Influence of dispersive effects on large-signal models based on differential parameter integration

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***Abstract* — In the paper is shown that, when devices are strongly affected by low-frequency dispersion effects, simply deriving the *i/v* characteristics through direct integration of bias-dependent differential parameters leads to results which are only locally self-consistent and coherent with empirical data. Dynamic drain current deviations due to charge trapping and self-heating phenomena in FETs, introducing a more complex dependence on average applied voltages and power in the dynamic *i/v* characteristics, must be taken into account by means of accurate approaches, such as the recently proposed equivalent voltage model [2]. Instead, integration of the low-frequency differential parameters in a suitably modified equivalent voltage domain is shown to be consistent with device physics. Experimental evidence is provided in the paper.**

I. Introduction

Large-signal, low-frequency[1] dynamic modelling of electron devices strongly influences the ability to obtain accurate performance predictions also at typical working frequencies of applications in the microwave and millimetre-wave range. This assumption, sometimes neglected in the past years, is of paramount importance in the context of empirical modelling of electron devices for MMIC circuit design. Unfortunately, large-signal modelling of low-frequency dynamic behaviour of electron devices can not be simply based on quasi-stationary *i/v* characteristics, when accurate predictions are required [1]-[5]. In fact, dispersion phenomena due to self-heating and/or charge trapping effects in III-V devices cause important deviations in the low-frequency dynamic drain current characteristics. For this reason, many widely used models are based on direct integration of low-frequency bias-dependent differential parameters (low-frequency trans- and output conductance $g_m^{AC}[V_{G_0}, V_{D_0}]$ and $g_d^{AC}[V_{G_0}, V_{D_0}]$, respectively) in order to obtain dynamic *i/v* characteristics. In this paper, it is shown that this approach leads to highly inaccurate results, which are only locally self-consistent and coherent with empirical data. From a mathematical point of view, a necessary and sufficient condition for the existence of an *integral* of a two-variable linear differential form such as:

$$g_m^{AC}[V_{G_0}, V_{D_0}] \cdot dV_{G_0} + g_d^{AC}[V_{G_0}, V_{D_0}] \cdot dV_{D_0} \qquad (1)$$

is that, for any couple of paths γ and λ, connecting the same two extreme points *A* and *B* in the voltage domain $\{V_{G_0}, V_{D_0}\}$, the following relation:

$$\int_\gamma \left(g_m^{AC}[V_{G_0}, V_{D_0}] \cdot dV_{G_0} + g_d^{AC}[V_{G_0}, V_{D_0}] \cdot dV_{D_0} \right) = \\ = \int_\lambda \left(g_m^{AC}[V_{G_0}, V_{D_0}] \cdot dV_{G_0} + g_d^{AC}[V_{G_0}, V_{D_0}] \cdot dV_{D_0} \right) \qquad (2)$$

is always verified. In the present paper, it is shown that *(2) is usually not verified in practice* over paths having not negligible lengths, when dealing with devices strongly affected by dispersion effects. As also shown in [2], the low-frequency dynamic drain current is not only dependent on the instantaneous applied voltages but also on other variables, such as the average values of the voltages[2].

Thus, bias-dependent low-frequency differential parameters are in this case not suitable to describe the problem complexity since they are only locally representative of the electron device dynamic behaviour (in the bias-region where the small-signal condition holds). Instead, in the paper is also shown that by means of a recently proposed modelling approach, namely the Equivalent Voltage Model [2], a non-dispersive device, associated to the real one and controlled by suitable modified voltages, can be properly introduced. Integration of differential parameters in the modified voltage domain is feasible and it can be exploited in order to obtain consistent modelling of the dynamic deviation of the drain current due to dispersion effects related to charge trapping and self-heating phenomena. The identification of the modified voltage domain along with the associated non-dispersive device is obtained on the base of standard DC and small-signal parameter measurements without need for non-linear optimisation procedures. Experimental results and numerical integral evaluations are provided in the paper, confirming the validity of the proposed approach.

