Endogenous Choice of Capacity and Product Innovation in a Di¤erentiated Duopoly[¤]

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Abstract

We model a symmetric duopoly where …rms choose whether to be quantity setters or price setters by deciding the optimal capacity; undertake R&D activity to determine the degree of di¤erentiation; and …nally compete in the market. Two games are proposed, where investment decisions follow di¤erent sequences. We assess price and quantity decisions, …nding a set of equilibria where the choice of the market variable is a¤ected by both technological commitments. As a result, the acquired wisdom that quantity setting is a dominant strategy for …rms, while price setting is a dominant strategy from a social standpoint, may not be con…rmed.

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1 Introduction

The interplay between technological choices and market behaviour in oligopoly models has been studied along two alternative routes. The …rst has emphasized the link between the shape of competition prevailing on the market and …rms' incentives to invest either in process or in product innovation. The second concerns the in‡uence of capacity constraints on market equilibrium.

Most literature on R&D races in oligopoly deals with the evaluation of incentives to undertake cost reducing investments as the number of …rms changes. This Schumpeterian approach holds that a major factor determining the pace of technological progress is market structure (amongst the countless contributions in this vein, see Arrow, 1962; Loury; 1979; Lee and Wilde, 1980; Dasgupta and Stiglitz, 1980; Delbono and Denicolò, 1991; for an overview see Reinganum, 1989).

An established result on cost reducing investment in oligopolistic markets under perfect certainty states that there is excess expenditure in R&D under Cournot competition, and conversely under Bertrand competition, due to the opposite slopes of reaction functions at the market stage (Brander and Spencer, 1983; Dixon, 1985). An extension of these results to the case of differentiated products can be found in Bester and Petrakis (1993). They maintain that the incentive to invest in cost reducing innovation depends upon the degree of product substitutability. Under both Cournot and Bertrand competition, underinvestment, as compared to the social optimum, obtains when products are fairly imperfect substitutes, while the opposite may occur when products are su¢ciently similar. Cournot competition provides a lower (respectively, higher) incentive to innovate than Bertrand competition if substitutability is high (respectively, low). As a result, social welfare may be higher under Cournot than under Bertrand competition (Delbono and Denicolò, 1990; Qiu, 1997).

While the R&D literature investigates the in‡uence of market competition on the optimal investment, contributions on capacity constraints follow a reverse route. They analyse how plant size determines the intensity of market competition (Levitan and Shubik, 1972; Kreps and Scheinkman, 1983; Osborne and Pitchik, 1986; Davidson and Deneckere, 1986). When …rms have enough capacity to serve the whole market a Bertrand equilibrium obtains. If capacity is binding, the Cournot outcome emerges. Deneckere and Kovenock (1996) prove that, under cost asymmetry, there is an incentive for the more efficient firm to drive the opponent out of business. This prevents the market from reaching a Cournot equilibrium.[1]

The main purpose of this paper is to bridge these two streams of literature in order to investigate the effect of technological choices on the shape of market competition, i.e., on firms' incentives to set either prices or quantities.

As far as R&D is concerned, a priori, there is no clearcut intuition as to how the shape of market competition can affect product innovation, i.e., investment aimed at reducing product substitutability. A preliminary result can be found in Lambertini and Rossini (1998), who show that a Prisoner's Dilemma can be responsible for product homogeneity in a binary model of investment in product innovation, followed by either Bertrand or Cournot competition. Here we extend the analysis to the case in which both R&D and capacity are continuous variables. The choice of capacity strategically anticipates the equilibrium output set according to the market variable chosen. As a result, in our model, capacity never binds, since it is endogenously set. Therefore, the optimal choice of capacity plays the role of a commitment of firms as to the market variable. We propose two alternative games. The first describes a symmetric duopoly where firms choose whether to be quantity setters or price setters, i.e., they fix capacity, then determine the reciprocal degree of differentiation through R&D, and finally compete in the market. In the second game, the first two stages follow a reverse order.

We are then able to reassess the choice between price and quantity. So far, the established wisdom in this respect (Singh and Vives, 1984) maintains that firms prefer to play a Cournot game because quantity setting is a dominant strategy for any given degree of substitutability. By introducing two additional stages, where firms undertake irreversible commitments, we find a richer set of equilibria, where the choice of the market variable is affected by technology. In neither of the two games Singh and Vives's result necessarily holds. When the R&D decision is taken at the second stage, goods are characterised by different degrees of substitutability in each subgame. There are parameter ranges wherein symmetric Cournot behaviour either is not an equilibrium or is not unique. When, instead, the R&D effort takes

[1] A subset of this literature concerns repeated games with capacity constraints (Brock and Scheinkman, 1985; Benoit and Krishna, 1987; Lambson, 1987; Davidson and Deneckere, 1990).

place at the first stage, it does not influence the choice of the market variable, uniquely determined by the tradeoff between revenue and the cost of capacity.

