Vertical Differentiation with a Positional Good

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Abstract

We investigate the role of positional effects in a market for vertically differentiated goods. We consider two alternative settings, a single-product monopoly and a single-product duopoly. We evaluated the performance of both regimes against social planning. Contrary to conventional wisdom, we establish that it can be socially inefficient to expand the product range.
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1 Introduction

The purchase of some goods can be due to social motivation rather than intrinsic characteristics. Consumption behaviours can be classified according to the distinction between material and positional goods (Hirsch, 1976; Frank, 1985a,b), or between functional and non-functional motivation (Leibenstein, 1950).

In the quest for social distinction, individuals try to consume goods which, for their quality and price or for their limited supply, cannot be purchased by all. In the existing literature on positional or snob goods, the external effect in consumption due to social concerns is generally modeled through a relative-consumption mechanism. The utility derived from consumption is a function of the quantity purchased relative to the average of the society or the reference group to whom the consumer compares. This approach tends to ignore that very often the social distinction accrues not from the quantity purchased, but from the quality of the chosen good. The choice can be viewed as dichotomous: some consumers buy the positional good, some others cannot. In a partial equilibrium framework, the analysis can be carried out making different assumptions about the market structure. There can be a monopolist selling a positional good, and the consumers who are not served simply do not buy.[1] Alternatively, we can imagine a duopoly, with one firm selling the positional good and the other one selling a lower quality non-positional substitute, which is bought only for its intrinsic (material or functional) characteristics.

The utility derived from positional consumption by each individual is inversely related to the number of other consumers buying the same good. If consumers are sensitive to the size of demand, the producer's choice of the output level can be compared to the provision of product quality, in that restricting output is essentially like providing consumers with a good characterised by a superior quality. Therefore, the existence of social effects in markets where intrinsic quality is endogenous prompts for a reassessment of the welfare implications of market power against either perfect competition or social planning.

In order to provide a theoretical framework apt to evaluate this issues, we first analyze the monopoly case, building upon Spence's (1975) seminal paper on vertical differentiation, introducing a positional effect into consumer preferences. We investigate in detail a model with standard assumptions concerning consumer distribution and technology. A single-product monopolist supplies a good whose production entails a variable cost, which is assumed to be convex in the quality level. Consumer distribution is uniform. In this setting, it is known that, if positional effects were absent, a profit-maximising monopolist would supply the same quality as a social planner, as long as partial market coverage obtains (Spence,

[1] The behaviour of a monopolist in a market where buyers's satisfaction increases in the number of consumers excluded from purchase is analysed in Basu (1987), who suggests that the positional concern may be traced back to either quality signalling by the firm or status seeking by consumers.

1975).[2] This allows us to describe the behaviour of equilibrium prices, qualities and output levels, and the resulting welfare implications. As a first result, we find that the monopolist's choices depend on the consumers' marginal willingness to pay for intrinsic quality. If buyers are very interested in the quality of the good they purchase, then an elitarian equilibrium emerges, with few buyers of a high quality good. On the other hand, if consumers are only marginally interested in the intrinsic quality, the monopolist exploits the positional attribute of the good, supplying a low quality to an increasing number of consumers as the positional externality grows larger. In this case, the incentive to expand output increases in the extent of the externality and, as a result, the welfare loss due to monopoly power shrinks as the role of positional externalities in determining consumers' satisfaction becomes more relevant. Profits and social welfare are always increasing in the amount of positional externality.

Then, under the same assumptions on technology and consumers' distribution, we analyze the duopoly case under full market coverage. Consumers have to choose among a high-quality (positional) good, and a low-quality (non-positional) good. The existence of a pure-strategy subgame perfect equilibrium in qualities and prices is proved. We show that the profits of the firm supplying the lower quality (non positional) good are non-monotone (i.e., first increasing and then decreasing) in the amount of the positional externality. This is due to the fact that unit profits of the low-quality firm are non-monotone while its market share is monotonically decreasing in the positional externality. The same non-monotonicity obtains in the surplus of consumers purchasing the positional good. On the contrary, the surplus of customers of the non-positional good is always decreasing. However, social welfare in the duopoly equilibrium is increasing in the extent of the externality, due to the fact that the profits of the high-quality firm selling the positional good are everywhere increasing. However, opposite to the monopoly case, the scope for regulation widens as the externality becomes more important.

The paper is organized as follows. Section 2 presents the monopoly model, showing the monopoly equilibrium and the social optimum, which are then comparatively evaluated. Section 3 presents the duopoly model with single-product firms, whose welfare performance is assessed against the behaviour of a planner providing both products. Section 4 contains concluding remarks.

2 Monopoly equilibrium

Consider a monopoly market for a good whose utility depends both on intrinsic characteristics, which are represented by quality q, and by the social status which

[2] See also Sheshinski (1976). Spence's analysis has been extended to the multiproduct case by Mussa and Rosen (1978) and Itoh (1983), where a continuum of qualities is considered. Gabszewicz *et al.* (1986) characterise the optimal product mix of the monopolist when technology involves fixed costs.

its purchase confers to consumers. Social distinction arises because the market is covered only partially, i.e. there are some agents who do not consume. Therefore, preferences exhibit a positional externality, whose amount depends inversely on market demand x. Consumers are characterised by parameter θ, representing the individual marginal willingness to pay for quality. Consumers are uniformly distributed over the interval $[\bar{\theta}-1,\bar{\theta}]$, with $\bar{\theta}\geq 1$. The number of individuals is normalised to 1. Each consumer buys one unit of the good maximizing the net surplus he obtains, provided that it is non-negative:

$$U=\theta q+\alpha(1-x)-p \tag{1}$$

where p is the price charged by the monopolist, while α (the same for all the agents) is a positive coefficient representing the weight of the positional externality in the utility function, and x is the market demand for the good, so that $1-x$ is the number of consumers who are not served. This implies that, *ceteris paribus*, an additional purchase by a consumer previously priced out of the market produces a negative externality on the group of individuals who were already being served.