[1] The *low-frequency* term is used in the paper for referring to a frequency range between the cut-off of dispersion phenomena and the lowest frequency where reactive phenomena related to charge storage variations and finite transit time of charges cannot be neglected.

[2] A dependence on the average dissipated power under dynamic operation must also be considered, when not negligible self-heating is present.

CHARGE-CONTROLLED	VOLTAGE-CONTROLLED
$\begin{cases} \underline{i}(t) = \underline{\Phi}\{\underline{q}(t)\} + \dfrac{d\underline{q}(t)}{dt} \\ \underline{q}(t) = \underline{\Psi}\{\underline{v}(t)\} \end{cases}$	$\underline{i}(t) = \underline{F}\{\underline{v}(t)\} + \underline{C}\{\underline{v}(t)\} \cdot \dfrac{d\underline{v}(t)}{dt}$ *where:* $\underline{F}\{\underline{v}\} = \underline{\Phi}\{\underline{\Psi}\{\underline{v}\}\}$ *and* $\underline{C}\{\underline{v}\} = \dfrac{d\underline{\Psi}\{\underline{v}\}}{d\underline{v}}$

TABLE I
VECTOR QUASI-STATIC MODEL EQUATIONS FOR A 2-PORT DEVICE

II. THE EQUIVALENT VOLTAGE MODEL

As proposed in [2], we consider first an ideal intrinsic FET where no low-frequency dispersion phenomena are present, so that a purely algebraic non-linear relationship can be assumed between charges and voltages. In particular, the quasi-static model formulation shown in Tab.I can be adopted, where: $\underline{i}=[i_S \ \ i_D]^T$, $\underline{q}=[q_{GS} \ \ q_{GD}]^T$, $\underline{v}=[v_{GS} \ \ v_{GD}]^T$ represent the source/drain currents, the gate-source/gate-drain charges (state-variables) and the intrinsic port voltages respectively. Moreover, $\underline{\Phi}\{\cdot\}=[\Phi_1\{\cdot\} \ \ \Phi_2\{\cdot\}]^T$, $\underline{\Psi}\{\cdot\}=[\Psi_1\{\cdot\} \ \ \Psi_2\{\cdot\}]^T$ and $\underline{F}\{\underline{v}\}$, $\underline{C}\{\underline{v}\}$ are vectors and matrices of purely-algebraic non-linear functions. The presence of low-frequency dispersion effects due to traps causes modifications in the charge-based state variables, introducing dependence with relatively long memory duration of charges on past values of voltages. Thus, the dispersive *q/v* model equation becomes:

$$\underline{q}(t) = \underline{\Psi}\{\underline{v}(t)\} + \underline{\Delta q}(t) \qquad (3)$$

where the components of $\underline{\Delta q}(t)$ can also be strongly dependent on voltages. However, an equivalent result can be obtained by still using the non-dispersive *q/v* equation, now referring to an *associated "non-dispersive" device*, provided that the actual port voltages $\underline{v}(t)$ are replaced by *equivalent port voltages* $\underline{\tilde{v}}(t)$. These must clearly satisfy the equivalence condition:

$$\underline{\tilde{v}}(t) = \underline{\psi}^{-1}\left\{\underline{\psi}\{\underline{v}(t)\} + \underline{\Delta q}(t)\right\} \qquad (4)$$

where we assume that, owing to the typically monotonic shape of the *q/v* characteristics, the inverse function of $\Psi_1\{\cdot\}$, $\Psi_2\{\cdot\}$ can be defined. Moreover, the non-dispersive *i/v* equation can be formally adopted to describe the associated non-dispersive device in the equivalent port voltage domain: $\underline{\tilde{v}} \doteq \underline{v} + \underline{\Delta v}$ according to Fig.1.