Some unconventional results in terms of social welfare are due to the capacity-intensive character of Bertrand competition. This may relieve the Cournot equilibrium from its social inefficiency. Moreover, we identify parameter sets, where the duopoly equilibrium may also be socially optimal.

The remainder of the paper is organized as follows. The basic model and the market stage are described in section 2. The two games and optimal investment behaviour are dealt with in section 3. In section 4, the welfare analysis is carried out. Concluding remarks are in section 5.

2 The model and the market stage

We describe a duopoly where firms noncooperatively play a one-shot three-stage game where they first choose the market variable, then determine their respective efforts in product innovation, and, finally, noncooperatively optimize w.r.t. the market variable chosen at the first stage. Firms play simultaneously in every stage. They may compete symmetrically either in quantities or in prices, or asymmetrically, one being a quantity setter while the other is a price setter. We also consider an alternative ordering of stages, where the determination of the R&D effort occurs at the first stage and the choice of the market variable is made at the second. As in the previous case, the third stage describes the market game.

The demand side is a simplified version of Dixit (1979) and Singh and Vives (1984). The symmetric demand functions under Cournot and Bertrand competition are, respectively

$$p_i = 1 - q_i - \gamma q_j \tag{1}$$

$$q_i = \frac{1}{1+\gamma} - \frac{p_i}{1-\gamma^2} + \frac{\gamma p_j}{1-\gamma^2} \tag{2}$$

In the asymmetric case, where firm i is a quantity setter, while firm j is a price setter, demand functions are:

$$p_i = 1 - q_i + \gamma(p_j + \gamma q_i - 1) \tag{3}$$

$$q_j = 1 - p_j - \gamma q_i \tag{4}$$

Parameter $\gamma \in (0,1]$ represents product substitutability as perceived by consumers, depending upon the products firms supply. Without loss of generality, we assume that unit cost is constant and equal to zero, so that individual profit gross of investment coincides with revenue $R_i^{IJ} = p_i q_i$, where $IJ \in \{PP, QQ, PQ, QP\}$ indicates the kind of competition prevailing at the market stage. The binary choice between a price or a quantity strategy entails setting up a capacity at the cost $x^{IJ} \in \left\{x^{PP}, x^{QQ}, x^{QP}, x^{PQ}\right\}$, i.e., choosing a plant size which is optimal in any possible market subgame, which we now solve by backward induction. Straightforward calculations lead to:

$$q_i^{QQ} = \frac{1}{2+\gamma} \in \left[\frac{1}{3},\frac{1}{2}\right); \quad q_i^{PP} = \frac{1}{2+\gamma-\gamma^2} \in \left[\frac{4}{9},\frac{1}{2}\right] \quad \forall \gamma \in (0,1]; \tag{5}$$

$$q_i^{PQ} = \frac{2-\gamma-\gamma^2}{4-3\gamma^2} \in \left[0,\frac{1}{2}\right); \quad q_i^{QP} = \frac{2-\gamma}{4-3\gamma^2} \in [0.454, 1]; \tag{6}$$

$$R_i^{QQ} = \frac{1}{(2+\gamma)^2} \in \left[\frac{1}{9},\frac{1}{4}\right); \tag{7}$$

$$R_i^{PP} = \frac{1-\gamma}{(2-\gamma)^2(1+\gamma)} \in \left[0,\frac{1}{4}\right); \tag{8}$$

$$R_i^{QP} = \frac{(\gamma-2)^2(1-\gamma^2)}{(3\gamma^2-4)^2} \in \left[0,\frac{1}{4}\right); \quad R_j^{PQ} = \frac{(\gamma-1)^2(\gamma+2)^2}{(3\gamma^2-4)^2} \in \left[0,\frac{1}{4}\right). \tag{9}$$

On the basis of the above quantities, the following chain of inequalities can be established:

$$q_i^{QP} > q_i^{PP} > q_i^{QQ} > q_i^{PQ} \ \forall \gamma \in (0,1]. \tag{10}$$

This reveals that, given the variable chosen by firm j, the output of firm i as a quantity setter is larger than her output as a price setter. Moreover, industry output under symmetric Bertrand behaviour is higher than in the alternative settings. Accordingly, the same sequence of inequalities must hold for capacities and their corresponding cost:

$$x_i^{QP} > x_i^{PP} > x_i^{QQ} > x_i^{PQ} \ \forall \gamma \in (0,1]; \ 2x^{PP} \geq x^{QP} + x^{PQ} > 2x^{QQ} \ \forall \gamma \in (0,1]. \tag{11}$$

As to revenues it is known from Singh and Vives (1984) that $R_i^{QP} \geq R_i^{PP}$ and $R_i^{QQ} > R_i^{PQ}$ $\forall \gamma \in (0,1]$, implying that quantity setting is the dominant strategy if the choice between price and quantity relies on revenue only.

Hence, there exists a trade-off between the cost of installing the plant and the revenues associated with size itself. The above discussion can be summarized in the following:

Lemma 1 Quantity setting revenue-dominates price setting, while price setting capacity-dominates quantity setting.

