PRICE AND QUANTITY PATTERNS WHEN THE CONSUMERS' INCOME IS UNCERTAIN

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ABSTRACT - The paper shows that the co-movements of optimal price and output in a monopolistic market can be a case of spurious correlation, price and quantity variations being affected by the degree of uncertainty in the consumers' incomes. The pattern of price and quantity changes depends on the shape of the distribution function of the stochastic variable "income of the consumers", as well as the size of the firm's costs.

JEL Classification: D80, E39

KEYWORDS: Monopoly, Markup, Uncertainty.

ACKNOWLEDGMENTS: We wish to thank Guido Candela, Vincenzo Denicolò, Paolo Garella, Luca Lambertini, Carlo Scarpa, Antonello Scorcu and Stefano Zamagni for insightful suggestions and discussion. The usual disclaimer applies.

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1. *Introduction*

This paper studies the theoretical relationship between prices and production within a monopoly model where market demand derives from consumers whose individual income is uncertain.

The issue of the firm's behaviour in an uncertain environment has been widely discussed in the literature, both as a microeconomic problem *per se*, and as a problem underlying some key macroeconomic questions. One could recall the lively debate at the turn of the Seventies, about the optimal behaviour of a monopolist facing *uncertain demand*: prominent contributions were put forth, among others, by Nevins (1966), Smith (1969), Zabel (1970, 1972) and Leland (1972). The results in this respect were mainly negative, and can be put in a nutshell as follows: it is often impossible to offer complete analytical solutions; when such exists, the equilibrium behaviour depends heavily on how uncertainty is assumed to bear on demand; in general, the conclusions about the sign of equilibrium price-quantity comovements as could been reached, relied heavily on particular assumptions about parameters.

An important point should however be stressed, which also emerged in that early discussion: it turned out that the direction of price-quantity co-movements could be traced back to something other than the demand shock triggering them. For instance, Zabel (1970) argued that the sign of the price-quantity correlation depends on the stock the firm holds at the beginning of its planning horizon, which in turn is obviously related to the cost the firm incurs for holding it. A similar result, albeit in a quite different framework, is offered by the more recent work by Blinder (1982). He shows that how strongly quantity and price react to a demand shock, depends on the persistence of the shock itself and the marginal costs of holding inventories.

Macroeconomists as well have focussed on somewhat related issues. Two separate traditions can be identified: on the one hand, Real Business Cycle theorists studied quantity decisions *vs* uncertainty, to account for persistence in observed deviations of aggregate output from its trend; on the other hand, New Keynesian economists worked on the price-quantity link in imperfect competition (on the main, it should be added, disregarding uncertainty), to enquire on the pro- or anti-cyclical behaviour of markup —which, as is well known, bears on the effectiveness of real demand policies. The markup pattern, it should be noticed, has also been tackled by a number of econometric papers, but mixed results were the rule. For example, Domowitz, Hubbard and Peterson (1986) found evidence of procyclical markup, while empirical work by Bils (1987) and Rotenberg and Woodford (1991) reached the opposite conclusion.

We propose to study the price-quantity link within a monopoly model with a *continuum* of consumers with uncertain income. One feature of our approach is that demand is explicitly derived from first principles —which allows us to make a clear distinction between uncertainty on individual incomes, and perfectly observable aggregate demand shocks. In fact the *continuum* assumption makes uncertain individual demand (income) consistent with perfectly observable market demand. One implication of this approach is that one can vindicate a quite appealing intuition: a prime candidate as a source of spurious correlation between quantity and price is uncertainty itself. Indeed, in our framework —a static partial equilibrium model— equilibrium price and quantity may move in the same or in the opposite direction, depending on the level of uncertainty about individual incomes. Indeed, we show that though market demand is fully observable, different degrees of individual income uncertainty make for different price elasticities, and hence different optimal choices on the firm's part. Besides this specific result, the way we formalize demand provides a simple setting for the analysis of macroeconomic shocks interacting with microeconomic uncertainty.

The paper is organized as follows. In section 2 we develop our model when no particular restrictions are imposed on the relevant functional forms. In section 3 we explicitely solve the model for a given analytical specification. Section 4 offers some concluding remarks.

2. *The basic model*

In this section we present a general framework that allows us to study the relationship among uncertain individual incomes, aggregate demand, and the choices of a monopolist firm.

