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FROM A REGRET TO AN EXPECTED UTILITY MODEL: A LEARNING PROCESS

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From a Regret to an Expected Utility Model: a learning process

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Abstract

This paper identifies in a feed-forward neural network the mathematical algorithm which can catch the learning process highlighted by econometric works that makes people assess the satisfaction arising in each single contingency so that they are better depicted in their decision making by an Expected Utility rather than by a Regret Model. Evidence from experimental economics are also accounted for, since the network does not manage to extrapolate the former from the latter model when probabilities are extreme.

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1. Introduction

The standard Subjective Expected Utility Theory has been challenged on several grounds and alternative theories have been suggested: Ratio Form Theory (Chew and MacCrimmon, 1979), Three Moments of Utility Theory (Hagen, 1979), Prospect Theory (Kahneman and Tversky, 1979), Regret Theory (Loomes and Sudgen, 1982; 1987), Skew-Symmetric Bilinear Utility Theory (Fischburn, 1982; 1984), Generalized Expected Utility Theory (Machina, 1982).

Camerer (1989) claims that "there are substantial violations of the Subjective Expected Utility Theory, but no single theory can explain the pattern of violations". Regret Theory, however, seems to be the most flexible one (Starner and Sugden, 1989): if you can assume the statistic independence, it accounts for the common ratio, common consequence and juxtaposition effects; if you can not, the preference reversal phenomenon can also be predicted. Moreover, it is well supported by evidence (Hey, 1991).

Hammond (1986) claims that "consequential reasoning taking into account all the relevant considerations will push us in the direction of the expected utility maximisation".

Through an econometric analysis based on panel data from rural India, Zagonari (1995) shows that the Subjective Expected Utility Model has the same explanatory power as other models when people repeatedly deal with the same kind of uncertainty in the same kind of framework.

The purpose of this paper is to depict the learning process that can make people assess the satisfaction arising in each single contingency so that they are better depicted in their decision making by an Expected Utility rather than by a Regret Model.

Hey and Di Cagno (1990) carried out an experiment on 68 people by asking them 60 questions about pairs of gambles; through a limited dependent variable econometric analysis based on the recorded preferences, they then estimated utility levels of each single or pair of outcomes.

Neural networks have often been used to solve gathering and processing information issues, such as learning and optimization.

We address our question by conducting a numerical experiment based on a feed-forward neural network where those levels of utilities are used as inputs.

We find that a neural network with 8 neurons in the input, 2 in the hidden and 1 in the output layer is suitable to represent the learning process suggested by the econometric analysis as well as it is in a position to depict the phenomenon according to which violations of the Expected Utility Model are more common for extreme probability distributions over events.

The structure of the paper is as follows.

In section 1 we depict the Regret Model, the Expected Utility Model and the main characteristics which they share; data for simulations are provided. Section 2 contains a short introduction to neural networks in general and a brief description of the feed-forward network in particular. Section 3 provides the simulation procedures and results. The conclusion appears in section 4.

2. The theoretical framework

We define a prospect as a list of consequences with an associated list of probabilities, one for each consequence, such that these probabilities sum to 1 and we assume consequences to be mutually exclusive and in finite number possibilities.

We consider an individual who reveals preferences over the set of all conceivable prospects which obey the ordering (completeness and reflexivity) and the continuity axioms. Thus, a prospect p is preferred to a prospect q according to the Regret Model (RM) if and only if:

$$\sum_i \sum_j p_i q_j M(x_i, x_j) > \sum_i \sum_j p_i q_j M(x_j, x_i)$$

where $M(.)$ is the modified utility, i.e. the intrinsic utility of what is modified by regret or rejoicing. If we define $\psi(.)$ by:

$$\psi(x_i, x_j) \equiv M(x_i, x_j) - M(x_j, x_i)$$

since $\psi(.)$ is skew-symmetric (i.e. $\psi(x_i,x_j) = -\psi(x_j,x_i)$), a prospect p is preferred to a prospect q according to the RM if and only if:

$$\sum_i \sum_j \psi(x_i, x_j) > 0$$

The decision maker in the RM is therefore assumed to take into account not simply the expected utility of the outcomes *per se*, but the expected utility modified by anticipated feeling of regret or rejoicing. Formally, $\psi(x_i,x_j)$ identifies not only the extra utility obtained from getting x_i *per se* rather than x_j *per se*, but also the utility or disutility obtained from rejoicing or regretting the fact of not getting x_j.