We are now in a position to tackle the issue of the R&D technology. A priori, parameter γ can be considered as an empty box to be filled by firms' choices.[2] At the second stage, firms may either costlessly produce perfect substitutes (with $\gamma = 1$), or differentiated products if at least one of them undertakes R&D activity. We adopt a general characterization of the R&D technology, $\gamma = \gamma(k_i; k_j)$; where k_i is the R&D investment of firm i, with $k_i \in [0; k_{max})$; k_{max} defining the amount of investment giving rise to two independent monopolies. For simplicity of notation, in the remainder of the paper we use γ_i and γ_{ii} to indicate $\partial\gamma/\partial k_i$ and $\partial^2\gamma/\partial k_i^2$; respectively. The R&D function is symmetric between the two firms, and $\gamma_i \leq 0$; $\gamma_{ii} \geq 0$: Profits are $\pi_i^{!J} = R_i^{!J} - k_i - x_i^{!J}$: We assume that $\pi_i^{!J}$ is continuous and twice differentiable w.r.t. k_i for all $\gamma \in (0; 1]$: Under the assumption that, initially, $\gamma = 1$; in order for investment to take place, it must be $\partial\pi_i^{!J}/\partial k_i > 0$ for at least one firm. An interior solution of the Nash game at the investment stage exists if there is at least a pair $(k_i; k_j)$ such that $\gamma \in (0; 1)$; $\partial\pi_i^{!J}/\partial k_i = 0$ and $\partial^2\pi_i^{!J}/\partial k_i^2 \leq 0$ for both firms. A sufficient condition for asymptotic stability is that $(\partial^2\pi_i^{!J}/\partial k_i^2)^2 \geq (\partial^2\pi_i^{!J}/\partial k_i \partial k_j)^2$:

3 Two alternative games

Here we describe two alternative games where

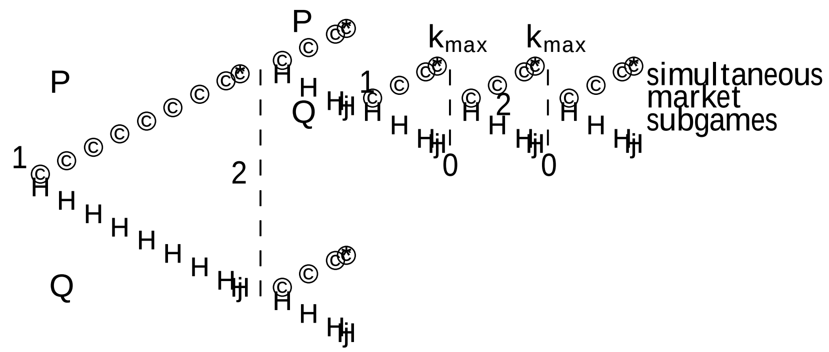
- At the first stage, by choosing the strategic market variable from the set $V = \{P; Q\}$; firms set capacity $x_i^{!J}$. At the second stage they choose the optimal R&D effort k_i: At the last stage they compete in the market.
- At the first stage, firms undertake R&D activity. At the second stage they choose capacity, while the third stage remains the same.

[2] The unit interval assumed for γ describes a symmetric horizontal differentiation equivalent to that arising in Hotelling's (1929) linear city, as long as, in the latter, firms play simultaneously. See the discussion in Harrington (1995).

3.1 The first game

The tree for the first game is illustrated in figure 1.

Figure 1 : Game I



The explicit solution of every subgame of the third stage is already known from the previous section. Here we deal with the second stage of the game, where firms decide the optimal level of investment k_i in product innovation.

3.1.1 Incentive to innovate under Cournot competition

When both firms are quantity setters, the relevant revenue functions are given by (7). First and second order conditions w.r.t. k_i are:

$$\frac{\partial \pi_i^{QQ}}{\partial k_i} = -\left(1 + \frac{2\gamma_i}{(2+\gamma)^3}\right) = 0; \quad (12)$$

$$\frac{\partial^2 \pi_i^{QQ}}{\partial k_i^2} = 3\gamma_i^2 - \gamma_{ii}(2+\gamma) \le 0. \quad (13)$$

We can implicitly define optimal investment behaviour by solving (12), obtaining $\gamma_i = -(2+\gamma)^3/2$, which is negative for all $\gamma \in (0,1]$. Substituting and rearranging, (13) simplifies to $\gamma_{ii} \ge 3(2+\gamma)^5/4$. If we consider $\partial \pi_i^{QQ}/\partial k_i$ at both the lower and the upper bound of the admissible investment range, defined respectively by $k_i = 0$ (and $\gamma = 1$) and k_{max} ($\gamma = 0$), we characterize the sufficient conditions for an interior solution to exist. When $\gamma = 1$, there is an incentive to start investing in differentiation if $\partial \pi_i^{QQ}/\partial k_i|_{\gamma=1} > 0$, i.e., $|\gamma_i| > 27/2$. If this does not hold, a corner solution may obtain at $k_i = 0$, involving $\gamma = 1$. If R&D productivity is not sufficiently high, an externality effect prevents firms from investing.

