Equilibrium Locations in a Spatial Model with Sequential Entry in Real Time[¤]

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Abstract

I describe the entry process in a spatial market over an in…nite time horizon. I show that, as long as the strategy space at the location stage is unbounded, the results derived from the single-period Stackelberg model of entry coincide with those obtained in the in…nite horizon model. On the contrary, if the strategy space is bounded, then the later the follower enters, the closer to the center of the market the leader locates at the initial date.

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1 Introduction

The issue of entry and entry deterrence has received wide attention in the literature on spatial competition, the research carried out in this field being focussed upon three main topics, namely, (i) the analysis of sequential entry by single-product firms (Hay, 1976; Prescott and Visscher, 1977; Lane, 1980; Eaton and Wooders, 1985; Neven, 1987); (ii) the existence of pure profits in the long-run (Eaton, 1976; Eaton and Lipsey, 1978); and (iii) the strategic use of capital and/or brand proliferation as a barrier to entry (Schmalensee, 1978; Eaton and Lipsey, 1980; Judd, 1985; Bonanno, 1987). Most of the existing literature on sequential entry makes use of a single-period setting where the sequential nature of the game is captured by the assumption that earlier movers act as Stackelberg leader, while later movers act as Stackelberg followers. Hence, the relevance of time in determining firms' decisions is summarised by the fact that, if a firm moves (i.e., enters) first and another moves (i.e., enters) second, the former can use strategically the information contained in the best reply function of the latter.

A well known result derived by this approach in the linear model of spatial differentiation with single-product firms is that a monopolist locates in the middle of the product space. This position is also optimal when a second firm enters, as long as the resulting degree of product differentiation is sufficient to ensure that no relocation incentive appears for the incumbent.[1] Otherwise, the leader should anticipate this event and either (i) choose a different location from the outset, if relocation costs are prohibitively high; or (ii) relocate as soon as the rival enters, if this can be done costlessly.

The aim of this note is to highlight the influence of time in determining the first entrant's decision as to location in the product space, under the assumption that any relocation after the rival's entry is impossible. I show that, if firms can choose any amount of reciprocal product differentiation, time is irrelevant and the single-period Stackelberg model offers a good description of the entry process. If, instead, the amount of differentiation that can be supplied at equilibrium is bounded, and coincides with the space of

[1] For the model with quadratic transportation costs, see Bonanno (1987); Neven (1987); Tabuchi and Thisse (1995); and Lambertini (1997). The same analysis is carried out by Anderson (1987) under the assumption of linear transportation costs.

consumer preferences, then the first entrant's equilibrium location is closer to the optimal monopoly location, the later is the date at which a second firm is expected to enter the market; conversely, the leader's location is closer to the simultaneous one-shot equilibrium location, the earlier the rival is expected to enter. This, in turn, entails that the degree of differentiation at equilibrium is negatively related with the amount of time along which the first entrant expects to remain a monopolist.

The remainder of the paper is organised as follows. The basic model and the price stage are described in section 2. The entry process is laid out in section 3. Finally, concluding remarks are in section 4.

2 Setup and price behaviour

The basic model shares many features with that introduced by d'Aspremont, Gabszewicz and Thisse (1979). I consider a market for horizontally differentiated products where consumers are uniformly distributed with unit density along the unit interval $[0;1]$. Let this market exist over $t \in [0;\infty)$. Two profit-maximising firms, labelled as 1 and 2, sequentially choose locations x_1 and x_2 and compete in prices simultaneously as soon as both are in the market. I will consider both the case where firms are to locate within the unit interval, and the case where they are free to locate also outside $[0;1]$. Accordingly, in the remainder of the paper, on the basis of the symmetry of the model, I assume alternatively that $x_1 \in [0;1/2]$ and $x_2 \in [1/2;1]$; or $x_1 \in [-\infty;1/2]$ and $x_2 \in [1/2;\infty]$.[2] Firm 1 enters at $t = 0$, while firm 2 enters at $\tau \in [0;\infty)$. I assume that τ is certain and known from the outset to both players. Entry entails a sunk cost F, such that location must be chosen once and for all at the time of entry in order to maximise the discounted flow of profits from that instant onwards. This fixed cost may be thought of as an irreversible R&D investment undertaken at $t = 0$. Such R&D activity results in a new product immediately for firm 1 and at time τ for firm 2. Unit production cost is assumed to be constant and equal across varieties. Without further loss of generality, I normalise it to zero. Throughout the time horizon considered, both firms have the same discount rate ρ.