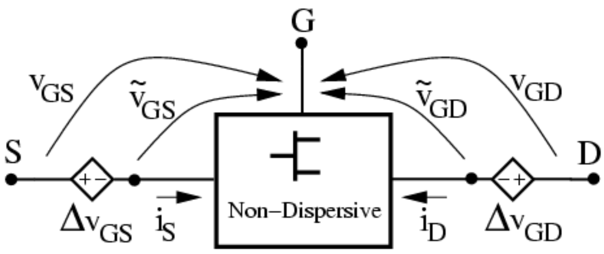

Fig. 1 - Intrinsic device circuit schematic describing the equivalent-voltage approach for low-frequency dynamic phenomena modelling

Thus, *the non-linear modelling of the dispersive device is here transformed into the modelling of the associated non-dispersive device, once the controlled voltage-sources $\underline{\Delta v}$ have been identified.* Since the trap filling is "frozen" in a particular state into the associated non-dispersive device, any conventional non-linear model for high-frequency performance predictions can be accurately identified in the equivalent voltage domain. Moreover, using the voltage perturbations $\underline{\Delta v}$ instead of charges $\underline{\Delta q}$ is a convenient choice, since the $\underline{\Delta v}$ terms involve weak dependence on the actual voltages *v*. In particular, we verified by experimental evidence that a simple linear dependence of $\underline{\Delta v}$ on port voltages $\underline{v}$ (with long memory time constant associated to charge trapping phenomena) provides good accuracy. Accordingly, we assume the frequency-domain relationship: $\underline{\Delta V} = \underline{A}(\omega)\cdot\underline{V}(\omega)$, where $\underline{\Delta V}$, $\underline{V}$ are the Fourier-transforms of $\underline{\Delta v}$, $\underline{v}$ respectively and $\underline{A}$ is a suitable matrix of low-pass transfer functions having cut-off frequencies coherent with dispersion phenomena. Thus, for operation at microwave frequencies we have: $\underline{\Delta V} = \underline{A}_0\cdot\underline{V}_0$, where: $\underline{A}_0 = \underline{A}(0)$ and $\underline{V}_0 = \underline{V}(0)$ are the DC components of $\underline{A}(\omega)$ and $\underline{V}(\omega)$, respectively. In order to embed the equivalent voltage approach in circuit-oriented models, the voltage perturbations $\underline{\Delta V}$ may be transformed from the common gate to the common-source voltage domain (the symbol *"S"* denotes common-source quantities in the following). This is a convenient choice, since all the dynamic drain current characteristics give $i_D=0$ for any v_{GS} when $v_{DS}=0$, thus forcing the constraint:

$$A^S_{210} = A^S_{220} = 0 \qquad (5)$$

where A^S_{ij0} $(i,j=1,2)$ are the DC components of the $\underline{A}^S$ matrix elements. Thus, the circuit equations become:

$$\begin{cases} \tilde{v}_{GS}(t) = v_{GS}(t) + A^S_{110}V_{G0} + A^S_{120}V_{D0} \\ \tilde{v}_{DS}(t) = v_{DS}(t) \end{cases} \qquad (6)$$

where V_{G0}, V_{D0} are the average values of $v_{GS}(t)$, $v_{DS}(t)$ and A^S_{110}, A^S_{120} are two model parameters to be identified.

III. INTEGRATION OF DIFFERENTIAL PARAMETERS

The equivalent voltage approach has been applied for the modelling of a 600µm GaAs PHEMT, which exhibits almost negligible self-heating effects. The two model parameters A^S_{110}, A^S_{120} have been identified by means of a least square solution of a multi-bias, over-determined, linear system of equations describing suitable congruence conditions on DC and AC (@1MHz) device conductances. In Fig. 2 the equivalent voltage $\tilde{v}_{GS}$ is shown versus v_{GS} for different v_{DS} values. Once the controlled voltage source is known, the drain current characteristics of the associated non-dispersive device can be easily evaluated on the basis of suitably corrected static device characteristics [2]. Results obtained with this procedure are shown in Fig.3. This identification method ensures theoretical coincidence between model prediction of the current static characteristics and corresponding measured data.

