Sequential vs Simultaneous Equilibria in a Differentiated Duopoly*

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Abstract

This paper tackles the issue of choosing roles in differentiated duopoly games. First, it is shown that the leader is not necessarily better off than in the simultaneous equilibrium. Second, it is proved that sequential equilibria obtain only if both firms are better off under sequential play than under simultaneous play. Finally, a duopoly game under vertical differentiation and Bertrand competition is illustrated, where the price leader can indeed happen to be worse off than in the simultaneous equilibrium.

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1 Introduction

The way firms can be expected to conduct oligopolistic competition has represented a relevant issue in the economists' research agenda for a long time. The earliest literature in this field treated a relevant feature such as the choice between simultaneous and sequential moves as exogenous (Stackelberg, 1934; Fellner, 1949). Later contributions considered as a sensible approach to investigate the preferences of firms over the distribution of roles in price or quantity games (Gal-Or, 1985; Dowrick, 1986; Boyer and Moreaux, 1987a,b). The preference for leadership (respectively, followership) in quantity (price) games can be established on the basis of the slope of firms' reaction functions or, likewise, noting that products are strategic substitutes (complements).[1] Other authors have taken into account the possibility that cost asymmetry or uncertainty may lead to Stackelberg equilibria (Ono, 1982; Albaek, 1990). Finally, some authors have analysed the choice between price and quantity as a strategic variable, taking into account only simultaneous equilibria (Singh and Vives, 1984; Cheng, 1985).

Recent literature explicitly models the strategic choice of timing, which is often possible in reality. Robson (1990a) has proposed an extended duopoly model where price competition takes place in a single period, preceded by firms' scattered price decisions, which cannot be altered. Only Stackelberg equilibria emerge from such a game. In an influential paper, Hamilton and Slutsky (1990) investigated the endogenous choice of roles, i.e., the endogenous arising of Stackelberg or Cournot equilibria, in noncooperative two-person games (typically, duopoly games), by analysing an extended game where players (say, firms) are required to set both the actual moves or actions and the time at which such actions are to be implemented. Their approach is close in spirit to Robson's, though they also consider Cournot competition and the mixed case where one firm sets her price and the other firm decides her output level. When firms choose to act at different times, sequential equilibria obtain, while if they decide to move at the same time, simultaneous Nash equilibria are observed. The choice of the timing occurs in a preplay stage which does not take place in real time, so that there is no

[1] The concept of strategic substitutability/complementarity is due to Bulow, Geanakoplos and Klemperer (1985).

discounting associated with waiting and payoffs are the same whether firms choose to move as soon as possible or they delay as long as they can. The decision to play early or at a later time is not sufficient *per se* to yield sequential play, since an analogous decision taken by the rival leads to simultaneous play.

Building on a lemma stating that the leader's payoff must be at least as high as that associated with simultaneous play, Hamilton and Slutsky (HS, henceforth) show that a Stackelberg equilibrium with sequential play is selected as a subgame perfect equilibrium of the extended game with observable delay if and only if the outcome of sequential play Pareto-dominates the outcome associated with simultaneous play (HS, 1990, Theorems III and IV). Otherwise, if firms are better off playing simultaneously rather than accepting the follower's role, the subgame perfect equilibrium involves simultaneous play (HS, 1990, Theorem II).[2]

Pal (1996) explicitly takes into account mixed strategies. He considers an extended quantity-setting game with two identical firms and two production periods before the market-clearing instant. He shows that only three outcomes are possible: (i) both firms produce in the second period, so that a simultaneous Cournot equilibrium obtains; (ii) firms produce in different period, yielding a Stackelberg-like equilibrium (see also Robson, 1990b); (iii) Stackelberg warfare may arise when firms produce in the first period, but both produce more than in the Cournot-Nash equilibrium.