Let $\widehat{\theta}(\alpha,p,q)=(p+\alpha\bar{\theta}-\alpha)/(q+\alpha)$ define the marginal willingness to pay of the consumer who is indifferent between buying and not buying. Hence, under partial market coverage and full market coverage, respectively, market demand is

$$x=\bar{\theta}-\widehat{\theta}(\alpha,p,q)=\frac{\bar{\theta}q+p-\alpha}{q+\alpha}\text{ for all }\{p,q,\alpha\}\text{ such that }\hat{\theta}\in(\bar{\theta}-1,\bar{\theta}]\quad(pmc); \tag{2}$$

$$x=1\text{ for all }\{p,q,\alpha\}\text{ such that }\hat{\theta}\leq\bar{\theta}-1\quad(fmc). \tag{3}$$

When $\max\{\bar{\theta}-1,\widehat{\theta}\}=\bar{\theta}-1$, full market coverage (fmc) obtains, i.e., $x=1$, and the positional externality disappears. On the supply side, production involves total costs $C=q^2x$. There are constant returns to scale and unit production costs $c(q)$ are convex in quality. The profit function is then

$$\Pi^M=\left(p-q^2\right)\cdot x\,. \tag{4}$$

Except for the presence of positional effects, this setup replicates the model introduced by Spence (1975). He proves that, for a given output level, if consumers are uniformly distributed and the market is covered only partially, the monopolist supplies the social optimum quality. In the following we will show that this is not the case when a positional concern is present.

2.1 Profit maximization

Since we are focusing on the effects of social distinction on market behaviour, we confine our analysis to the case of partial market coverage, i.e., $\hat{\theta}\in(\bar{\theta}-1,\bar{\theta}]$. Monopoly profits are given by:

$$\Pi^M = (p - q^2) \cdot \left[\frac{\bar{\theta}q + p - \alpha}{q + \alpha} \right] . \tag{5}$$

This expression has to be maximized with respect to the two choice variables: price and quality.[3] The first order conditions (FOCs) with respect to price and quality are:

$$\frac{\partial \Pi^M}{\partial p} = \frac{\alpha + \bar{\theta}q - 2p + q^2}{q + \alpha} = 0 ; \tag{6}$$

$$\frac{\partial \Pi^M}{\partial q} = \frac{-2\overline{\theta}q^3 + q^2(p - \alpha - 3\alpha\overline{\theta}) + 2\alpha\overline{\theta}(p - \alpha) + p^2 + \alpha\overline{\theta}p - \alpha p}{(q + \alpha)^2} = 0 . \tag{7}$$

Solving the system (6-7) yields:

$$p^M = \frac{3\alpha + 4\alpha^2 - 2\alpha\bar{\theta} + \bar{\theta}^2 + (\bar{\theta} - \alpha) \cdot k}{9} ; \tag{8}$$

$$q^M = \frac{\bar{\theta} - 4\alpha + k}{6} , \tag{9}$$

where $k = \sqrt{\overline{\theta}^2 + 16\alpha\overline{\theta} + 16\alpha^2 - 12\alpha}$. Observe that, positional concern being absent ($\alpha = 0$), $q^M = \overline{\theta}/3$ and $p^M = 2\overline{\theta}^2/9$ (see Lambertini, 1997). Market demand in equilibrium amounts to:

$$x^M = \frac{\overline{\theta}^2 - 8\alpha\overline{\theta} + 12\alpha - 8\alpha^2 + (2\alpha + \overline{\theta})k}{3(\overline{\theta} + 2\alpha + k)} . \tag{10}$$

Consumer surplus is

$$CS^M = \int_{\widehat{\theta}}^{\overline{\theta}} [\theta q^M + \alpha(1 - x^M) - p^M] d\theta . \tag{11}$$

Social welfare corresponds to $SW^M = CS^M + \Pi^M$. The comparative statics properties of the positional externality on price, quality and output are summarised by

Lemma 1 *Monopoly price is always increasing in α. Moreover, $x^M < 1$ for all $\overline{\theta} \in \left[1, 2 + \sqrt{4\alpha + 1}\right)$.*

- $\lim_{\alpha \to 0} x^M = \overline{\theta}/3$; $\lim_{\alpha \to \infty} x^M = 1/2$.

[3] The choice between price and quantity is obviously irrelevant, with the same results obtaining in the two cases.

- *For all* $\overline{\theta} \in [1, 3/2)$, $\partial x^M / \partial \alpha > 0$ *and* $\partial q^M / \partial \alpha < 0$.
- *For all* $\overline{\theta} \in \left(3/2, 2 + \sqrt{4\alpha + 1}\right)$, $\partial x^M / \partial \alpha < 0$ *and* $\partial q^M / \partial \alpha > 0$.
- *For* $\overline{\theta} = 3/2$, $\partial x^M / \partial \alpha = \partial q^M / \partial \alpha = 0$, *with* $x^M = q^M = 1/2$.