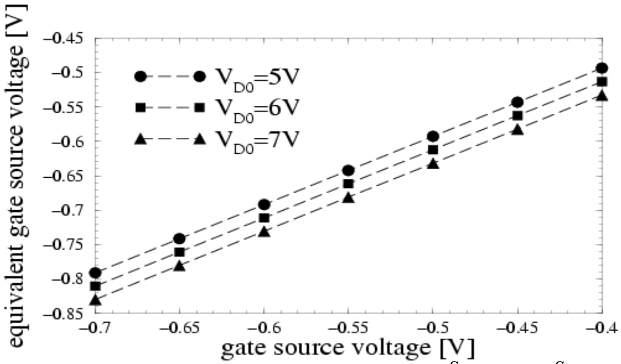

Fig. 2 – Equivalent voltage $\tilde{v}_{GS} = v_{GS} + A^S_{11_0} V_{G0} + A^S_{12_0} V_{D0}$ versus v_{GS} for a 600μm GaAs pHEMT. Model parameters $A^S_{11_0}$, $A^S_{12_0}$ have been identified by means of a least square solution of a multi-bias, over-determined, linear system describing suitable congruence of DC and AC (@1Mhz) device conductances

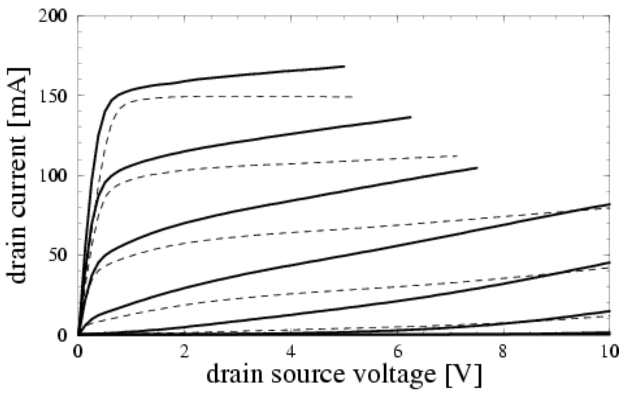

Fig. 3 – Drain current characteristics of the "non-dispersive" device (—) associated with a 600μm GaAs pHEMT, valid for DC and low-frequency conditions. Static device characteristics are also shown (- -).

Low-frequency small-signal parameter predictions based on the proposed approach are shown in Tab. II. In Fig.4 dynamic large-signal drain current prediction versus measurements at 2 MHz in a 50Ω environment