The aim of this paper is twofold. First, I shall prove that the lemma adopted by HS is questionable and redundant, in that their results holds even if that lemma does not apply. Second, I will show that if a multistage duopolistic competition is considered, where firms first choose the timing of moves, then product quality and finally compete *à la* Bertrand, the subgame perfect equilibrium of such an extended game drastically differs from that observed under price competition when firms cannot endogenously differentiate their respective goods. Specifically, in the game I present, simultaneous play emerges when firms bear variable production costs, due to the fact that the price leader's profit is lower than simultaneous play profit, so that both duopolists play at the latest opportunity in order to avoid being first. Otherwise, when costs take the form of R&D efforts, a sequential equilibrium

[2]HS (1990, section IV) also consider an extended game with action commitment in the spirit of Dowrick (1986), where each firm must commit to a particular action irrespectively of the rival trying to lead or follow. This yields multiple equilibria with both simultanous and sequential play.

emerges with the low-quality firm taking the lead.

The remainder of the paper is structured as follows. HS's lemma is discussed in section 2. Section 3 is devoted to the description of the duopoly game under vertical differentiation. Finally, section 4 contains concluding remarks.

2 The extended game when the leader is worse off

Consider the simplest extended game game where firms can set a single strategic variable (price or quantity) and must choose between moving first or second. I shall adopt here a symbology which largely replicates that in HS (1990, p.32). Define $\Gamma^1 = (N, \Sigma^1, P^1)$ the extended game with observable delay. The set of players (or firms) is $N = \{A, B\}$, and α and β are the compact and convex intervals of $\mathbb{R}^1$ representing the actions available to A and B in the basic game. Payoffs depend on the actions undertaken in the latter, according to the following functions, $a : \alpha \times \beta \rightarrow \mathbb{R}^1$ and $b : \alpha \times \beta \rightarrow \mathbb{R}^1$. The set of times at which firms can choose to move is $T = \{F, S\}$, i.e., *first* or *second*. The set of strategies for player i is $\Sigma_i^1 = \{F, S\} \times \Phi_i$, where Φ_i is the set of functions that map $\{T \times \beta(\text{or } \alpha)\}$ into $\alpha(\text{or } \beta)$. If both firms choose to move at the same time, they obtain the payoffs associated with the simultaneous Nash equilibrium, (a_n, b_n), otherwise they get the payoffs associated with the Stackelberg equilibrium, e.g., (a_l, b_f) if A moves first and B moves second, or *viceversa*. The game can be described in normal form as in matrix 1 (cfr. HS, 1990, p.33).

		B	
		F	S
A	F	a_n, b_n	a_l, b_f
	S	a_f, b_l	a_n, b_n

Matrix 1

I shall start by considering the cases where both players rank the payoffs according to the same sequence. To begin with, assume payoffs are ranked as follows:

$$a_l > a_n > a_f; b_l > b_n > b_f. \tag{1}$$

This ordering holds, e.g., in a standard Cournot duopoly with substitute goods and in a Bertrand duopoly with complements (see Singh and Vives, 1984; Boyer and Moreaux, 1987b). In such a case, both firms move at the first occasion in order to avoid playing the follower's role, and the subgame perfect equilibrium of the extended game involves simultaneous play (HS, 1990, Theorem II). If instead the payoff ranking is

$$a_f > a_l > a_n; b_f > b_l > b_n, \tag{2}$$

as in a standard Bertrand duopoly with substitute goods or in a Cournot duopoly with complements (see Singh and Vives, 1984; Boyer and Moreaux, 1987b), the extended game exhibits multiple equilibria in that both sequential equilibria of the market stage are subgame perfect (HS, 1990, Theorem III). Moreover, there are also a correlated equilibrium and a mixed-strategy equilibrium where firms randomizes over moving first or second.