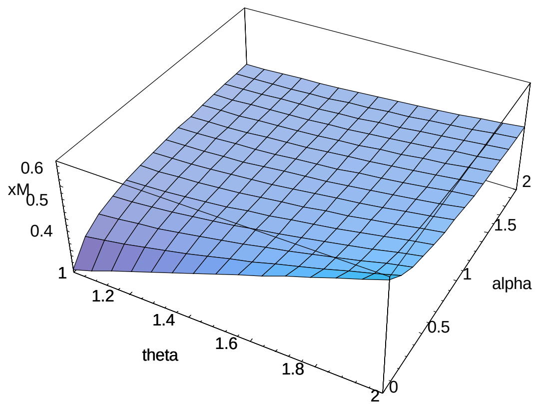
The proof of Lemma 1 is a matter of tedious calculations. To verify the behaviour of x^M as α changes, it suffices to examine:

$$\frac{\partial x^M}{\partial \alpha} = 4[30\alpha^2 - 18\alpha - 16\alpha^3 + 30\alpha\overline{\theta} - 24\alpha^2\overline{\theta} + 3\overline{\theta}^2 - 12\alpha\overline{\theta}^2 - 2\overline{\theta}^3 +$$

$$+k(4\alpha^2 - 6\alpha + 3\overline{\theta} + 4\alpha\overline{\theta} - 2\overline{\theta}^2)]/[3k(2\alpha + \overline{\theta} + k)]^2 \,, \quad (12)$$

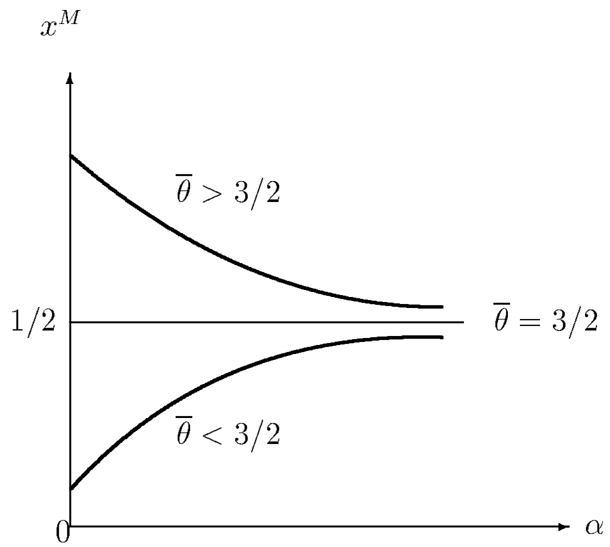
whose sign changes along $\overline{\theta} = 3/2$. The behavior of market demand in the monopoly equilibrium over the parameter space $\{\overline{\theta}, \alpha\}$ is drawn in figure 1.

Figure 1 : Monopoly output in $\{\overline{\theta}, \alpha\}$



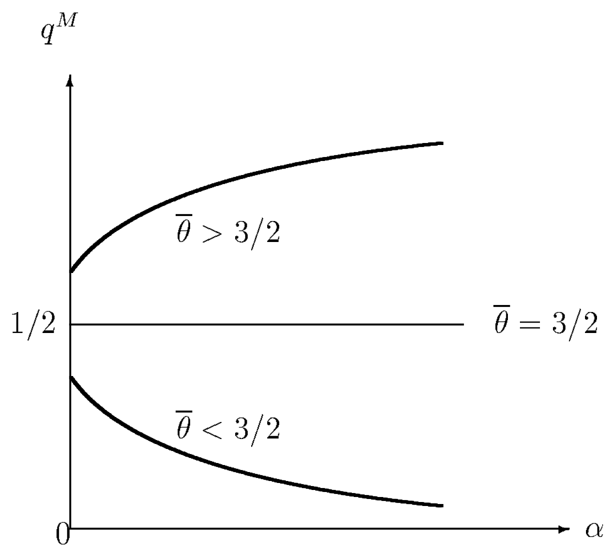
The behaviour of x^M as α varies, for given levels of $\overline{\theta}$, is represented in figure 2, which obtains by taking a section of the surface in figure 1 along three specific values of $\overline{\theta}$.

Figure 2 : Monopoly output



Notice that, depending on the level of $\overline{\theta}$, equilibrium quality levels tend to diverge as the amount of positional externality α increases. This phenomenon is described in figure 3.

Figure 3 : Monopoly quality



Overall, the monopolist's behaviour can be interpreted in the following terms:

- $\overline{\theta} = 3/2$. Here, $q^M = x^M = 1/2$, and the net surplus of the marginal consumer at $\widehat{\theta}$ is $U_{\widehat{\theta}} = \widehat{\theta} q^M - p^M + \alpha(1 - x^M) = (1 - 2p^M + \alpha)/2 = 0$ at $p^M = (1 + \alpha)/2$. This entails that, as p^M increases in α as fast as the positional effect $\alpha(1 - x^M)$, along $\overline{\theta} = 3/2$ the identity of the marginal consumer, and therefore the size of demand, is independent of α.