As products tend to become completely independent, i.e., γ tends to zero, we have

$$\lim_{\gamma \to 0} \frac{\partial \pi_i^{QQ}}{\partial k_i} = -2(\gamma_i + 4) < 0 \Rightarrow |\gamma_i| < 4. \tag{14}$$

If the above condition is violated, a corner solution may arise as k_i tends to k_{max} and γ tends to zero. When $\partial \pi_i^{QQ}/\partial k_i|_{\gamma=1} > 0$ and $\lim_{\gamma \to 0} \partial \pi_i^{QQ}/\partial k_i > 0$, a sufficient condition for an interior solution to exist is $\pi_i^{QQ}|_{\gamma=1} \geq \lim_{\gamma \to 0} \pi_i^{QQ}$, which holds if $k_{max} \geq 5/36$.

3.1.2 Incentive to innovate under Bertrand competition

Under symmetric price setting behaviour, the relevant revenue functions are given by (8). First and second order conditions w.r.t. k_i are:

$$\frac{\partial \pi_i^{PP}}{\partial k_i} = \frac{3\gamma(\gamma - 1)\gamma_i}{(\gamma - 2)^3(1 + \gamma)^2} - \frac{\gamma_i}{4 - 3\gamma^2 + \gamma^3} - 1 = 0, \tag{15}$$

$$\frac{\partial^2 \pi_i^{PP}}{\partial k_i^2} = 2\left[3\gamma_i(1 - 3\gamma + \gamma^2 - \gamma^3) + \gamma_{ii}(\gamma - 2 - 2\gamma^3 + \gamma^4)\right] \leq 0. \tag{16}$$

From (15) we obtain

$$\gamma_i = \frac{(\gamma - 2)^3(\gamma + 1)^2(4 - 3\gamma^2 + \gamma^3)}{3\gamma(\gamma - 1) - (\gamma - 2)^3(\gamma + 1)^2} < 0 \quad \forall \gamma \in (0, 1]. \tag{17}$$

Plugging (17) into (16), the second order condition (SOC) simplifies as follows:

$$\gamma_{ii} \geq \frac{3(\gamma - 2)^4(\gamma + 1)^2(1 - 3\gamma + \gamma^4 - \gamma^3)}{7\gamma - 8 - 7\gamma^3 + 2\gamma^4 + 6\gamma^5 - 5\gamma^6 + \gamma^7} > 0 \quad \forall \gamma \in (0.3611, 1]. \tag{18}$$

The above condition implies that the SOC is always met for $\gamma \in (0, 0.3611]$.

We now characterize the sufficient conditions for an interior solution to exist. Consider $\partial \pi_i^{PP}/\partial k_i$ at $k_i = 0$ (and $\gamma = 1$) and k_{max} ($\gamma = 0$). At the outset, under complete substitutability, firm i starts investing in differentiation if $\partial\pi_i^{PP}/\partial k_i|_{\gamma=1} > 0$; i.e., $|\gamma_i| > 2$. Otherwise, a corner solution may obtain at $k_i = 0$, involving $\gamma = 1$.

As products tend to become completely independent, i.e., γ tends to zero, we have that $\lim_{\gamma \to 0} \partial\pi_i^{PP}/\partial k_i < 0$ if $|\gamma_i| < 4$. This obviously coincides with (14), since the requirement for products to become completely independent is unrelated with the strategic variable. Again, if this condition is not met, a corner solution may arise as k_i tends to k_{max} and γ tends to zero. When $\partial\pi_i^{PP}/\partial k_i|_{\gamma=1} > 0$ and $\lim_{\gamma \to 0} \partial\pi_i^{PP}/\partial k_i > 0$, a sufficient condition for an interior solution to obtain is $\pi_i^{PP}|_{\gamma=1} \geq \lim_{\gamma \to 0} \pi_i^{PP}$, which holds if $k_{max} \geq 1/4$.

3.1.3 Incentive to innovate in the mixed setting

We now deal with the mixed case where, say, firm i is a quantity setter and firm j is a price setter. The FOCs at the investment stage are, respectively:

$$\frac{\partial\pi_i^{QP}}{\partial k_i} = \frac{8(8 + 2\gamma_i - 3\gamma\gamma_i) - \gamma^2(27\gamma^4 - 108\gamma^2 + 12\gamma^2\gamma_i - 34\gamma\gamma_i + 12\gamma_i + 144)}{(3\gamma^2 - 4)^3} = 0 \tag{19}$$

$$\frac{\partial\pi_j^{PQ}}{\partial k_j} = \frac{12\gamma(\gamma^2 + \gamma - 2)^2\gamma_j}{(4 - 3\gamma^2)^3} + \frac{2(1 + 2\gamma)(\gamma^2 + \gamma - 2)\gamma_j}{(4 - 3\gamma^2)^2} - 1 = 0. \tag{20}$$