HS (1990, Lemma I) claim that each player's (firm's) leadership payoff must exceed his payoff in simultaneous play because if he is the leader, he is obviously able to choose the best position along the follower's reaction function, so that the Nash equilibrium point is feasible for him. If he accepts to move first (and chooses a point which differs from the Nash equilibrium one), it must be true that he is at least as well off as in the simultaneous equilibrium. *Per se*, this argument appears intuitive and unquestionable. Though, intuition also suggests that analogous considerations must hold for the follower as well. Consider a firm that is contemplating the opportunity of moving second. Provided that by moving at the first occasion, she can at least obtain the Nash payoff, she will accept to move late only if she is better off as a follower than in any other situation. Incidentally, notice that this is precisely what emerges from HS's Theorems II, III and IV. Accordingly, one can state the following:

Lemma 1 *If the game is symmetric and each player's followership payoff does not exceed his payoff in simultaneous play, sequential play in pure strategies will never emerge at equilibrium.*

Let us now turn to the possibility that the leader's payoff be lower than in the simultaneous equilibrium. Consider the following ranking of the payoffs in matrix 1:

$$a_f > a_n > a_l; b_f > b_n > b_l. \tag{3}$$

Here, followership is preferred to simultaneous play, which in turn is preferred to leadership. Moving second is a dominant strategy, and the equilibrium of such a game involves simultaneous moves at the latest occasion, so to avoid being the leader. Finally, consider the following sequence:

$$a_n > a_l > a_f; b_n > b_l > b_f. \tag{4}$$

In such a case, both firms move simultaneously, either at the first or at the second chance, so that two equilibria in pure strategies exist. Moreover, there's an equilibrium in mixed strategies where players randomise over F and S, and a correlated equilibrium. The mixed-strategy equilibrium entails sequential play with positive probability. Notice that this situation is the mirror image of that arising under Bertrand competition in substitute goods, where there exists an equilibrium in mixed strategies in which firms play simultaneously with a positive probability.[3]

Accordingly, HS's Lemma 1 can be reformulated as follows:

Lemma 2 *If the game is symmetric and each player's leadership payoff does not exceed his payoff in simultaneous play, sequential play in pure strategies will never emerge at equilibrium.*

Turn now to asymmetric cases, where the order of payoffs is different across players. Consider first the following rankings:

$$a_f > a_n > a_l; b_f > b_l > b_n. \tag{5}$$

Player A has a strictly dominant strategy, i.e., playing late (S). It follows that such a game has a unique equilibrium defined by the strategy combination (S-F). If A's ranking were $a_l > a_n > a_f$, given the ranking for B as specified in (5), the equilibrium would still involve sequential play, with A leading and B following. This would happen, e.g., if A were a quantity-setter and B a price-setter (see HS, 1990, p.40).

Finally, when the payoffs are ordered as follows:

$$a_f > a_n > a_l; b_n > b_l > b_f, \tag{6}$$

the equilibrium is unique and simultaneous, with both A and B moving at the second chance.[4] Hence, the following holds:

[3]Multiple equilibria would also occur if the follower's payoff were smaller than the simultaneous play payoff, but larger than the leader's payoff.

[4]Simultaneous play would also obtain if, given the payoff ranking specified in (6) for B, A's ranking were $a_l > a_n > a_f$. In such a case, though, both would move at the first occasion.

Lemma 3 *If the game is asymmetric and no player is better off playing simultaneously than playing sequentially, the extended game with observable delay has a unique equilibrium which is in pure strategies and involves sequential play. Otherwise, it exhibits a unique equilibrium that involves simultaneous moves.*

Together with the above Lemma 1 and Lemma 2, this entails what follows:

Proposition 1 *The subgame perfect equilibrium of the extended game with observable delay involves sequential moves if and only if the basic game exhibits at least one Stackelberg equilibrium that Pareto-dominates the simultaneous Nash equilibrium.*

In other words, the method for equilibrium selection proposed by HS holds with no specific requirement on the sequence of profits associated with the roles firms can play in the basic game.

A natural question now arises, i.e., whether a situation where the leader is worse off than under simultaneous play may be oserved in a duopoly setting, so that a payoff ranking as in (3) or (5) obtains. I am going to show in the next section that this is precisely the case when firms can endogenously differentiate products before engaging price competition on the market.