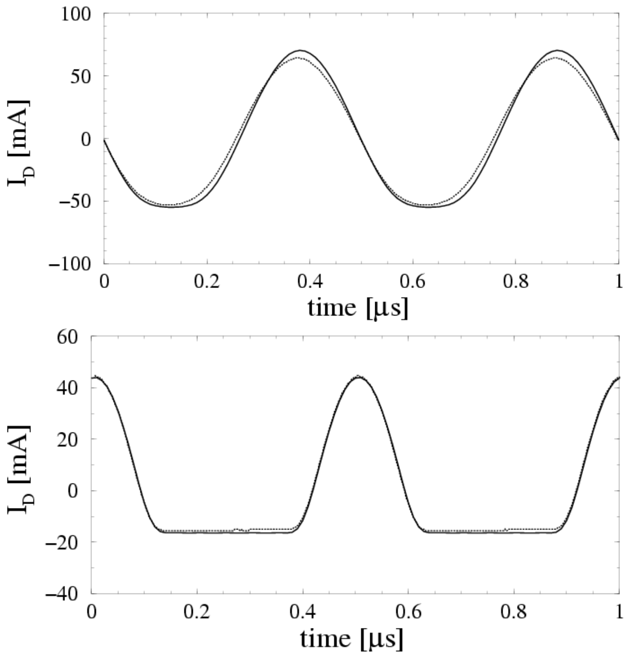

Fig. 4 - Dynamic drain current measured (dotted) and predicted by means of the EVM (solid) vs. time for a 600μm GaAs pHEMT (50Ω terminations). Sinusoidal voltage excitation @2MHz with amplitude 0.5V (large-signal) has been applied. Bias point considered: V_{gs}=-0.55V, V_{ds}=6.5V (top); V_{gs}=-1V, V_{ds}=4V (bottom).

(sinusoidal excitation) are also presented confirming the good prediction capabilities of the proposed model.

As previously stated in Sec.I, direct integration of (1) in the voltage domain $\{V_{G0}, V_{D0}\}$ leads to strong integral path-dependence. In Tab. III, direct integration results of (1) are provided over two different paths according to fig. 5. The integral extremes have been chosen in order to have a non-negligible length path.

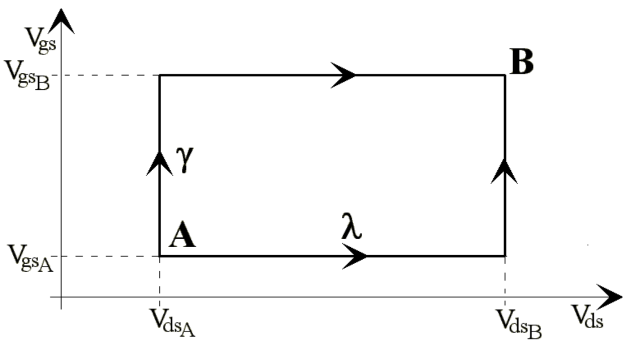

Fig. 5 – Different paths adopted for the integration tests on device differential parameters (see text).

g_m [mS] V_{GS} / V_{DS}		-0.7V	-0.55V	-0.4V
5V	meas.(dc)	147.573	194.071	213.271
	meas.(ac)	144.914	190.635	204.513
	sim.(ac)	144.971	189.035	203.040
6V	meas.(dc)	152.284	195.685	210.276
	meas.(ac)	148.782	189.949	201.266
	sim.(ac)	148.649	189.236	202.112
7V	meas.(dc)	157.684	196.852	205.506
	meas.(ac)	152.755	189.374	198.069
	sim.(ac)	152.872	188.987	197.872

g_d [mS] V_{GS} / V_{DS}		-0.7V	-0.55V	-0.4V
5V	meas.(dc)	1.949	2.284	2.282
	meas.(ac)	5.099	6.447	6.834
	sim.(ac)	5.080	6.207	6.256
6V	meas.(dc)	2.034	2.537	2.404
	meas.(ac)	5.008	6.138	6.393
	sim.(ac)	5.224	6.402	6.275
7V	meas.(dc)	2.436	2.889	2.455
	meas.(ac)	4.975	5.890	6.011
	sim.(ac)	5.372	6.689	6.227

TABLE II

STATIC AND DYNAMIC (@1MHZ) CONDUCTANCES FOR A 600μm PHEMT.

		600 μm pHEMT			1mm pHEMT		
		Path λ	*Path γ*	*Error %*	*Path λ*	*Path γ*	*Error %*
I_d [mA]	*Direct integration of (1)*	110.5	175.0	45.2%	150.4	228.5	41.2%
	Integration of (8) EVM	110.4	116.9	5.8%	155.6	154.4	0.7%

TABLE III

NUMERICAL INTEGRATION RESULTS OF MEASURED LOW-FREQUENCY DIFFERENTIAL CONDUCTANCES OVER TWO DIFFERENT PATHS IN THE INTRINSIC DEVICE VOLTAGE DOMAIN (*UPPER*) AND IN THE EQUIVALENT VOLTAGE DOMAIN (*LOWER*) FOR DIFFERENT MANUFACTURER pHEMT DEVICES (LEFT - VOLTAGE EXTREMES: V_{GA}=-1.4V, V_{GB}=-0.3V, V_{DA}=0.125V, V_{DB}=8V; RIGHT - VOLTAGE EXTREMES: V_{GA}=-2.5V, V_{GB}=-0.65V, V_{DA}=0.125V, V_{DB}=7V).