From (20) it can be verified that $\partial\pi_j^{PQ}/\partial k_j|_{\gamma=1} = -1$, which implies that the price setting firm initially does not invest. As to the quantity setting firm, $\partial\pi_i^{QP}/\partial k_i|_{\gamma=1} > 0$ for all $|\gamma_i| > 1/2$. This entails that, at the outset, the quantity setter finds it easier to start investing if the rival is a price setter, than in the symmetric situation where both firms are quantity setters. This can be given the following interpretation. On the one hand, the quantity setter has a higher incentive since $R_i^{QP} > R_j^{PQ}$ for all $\gamma \in (0,1)$. Intuitively, when $\gamma = 1$, the quantity setter is unable to internalize the strategic advantage implicit in setting an output level in the market stage, because product homogeneity drives the price down to marginal cost and consequently the output to the level corresponding to perfect competition and zero profits. On the other hand, the price setter never starts investing in differentiation because she exploits the positive spillover exerted by the R&D effort of the quantity setter.

The SOC for the quantity setter is

$$\frac{\partial^2 \pi_i^{QP}}{\partial k_i^2} = 3(\gamma_i)^2 \left[16 - 32\gamma - 8\gamma^2 + 56\gamma^3 - 51\gamma^4 + 12\gamma^5\right] +$$
$$+\gamma_{ii}\left[48\gamma - 32 + 48\gamma^2 - 104\gamma^3 + 6\gamma^4 + 51\gamma^5 - 18\gamma^6\right] \leq 0. \qquad (21)$$

Simple calculations suffice to establish that

$$\frac{\partial^2 \pi_i^{QP}}{\partial k_i^2}\Big|_{\gamma=1} = -21(\gamma_i)^2 - \gamma_{ii} \leq 0; \qquad (22)$$

which is always true.

To complete the characterization of the interior solution for the quantity setter, observe that obviously $\lim_{\gamma \to 0} \partial\pi_i^{QP}/\partial k_i < 0$ if $|\gamma_i| < 4$. Finally, when $\partial\pi_i^{QP}/\partial k_i|_{\gamma=1} > 0$ and $\lim_{\gamma \to 0} \partial\pi_i^{QP}/\partial k_i > 0$, a sufficient condition for an interior solution to obtain is $\pi_i^{QP}|_{\gamma=1} \geq \lim_{\gamma \to 0} \pi_i^{QP}$, which holds if $k_{max} \geq 1/4$.

3.1.4 The reduced form

We are now in the position to solve the first stage of the game, whereby each firm chooses to act either as a quantity setter or as a price setter. The reduced form of the game is described by matrix 1.

		j: P	j: Q
i	P	$\pi_i^{PP}; \pi_j^{PP}$	$\pi_i^{PQ}; \pi_j^{QP}$
	Q	$\pi_i^{QP}; \pi_j^{PQ}$	$\pi_i^{QQ}; \pi_j^{QQ}$

Matrix 1

Suppose that at least one of the subgames represented in the above matrix yields an interior solution where either one or both firms invest in product differentiation, i.e., exclude the situations where $\gamma = 1$ or $\gamma = 0$ in all subgames. The choice of market variable is driven by the incentive to invest in product differentiation, which in turn depends on the marginal productivity of capital, γ_i and γ_j. As to the investment behaviour of firm i, the relevant parameter space can be divided into three regimes:

1. $|\gamma_i| > 27/2$: In this region, at least one firm invests in product differentiation, independently of downstream competition. The efficiency of technology is high enough to trigger investment regardless of the level of spillover exerted on the rival.

2. $|\gamma_i| \in (2; 27/2]$: In this region, investment is observed in all cases but in the symmetric Cournot setting.

3. $|\gamma_i| \in (1/2; 2]$: In this region, mixed cases give rise to investment at least by the quantity setter. When $\gamma = 1$; the price setter does not invest. However, we cannot exclude that the behaviour of the quantity setter triggers investment by the price setter for some $\gamma \in (0; 1)$:

4. $|\gamma_i| \in (0; 1/2]$: In this region no firm invests. As a result, market competition takes place in homogeneous products, due to extremely low marginal productivity of the R&D activity.

A straightforward consequence of the above discussion is:

Lemma 2 At the outset, under product homogeneity, the incentive to invest of a quantity setter is higher when the rival is a price setter. The opposite holds for the price setter.