3 A differentiated duopoly game

Consider a duopolistic market where firms supply a vertically differentiated good, whose quality is denoted by q_i, $i = H, L$, with $q_H \geq q_L > 0$. They employ the same productive tecnology, which can alternatively involve variable costs of quality improvements,

$$C_i = tq_i^2 x_i, \tag{7}$$

where x_i denotes the output of firm i and t is a positive parameter;[5] or fixed costs of quality improvements,

$$C_i = tq_i^2, \tag{8}$$

[5]Motta (1993) deals with the case where t=1/2, to model Bertrand and Cournot competition, with fixed or variable costs.

which may be the case when the cost of increasing the quality level falls on R&D investments and is not related to the scale of production (see, *inter alia*, Shaked and Sutton, 1982, 1983).

Consumers are uniformly distributed over the interval $[0, \overline{\theta}]$. Parameter θ represents each consumers' marginal willingness to pay for quality, and it can be thought of as the reciprocal of the marginal utility of nominal income or money (cfr. Tirole, 1988, p.96). As $\overline{\theta}$ increases, the size of the market increases. Consumers' density can be normalised to one, so that total population is also equal to one. The indirect utility function of the generic consumer is:

$$U = \theta q_i - p_i. \tag{9}$$

If the consumer buys, he buys just one unit of the product from the firm that offers the price-quality ratio ensuring the highest utility. Let h and k denote the marginal willingness to pay characterizing, respectively, the consumer who is indifferent between the high and the low-quality good, and that who is indifferent between buying the low-quality good or nothing at all:

$$h = \frac{p_H - p_L}{q_H - q_L}; k = \frac{p_L}{q_L}. \tag{10}$$

Then, the market demands for the two varieties are, respectively,

$$x_H = \overline{\theta} - h \tag{11}$$

if $h \in]k, \overline{\theta}[$, and

$$x_L = h - k \tag{12}$$

if $k \in]0, h[$.

Competition takes place in two stages, the first played in the quality space, the second in the price space. I consider the equilibria arising in such a game, if firms can choose between sequential and simultaneous play in the price (or market) stage, after having played simultaneously in the first stage describing quality competition. The following subsection deals with the case of the technology involving variable costs.

3.1 Variable costs of quality improvement

Assume production costs are described by (7). Firms' objective functions are defined as follows:

$$\pi_H = (p_H - tq_H^2)x_H; \tag{13}$$

and

$$\pi_L = (p_L - tq_L^2)x_L. \tag{14}$$

The game is solved by backward induction. Consider first the fully simultaneous game. The first order conditions (FOCs) at the price stage are:

$$\frac{\partial \pi_H}{\partial p_H} = \frac{p_L - 2p_H + \overline{\theta} q_H - \overline{\theta} q_L + tq_H^2}{q_H - q_L} = 0; \tag{15}$$

$$\frac{\partial \pi_L}{\partial p_L} = \frac{p_H q_L - 2p_L q_H + tq_H q_L^2}{q_L(q_H - q_L)} = 0. \tag{16}$$

Solving the system (15-16), I obtain the equilibrium prices:

$$p_H^n = \frac{q_H}{4q_H - q_L}[2\overline{\theta}(q_H - q_L) + 2t(q_H^2 + q_L^2)]; \tag{17}$$

and

$$p_L^n = \frac{q_L}{4q_H - q_L}[\overline{\theta}(q_H - q_L) + tq_H(q_H + 2q_L)], \tag{18}$$

where superscript n stands for Nash equilibrium.

The solution of the first stage of the game involves numerical calculations. Normalising both $\overline{\theta}$ and t to one, it can be shown that $q_H^n = 0.40976$ and $q_L^n = 0.19936$. The corresponding equilibrium profits amount to $\pi_H^n = 0.0164$ and $\pi_L^n = 0.0121$. Further numerical computations show that

$$q_i^n(\overline{\theta}, t) = \frac{\overline{\theta}}{t} q_i^n(1, 1); \tag{19}$$

$$\pi_i^n(\overline{\theta}, t) = \frac{\overline{\theta}^3}{t} \pi_i^n(1, 1). \tag{20}$$

This holds independently of the timing of moves firms adopt in the market stage. The leader's problem in the price stage can be described as follows:

$$\max_{p_i} \pi_i \tag{21}$$

$$s.t. : \frac{\partial \pi_j}{\partial p_j} = 0, i \neq j, \tag{22}$$

for both firms, i.e., it consists in the maximization of the leader's profit under the constraint represented by the follower's reaction function, implicitly given by the derivative of her profit function w.r.t. her price, as in (22).[6] The equilibrium prices that obtain in the two problems can be found in Appendix A.