The market is populated by a continuum of consumers, each of whom belongs to a given income class i and receives a random income e_i. His preferences are assumed to yield the following simple rule: for any given price $p>0$, one unit of the commodity is bought whenever (realized) income $e_i \geq p$, while nothing is consumed otherwise.[1] All this implies that a consumer is effectively identified by his income class, which we describe as the distribution from which his income is drawn: an individual of type i draws his income e_i from a distribution with density $f(e_i;i,\theta_i)$, whose cumulative we denote by $F(e_i;\cdot,\cdot)$. The parameter $\theta_i \in \Theta$ is a mean preserving spread,[2] which measures individual uncertainty and is the same for all consumers in class i; $F(e_i;\cdot,\cdot)$ is defined over some support $[e_i^m, e_i^M]$, the boundaries of which satisfy $0<e_i^m<e_i^M<\infty$, and may depend on θ_i. A consumer of type i obtains expected income equal to

$$\bar{e}(i) = \int_{e_i^m}^{e_i^M} e_i f(e_i;i,\theta_i)\ \mathrm{d}e_i, \tag{1}$$

independent of θ_i.

We treat i as an index of first order stochastic dominance: in formal terms, this means that

$$\frac{\partial F(e_i;i,\theta_i)}{\partial i} \leq 0, \tag{2}$$

for all e_i (with strict inequality holding somewhere), which implies $\mathrm{d}\bar{e}(i)/\mathrm{d}i>0$.[3] In more substantial terms, this may be interpreted as saying that whenever $i > j$, it must be that class i is higher in the income scale than class j – where "higher in the income scale" means that (*a*) $e_i^m \geq e_j^m$ and $e_i^M \geq e_j^M$, and (*b*) for any given income level $e \in [e_j^m, e_i^M]$, a class-i-individual faces a higher probability than a class-j-individual of ending up with an actual income higher than e.

Income classes are themselves distributed according to a cumulative function $G(i;\cdot)$, defined over some support Γ: $G(i;\cdot)$ is then the proportion of consumers whose expected income is less than (or equal to) that of class i, while the associated density $g(i;\cdot)$ is the proportion of consumers facing an expected income $\bar{e}(i)$.[4] We write these out more precisely as $G(i,\gamma)$ and $g(i,\gamma)$, where the parameter γ is an index of first order stochastic dominance, which we interpret as a positive shift to *aggregate* income. Indeed, the latter is defined by

$$Y(\gamma) = \int_{\Gamma} \bar{e}(i) g(i,\gamma) \; \mathrm{d}i, \tag{3}$$

which is strictly increasing in γ if $\partial G/\partial\gamma < 0$: as γ gets higher, consumers move up in the income scale in the sense that the probability of belonging to a higher income class goes up.

We are now able to derive the demand curve faced by the monopolist firm – actually an expected demand function. Given that an infinity of consumers of nul measure are present, endowed with uncorrelated individual incomes, expected demand is in fact equal to realized demand, and the firm's problem is non-stochastic (see e.g. Sheshinski and Drèze, 1976, p.737)

Suppose the firm quotes a price $p > 0$. A consumer of type i will buy with probability $1 - F(p;i,\theta_i)$, so that the demand function will take the general form

$$Q(p,\gamma) = \int_{\Gamma} [1 - F(p;i,\theta_i)] g(i;\gamma) \; \mathrm{d}i. \tag{4}$$

When p is sufficiently low, the whole population will purchase with probability one; on the other hand, if p is very high demand will be nil. Formally, under our assumptions there is some pair $(p_H,\ p_L)$, $p_H > p_L \geq 0$, such that[5]

$$Q(p,\gamma) = 1, \qquad \text{for } p \leq p_L,$$

$$Q(p,\gamma) = 0, \qquad \text{for } p \geq p_H.$$

Moreover, the demand curve is continuously differentiable and satisfies the following conditions over the interval (p_L, p_H):

$$\frac{\partial Q(p,\gamma)}{\partial p} = -\int_{\Gamma} f(p;i,\theta_i)g(i;\eta)\ \mathrm{d}i < 0, \tag{5a}$$

$$\frac{\partial Q(p,\gamma)}{\partial \gamma} = \int_{\Gamma} \frac{\partial G(i;\gamma)}{\partial \gamma} \cdot \frac{\partial F(p;i,\theta_i)}{\partial i}\ \mathrm{d}i > 0. \tag{5b}$$

Finally, let us notice that we are interested in how the firm's reactions to a change in γ may be affected by variations in uncertainty, as measured by the parameter θ_i. Strictly speaking, the latter should be seen as a function from Γ to Θ: variations in individual uncertainty would then be captured by changes in the form of that function. However, we may parametrize a common component of the single θ_i's by a (nonnegative) parameter θ, that is $\theta_i = \theta_i(\theta, i)$, such that an increase in θ translates into an increase in the uncertainty of each consumer's individual income $(\partial\theta_i/\partial\theta > 0)$. This implies that we can write the demand curve as $Q(p,\theta,\gamma)$. In this case,

$$\frac{\partial Q(p,\theta,\gamma)}{\partial \theta} = -\int_{\Gamma} \frac{\partial F(p;i,\theta_i)}{\partial \theta_i} \cdot \frac{\partial \theta_i}{\partial \theta} \cdot g(i,\gamma)\ \mathrm{d}i,$$

which cannot be signed under our assumptions.

