinear Dynamics and Chaos*, New York, Wiley & Sons, chap. 8.
Verdoorn P. J. (1949), Fattori che regolano lo sviluppo della produttività del lavoro, *L'Industria*, no. 1, pp. 45-53.
Young A. A. (1928), Increasing Returns and Economic Progress, *Economic Journal*, Vol. 38, no. 152, December, pp. 527-542.

Appendix

As we have seen in section IV the stability of our map requires that $\omega + \gamma < 1$. In order to reach this result we need first of all to calculate the eigenvalues of the Jacobian. To do this we compute the eigenvalues μ by setting to zero the characteristic equation:

$$\det\begin{vmatrix} \dfrac{\partial W_{t+1}}{\partial W_t} - \mu & \dfrac{\partial W_{t+1}}{\partial Z_t} \\ \dfrac{\partial Z_{t+1}}{\partial W_t} & \dfrac{\partial Z_{t+1}}{\partial Z_t} - \mu \end{vmatrix} = 0$$

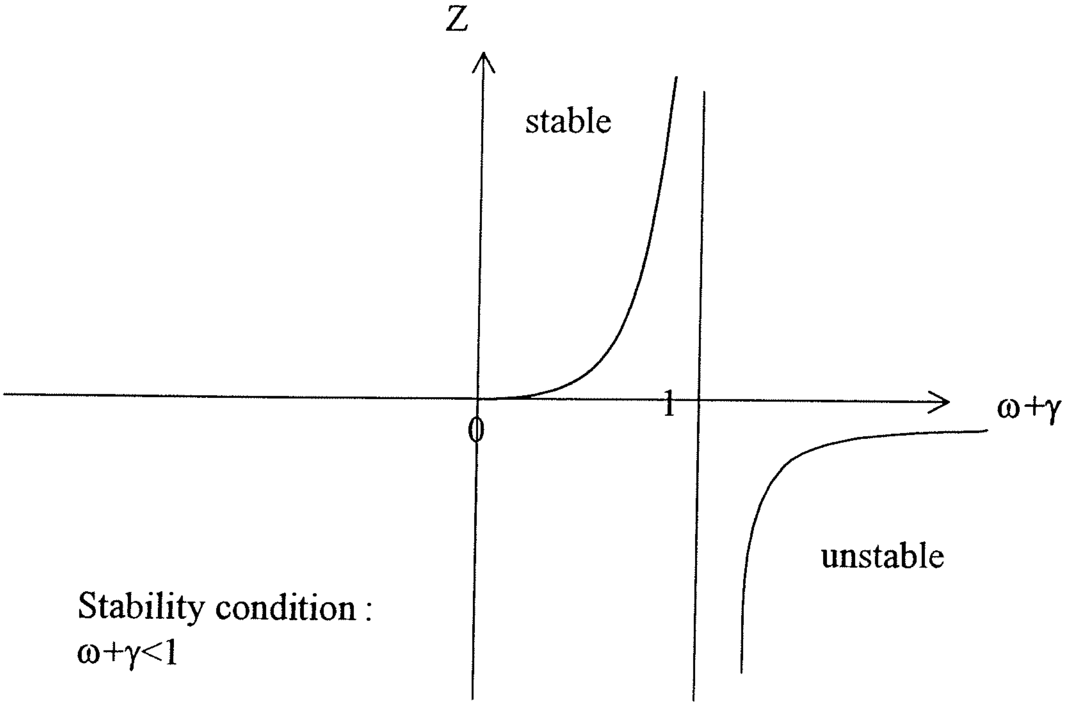
$$(\omega - \mu)(\gamma - \mu) - \omega\gamma = 0$$
$$\mu(\mu - \omega - \gamma) = 0$$

Solving the equation we get for the two eigenvalues the simple expressions:

$$\mu_1 = 0$$
$$\mu_2 = \omega + \gamma$$

The condition $|\mu_1| < 1$ is always true, while for the second eigenvalue, for a positive γ and ω, it reads simply $\omega + \gamma < 1$

In the following figure we show a sketch of the behaviour of Z as a function of $\gamma+\omega$.