- $\overline{\theta} \in \left(3/2, 2 + \sqrt{4\alpha + 1}\right)$. In this range demand is larger, but, as α increases, the monopolist supplies a better quality serving fewer consumers. When the marginal willingness to pay for quality is high, i.e. the market is rich, the presence of a positional effect leads to an elitarian equilibrium, with few buyers of an expensive good characterised by a high quality.

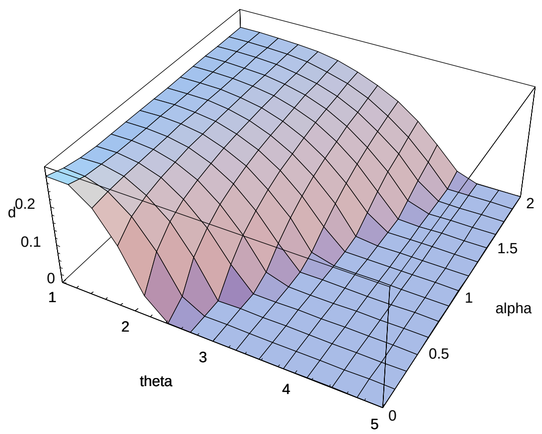
- $\overline{\theta} \in [1, 3/2)$. In this range demand is low, but, as α increases, the monopolist lowers the quality level, serving more consumers. When the marginal willingness to pay for quality is low, i.e. the market is relatively poor, the presence of a positional effect is simply exploited by the monopolist so as to increase price in spite of the reduction in quality and the expansion in demand.

As to social welfare and its components, the following holds:

Proposition 1 *Monopoly profits and social welfare are always increasing in the extent of the positional externality.*

Proof. The solution to $\partial \Pi^M / \partial \alpha = 0$ is $\overline{\theta} = 2 + \sqrt{4\alpha + 1}$, which is the upper bound of the range for $\overline{\theta}$ wherein partial market coverage obtains. For all $\overline{\theta} \in \left[1, 2 + \sqrt{4\alpha + 1}\right)$, $\partial \Pi^M / \partial \alpha > 0$. Consumer surplus is initially increasing and then decreasing in α. As to the overall effect of the positional externality on social welfare, we observe that $\partial SW^M / \partial \alpha$ is initially positive and then negative as α increases (see figure 4).

Figure 4 : The derivative $d = \partial SW^M / \partial \alpha$ in the space $\{\alpha, \overline{\theta}\}$



Numerical calculations show that $\partial SW^M/\partial\alpha = 0$ along $\widetilde{\alpha} = 0.263\cdot\overline{\theta}^2 - 0.686\cdot\overline{\theta} + 0.213$,with $\widetilde{\alpha} > \widehat{\alpha} = (\overline{\theta}^2 - 4\overline{\theta} + 3)/4$, along which full market coverage obtains at the margin. Hence, for all admissible α, $\partial SW^M/\partial\alpha > 0$. ■

Observe that the main result we have derived is that any increase in the positional externality entails a welfare improvement, *independently* of the marginal willingness to pay. This fact can be given the following interpretation. When the market is poor, i.e., $\overline{\theta} \in [1, 3/2)$, $\partial x^M/\partial\alpha > 0$, so that obviously welfare must increase as α gets larger. When consumers are rich, i.e., $\overline{\theta} \in \left(3/2, 2 + \sqrt{4\alpha+1}\right)$, $\partial x^M/\partial\alpha < 0$, but the output restriction as α becomes larger is more than compensated by higher profits and consumer surplus due to the elitarian effect operating in the individual surplus function.

2.2 Welfare maximization

Consider now the behaviour of a benevolent social planner maximising social welfare with respect to price and quality. The first order condition for welfare maximisation w.r.t. price is:

$$\frac{\partial SW^{SP}}{\partial p} = \frac{\alpha^2 + \alpha\overline{\theta}q - 2\alpha p - pq + \alpha q^2 + q^3}{(q+\alpha)^2} = 0\ , \tag{13}$$

where superscript SP stands for *social planning*. This yields $p^{SP} = (\alpha^2 + \alpha\overline{\theta}q + \alpha q^2 + q^3)/(2\alpha + q)$. Then, solving $\partial SW^{SP}/\partial q = 0$ yields optimal quality:

$$q^{SP} = \frac{\overline{\theta} - 8\alpha + h}{6}\ , \tag{14}$$

where $h = \sqrt{\overline{\theta}^2 + 32\alpha\overline{\theta} - 12\alpha + 64\alpha^2}$. It is easy to verify that $\partial q^{SP}/\partial\alpha > 0$ always, and $\lim_{\alpha\to\infty} q^{SP} = (4\overline{\theta} - 1)/8$. The associated price rewrites as

$$p^{SP} = \frac{\left[36\alpha(6\alpha + \overline{\theta}) + 6\alpha + 1\right](\overline{\theta} - 8\alpha + h)}{36(\overline{\theta} + 4\alpha + h)}\ . \tag{15}$$

It can be verified that $\partial p^{SP}/\partial\alpha > 0$, and $\lim_{\alpha\to\infty} p^{SP} = \infty$.