As to the choice of the market variable, one may wonder whether the well known result of Singh and Vives (1984) holds true in the present setting. We prove the following:

Theorem 1 (A) Suppose $|\gamma_i| \in (0; 1/2]$: Hence, $\gamma = 1$; and the first stage of the game (i) is a Prisoner's Dilemma with a unique Nash equilibrium $(P; P)$ if $x_i^{QQ} - x_i^{PQ} > 1/9$; (ii) is a coordination game with two Nash equilibria, $(P; P)$ and $(Q; Q)$, if $x_i^{QQ} - x_i^{PQ} < 1/9$: (B) Suppose (i) $|\gamma_i| \in (1/2; 2]$; (ii) $\gamma_j \in [0; 2.3326]$; and (iii) $R_i^{QP} - k_i > x^{QP} - x^{PP}$: The reduced form of the game is a chicken game with two Nash equilibria, $(P; Q)$ and $(Q; P)$; if $R_i^{PQ} - 1/9 > x^{PQ} - x^{QQ}$: A sufficient condition for this to obtain is $\gamma \in (0; 2/3)$:

Proof. (A) If $|\gamma_i| \in (0; 1/2]$; no firm invests in product differentiation independently of the shape of market competition, which takes place with homogeneous products. The reduced form is represented by matrix 2.

		j: P	j: Q
i	P	$-x^{PP}, \; -x^{PP}$	$-x^{PQ}, -x^{QP}$
	Q	$-x^{QP}, -x^{PQ}$	$1/9 - x^{QQ}, \; 1/9 - x^{QQ}$

Matrix 2

First, observe that $1/9 - x^{QQ} > -x^{PP}$ and $-x^{PP} > -x^{QP}$. This excludes a chicken game, giving rise to either a Prisoner's Dilemma or a coordination game. The former obtains if $x_i^{QQ} - x_i^{PQ} > 1/9$, yielding price setting as a strictly dominant strategy generating (P, P) as the unique equilibrium. The latter obtains if $x_i^{QQ} - x_i^{PQ} < 1/9$. In this case, no dominant strategy exists and both (P, P) and (Q, Q) are equilibria, with (Q, Q) obviously dominating (P, P).

(B). Suppose (i) $|\gamma_i| \in (1/2, 2]$ and (ii) the reduction in γ resulting from the quantity setter's investment does not induce the price setter to undertake R&D activity. To ensure this, consider the FOC (20) for the price setter at the R&D stage, and solve it w.r.t. γ_j to obtain:

$$\gamma_j^* = \frac{(4 - 3\gamma^2)^3}{2(\gamma^2 + \gamma - 2)\left[6\gamma(\gamma^2 + \gamma - 2) + (1 + 2\gamma)(4 - 3\gamma^2)\right]} \qquad (23)$$

which takes its minimum at $\gamma = 0.91693$, where $|\gamma_j^*| = 2.3326$. As a consequence, $|\gamma_j| \in [0, 2.3326]$ is a sufficient condition for the price setter not to invest. This case is shown in matrix 3.

		j: P	j: Q
i	P	$-x^{PP}, \; -x^{PP}$	$R_i^{PQ} - x^{PQ}, \; R_j^{QP} - k_j - x^{QP}$
	Q	$R_i^{QP} - k_i - x^{QP}, \; R_j^{PQ} - x^{PQ}$	$1/9 - x^{QQ}, \; 1/9 - x^{QQ}$

Matrix 3

If $R_i^{QP} - k_i > x^{QP} - x^{PP} > 0$ and $R_i^{PQ} - 1/9 > x^{PQ} - x^{QQ}$, a chicken game obtains. Notice that, as $x^{PQ} - x^{QQ} < 0$, a sufficient condition for $R_i^{PQ} - 1/9 > x^{PQ} - x^{QQ}$ to hold is $R_i^{PQ} - 1/9 = (\gamma - 1)^2(\gamma + 2)^2/(3\gamma^2 - 4)^2 - 1/9 > 0$, which is true for all $\gamma \in (0, 2/3)$. ■

The asymmetry observed at equilibrium in terms of capacity in case B is driven by the expectation on the part of the price setter of receiving a positive externality from the quantity setter's decision to invest in product differentiation at the ensuing R&D stage.[3] The above discussion is sufficient to show that there may exist a non trivial parameter range where quantity setting is not a dominant strategy, so that the choice of the market variable may not follow the rule traced by Singh and Vives (1984). Analogous considerations hold in the remainder of the range, i.e., for $|\gamma_i| > 2$. E.g., suppose that $|\gamma_i| \in (2, 27/2]$; in this range, investment is observed in all cases but the symmetric Cournot one. For the latter to be the unique outcome of the game, as a result of an equilibrium in dominant strategies, the following inequalities must simultaneously hold:

$$\pi_i^{QP} - \pi_i^{PP} \geq 0; \quad \frac{1}{9} - \pi_i^{PQ} \geq x^{QQ}. \tag{24}$$

If both inequalities above are violated the unique equilibrium is (P, P). If only one of the two inequalities holds we end up with either a chicken game or a coordination game. Our conclusions impinge upon the endogenisation of the differentiation parameter which is influenced by the nature of downstream competition. The different incentives to invest in product differentiation determine a result which is related to the relative size of revenues and/or capacities.