The generic consumer located at $a \in [x_1;x_2]$ buys one unit of the good, enjoying the following net surplus:

[2] This assumption is meant to exclude the possibility of leapfrogging by either firm.

$$U = s - p_i - c(x_i - a)^2 \geq 0; \qquad i = 1; 2; \tag{1}$$

where x_i and p_i are firm's i location and mill price, respectively, and $c > 0$ is the transportation cost rate. In the remainder of the paper, I suppose that the reservation price s is never binding, so that full market coverage always obtains. To ensure this, it must be assumed that $s \geq 3c$ (cf. Bonanno, 1987; Harter, 1993). Let the demand function for firm i be $y_i^k \in [0; 1]$ in each period t; with $y_i^k = 1 - y_j^k$: Then, firm i's instantaneous profit function is $\pi_i^k = p_i^k y_i^k$; where superscript $k = m; d$ indicates the relevant market regime (monopoly or duopoly).

As long as the first firm to enter the market (firm 1) remains a monopolist, her per period demand function is $y_1^m = 1$: Hence, her profit function is $\pi_1^m = p_1^m$; and she finds it optimal to set a price such that the farthest consumer's net surplus at equilibrium is nil. Given the above assumption about the strategy space in locations, such consumer is at the right extreme of the segment, and the monopoly price and profits are

$$p_1^m = s - c(1 - x_1)^2 = \pi_1^m(x_1) : \tag{2}$$

Consider now the duopoly setting. One can easily derive from (1) the location of the consumer who is indifferent between the two goods at generic price and location pairs,

$$s - p_1^d - c(a - x_1)^2 = s - p_2^d - c(x_2 - a)^2; \tag{3}$$

as well as the demand functions, provided $a \in [x_1; x_2]$:[3]

$$y_1^d = \frac{p_2^d - p_1^d + c(x_2^2 - x_1^2)}{2c(x_2 - x_1)}; \qquad y_2^d = 1 - y_1: \tag{4}$$

Suppose firms set prices simultaneously. The Nash equilibrium in prices, for a given pair of locations, is given by

$$p_1^d = \frac{c(x_2^2 + 2x_2 - 2x_1 - x_1^2)}{3} ; \; p_2^d = \frac{c(x_1^2 - 4x_1 + 4x_2 - x_2^2)}{3} : \tag{5}$$

As a result, per period individual profit functions simplify as follows:

$$\pi_1^d(x_1; x_2) = \frac{c(x_2 - x_1)(x_1 + x_2 + 2)^2}{18} ; \; \pi_2^d(x_2; x_1) = \frac{c(x_2 - x_1)(x_1 + x_2 - 4)^2}{18} : \tag{6}$$

[3] If this condition is not met, e.g., if the indifference condition is written under the assumption that $a \in (x_2; 1]$; then it can be immediately verified that the location of the indifferent consumer is undefined.

3 The entry game

I assume that the game takes place in continuous time. Firm 2 enters at date τ, in a location $x_2 \geq x_1$, which, in turn, must be chosen once and for all by firm 1 at date 0. In describing firm's choice of location, the fixed R&D cost F can be disregarded. The objective function of firm 1 is

$$\Pi_1 = \int_0^{\tau} [\pi_1^m(x_1) \cdot e^{-\rho t}]dt + \int_{\tau}^{\infty} [\pi_1^d(x_1; x_2) \cdot e^{-\rho t}]dt ; \tag{7}$$

while the objective function of firm 2 is

$$\Pi_2 = \int_{\tau}^{\infty} [\pi_2^d(x_2; x_1) \cdot e^{-\rho t}]dt . \tag{8}$$