Let us consider now the firm's behaviour. We assume it to face a linear cost function $C(Q) = cQ$, with $c > 0$. Profit is then given by

$$\Pi(p,\theta,\gamma)=(p-c)\cdot Q(p,\theta,\gamma), \tag{6}$$

maximization of which with respect to p yields the obvious first order condition:

$$Q(p,\theta,\gamma)+\frac{\partial Q(p,\theta,\gamma)}{\partial p}\cdot(p-c)=0.$$

The second order condition requires $2Q_p+(p-c)Q_{pp}<0$ (where subscripts denote partial or cross derivatives).

Let us focus on the first order condition. Implicit differentiation then allows us to write down optimal (starred) variables as a function of the exogenous parameter γ, such that

$$\frac{\mathrm{d}p^*}{\mathrm{d}\gamma}=-\frac{Q_\gamma+(p-c)Q_{p\gamma}}{2Q_p+(p-c)Q_{pp}}, \tag{7a}$$

$$\frac{\mathrm{d}Q^*}{\mathrm{d}\gamma}=Q_\gamma-Q_p\cdot\frac{Q_\gamma+(p-c)Q_{p\gamma}}{2Q_p+(p-c)Q_{pp}}\;=Q_\gamma-\frac{\mathrm{d}p^*}{\mathrm{d}\gamma}, \tag{7b}$$

Clearly, these expressions cannot be unambigously signed in this general framework. This is in line with the result (Zabel, 1970) which we already mentioned in the introduction. Notice however that if $Q_{p\gamma}>0$, then $\mathrm{d}p^*/\mathrm{d}\gamma>0$, which leaves the sign of $\mathrm{d}Q^*/\mathrm{d}\gamma$ ambiguous. By contrast, if $\mathrm{d}p^*/\mathrm{d}\gamma<0$, then equilibrium quantity increases unambiguously with γ.

With a slight *abus de language*, we might say that we have procyclical markup whenever both (7*a*) and (7*b*) have the same sign, and that the markup is anticyclical if their signs differ. Our contention is that there exist situations in which either of these may take place, *depending only on the value of* θ. The mark-up may move *systematically* in the same (or in the opposite) direction as produced quantities, according to the value of the uncertainty index. In this case, the "spurious correlation" we posited emerges very clearly. Variations in aggregate income cause variations in the optimal price and in the expected demand (i.e, in production). Depending on the amount of uncertainty, the directions of these co-movements can be the same or the opposite, with an apparent pro- or anti-cyclical behaviour of price (i.e., of markup). We now proceed to give a specific example where this is indeed the case.

3. *An example*

In this section we are going to specialize our assumptions in such a way that the monopolist's problem can be explicitly worked out.

Assume that the income of an individual belonging to the income class i can span over the interval $[\bar{e}(i)-\theta_i\ ,\ \bar{e}(i)+\theta_i]$ and that the uncertainty parameter θ_i takes the form $\theta_i = i\cdot\theta$, $0<\theta<1$, which implies that higher income class consumers face a wider range of possible income levels. Assume also that each individual density $f(e;i,\theta)$ is uniform, so that $e\in[(1-\theta)i,$ $(1+\theta)i]$ and the following hold:

$$f(e;i,\theta) = \frac{1}{2i\theta}$$

$$F(e;i,\theta) = \frac{1}{2} + \frac{e-i}{2i\theta}$$

$$\bar{e}(i) = i\,.$$

Notice that an increase in θ widens the range of possible realizable incomes for each income class, without affecting its mean; it is of course true, as from (2), that $\frac{\partial F}{\partial i}<0$ and $\mathrm{d}\bar{e}(i)/\mathrm{d}i>0$. The assumption on θ_i means that the standard deviation of individual income is linear in average income – so that the standard deviation relative to the mean, i.e. the coefficient of variation (σ/i), is the same for all income classes.

As far as the distribution $g(\cdot,\gamma)$ is concerned, we assume it is itself uniform and positive over $i\in[1,\gamma]$, $\gamma>1$; this means that a higher γ extends the range of populated income classes. In other words, γ is an index of first order stochastic dominance, and indeed $\frac{\partial G}{\partial \gamma} < 0$. Given our assumption of uniform distribution, the following holds trivially for $i \in [1, \gamma]$:

$$g(\cdot, \gamma) = \frac{1}{\gamma - 1} \;,$$

$$G(i, \gamma) = \frac{i-1}{\gamma - 1}.$$

Using (3), aggregate income is $Y(\gamma) = (\gamma + 1)/2$.