The RM is quite comprehensive [1]: if prospects are statistically independent, it accounts for the common ratio, common consequence and juxtaposition effects; if they are not, the preference reversal phenomenon can be predicted, but the common consequence effect is not caught. In order to grasp this point and to highlight similarities between the RM and the Expected Utility

[1] The Skew-Symmetric Bilinear Utility Theory by Fishburn (1982, 1984) is very similar to the Regret Theory, even if he arrived at his theory in a pure axiomatic way, by weakening the axioms of the Ratio Form Theory by Chew and MacCrimmon (1979).

Model (EUM), we will assume statistic independence.

Hey and Di Cagno (1990) carried out an experiment on 68 people by answering them 60 preference questions involving a pairwise choice between two gambles: outcomes involved the amounts 0, 10, 20, 30. This procedure led them to identify two groups of people: the P group consisted of those 36 subjects who always expressed a strict preference; the I group of those 32 subjects who somewhere expressed indifference. By applying a limited dependent variable econometric analysis to the data so recorded (probit models), they provided estimation of the $\psi(.)$ values for both these groups.

To make use of the results of this work, we will focus on four consequences prospects (30, 20, 10, 0). Hence the various values which $\psi(.)$ can take are given by: ψ(30,0), ψ(30,10), ψ(30,20), ψ(30,30), ψ(20,0), ψ(20,10), ψ(20,20), ψ(10,0), ψ(10,10), ψ(0,0).

Since consequences are assumed to be independent and a single prospect is under consideration, the related probabilities are: $p_{30}(1-p_0), p_{30}(1-p_{10}), p_{30}(1-p_{20}), p_{30}(1-p_{30}), p_{20}(1-p_0), p_{20}(1-p_{10}), p_{20}(1-p_{20}), p_{10}(1-p_0), p_{10}(1-p_{10}), p_0(1-p_0)$, where $\sum_i p_i = 1$.

The utility levels we will use for simulations are reported in Table 1 and 1' for I and P groups, respectively.

$\psi(30,0) = 1$	$\psi(30,10) = .717$	$\psi(30,20) = .237$	$\psi(30,30) = 0$
$\psi(20,0) = .717$	$\psi(20,10) = .227$	$\psi(20,20) = 0$	
$\psi(10,0) = .533$	$\psi(10,10) = 0$		
$\psi(0,0) = 0$			

Table 1. RM Utility Levels: Indifference Case

$\psi(30,0) = 1$	$\psi(30,10) = .901$	$\psi(30,20) = .394$	$\psi(30,30) = 0$
$\psi(20,0) = .492$	$\psi(20,10) = .182$	$\psi(20,20) = 0$	
$\psi(10,0) = .213$	$\psi(10,10) = 0$		
$\psi(0,0) = 0$			

Table 1'. RM Utility Levels: Preference Case

As shown by econometric analysis, a learning process may allow individuals to identify the utility they get from each single final outcome (Zagonari 1995). If this is the case, the difference between the modified utilities in the RM may be equal to the difference between the unmodified utilities in the EUM [2]. Formally:

$$\psi(x_i, x_j) = u(x_i) - u(x_j) \ \forall i \ \forall j$$

Thus, a prospect p is preferred to a prospect q according to the EUM if and only if:

[2] When there are less than 4 pure consequences and statistic independence is assumed, the RM becomes the Ratio Form Model (Chew and MacCrimmon, 1979) if one introduces a transitive ordering of pure preferences and the assumptions of increasingness and convexity.

$$\sum_i u(x_i)p_i > \sum_i u(x_i)q_i$$

It is easy to show that now transitivity and independent axioms hold.

To make use of the results obtained by Hey and Di Cagno (1990), we focus again on four consequences prospects (30, 20, 10, 0) so that the possible cases are given by: $u(0)$, $u(10)$, $u(20)$, $u(30)$. Since consequences are independent, the related probabilities are: p_{30}, p_{20}, p_{10}, p_0, where $\sum_i p_i = 1$.

3. Neural networks and individual thinking

Neural networks have often been used to solve gathering and processing information issues, such as learning and optimization: they are, therefore, good candidates for explaining the process we are interested in.

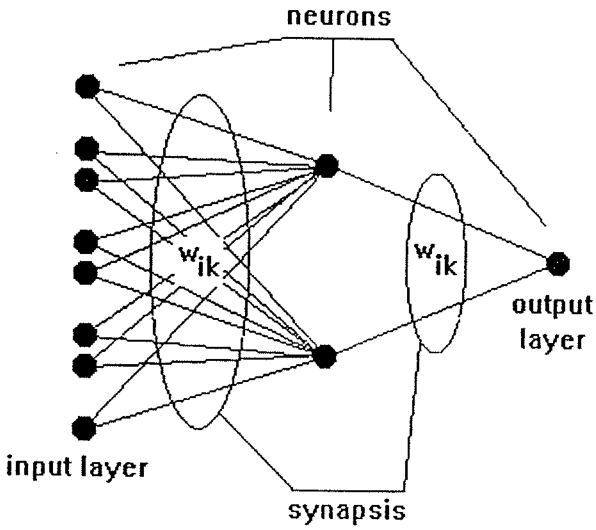

Figure 1: The Neural Network

In mathematical terms, a neural network is a direct graph with the following properties (See Figure 1):