3.2 The second game

Consider the alternative game, where the choice of the R&D effort takes place at the first stage of the game, while the choice between P and Q, i.e., the choice of capacity, is located at the second stage. Suppose, at the first stage, firms set a specific equilibrium level of k_i and k_j. In so doing, they fix the numerical value of γ in the unit interval. Then, the choice between P and Q is described by matrix 4.

[3] This resembles the analysis carried out by Fudenberg and Tirole (1984) and Bulow, Geanakoplos and Klemperer (1985) concerning entry deterrence.

		j: P	j: Q
i	P	$R_i^{PP} - x^{PP};\ R_j^{PP} - x^{PP}$	$R_i^{PQ} - x^{PQ};\ R_j^{QP} - x^{QP}$
	Q	$R_i^{QP} - x^{QP};\ R_j^{PQ} - x^{PQ}$	$R_i^{QQ} - x^{QQ};\ R_j^{QQ} - x^{QQ}$

Matrix 4

Observe that the investment in product differentiation is omitted from matrix 4, as it is common to all payoffs accruing to firm i; and consequently it is irrelevant as to the choice between P and Q. The inspection of matrix 4 reveals that the equilibrium of this game depends on the interplay between the capacity-dominance of price-setting and the revenue-dominance of quantity setting. Singh and Vives's (1984) conclusion, that symmetric Cournot behaviour should obtain in the unique equilibrium of their game, draws upon the revenue-dominance of quantity setting, which can be offset by the costly acquisition of capacity. As to the literature on Cournot equilibria under capacity constraints (Levitan and Shubik, 1972; Kreps and Scheinkman, 1983; Davidson and Deneckere, 1986; Osborne and Pitchik, 1986), the above analysis highlights the opportunity cost of building up capacity, and its role in shaping endogenously market competition. As an illustration, consider the following inequalities:

$$x^{QP} - x^{PP} > R^{QP} - R^{PP} > 0; \quad x^{QQ} - x^{PQ} > R^{QQ} - R^{PQ} > 0. \qquad (25)$$

If the above inequalities hold, the unique equilibrium is (P, P). Otherwise the game in matrix 4 can have (Q, Q) as the unique equilibrium or be either a chicken game, or a coordination game.

4 Welfare analysis

The foregoing analysis of firms' behaviour opens the question whether the well established result that Bertrand competition yields the highest welfare level should be expected to carry over to a setting where firms invest in R&D and productive capacity. Indeed, some contributions point out that this may not happen when firms' R&D efforts are directed towards the attainment of a process innovation (Delbono and Denicolò, 1990; Qiu, 1997). We show below that similar results emerge when capacity is endogenously decided.

De…ne social welfare as the sum of consumer surplus plus net pro…ts, i.e., $SW^{IJ} = CS^{IJ} + \Pi^{IJ}$; where $\Pi^{IJ} = \pi_i^{IJ} + \pi_j^{IJ}$: The established wisdom states that price setting behaviour enhances consumer surplus, with $CS^{PP} \geq CS^{PQ} = CS^{QP} > CS^{QQ}\ \forall\gamma \in (0;1]$. However, from (5-6), we know that

$$2q^{PP} \geq q^{QP} + q^{PQ} > 2q^{QQ}\ \forall\gamma \in (0;1]; \tag{26}$$

implying

$$2x^{PP} \geq x^{QP} + x^{PQ} > 2x^{QQ}\ \forall\gamma \in (0;1]: \tag{27}$$

As a consequence, we can state the following

Lemma 3 From a social standpoint, price setting surplus-dominates quantity setting, while quantity setting capacity-dominates price setting.

Although we cannot derive explicit conclusions for both games and all the relevant parameter subsets, we can unambiguously determine that there exists at least one case where symmetric Bertrand behaviour is not socially e¢cient. Consider the …rst game, where the choice of capacity takes place at the …rst stage. The following obtains:

Proposition 1 Suppose $|\gamma_i| \in (0;1/2]$: A necessary and su¢cient condition for the social optimality of symmetric Bertrand competition is $(x^{PP} - x^{QQ}) \in (0;1/36)$: Otherwise, symmetric Cournot competition is socially optimal.

Proof. If $|\gamma_i| \in (0;1/2]$; we know from the previous section that …rms compete in homogeneous products, irrespective of the strategic variables chosen at the …rst stage. Given $\gamma = 1$, industry output is equal to one in all cases except the symmetric Cournot; as a consequence, the cost of the overall capacity used in the industry must be the same everywhere except in the Cournot setting. Hence, from the social standpoint, relevant welfare levels are represented in matrix 5.

		j: P	j: Q
i	P	$1/2 - 2x^{PP}$	$1/2 - 2x^{PP}$
	Q	$1/2 - 2x^{PP}$	$4/9 - 2x^{QQ}$

Matrix 5

To prove the above Proposition, it suffices to observe that

$$\frac{1}{2} - 2x^{PP} > \frac{4}{9} - 2x^{QQ} \iff x^{PP} - x^{QQ} < \frac{1}{36}. \tag{28}$$