In order to write the demand curve which holds in this setting, we note that the lowest possible income realization is $e_1^m = 1 - \theta$, while the highest income realization is $e_\gamma^M = (1+\theta)\gamma$: this being so, the demand function (4) takes the form:

$$Q(p;\theta,\gamma) = 1 \qquad\qquad p \le 1 - \theta \equiv p_L$$

$$Q(p;\theta,\gamma) = \frac{2\gamma\theta - 1 - \theta}{2\theta(\gamma-1)} + \frac{p}{2\theta(\gamma-1)} \cdot \left(1 - \ln\left(\frac{p}{1-\theta}\right)\right) \qquad 1 - \theta \le p \le \min\{(1+\theta), (1-\theta)\gamma\}$$

$$Q(p;\theta,\gamma) = \alpha - \beta p \qquad\qquad \min\{(1+\theta), (1-\theta)\gamma\} \le p \le \max\{(1+\theta), (1-\theta)\gamma\}$$

$$Q(p;\theta,\gamma) = \frac{(1+\theta)\gamma}{2\theta(\gamma-1)} - \frac{p}{2\theta(\gamma-1)} \cdot \left(1 - \ln\left(\frac{p}{(1+\theta)\gamma}\right)\right) \qquad \max\{(1+\theta), (1-\theta)\gamma\} \le p \le (1+\theta)\gamma$$

$$Q(p;\theta,\gamma) = 0 \qquad\qquad (1+\theta)\gamma \equiv p_H \le p$$

This demand curve is continuous and continuosly differentiable for any $p > 0$; it is flat (and equal to 1) in the first portion, concave in the second, linear in the third, then it becomes convex and finally flat (and equal to zero). In the linear portion the parameters α and β take on different values, according as $(1+\theta)$ is greater or smaller than $(1-\theta)\gamma$; in particular we have

$$\alpha = \frac{1+\theta}{2\theta}, \qquad \beta = \frac{\ln\gamma}{(\gamma-1)2\theta} \qquad \text{for } 1+\theta > (1-\theta)\gamma, \qquad (\textit{case (a)})$$

$$\alpha = \frac{\gamma}{\gamma-1}, \qquad \beta = \frac{1}{(\gamma-1)2\theta}\cdot\ln\left(\frac{1+\theta}{1-\theta}\right) \qquad \text{for } 1+\theta < (1-\theta)\gamma, \qquad (\textit{case (b)})$$

It can be proved numerically that, for any value of the marginal cost c, $c \in (p_L, p_H)$, this demand function gives rise to a unimodal profit function $\Pi(p,\ \theta,\gamma)$.[6] Notice that the inequality of case (b) above amounts to stating that the maximum possible income of the consumers in the lowest income class (e_1^M) is lower than the minimum possible income of the consumers in the highest income class (e_γ^m); this being a more reasonable hypothesis than the alternative, we shall adopt it in the sequel. Some examples of the demand function, corresponding to different numerical values of parameters, are plotted in figure 1, with $1+\theta < (1-\theta)\gamma$. The corresponding profit functions are shown in figure 2.

INSERT FIGURES 1 AND 2 HERE

For the ease of notation, we define $H(\theta) = \frac{1}{2\theta}\cdot\ln\left(\frac{1+\theta}{1-\theta}\right)$, which is increasing in θ and such that $\lim_{\theta\to 0} H(\theta) = 1$ and $\lim_{\theta\to 1} H(\theta) = \infty$. This allows us to write the linear portion of the demand curve (case (b)) as:

$$Q(p;\gamma,\theta) = \frac{\gamma}{\gamma-1} - \frac{H(\theta)}{\gamma-1}\cdot p.$$

We also impose some restrictions on c, such that the optimal price-quantity pair lies indeed in the linear part of the demand curve: that is, we assume that γ is sufficiently high so that

$$1+\theta < \frac{\gamma}{2H(\theta)},$$

$$c < \left(2(1-\theta) - \frac{1}{H(\theta)}\right)\gamma .$$

Some straightforward calculations show that the former inequality ensures the optimum price-quantity pair never lies in the concave region, while the latter rules out that it lie in the convex one. Given these restrictions on parameters, the demand elasticity in the linear region takes the form