Derivation of the dynamic *i/v* model equation in the equivalent voltage domain leads to simple relationships between the low-frequency trans- and output differential conductances at the intrinsic ports of the electron device and the corresponding quantities (denoted with tilde) in the equivalent voltage domain:

$$\begin{cases} \tilde{g}_m^{AC}\left[\tilde{V}_{G_0}, V_{D_0}\right] = g_m^{AC}\left[V_{G_0}, V_{D_0}\right] = g_m^{AC}\left[\dfrac{\tilde{V}_{G_0} - A_{12_0}^S V_{D_0}}{1 + A_{11_0}^S}, V_{D_0}\right] \\ \tilde{g}_d^{AC}\left[\tilde{V}_{G_0}, V_{D_0}\right] = g_d^{AC}\left[V_{G_0}, V_{D_0}\right] = g_d^{AC}\left[\dfrac{\tilde{V}_{G_0} - A_{12_0}^S V_{D_0}}{1 + A_{11_0}^S}, V_{D_0}\right] \end{cases} \tag{7}$$

Owing to (7), the linear differential:

$$dF\left[\tilde{V}_{G_0}, V_{D_0}\right] = \tilde{g}_m^{AC}\left[\tilde{V}_{G_0}, V_{D_0}\right] d\tilde{V}_{G_0} + \tilde{g}_d^{AC}\left[\tilde{V}_{G_0}, V_{D_0}\right] dV_{D_0} \tag{8}$$

can be integrated *in the equivalent voltage domain* $\{\tilde{V}_{G_0}, V_{D_0}\}$ adopting the same paths and extremes described in fig. 5. In this domain, the integral results are much more independent on the considered path as shown in Table III, indirectly confirming the good capabilities of the equivalent voltage approach for the prediction of low frequency dispersion effects. The same test, also shown in tab. III, has been performed for a larger device with different characteristics (1mm pHEMT), confirming the validity of the theoretical developments.

Presented results also suggest that, since (8) corresponds almost to a *total differential form*, integration of the differential parameters in the equivalent voltage domain $\{\tilde{V}_{G_0}, V_{D_0}\}$ could lead to an alternative method for the identification of the function $F\{.\}$, corresponding to the associated non-dispersive device *i/v* characteristics. In this case, even better prediction of the device low-frequency dynamic behaviour could be obtained, but, as a drawback, only approximated (i.e. not theoretically coincident with measures [2]) model prediction of the static device characteristics would be obtainable.

IV. CONCLUSIONS AND FUTURE WORK

In this paper it has been shown that, when devices are strongly affected by low-frequency dispersion effects, simply deriving the *i/v* characteristics through direct integration of bias-dependent low-frequency differential parameters leads to results which are only locally self-consistent and coherent with empirical data.

Instead, low-frequency dispersion effects must be taken into account by means of accurate approaches, such as the equivalent voltage model [2]. Experimental results have shown that integration of the low-frequency differential parameters in a suitably modified equivalent voltage domain is feasible and consistent with device physics. This could be exploited as a basis for an alternative identification procedure of an associated non-dispersive device.

Future work will be oriented toward investigations on low-frequency dispersion on electron device capacitances [5-6]. In particular, the application of the equivalent voltage concept to the modelling of imaginary-part low-frequency differential parameters could lead to interesting approaches for the identification of physically consistent dynamic *q/v* characteristics by integration of device capacitances on a suitably modified voltage domain.

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