If the opposite obtains, i.e., $x^{PP} - x^{QQ} > 1/36$, the tradeoff between consumer surplus and the cost of capacity favours Cournot against Bertrand competition. ■

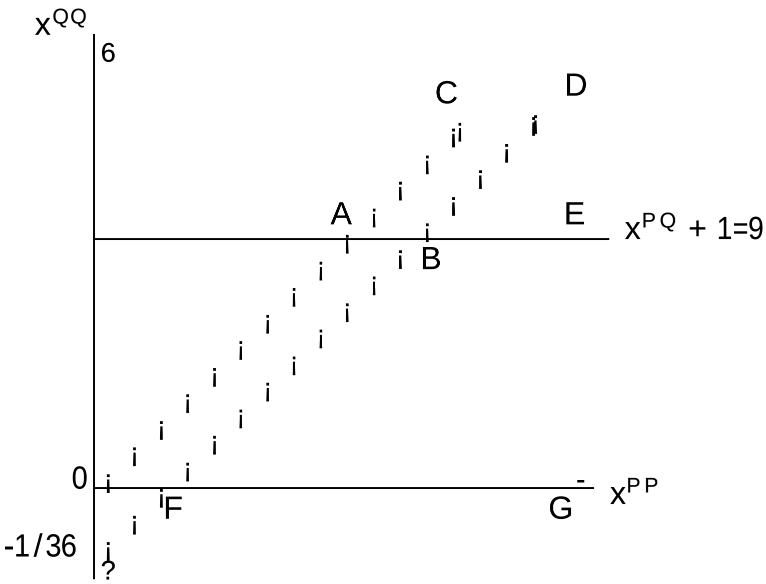
Our next step in the welfare analysis consists in verifying whether there exists any subset of the space defined by the cost of capacity, wherein the privately optimal equilibrium chosen by firms is socially efficient. We prove the following

Proposition 2 Suppose $|\gamma_{-}| \in (0, 1/2]$ and $x^{QQ} < x^{PP}$. If (i) $x^{QQ} > x^{PP} - 1/36$, and (ii) $x^{QQ} > x^{PQ} + 1/9$, the unique duopoly equilibrium (P, P) is also socially efficient. If (i) $x^{QQ} < x^{PP} - 1/36$, and (ii) $x^{QQ} > x^{PQ} + 1/9$, the unique duopoly equilibrium (Q, Q) is socially inefficient. If $x^{QQ} < x^{PQ} + 1/9$, firms play a coordination game with two equilibria, (P, P) and (Q, Q). When $x^{QQ} > x^{PP} - 1/36$, (P, P) is socially efficient; otherwise (Q, Q) is socially efficient.

Proof. To prove the above Proposition, we resort to a graphical exposition. Figure 2 below is drawn in the space $\left(x^{PP}, x^{QQ}\right)$, for a given level of x^{PQ}.

- In region ABCD, the duopoly equilibrium $\{P, P\}$ coincides with the social optimum.
- In region BDE, the duopoly equilibrium is again $\{P, P\}$, while the social optimum is $\{Q, Q\}$.

Figure 2 : Equilibrium analysis



- In region 0ABF, the duopoly game has two equilibria, fP; Pg and fQ; Qg; the former being socially optimal.
- In region BEFG, the duopoly game has two equilibria, fP; Pg and fQ; Qg; the latter being socially optimal. ■

Consider now the second game where the choice of capacity takes place at the second stage. On the basis of Lemma 1 and Lemma 3, it can be established that a con‡ict between social optimality and private optimality exists as far as the market variable is concerned.

5 Conclusions

Adopting the same demand structure as in Singh and Vives (1984), we have endogenised both the choice of capacity and the degree of product substitutability by introducing two stages where …rms noncooperatively undertake technological commitments before competing on the market. The consequences are twofold. First, in both cases the investment behaviour of any given ...rm depends upon whether the rival is a quantity or a price setter. Speci...cally, we have established that (i) given the market variable selected by the rival, being a quantity setter is costlier than being a price setter; and (ii) a quantity setter's incentive to innovate is higher when the rival is a price setter, while the opposite holds for a price setter. Second, a set of equilibria emerges from the reduced form of the game, where ...rms choose between price and quantity. We have shown that, when the productivity of investment is relatively low, the game exhibits two asymmetric equilibria in which one ...rm is a price setter and the other is a quantity setter, and only the latter invests in di¤erentiation. In the remaining scenarios of innovation technology, the equilibria are determined by the interplay between the level of investment and the resulting level of di¤erentiation, vis à vis the comparison between revenues and the cost of capacity.

Social welfare analysis suggests two considerations. First, symmetric price competition is more expensive in terms of installed capacity. This implies that Bertrand behaviour may not be socially e¢cient. Second, there are conditions under which private incentives may lead ...rms to play a socially optimal equilibrium.

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