Obviously, it would be in firm 2's interest to enter as early as possible (conversely, firm 1 would like to remain a monopolist as long as possible). However, an entry date $\tau > 0$ can result either from unsuccessful R&D activity by firm 2 between 0 and τ, or from a broad patent protection sheltering the monopoly power of firm 1 over the same time span. I intentionally leave the explicit modelization of these elements out of the picture, in order to focus upon the descriptive power of single-period Stackelberg games versus multi-period entry games. It is reasonable to assume that, in maximising (7), firm 1 acts as a Stackelberg leader by taking into account the best reply of firm 2, implicitly defined by the first order condition for the maximisation of (8) w.r.t. x_2. Hence, the leader's problem consists in maximising (7) under the following constraint:

$$3x_2^2 - x_1^2 - 16x_2 + 2x_1x_2 + 16 = 0; \tag{9}$$

i.e., either $x_2 = 4 - x_1$ or $x_2 = (x_1 + 4)/3$. As is well known (Anderson, 1988; Lambertini, 1997), products behave as strategic complements, and this suffices to establish that the correct best reply of firm 2 is $x_2 = (x_1 + 4)/3$.

Consider first the unconstrained case. I am going to prove the following:

Theorem 1 Assume (i) $x_1 \in [-\infty; 1/2]$ and $x_2 \in [1/2; \infty]$ and (ii) firm 1 acts as a Stackelberg leader in locations. Then, in equilibrium, $x_1 = 1/2$ and $x_2 = 3/2$ for all $\tau \in [0; \infty)$.

Proof. Observe that, in the unconstrained location game considered here, firms' behaviour is dictated by first order conditions which replicate those of the well known single-period problem, in that, when firm 2 enters at time τ; we know that $x_2 = (x_1 + 4)/3$ and

$$\text{sign}\,\frac{\partial \Pi_1}{\partial x_1} = \text{sign}\,\frac{\partial \pi_1}{\partial x_1}\,. \tag{10}$$

As a result, firm 1 locates in $x_1 = 1/2$ at date 0, which is optimal independently of τ. Firm 1's equilibrium per period profits are $\pi_1^m = s - c/4$ for all $t \in [0; \tau)$ and $\pi_1^d = 8c/9$ for all $t \in [\tau; \infty)$. Firm 2's equilibrium per period profits are $\pi_2^d = 2c/9$ for all $t \in [\tau; \infty)$. Hence, this model replicates the static one (see Tabuchi and Thisse, 1995; Lambertini, 1997). ■

Consider now the constrained case. The following holds:

Theorem 2 Assume (i) $x_1 \in [0; 1/2]$ and $x_2 \in [1/2; 1]$ and (ii) firm 1 acts as a Stackelberg leader in locations. Then, in equilibrium,

$$x_1 = \frac{13 - 18e^{\rho\tau} + 2\sqrt{13 - 90e^{\rho\tau} + 81^{2\rho\tau}}}{3} \text{ for all } \tau \in \left(\frac{\ln(13/12)}{\rho};\, \frac{\ln(107/72)}{\rho}\right);$$

$$x_1 = 0 \quad \text{for all} \quad \tau \in \left[0;\, \frac{\ln(13/12)}{\rho}\right];$$

$$x_1 = \frac{1}{2} \quad \text{for all} \quad \tau \in \left[\frac{\ln(107/72)}{\rho};\, \infty\right);$$

$$x_2 = 1 \quad \text{for all} \quad \tau \in [0; \infty)\,.$$

Proof. Given the boundaries to the strategy space, $x_2 = 1$. The first order condition for firm 1 at the location stage is then

$$\frac{\partial \Pi_1}{\partial x_1} = \frac{c(36e^{\rho\tau} + 2(13 - 18e^{\rho\tau})x_1 - 3x_1^2 - 39)}{18\rho e^{\rho\tau}} = 0\,; \tag{11}$$

whose roots are

$$x_{1A} = \frac{13 - 18e^{\rho\tau} - 2\sqrt{13 - 90e^{\rho\tau} + 81e^{2\rho\tau}}}{3}\,; \tag{12}$$

$$x_{1B} = \frac{13 - 18e^{\rho\tau} + 2\sqrt{13 - 90e^{\rho\tau} + 81e^{2\rho\tau}}}{3}\,. \tag{13}$$