The equilibrium output is

$$x^{SP} = \frac{2\left[\overline{\theta}^2 - 16\alpha\overline{\theta} + 12\alpha - 32\alpha^2 + (4\alpha + \overline{\theta})h\right]}{3(4\alpha + \overline{\theta} + h)}\ , \tag{16}$$

with full market coverage obtaining at $\alpha = \alpha' = (4\overline{\theta}^2 - 8\overline{\theta} + 3)/16$ which is larger than $\widehat{\alpha}$ for all $\alpha > 0$ and $\overline{\theta} > 1$. Moreover, $\partial x^{SP}/\partial\alpha < 0$ always.

This proves the following lemma:

Lemma 2 *The socially optimal policy as to the extent of market coverage establishes that*

- *the planner prices some consumers out of the market for all* $\alpha > \alpha'$;
- *the planner serves all consumers for all* $\alpha \leq \alpha'$.

In combination with the behaviour of the monopoly output x_M described in lemma 1, this entails

Lemma 3 *Given* $\alpha' > \widehat{\alpha}$,

- *for all* $\alpha > \alpha'$, *partial market coverage obtains under both monopoly and social planning;*
- *for all* $\alpha \in (\widehat{\alpha}, \alpha']$, *the monopolist excludes some consumers from purchase, while full market coverage obtains under social planning;*
- *for all* $\alpha \leq \widehat{\alpha}$, *all consumers are served under both regimes.*

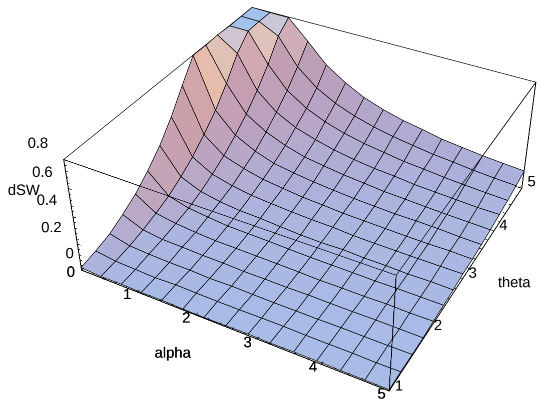
The overall behaviour of social welfare as α varies remains to be evaluated. Again, resorting to numerical calculations shows that $\partial SW^{SP}/\partial\alpha = 0$ along the locus $\alpha = 0.25 \cdot \overline{\theta}^2 - 0.5 \cdot \overline{\theta} + 0.187 < \alpha'$ for all $\alpha > 0$. Moreover, $\partial SW^{SP}/\partial\alpha > 0$ for all $\alpha > \alpha'$. As a result, we can state

Proposition 2 *Under social planning, social welfare is everywhere increasing in the amount of the positional externality.*

2.3 Monopoly vs social planning

Propositions 1 and 2 prompt for a comparison of welfare levels under the alternative regimes of monopoly and social planning as the extent of positional externality changes. Define $\Delta SW = SW^{SP} - SW^{M} > 0$. Numerical calculations show that $\partial SW^{M}/\partial\alpha > \partial SW^{SP}/\partial\alpha$. Therefore, as illustrated in figure 5, ΔSW is decreasing in α. The ultimate implication of this result is that the need for regulating the monopolist's behaviour tends to disappear as the externality becomes more relevant.

Figure 5 : $\Delta SW = SW^{SP} - SW^{M}$



3 Duopoly equilibrium

Here, we consider the case of a duopoly consisting in two single-product firms offering qualities $q_H \geq q_L \geq 0$, following Cremer and Thisse (1994). In particular, we focus upon the case where the positional externality is associated with q_H, and all consumers who are unable to purchase the positional high-quality good buy an alternative low-quality good which does not confer any *status*. Hence, we confine our attention to the full market coverage setting. The net surplus of a consumer who buys the high quality good is

$$U_H = \theta q_H + \alpha(1 - x_H) - p_H \ , \tag{17}$$

where $1 - x_H = x_L$, while that of a consumer buying q_L is $U_L = \theta q_L - p_L$. Define as θ' the location along $\left[\overline{\theta} - 1, \overline{\theta}\right]$ of the consumer who is indifferent between q_H and q_L at generic prices $\{p_H, p_L\}$. Therefore, $1 - x_H = x_L = 1 - \overline{\theta} + \theta'$ and θ' obtains from the solution of $U_H = U_L$:

$$\theta' = \frac{\alpha(\overline{\theta} - 1) + p_H - p_L}{\alpha + q_H - q_L} \ . \tag{18}$$

Consumer surplus in the two market segments is, respectively:

$$CS_H = \int_{\theta'}^{\overline{\theta}} U_H d\theta \ ; \ CS_L = \int_{\overline{\theta}-1}^{\theta'} U_L d\theta \ . \tag{19}$$

Firm i's profit function is $\pi_i = (p_i - q_i^2)x_i$. Therefore, social welfare amounts to

$$SW = CS_H + CS_L + \pi_H + \pi_L \ . \tag{20}$$

3.1 Profit-maximising duopoly

Strategic interaction between firms takes place in two stages, with firms moving simultaneously in both stages. In the first, firms choose qualities; in the second, they choose prices. As usual, we solve the game by backward induction, the solution concept being subgame perfection.