$$\eta_{Q,p} = \frac{H(\theta)\cdot p}{\gamma - H(\theta)\cdot p} \;, \tag{8}$$

while the optimal price and quantity are respectively

$$p^*(\theta,\gamma) = \frac{\gamma + cH(\theta)}{2H(\theta)}, \tag{9}$$

$$Q^*(\theta,\gamma) = \frac{\gamma - cH(\theta)}{2(\gamma-1)}. \tag{10}$$

Let us now consider the effect of a positive income shock. An increase in γ has an unambigous effect on p^*: it raises the optimal price as it amounts to an increase in the consumers' willingness to pay. On the other hand, higher γ may raise or lower Q^*, depending on the value of θ; indeed,

$$Q^*_\gamma(\theta,\gamma) \equiv \frac{\partial Q^*(\theta,\gamma)}{\partial\gamma} = \frac{cH(\theta)-1}{2(\gamma-1)^2}, \tag{11}$$

which for given c is positive or negative according as $H(\theta)$ is higher or lower than $\frac{1}{c}$. In other words, we can put forward the following

RESULT: *Suppose* $1 > c > \dfrac{2(\gamma-1)}{(\gamma+1)\cdot\ln\gamma}$. *Then there exists one* $\theta^* = \theta(c,\gamma)$, *such that* $Q^*_\gamma(\theta,\gamma) > 0$ *for* $\theta > \theta^*$ *and* $Q^*_\gamma(\theta,\gamma) < 0$ *for* $\theta < \theta^*$.

PROOF: We are assuming case (b) above, that is $1+\theta < (1-\theta)\gamma$. Hence,

$\theta \leq \widehat{\theta}(\gamma) \equiv \frac{\gamma - 1}{\gamma + 1} < 1$. Clearly, $\lim_{\theta \to 0} Q^*_\gamma(\theta, \gamma) < 0$, since $\lim_{\theta \to 0} H(\theta) = 1$. On the other hand, $\lim_{\theta \to \widehat{\theta}} Q^*_\gamma(\theta, \gamma) > 0$, as $\lim_{\theta \to \widehat{\theta}} H(\theta) = H(\widehat{\theta}) = \frac{\gamma + 1}{2(\gamma - 1)} \cdot \ln\gamma$, and $cH(\widehat{\theta}) > 1$. Since $H(\theta)$ is monotonically increasing, there exists one $\theta^* \in (0, \widehat{\theta})$ such that $Q^*_\gamma(\theta, \gamma) = 0$, $Q^*_\gamma(\theta, \gamma) > 0$ for $\theta > \theta^*$ and $Q^*_\gamma(\theta, \gamma) < 0$ for $\theta < \theta^*$.□

In order to capture the economic intuition behind this result let us concentrate on the two separate effects of γ and θ. As to the former, consider the trivial case with no uncertainty (θ tends to zero), in which $Q^*_\gamma(\theta, \gamma)$ is greater (smaller) than zero according as c is greater (smaller) than one. An increase in γ amounts to an outward shift of the demand curve and hence an increase in each consumer's willingness to pay. For any given price, this creates *new* selling opportunities for the firm, *and* widens its customers' consumers' surplus. The former tendency pushes the firm towards increasing the produced quantity; the latter will induce it to get hold of a larger amount of consumer surplus through substantial price increases – an incentive which prevails if the firm is serving more than half the market. Indeed, it can easily be checked that an increase in γ raises the marginal revenue for $Q > \frac{1}{2}$ and decreases it for $Q < \frac{1}{2}$: the marginal revenue curve rotates on the marginal revenue-quantity pair $(1, \frac{1}{2})$, where elasticity is one for any γ. Thus, the equilibrium quantity reacts positively to γ if $c > 1$, and negatively otherwise.[7]

As to the role of θ, it is useful to recall that the *inverse* demand curve in its linear region obeys the equation

$$p(Q) = \frac{1}{H(\theta)} \cdot [\gamma - (\gamma - 1)Q],$$

so that the marginal revenue function is obviously $MR(Q) = \frac{1}{H(\theta)} \cdot [\gamma - 2(\gamma - 1)Q]$. The inverse demand curve shows that an increase in θ, *viewed from the firm's perspective*, is equivalent to a reduction of the consumers' willingness to pay by a factor $\frac{1}{H(\theta)}$, and hence to an inward shift of the linear portion of the demand curve. This can be euristically shown as follows. Assume that the firm is setting a price p; at that price, an increase in θ raises the probability that the realized income of consumers with expected income $\bar{e}(i)=i>p$ be actually less than p, *more than* it makes it more likely that realized income of the symmetric consumers with expected income $\bar{e}(j)=j<p$ be higher than p.[8] Formally:

$$\frac{\partial}{\partial\theta}\ \Pr\{e_i<p\,|\,\bar{e}(i)=p+h\}>\ \frac{\partial}{\partial\theta}\ \Pr\{e_j>p\,|\,\bar{e}(j)=p-h\}>0$$

for any given p and $h>0$ such that $p\pm h$ belong to the linear portion. In this region an increase in θ reduces demand *ceteris paribus*.