The above roots are real if

$$13 - 90e^{\rho\tau} + 81e^{2\rho\tau} \geq 0\,; \tag{14}$$

i.e., if $\rho\tau \geq \ln[(5+2\sqrt{3})/9]$, which is always true in that $(5+2\sqrt{3})/9 < 1$. It is immediate to check that the leader's location is given by x_{1B}; in that $\partial^2\Pi_1/\partial x_1^2 < 0$ in x_{1B} only, and

$$\lim_{\tau\to 0} x_{1B} = -\frac{1}{3} \tag{15}$$

which coincides with the leader's best reply to $x_2 = 1$ in the single-period unconstrained model. For any given ρ, x_{1B} is increasing and concave in τ; with $x_{1B} = 0$ when $\tau = \ln(13/12)/\rho$, and $x_{1B} = 1/2$ when $\tau = \ln(107/72)/\rho$. This entails that $x_1 = 0$ for all $\tau \in [0\,;\ \ln(13/12)/\rho]$; $x_1 = x_{1B}$ for all $\tau \in [\ln(13/12)/\rho\,;\ \ln(107/72)/\rho]$; and, finally, $x_1 = 1/2$ for all $\tau \in [\ln(107/72)/\rho\,;\ \infty)$. ■

Theorem 1 states that, as long as the strategy space at the location stage is unbounded, no loss of information on firms' behaviour is attached to the single-period Stackelberg metaphor usually adopted to describe the entry process. On the contrary, Theorem 2 shows that, if the strategy space is bounded, then time matters in determining the leader's location as a function of the date at which the follower enters the market. If such entry takes place early enough, the leader chooses not to exploit fully her own monopoly power over the period $[0;\tau)$; and locates at 0 from the outset; otherwise, if entry is late enough, the opposite happens. In general, the later the follower enters the market, the closer to 1/2 the leader locates at date 0. The straightforward implication of this result is that the single-period Stackelberg modelization of the sequential location choice offers a good description of the entry process if and only if the time span over which the first entrant remains a monopolist is long enough.

As Theorem 2 highlights, the extent of product differentiation $x_2 - x_1$ that we expect to observe at equilibrium is smaller, the later the follower enters the market. Then, a relevant corollary of Theorem 2 is the following:

Corollary 1 *If the strategy space at the location stage is bounded, then product differentiation at equilibrium is non-increasing in τ.*

Finally, the situation where the incumbent may relocate costlessly can be considered. If products can be redesigned freely at any point in time, which could be the case if no sunk investment were required, then the incumbent would (i) locate at 1=2 for all t 2 [0; ¿); and relocate to $x_1 = 0$ from ¿ onwards, if the location space is [0; 1]; (ii) locate at 1=2 forever, if …rms can choose any point along the real axis, also outside the boundaries of the linear city. Hence, without sunk costs, the single-period approach usually adopted in most of the existing literature involves no signi…cant loss of information on the optimal behaviour of …rms.

4 Concluding remarks

The foregoing analysis has stressed the importance of real time in explaining …rms' behaviour as to product characteristics and the resulting degree of di¤erentiation. In particular, the main …ndings are that (i) if the strategy space is unbounded, the in…nite horizon model and the single period model yield qualitatively similar results; (ii) if, instead, …rms are compelled to locate within the consumer preference space, then the …rst entrant's location is closer to the middle of the market, the later is the date of entry of the rival …rm, and conversely. Hence, if the …rst innovator expects to stand alone on the market place for a long time, the equilibrium con…guration of the location stage closely recalls the familiar Stackelberg equilibrium of a single-period model. Otherwise, when the entry date of the follower is early enough, equilibrium locations are the same as under simultaneous entry.

These results have some relevant consequences on our ability to describe such aspects of …rms' behaviour, as their R&D activities to discover and introduce new products. This issue is tackled by Harter (1993) in the quadratic transportation cost version of the linear model. He assumes that there are no externalities to the R&D, so that the …rms' discovery dates are independent. Each …rm bears a …xed cost, which is sunk at the beginning of the research activity, plus a constant cost in each period, during the time interval over which she continues to undertake her R&D e¤ort. In his analysis, Harter considers only location-speci…c R&D activities, and focuses upon the case where the …rst innovator locates in the middle point of the market, and the second innovator must evaluate whether to discontinue her R&D or not, and, if not, she locates at one of the extremes. In the light of the above results, this kind of investigation can be recast into a picture where R&D

is not location-speci…c and each …rm chooses location according to both the likelihood of entering …rst and the expected date at which the other …rm would innovate.

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