The first order conditions w.r.t. prices, given qualities q_H and q_L chosen at the first stage, are:

$$\frac{\partial \pi_H}{\partial p_H} = \frac{p_L - 2p_H + \overline{\theta}(q_H - q_L) + q_H^2 + \alpha}{q_H - q_L + \alpha} = 0 \; ; \tag{21}$$

$$\frac{\partial \pi_L}{\partial p_L} = \frac{p_H - 2p_L + (1 - \overline{\theta})(q_H - q_L) + q_L^2}{q_H - q_L + \alpha} = 0 \; , \tag{22}$$

whose simultaneous solution yields candidate equilibrium prices:

$$p_H^* = \frac{(1 + \overline{\theta})(q_H - q_L) + 2q_H^2 + q_L^2 + 2\alpha}{3} \; ; \tag{23}$$

$$p_L^* = \frac{(2 - \overline{\theta})(q_H - q_L) + q_H^2 + 2q_L^2 + \alpha}{3} \; . \tag{24}$$

Plugging (23-24) into the profit functions and simplifying, we obtain the relevant expressions for profits at the first stage of the game:

$$\pi_H = \frac{\left[(1 + \overline{\theta})(q_H - q_L) - q_H^2 + q_L^2 + 2\alpha\right]^2}{9(q_H - q_L + \alpha)} \; ; \tag{25}$$

$$\pi_L = \frac{\left[(2 - \overline{\theta})(q_H - q_L) + q_H^2 - q_L^2 + \alpha\right]^2}{9(q_H - q_L + \alpha)} \; . \tag{26}$$

Differentiating (25) and (26) w.r.t. q_H and q_L, respectively, we obtain the first order conditions at the first stage:

$$\frac{\partial \pi_H}{\partial q_H} = \frac{(1 + \overline{\theta})(q_H - q_L) - q_H^2 + q_L^2 + 2\alpha}{9(q_H - q_L + \alpha)} \cdot [2(1 + \overline{\theta} - 2q_H) + \\ - \frac{(1 + \overline{\theta})(q_H - q_L) - q_H^2 + q_L^2 + 2\alpha}{9(q_H - q_L + \alpha)}] = 0 \; ; \tag{27}$$

$$\frac{\partial \pi_L}{\partial q_L} = \frac{(2 - \overline{\theta})(q_H - q_L) + q_H^2 - q_L^2 + \alpha}{9(q_H - q_L + \alpha)} \cdot [2(2 - \overline{\theta} + 2q_L) + \\ + \frac{(2 - \overline{\theta})(q_H - q_L) + q_H^2 - q_L^2 + \alpha}{9(q_H - q_L + \alpha)}] = 0 \; . \tag{28}$$

The system (27-28) has seven critical points identifying quality pairs belonging to $\mathbb{R}$. Among them, we select the only pair of qualities which (i) satisfy second order conditions and (ii) when $\alpha = 0$, coincide with the equilibrium qualities observed in the well known model without externality (see Cremer and Thisse, 1994; Ecchia and Lambertini, 1997, *inter alia*):

$$q_H^* = \frac{12\overline{\theta} + 32\alpha\overline{\theta} + 3}{8(8\alpha + 3)} \; ; \; q_L^* = \frac{12\overline{\theta} + 32\alpha\overline{\theta} - 48\alpha - 15}{8(8\alpha + 3)} \; . \tag{29}$$

Qualities (29) and prices (23-24) define the subgame perfect equilibrium of the two-stage game if and only if the market is sufficiently rich to ensure full market coverage. This requires $\overline{\theta} \geq \theta$". The level of θ" is derived in appendix.

On the basis of (29), we can immediately observe what follows:

Remark 1 *The degree of differentiation is independent of the extent of the positional externality α, in that $q_H^* - q_L^* = 3/4$.*

Moreover, since $\partial q_H^*/\partial\alpha$ and $\partial q_L^*/\partial\alpha$ are both negative, we can also state:

Remark 2 *Both equilibrium qualities are everywhere decreasing in the extent of the positional externality.*

Observe that the monotonicity of qualities w.r.t. α, irrespective of $\overline{\theta}$, indicates that there may exist a scope for regulation under duopoly, possibly wider than that observed under monopoly. Equilibrium output levels and profits are:

$$x_H^* = \frac{32\alpha + 9}{6(8\alpha + 3)} \; ; \; x_L^* = \frac{16\alpha + 9}{6(8\alpha + 3)} \; ; \tag{30}$$

$$\pi_H^* = \frac{(4\alpha + 3)(32\alpha + 9)^2}{144(8\alpha + 3)^2} \; ; \; \pi_L^* = \frac{(4\alpha + 3)(16\alpha + 9)^2}{144(8\alpha + 3)^2} \; . \tag{31}$$

On the basis of (30) and (31), the following results can be quickly derived:

Remark 3 *$\partial x_H^*/\partial\alpha > 0$ and $\partial x_L^*/\partial\alpha < 0$, i.e., the market share of the high- (respectively, low-) quality firm increases (resp., decreases) as the positional externality becomes more relevant. $x_H^* \in [1/2, 2/3)$ for $\alpha \in [0, \infty)$.*

Remark 4 *$\partial\pi_H^*/\partial\alpha > 0$ everywhere. $\partial\pi_L^*/\partial\alpha < 0 \; \forall\alpha \in [0, 3(\sqrt{17} - 3)/32)$; $\partial\pi_L^*/\partial\alpha > 0 \; \forall\alpha > 3(\sqrt{17} - 3)/32)$.*

Remark 4 entails that the behaviour of the low-quality firm's profits is non-monotone in α, due to strategic complementarity in prices. As α increases, the increase in prices and the decrease in qualities jointly determine a larger unit profit. Initially, for low levels of α, this is more than offset by the negative effect on the output level x_L^*. Then, as α becomes sufficiently large, the opposite happens.