Notice that if $\theta\in(0,\widehat{\theta})$, an increase in γ raises marginal revenue for $Q<\frac{1}{2}$ and lowers it for $Q>\frac{1}{2}$, the corresponding curve rotating on the point with coordinates $\left(\frac{1}{2},\ \frac{1}{H(\theta)}\right)$ - see figure 3. An increase in uncertainty lowers vertically this pivot point. For given c such a point may happen to lie above or below c, depending on θ; accordingly, an increase in γ may induce positive or negative variations of the optimal produced quantity, depending on θ.

INSERT FIGURE 3 HERE

To understand the economic intuition behind the analytics, consider a firm whose set price is such that $Q^*=\frac{1}{2}$, given θ. We know that γ doesn't affect the optimal production in this case $-$the only effect being on price. Suppose that θ increases; at the same price, the expected demand falls below $\frac{1}{2}$. This reduction of the served market induces the firm to react to a positive shock on γ with an increase in the produced quantity, which requires an increase in price which is the lower, the higher is θ. In other words, different values of θ translate into different values of demand elasticity; when a positive shock on γ occurs, different θ's imply different equilibrium reactions to that shock, the firm choosing a different mixture between (let us say) exploiting the intensive and the extensive margins – when the individual uncertainty is high, the exploitation of the extensive margin prevails.

4 . *Conclusions*

In this paper the firm has been assumed to face a continuum of consumers with uncertain income. Thus, while the individual consumer's demand is uncertain, aggregate demand is perfectly observable. We have also assumed observable 'macroeconomic' shocks to aggregate demand. We believe that this setting is not unrealistic, as aggregate information on demand is arguably more easily collected than information on individual incomes.

Our main finding is that the amount of individual uncertainty affects the sign of the equilibrium price-quantity comovements. It is worth stressing that the result has been cast in quite a traditional framework. Indeed, in *The Economics of Imperfect Competition*, Joan Robinson wrote:

> An increase in the wealth is likely to make the demand of the individual buyer of any particular commodity less elastic. Thus an increase in demand due to an increase of wealth is likely to reduce the elasticity of the demand curve, and may reduce the elasticity so much that the slope of the curve is increased. (Robinson, 1969,p.70)

and

> If the higher demand curve is steeper than the lower, the marginal revenue curves may cut each other. Then if the marginal cost curve cuts the marginal revenue curves below their point of intersection, the ouput appropriate to the higher demand curve will be less than that appropriate to the lower demand curve. (p.66)

This effect is exactly what is induced in our model by an increase on aggregate income. Our microfoundation of market demand allows us to focus on a further relevant argument of the demand function, namely uncertainty. An increase in individual uncertainty has the effect of making market demand more elastic to price. Different levels of uncertainty generate a different balance between the elasticity-reducing effect of the aggregate demand shock (the equivalent of the 'wealth effect' described by J.Robinson) and the increasing-elasticity effect due to uncertainty itself, as is very clear from equation (9) in our example. In fact, one major advantage of our setting is the distinction between the effect of demand shocks and the effect of uncertainty – the latter being the key element shaping the pro- or anti-cyclical behaviour of equilibrium price and quantity.

Though we have focussed on a very simple, analytically tractable problem, we do believe that the general setting offered can be useful to address a wider range of questions. Two of these are in our research agenda: an intertemporal extension of the model, and the application to monopolistic competition or oligopoly with product differentiation.

FOOTNOTES

[1] This means that the individual demand function for a given realization e takes the form

$$q^d = 1 \text{ if } e \geq e_{cr}(p), \qquad q^d = 0 \text{ if } e < e_{cr}(p),$$

where $e_{cr}(p)$ is some critical level of income; without loss of generality one can set $e_{cr}(p) = p$. Models with such a structure of demand are very common in the literature: see for instance Anderson, De Palma and Thisse (1992) and Tirole (1988, chs. 2 and 7)

[2] The definition of mean preserving spread is well known (Rothschild and Stiglitz, 1970), and can be summarized as follows: given some cumulative distribution $S(x,\varrho)$ for the random variable $x \in [a,b]$, define $T(x,\varrho) \equiv \int_a^x \frac{\partial S(x,\varrho)}{\partial \varrho}\, \mathrm{d}x$. Then ϱ is a mean preserving spread if $T(x,\varrho) \geq 0$ for $x \in [a,b)$ and $T(b,\varrho) = 0$.

[3] This follows from solving by parts the integral in (1).