Obviously, equilibrium qualities are different in each of the games being considered, though the quality stage is always played simultaneously. The optimal qualities selected when the high-quality firm is appointed the leader's role in the ensuing price stage are:

$$q_H^l(\overline{\theta}, t) = \frac{\overline{\theta}}{t} q_H^l(1,1) = 0.41601\frac{\overline{\theta}}{t};\, q_L^f(\overline{\theta}, t) = \frac{\overline{\theta}}{t} q_L^f(1,1) = 0.21887\frac{\overline{\theta}}{t}, \tag{23}$$

and the corresponding equilibrium profits amount to:

$$\pi_H^l(\overline{\theta}, t) = \frac{\overline{\theta}^3}{t} \pi_H^l(1,1) = 0.01506\frac{\overline{\theta}^3}{t};\, \pi_L^f(\overline{\theta}, t) = \frac{\overline{\theta}^3}{t} \pi_L^f(1,1) = 0.01412\frac{\overline{\theta}^3}{t}. \tag{24}$$

It is immediate to verify that (i) both qualities increase as compared to the fully simultaneous game; and (ii) the follower's profit exceeds the simultaneous play profit, while the leader's does not.

Consider now the case where the low-quality firm acts as the price leader, which is perhaps hardly justifiable on both theoretical and empirical grounds, nevertheless needed to complete the picture. Equilibrium qualities and profits are:

$$q_H^f(\overline{\theta}, t) = \frac{\overline{\theta}}{t} q_H^f(1,1) = 0.39999\frac{\overline{\theta}}{t};\, q_L^l(\overline{\theta}, t) = \frac{\overline{\theta}}{t} q_L^l(1,1) = 0.19999\frac{\overline{\theta}}{t}; \tag{25}$$

$$\pi_H^f(\overline{\theta}, t) = \frac{\overline{\theta}^3}{t} \pi_H^f(1,1) = 0.018\frac{\overline{\theta}^3}{t};\, \pi_L^l(\overline{\theta}, t) = \frac{\overline{\theta}^3}{t} \pi_L^l(1,1) = 0.01199\frac{\overline{\theta}^3}{t}. \tag{26}$$

Observe that, as compared to the fully simultaneous equilibrium, (i) the high quality decreases, while the low quality increases; and (ii) again, as in the previous case, the leader is worse off than under simultaneous play.

Consider now the possibility that firms choose the timing of moves pertaining to the price stage before setting qualities in the first stage. The outcome of such a game is summarized by the following:

[6] Several others equilibria could be investigated, if firms were assumed to be able to play simultaneously also in the quality stage, or set quantities instead of prices in the market stage. For an analisys of such equilibria, see Lambertini (1996).

Proposition 2 *If firms can set the timing of moves in the price stage before deciding their respective quality levels, both will choose to move late, so that simultaneous play emerges.*

Proof. Since the size of both $\overline{\theta}$ and t exerts only a scale effect on profits, I can confine to the case of $\overline{\theta}$=10 and t=1. Then, the game can be described by matrix 2.