At the duopoly equilibrium, social welfare amounts to:

$$SW^* = \frac{144\overline{\theta}(\overline{\theta}-1)(9\alpha^2+48\alpha+64)+81+7680\alpha^2+8192\alpha^3}{576(8\alpha+3)^2} . \tag{32}$$

Differentiating (32) w.r.t. α, we obtain:

$$\frac{\partial SW^*}{\partial \alpha} = \frac{4096\alpha^3+4608\alpha^2+2016\alpha+243}{36(8\alpha+3)^3} > 0 \quad \forall \alpha \geq 0; \overline{\theta} \geq \theta" . \tag{33}$$

Therefore,

Proposition 3 *The level of social welfare associated with the duopoly equilibrium is everywhere increasing in the extent of the positional externality.*

Notice that this result stems from the possibility for the high-quality firm to extract more surplus from her customers as α increases. This, in turn, produces a positive externality on the low-quality firm in the same direction. Indeed, when we consider the behaviour of consumer surplus in the two market segments, it emerges that $\partial CS_L^*/\partial\alpha < 0$ for all $\alpha \geq 0$, while CS_H^* is non-monotone in α. In particular, CS_H^* is initially increasing in α, when the positional externality is very low, and then decreases. Hence, the behaviour of SW^* is to be imputed primarily to profits.

3.2 Social planning

A benevolent social planner chooses prices and qualities to maximise welfare (20). The price of the low-quality good is $p_L^{SP} = (\overline{\theta}-1)q_L$, while the price of the high-quality good is the solution of the first order condition $\partial SW/\partial p_H = 0$:

$$p_H^{SP} = \frac{\alpha[(\alpha-2q_L)+(q_H+q_L)(\overline{\theta}+q_H-q_L)]}{q_H-q_L+2\alpha} + \tag{34}$$

$$+\frac{q_Hq_L(\overline{\theta}-1-q_H-q_L)-q_L^2(\overline{\theta}-1)+q_H^3+q_L^3}{q_H-q_L+2\alpha} .$$

The system of first order conditions w.r.t. qualities, i.e., $\{\partial SW/\partial q_H = 0; \partial SW/\partial q_L = 0\}$ has five critical points, out of which

$$q_H^{SP} = \frac{4\overline{\theta}-1}{8};\ q_L^{SP} = \frac{4\overline{\theta}-3}{8} \tag{35}$$

is the only pair satisfying second order conditions. Notice that q_H^{SP} and q_L^{SP} do not depend on α. In particular, they coincide with the socially optimal qualities associated with the maximisation of social welfare in a standard model without positional externality (see Lambertini, 1997). Moreover, $q_H^{SP} - q_L^{SP} = 1/4$. This, compared with $q_H^* - q_L^* = 3/4$, reproduces the well known excess differentiation characterising the duopoly equilibrium. Finally, notice that $q_H^{SP} > q^{SP}$ for all $\alpha \in [0, \infty)$ (cf. expression (14) above).

Social welfare amounts to $SW^{SP}(2) = [16(\overline{\theta}^2 - \overline{\theta} + \alpha) + 5]/64$, where the number in brackets indicates that the planner supplies two varieties. Equilibrium prices are $p_H^{SP} = (4\overline{\theta}^2 - 5\overline{\theta} + 4\alpha + 2)/8$ and $p_L^{SP} = (\overline{\theta} - 1)(4\overline{\theta} - 3)/8$, while output levels are $x_H^{SP} = x_L^{SP} = 1/2$. Comparing these output levels with the quantity produced by the planner operating with a single product (expression (16)), the following emerges:

Lemma 4 *$x_H^{SP} = 1/2 < x^{SP}$ for all admissible values of α and $\overline{\theta}$.*

The above lemma states that the introduction of a non-positional good in the lower segment of the quality spectrum entails a reduction in the provision of the positional variety. Adding to the above information concerning optimal qualities the further observation that p_L^{SP} and x_i^{SP} are independent of α, we have the following:

Proposition 4 *The socially optimal allocation of consumers obtains when the richer (poorer) half of the population buys the high (low) quality good. To this aim, the planner imposes a tax on the positional good, whose size is the same as the positional externality.*

The proof is straightforward, by writing $p_H^{SP} = (4\overline{\theta}^2 - 5\overline{\theta} + 2)/8 + \alpha x_L^{SP}$. This discussion also enlightens two alternative directions along which a policy maker taking care of social welfare should regulate the profit-maximising duopoly. Consider first that $\partial q_L^* / \partial \alpha < 0$ and $\partial x_L^* / \partial \alpha < 0$ for all $\alpha > 0$. This entails that the low-quality firm increases the quality distortion downwards as the positional externality becomes more relevant, compared to the social optimum. This produces an increasing distortion in the allocation of consumers across qualities as α increases. This distortion can be corrected by either taxing the high-quality good,[4] or increasing the low quality level through the adoption of a minimum quality standard.[5] Obviously, a combination of both measures could also be adopted. The following result can be easily proved:

[4] The Pareto-improving nature of a tax applied to the positional good has been pointed out by Ireland (1994), where the positional good is wealth.