The branch in the positive plane is always stable, while the negative branch is always unstable. This is not a problem as a negative exponent Z is economically meaningless.

Figure 1

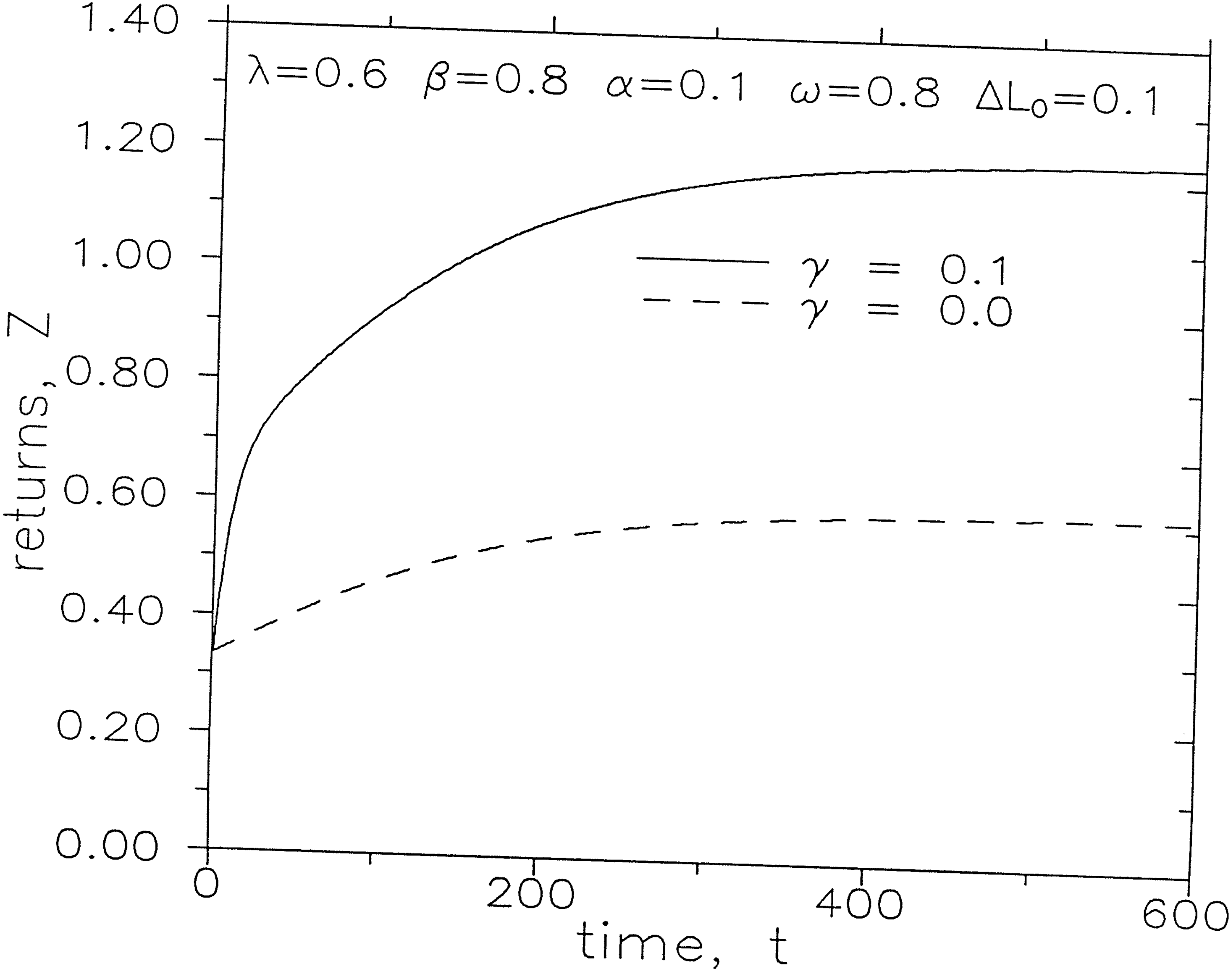