[4] Since $i \in \Gamma$ and by (2) the function $\bar{e}$ is strictly increasing, $\bar{e}$ will belong to some set $\widehat{\Gamma}$: this will be the support of some induced distribution $\widehat{G}$ of mean incomes. Suppose, as will be the case in the next section, that $\Gamma = [1,\gamma]$, $\gamma > 1$: then *actual* incomes will fall within the interval $[e_1^m, e_\gamma^M]$.

[5] Indeed, demand is unity for any $p \leq e_1^m$ (the lowest possible income for the poorest income class), for in that case $F(p;\theta_i,i) = 0$ for all i and $\int_\Gamma g(i,\gamma)\, \mathrm{d}i = 1$; demand is zero for any $p \geq e_\gamma^M$ (the highest possible income for the richest income class), for then $F(p;\theta_i,i) = 1$ for all i. Inequality (4) derives immediately from integration by parts.

[6] This cannot be proved analytically, as the first order conditions applying to the nonlinear portions of the demand curve are not algebraic equations.

[7] In terms of equations $7a,b$, this is a case where $Q_{p\gamma} > 0$, which, while ensuring that $\mathrm{d}p^*/\mathrm{d}\gamma > 0$, is consistent with $\mathrm{d}Q^*/\mathrm{d}\gamma$ taking either sign.

[8] Notice that this depends crucially on our hypothesis on θ_i. That is, on being $\theta_i = i\theta$. If θ was independent of i, it would exert no influence on the firm's choices.

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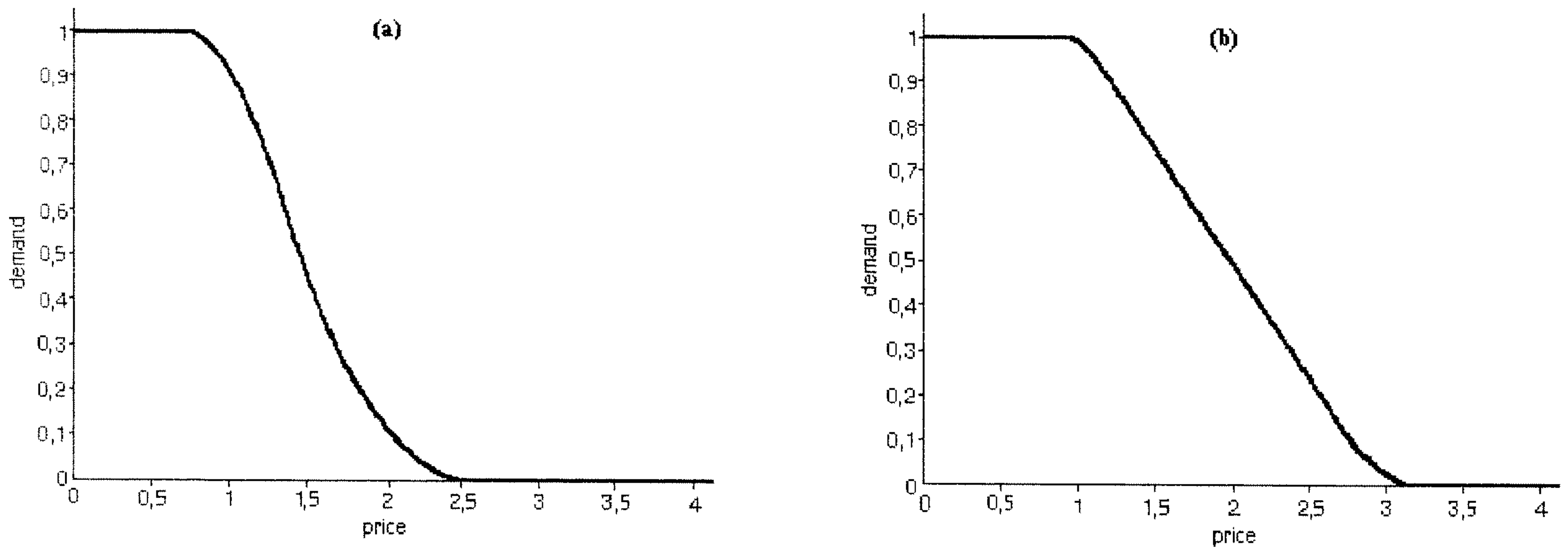
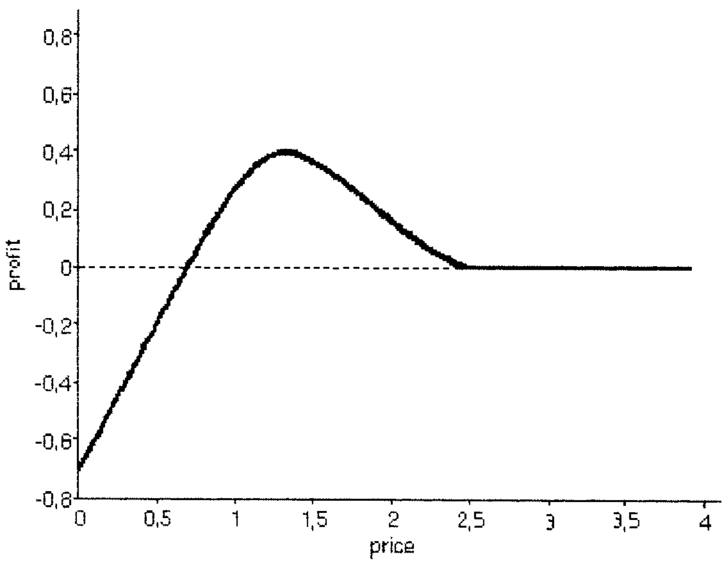