[5] As qualities are strategic complements (at least in the neighbourhood of the equilibrium), any standard imposing an increase to the low quality would also imply an increase in the high quality (see Crampes and Hollander, 1995; Ecchia and Lambertini, 1997).

Proposition 5 *The incentive for a benevolent policy maker to regulate the behaviour of the profit-maximising duopolists is increasing in the extent of the positional externality.*

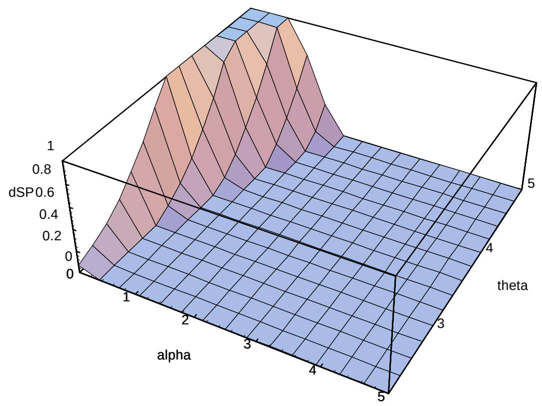
This can be shown by observing that

$$\Delta SW = SW^{SP} - SW^{*} = \frac{81 + 864\alpha + 528\alpha^2 + 256\alpha^3 + 1980\overline{\theta}(1 - \alpha^2 - \overline{\theta} + \alpha^2\overline{\theta})}{144(8\alpha + 3)^2} \tag{36}$$

is everywhere positive and increasing in α, for all $\overline{\theta} \geq \theta$".

In the light of the foregoing analysis, a further issue arises, namely, whether the conventional result, according to which the welfare performance of the planner would improve as the number of varieties increases (see Mussa and Rosen, 1978; Champsaur and Rochet, 1989; Lambertini, 1997), holds true in the present setting as well. In order to answer this question, it suffices to compare SW^{SP} against $SW^{SP}(2)$. Define $\Delta SP = SW^{SP} - SW^{SP}(2)$, which is plotted over $\{\alpha, \overline{\theta}\}$ in figure 6.

Figure 6 : $\Delta SP = SW^{SP} - SW^{SP}(2)$



Obviously, the viable region wherein it makes sense to carry out such comparison is $\alpha \in [0, \alpha')$. Figure 6 highlights that there is a wide subset of this region of the relevant parameters $\{\alpha, \overline{\theta}\}$ where it is not convenient for the planner to introduce a non-positional low-quality good. The explanation of this phenomenon is to be looked for in the behaviour of qualities and prices as the planner switches from one to two varieties. The decrease in the output and the associated increase in the quality level of the positional good seem to point to a welfare improvement for those consumers who purchase the positional variety independently of the number of products supplied by the planner. The welfare effect should be positive also for those consumers located in the lower part of the range of θ, who are in a position to buy only when two products are available. However, consider that $\Delta SP = SW^{SP} - SW^{SP}(2) > 0$ for low values of α. In this region, qualities are approximately linear in $\overline{\theta}$, while marginal production costs as well as prices are approximately quadratic in $\overline{\theta}$, so that gross consumer surplus θq increases less rapidly than marginal costs. Besides, the lower is α, the larger becomes the difference $q_H^{SP} - q^{SP}$, making heavier the cost burden associated with expanding the product range.

4 Concluding remarks

We have modeled the role of positional effects in a market for vertically differentiated goods. We have considered two alternative settings, namely, a single-product monopoly and a single-product duopoly. In the second case, the positional externality is associated with the high-quality good, while the low-quality variety is non-positional. Then, we have evaluated the welfare performance of both monopoly and duopoly against the behaviour of a social planner operating with the same product range.

A few qualitative results hold true independently of the specific setting. In particular, the equilibrium level of social welfare is always increasing in the extent of the positional externality, irrespective of the market regime. Moreover, the welfare loss due to monopoly power is decreasing in the extent of the positional externality, so that the scope for regulation shrinks as the positional effect becomes more relevant.

Finally, we have shown that the conventional wisdom establishing that the planner's welfare performance increases in the number of varieties being supplied may not hold in the presence of a positional concern.

Appendix

To calculate the critical level of $\overline{\theta}$, i.e., θ", above which all consumers are able to buy in equilibrium, consider the condition of the poorest consumer located at $\overline{\theta}-1:$

$$U_L(\overline{\theta}-1)=(\overline{\theta}-1)q_L-p_L= \tag{37}$$

$$=\frac{48(64\alpha^2+48\alpha+9)\overline{\theta}^2-4(1536\alpha^2+115\alpha+216)\overline{\theta}-4096\alpha^3-4608\alpha^2-1728\alpha-243}{192(8\alpha+3)^2}.$$

Solving $U_L(\overline{\theta}-1)=0$ w.r.t. $\overline{\theta}$, we obtain:

$$\overline{\theta}=\frac{3(256\alpha^2+192\alpha+36)\pm(8\alpha+3)\sqrt{3(4096\alpha^3+7680\alpha^2+4032\alpha+675)}}{12(8\alpha+3)^2}. \tag{38}$$

Notice that only the larger root $\overline{\theta}_+$ is acceptable, in that it tends to $9/4$ as α tends to zero (cf. Cremer and Thisse, 1994). Hence, full market coverage is observed in equilibrium for all $\overline{\theta}\geq\theta" =\overline{\theta}_+$.

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