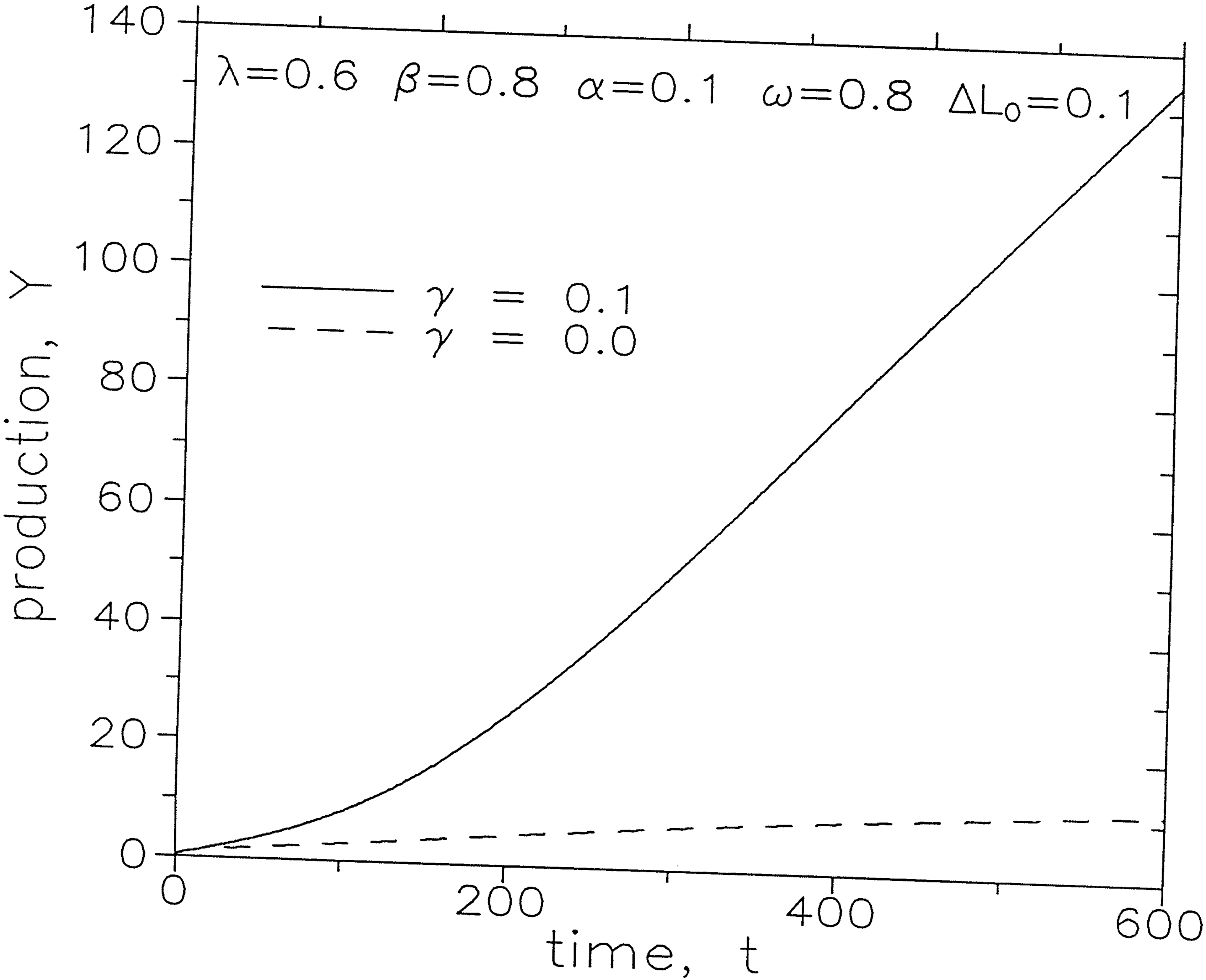
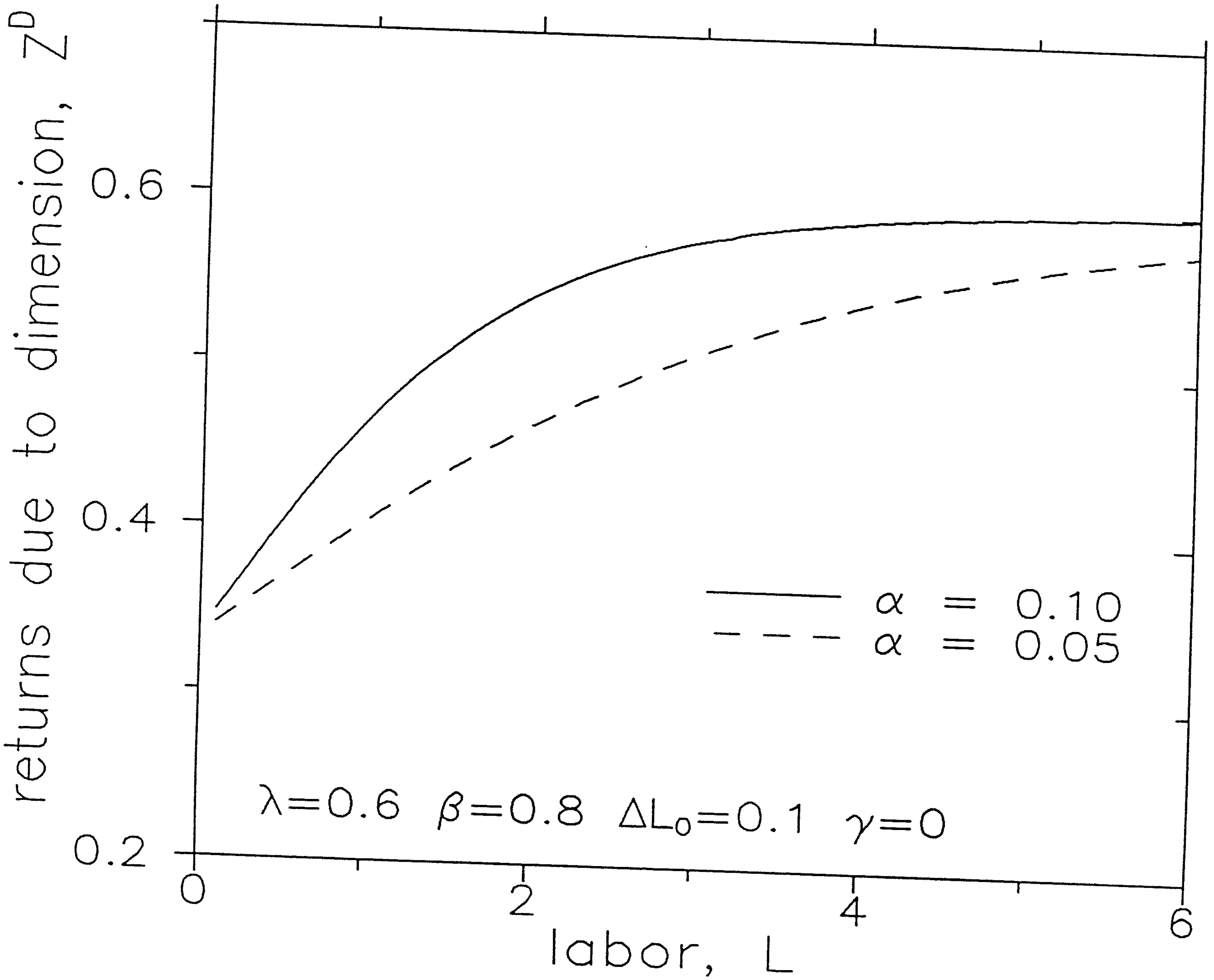