Figure 1

The demand function for (a) $\gamma = 2, \theta = .3, c = .7$; (b) $\gamma = 3, \theta = .1, c = .7$

(a)



(b)

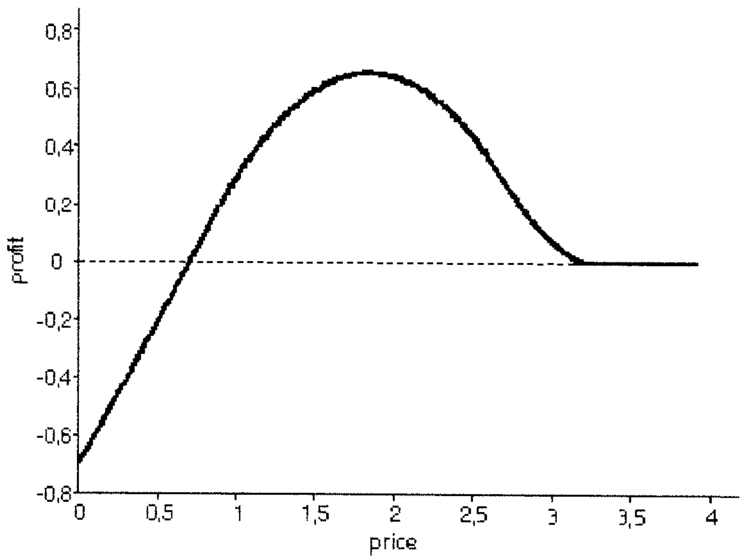


Figure 2

The profit function for (a) $\gamma = 2, \theta = .3, c = .7$; (b) $\gamma = 3, \theta = .1, c = .7$.

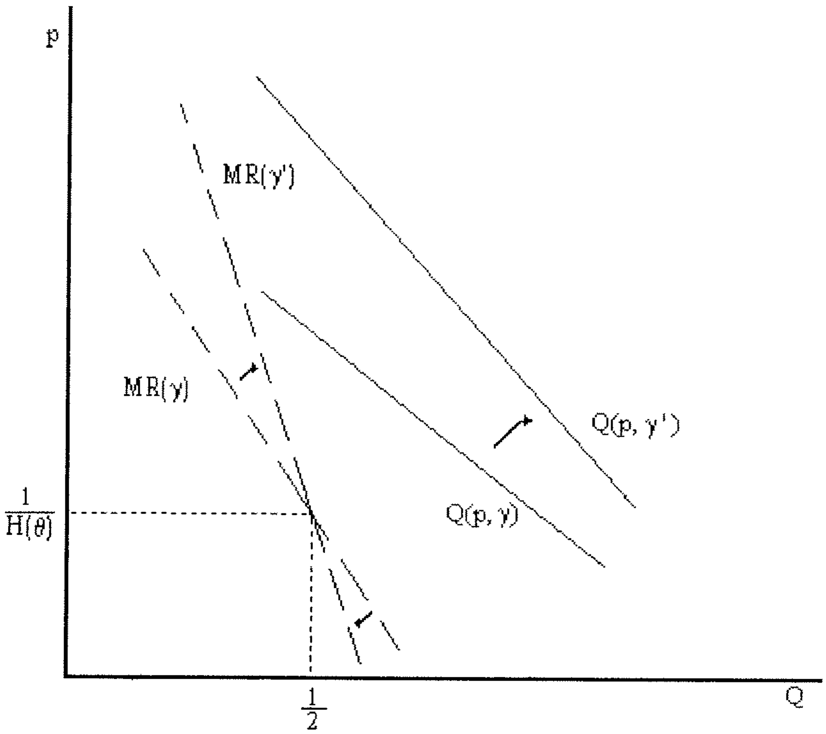


Figure 3
An increase from γ to γ'