		L	
		F	S
H	F	16.4; 12.1	15.06; 14.12
	S	18; 11.99	16.4; 12.1

Matrix 2

A quick inspection of matrix 2 suffices to verify that playing late (S) is a dominant strategy for both firms, so that simultaneous play emerges at equilibrium. ■

3.2 Fixed costs of quality improvements

Production costs are given by (8), and can be thought of as R&D efforts. Market demands correspond to (11-12). Firms' profit functions can be written as:

$$\pi_i = p_i x_i - t q_i^2, i = H, L. \tag{27}$$

Consider first the fully simultaneous game. Proceeding backwards, I calculate the FOCs pertaining to the price stage:

$$\frac{\partial \pi_H}{\partial p_H} = \overline{\theta} - h = 0; \tag{28}$$

$$\frac{\partial \pi_L}{\partial p_L} = h - 2k = 0. \tag{29}$$

Solving the sistem (26-27) yields the following equilibrium prices:

$$p_H^n = 2\overline{\theta} q_H \frac{(q_H - q_L)}{(4q_H - q_L)}; \tag{30}$$

$$p_L^n = \overline{\theta} q_L \frac{(q_H - q_L)}{(4q_H - q_L)}. \quad (31)$$

The corresponding profits at the quality stage are:

$$\pi_H = \frac{4(\overline{\theta} q_H)^2 (q_H - q_L)}{(4q_H - q_L)^2} - tq_H^2; \quad (32)$$

$$\pi_L = \frac{\overline{\theta}^2 q_H q_L (q_H - q_L)}{(4q_H - q_L)^2} - tq_H^2. \quad (33)$$

I am now in a position to look for the equilibrium qualities at the first stage. The FOCs of this problem are (cfr. Motta, 1993, pp. 116):

$$\frac{\partial \pi_H}{\partial q_H} = \frac{4\overline{\theta}^2 q_H (4q_H^2 - 3q_H q_L + 2q_L^2)}{(4q_H - q_L)^3} - 2tq_H = 0; \quad (34)$$

$$\frac{\partial \pi_L}{\partial q_L} = \frac{\overline{\theta}^2 q_H^2 (4q_H - 7q_L)}{(4q_H - q_L)^3} - 2tq_L = 0. \quad (35)$$

Manipulating appropriately (34-35), yields the following equilibrium quality levels:[7]

$$q_H^n(\overline{\theta}, t) = 0.12665 \frac{\overline{\theta}^2}{t}; q_L^n(\overline{\theta}, t) = 0.02412 \frac{\overline{\theta}^2}{t}, \quad (36)$$

where $q_H^n(1,1)$=0.12665 and $q_L^n(1,1)$=0.02412 are the qualities selected when both $\overline{\theta}$ and t are equal to one. The corresponding equilibrium profits are:

$$\pi_H^n = 0.01222 \frac{\overline{\theta}^4}{t}; \pi_L^n = 0.000764 \frac{\overline{\theta}^4}{t}. \quad (37)$$

Again, this applies independently of the order of moves at the price stage. I turn now to the setting where each firm is alternatively appointed the leadership in the price stage. The leader's problem is conceptually equivalent to the case of variable production costs, described by (21-22), so that I can confine to the relevant equilibrium magnitudes. The equilibrium prices can be found in Appendix B.

[7] From an inspection of (35), it can be noticed that, if costs were nil, the low quality would be $q_L^n = 4q_H^n/7$. See Choi and Shin (1992).

When the high-quality firm acts as the price leader, equilibrium qualities and profits are:

$$q_H^l(\overline{\theta}, t) = 0.12715\frac{\overline{\theta}^2}{t}; q_L^f(\overline{\theta}, t) = 0.02949\frac{\overline{\theta}^2}{t}, \tag{38}$$

$$\pi_H^l = 0.01145\frac{\overline{\theta}^4}{t}; \pi_L^f = 0.000942\frac{\overline{\theta}^4}{t}. \tag{39}$$

It can be quickly verified that (i) both qualities increase as compared to the fully simultaneous game; and (ii) the follower's profit exceeds the simultaneous play profit, while the leader's is lower than that associated with simultaneous play.