- each state variable n_i is associated with a node;

- the strength of each link between two nodes i and k is quantified by a real value w_{ik} called weight;

- each node i is associated with a bias θ_i;

- the state of each node n_i is defined by a transfer function $f_i(n_k,w_{ik},\theta_i)$, $k \neq i$, which depends on the state of nodes linked to it, the weights associated with these nodes and the value of its bias.

In what follows, nodes will be called neurons and links synapsis.

The transfer function takes usually the following form:

$$f_i(n_k, w_{ik}, \theta_i) = f_i(\sum_k w_{ik}\, n_k - \theta_i)$$

where $f_i(.)$ is a step, linear or a sigmoid function. The dynamics of the evolution of a neural network is defined as follows:

$$n_i(t+1) = f_i(\sum_k w_{ik} n_k(t) - \theta_i)$$

A neural network is said to have learned an input when the presentation of the latter drives the former to show the same set of states of neurons. The required change of the values of synapses and biases to achieve this result identifies two main classes of neural networks: unsupervised and supervised.

In the former, the learning process is not driven by elements outside the network: synapses autonomously conform to the presented inputs. In the latter, the neural network is provided with values of inputs and correspondent output and its synapses change until this output is reached: the learning process is driven by examples.

The perceptron we will use below belongs to the second class and it is characterized by the synaptic updating algorithm it relies on: this is based on the minimization of the difference between the desired and the network output and it is called error back propagation (Rumelhart, Hinton and Williams, 1986). The perceptron, therefore, turns out to be particularly suitable to identify a function or a relation between two sets of variables, i.e. a map from R^n to R^m.

4. Simulation procedures and results

The neural network used for simulation studies is a feed-forward network with 8 neurons in the input, 2 in the hidden and 1 in the output layer.

The neural network is first subject to a training procedure. Through the presentation of each pair of outcomes as inputs and the correspondent utility as the output, it is asked to learn to match the level of utility with the related pair of outcomes. The neural network is then required to generalize the information set acquired through the previous training procedure in order to identify the level of utility correspondent to each single outcome.

The input pattern for the training phase is defined in such a way that the presence of the outcome x_i is caught by the activation of the i-th neuron of input, whereas the absence of the outcome $\bar{x}_i$by that of the $(i+4)$-th one. The desired output during the training procedure is given by the level of utility as defined by $\psi(x_i,x_j)$. The two input patterns for people in I and P groups are provided in Table 2 and 2', respectively.

$\psi(x_i,x_j)$	Input Pattern	Output Pattern	$\psi(x_i,x_j)$	Input Pattern	Output Pattern
$\psi(0,0)$	10001000	0.000	$\psi(20,0)$	00101000	0.717
$\psi(0,10)$	10000100	-0.533	$\psi(20,10)$	00100100	0.227
$\psi(0,20)$	10000010	-0.717	$\psi(20,20)$	00100010	0.000
$\psi(0,30)$	10000001	-1.000	$\psi(20,30)$	00100001	-0.237
$\psi(10,0)$	01001000	0.533	$\psi(30,0)$	00011000	1.000
$\psi(10,10)$	01000100	0.000	$\psi(30,10)$	00010100	0.717
$\psi(10,20)$	01000010	-0.277	$\psi(30,20)$	00010010	0.237
$\psi(10,30)$	01000001	-0.717	$\psi(30,30)$	00010001	0.000

Table 2. Training Phase Pattern: Indifference Case

$\psi(x_i,x_j)$	Input Pattern	Output Pattern	$\psi(x_i,x_j)$	Input Pattern	Output Pattern
$\psi(0,0)$	10001000	0.000	$\psi(20,0)$	00101000	0.717
$\psi(0,10)$	10000100	-0.533	$\psi(20,10)$	00100100	0.227
$\psi(0,20)$	10000010	-0.717	$\psi(20,20)$	00100010	0.000
$\psi(0,30)$	10000001	-1.000	$\psi(20,30)$	00100001	-0.237
$\psi(10,0)$	01001000	0.533	$\psi(30,0)$	00011000	1.000
$\psi(10,10)$	01000100	0.000	$\psi(30,10)$	00010100	0.717
$\psi(10,20)$	01000010	-0.277	$\psi(30,20)$	00010010	0.237
$\psi(10,30)$	01000001	-0.717	$\psi(30,30)$	00010001	0.000