λ=0.6 β=0.8 α=0.1 ω=0.8 ΔL0=0.1
γ = 0.1
γ = 0.0
returns, Z
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
0
200
400
600
time, t

Figure 2



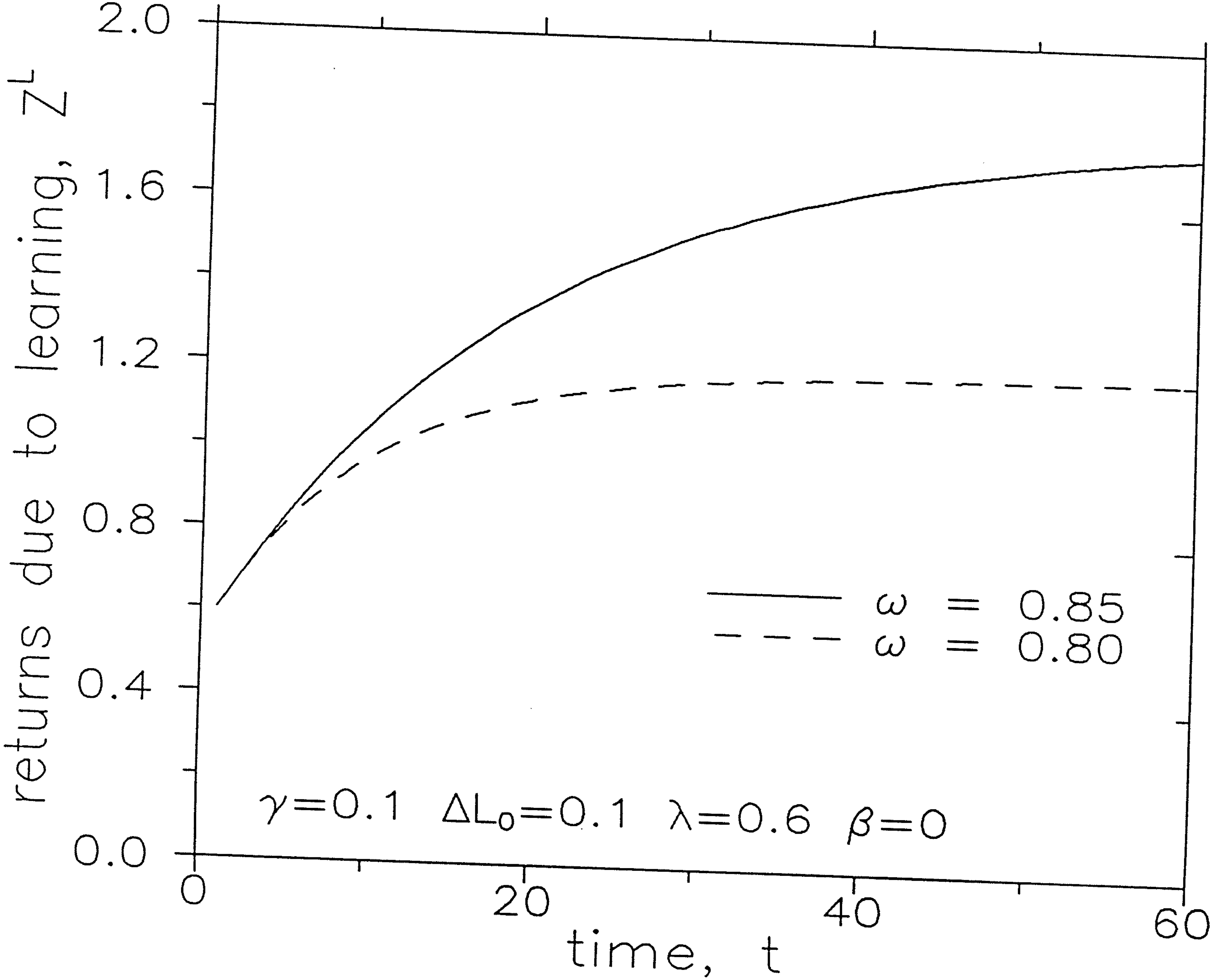
λ=0.6 β=0.8 α=0.1 ω=0.8 ΔL0=0.1
γ = 0.1
γ = 0.0
production, Y
time, t
0
20
40
60
80
100
120
140
0
200
400
600

Figure 3



returns due to dimension, Z^D
0.2
0.4
0.6
0
2
4
6
labor, L
α = 0.10
α = 0.05
λ=0.6 β=0.8 ΔL0=0.1 γ=0

Figure 4



returns due to learning, Z^L
2.0
1.6
1.2
0.8
0.4
0.0
0
20
40
60
time, t
$\omega = 0.85$
$\omega = 0.80$
$\gamma=0.1$ $\Delta L_0=0.1$ $\lambda=0.6$ $\beta=0$