Finally, the case where the low-quality firm is the price leader remains. The equilibrium levels of qualities and profits are:

$$q_H^f(\overline{\theta}, t) = 0.12613\frac{\overline{\theta}^2}{t}; q_L^l(\overline{\theta}, t) = 0.02425\frac{\overline{\theta}^2}{t}, \tag{40}$$

$$\pi_H^f = 0.01234\frac{\overline{\theta}^4}{t}; \pi_L^l = 0.000766\frac{\overline{\theta}^4}{t}. \tag{41}$$

Here, (i) the high quality decreases whereas the low quality increases as compared to the fully simultaneous game; and (ii) both the follower's and the leader's profits are higher than under simultaneous play.

Assume firms can decide the timing of their respective moves at the price stage before setting qualities in the first stage. The outcome of such an extended game is described by the following:

Proposition 3 *If firms can set the timing of moves in the price stage before deciding their respective quality levels, the high-quality firm will choose to move late whereas the low-quality firm whill choose to move first, so that a unique equilibrium in pure strategies exists, involving sequential play.*

Proof. Again, provided that the size of both $\overline{\theta}$ and t exerts only a scale effect on profits, I can confine to the case of $\overline{\theta}$=10 and t=1. Then, the game can be described by matrix 3.

		L	
		F	S
H	F	122.2; 7.64	114.5; 9.42
	S	123.4; 7.66	122.2; 7.64

Matrix 3

The inspection of matrix 3 reveals that playing late (S) is a strictly dominant strategy for the high-quality firm, so that it is optimal for the low-quality firm to play early (F), and the unique equilibrium of this game, identified by the combination of strategies (S-F) involves sequential play with the low-quality firm in the price leader's position. ■

4 Concluding remarks

In this paper I have analysed the nature of the equilibria that can be expected to arise in extended duopoly games where firms first set the timing of moves pertaining to one or more stages of the game, and then proceed to play. I have obtained three main results. First, I have shown that the criteria for equilibrium selection introduced by Hamilton and Slutsky (1990) hold even without the requirement that the leader's profit be at least as high as in the simultaneous equilibrium. Second, I have generalized Hamilton and Slutsky's results, establishing that sequential play will be observed if and only if both firms are at least weakly better off playing sequentially than playing simultaneously. Third, resorting to a model of endogenous differentiation followed by price competition, I have proved the existence of cases where the leader can indeed be worse off than under simultaneous play, and the extended game exhibits either several subgame perfect equilibria involving simultaneous moves if both firms are worse off in the leader's position, or a unique subgame perfect equilibrium involving sequential play if one firm is worse off leading than playing simultaneously while the opposite holds for the rival.

Appendix A: equilibrium prices under sequential play and variable production costs

i) The equilibrium prices when the high-quality firm is the price leader are $p_H^l = q_H[2\overline{\theta}(q_H - q_L) + 2tq_H^2 - tq_H q_L + tq_L^2]/[2(2q_H - q_L)]$ and $p_H^f = q_L[2\overline{\theta}(q_H - q_L) + 2tq_H^2 + 3tq_H q_L - tq_L^2]/[4(2q_H - q_L)]$.

ii) The equilibrium prices when the low-quality firm is the price leader are $p_H^f = q_H[\overline{\theta}(q_H - q_L) + tq_H^2 - tq_H q_L + tq_L^2]/(2q_H - q_L)$ and $p_H^l = [\overline{\theta} q_L(q_H - q_L) + tq_H^2 q_L + 2tq_H q_L^2 - tq_L^3]/[2(2q_H - q_L)]$.

Appendix B: equilibrium prices under sequential play and fixed production costs

i) The equilibrium prices when the high-quality firm acts as the price leader are $p_H^l = \overline{\theta} q_H(q_H - q_L)/(2q_H - q_L)$ and $p_L^f = \overline{\theta} q_L(q_H - q_L)/[2(q_H - q_L)]$.

ii) The equilibrium prices when the low-quality firm acts as the price leader are $p_H^f = \overline{\theta}(4q_H - q_L)(q_H - q_L)/[4(2q_H - q_L)]$ and $p_L^l = \overline{\theta} q_L(q_H - q_L)/[2(2q_H - q_L)]$.

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