Table 2'. Training Phase Pattern: Preference Case

All synapses and biases are fixed to 0 for initialization. The synapses updating process is based on the error back propagation method. This implies that for each input pattern the state variable n_i for input neurons is worked out according to the following rule:

$$f_i(n_k, w_{ik}, \theta_i) = f_i(\sum_k w_{ik}\, n_k - \theta_i)$$

where the chosen transfer functions are the identity for the first and the sigmoid functions for the second layer, respectively. The output neuron activation due to the presented pattern is compared with the desired value according to:

$$E_p = \frac{1}{2}(a_p - b_p)^2$$

where E_p is the error on the p-th pattern presented, a_p is the activation of the neuron and b_p the desired correspondent output. The minimization of E_p is based one the gradient method (Muller and Reinhart 1991), i.e. the synapsis values are corrected according to a fixed proportion of the error. The training process due to a successive presentation of patterns and the related updating mechanism of synapses drive the network to reach a minimum error condition here identified by a level below 0.002.

The generalization phase is performed by presenting a set of input patterns depicting the presence and the absence of each single outcome (x_i and $\bar{x}_i$ respectively) and asking the network to identify the correspondent level of utility as output. Results for I and P group are presented in Table 3 and 3', respectively.

x_i	Input Pattern	Output	$\bar{x}_i$	Input Pattern	Output
x_1	10000000	0	$\bar{x}_1$	00001000	0
x_2	01000000	.43424	$\bar{x}_2$	00000100	-.43893
x_3	00100000	.63443	$\bar{x}_3$	00000010	-.65580
x_4	00010000	1	$\bar{x}_4$	00000001	-1

Table 3. Generalisation Phase Pattern and Network Output: Indifference Case

x_i	Input Pattern	Output	$\bar{x}_i$	Input Pattern	Output
x_1	10000000	0	$\bar{x}_1$	00001000	0
x_2	01000000	.27791	$\bar{x}_2$	00000100	-.26309
x_3	00100000	.48786	$\bar{x}_3$	00000010	-.500111
x_4	00010000	1	$\bar{x}_4$	00000001	-1

Table 3'. Generalisation Phase Pattern and Network Output: Preference Case

Therefore, the suggested neural network seems to be in a position to represent the learning process that make people identify the utility level arising from each single event.

It should be noticed that the numerical experiments indicate subjects who are risk averse for small amounts but risk loving for larger amounts. A similar result is obtained by Hey and Di Cagno (1990) for the I group.

Since the learning process is applied to a quite comprehensive model such as the RM, one could conclude that the EUM is a limit case where that process has fully developed.

MacCrimmon and Larsson (1979) have shown, however, that the frequency of the inconsistencies with the EUM depends crucially on the probabilities: for extreme values the "violation level reached about 65 per cent. However, for smaller values, those more likely to actually encountered by subjects, the choices were quite consistent". To be a good representation of the learning process, the previously developed neural network must catch this phenomenon: it has to find difficulties in extrapolating an EUM from a RM when probabilities are extreme.

Carrying out the same numerical experiments depicted above with the probability to encounter the state 30 being 20 times all the others gives the results depicted in Table 4 and 4'.

x_i	Input Pattern	Output	$\bar{x}_i$	Input Pattern	Output
x_1	10000000	0	$\bar{x}_1$	00001000	0
x_2	01000000	.36662	$\bar{x}_2$	00000100	-.55284
x_3	00100000	.68065	$\bar{x}_3$	00000010	-.90445
x_4	00010000	1	$\bar{x}_4$	00000001	-1

Table 4. Generalisation Phase Pattern and Network Output: Indifference Case with larger p_{30}

x_i	Input Pattern	Output	$\bar{x}_i$	Input Pattern	Output
x_1	10000000	0	$\bar{x}_1$	00001000	0
x_2	01000000	.017652	$\bar{x}_2$	00000100	-.473883
x_3	00100000	.059078	$\bar{x}_3$	00000010	-.915820
x_4	00010000	1	$\bar{x}_4$	00000001	-1

Table 4'. Generalisation Phase Pattern and Network Output: Preference Case with larger p_{30}

We can conclude, therefore, that the EUM can be characterized as a limit case where the learning process has fully developed under a particular condition only: when probabilities of outcomes are relatively similar.

5. Conclusions

A learning process may make people assess the satisfaction arising in each single contingency so that they are better depicted in their decision making by an Expected Utility rather than by a Regret Model.

This result is obtained by applying a feed-forward neural network.

Indeed, not only does this account for results obtained by econometric works: it makes utilities for the presence of each outcome opposite to those for their absence; but also for evidence by experimental economics: it does not manage to extrapolate an Expected Utility Model from a Regret Model when probabilities are extreme.

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